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Is the Moon There If Nobody Looks: Bell Inequalities and Physical Reality

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Bell-CHSH inequalities are trivial algebraic properties satisfied by each line of an Nx4 spreadsheet containing ± 1 entries, thus it is surprising that their violation in some experiments allows us to speculate about the existence of non-local influences in nature and casts doubt on the existence of the objective external physical reality. Such speculations are rooted in incorrect interpretations of quantum mechanics and in a failure of local realistic hidden variable models to reproduce quantum predictions for spin polarization correlation experiments (SPCE). In these models, one uses a counterfactual joint probability distribution of only pairwise measurable random variables (A, A′, B, B′) to prove Bell-CHSH inequalities. In SPCE, Alice and Bob, using 4 incompatible pairs of experimental settings, estimate imperfect correlations between clicks registered by their detectors. Clicks announce the detection of photons and are coded by ± 1. Expectations of corresponding random variables—E (AB), E (AB′), E (A′B), and E (A′B′)—are estimated and compared with quantum predictions. These estimates significantly violate CHSH inequalities. Since variables (A, A′) and (B, B′) cannot be measured jointly, neither Nx4 spreadsheets nor a joint probability distribution of (A, A′, B, B′) exist, thus Bell-CHSH inequalities may not be derived. Nevertheless, imperfect correlations between clicks in SPCE may be explained in a locally causal way, if contextual setting-dependent parameters describing measuring instruments are correctly included in the description. The violation of Bell-CHSH inequalities may not therefore justify the existence of a spooky action at the distance, super-determinism, or speculations that an electron can be both here and a meter away at the same time. In this paper we review and rephrase several arguments proving that such conclusions are unfounded. Entangled photon pairs cannot be described as pairs of socks nor as pairs of fair dice producing in each trial perfectly correlated outcomes. Thus, the violation of inequalities confirms only that the measurement outcomes and 'the fate of photons' are not predetermined before the experiment is done. It does not allow for doubt regarding the objective existence of atoms, electrons, and other invisible elementary particles which are the building blocks of the visible world around us.

Keywords: quantum non-locality, counterfactual definiteness, local realism, non-invasive measurability, Tsirelson bound, EPR paradox, Bell-CHSH inequalities

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INTRODUCTION

External physical reality existed before we were able to probe it with our senses and experiments. From early childhood, we learn that the objects surrounding us continue to exist even when we stop looking at them.

Another notion imprinted in our genes is the notion of a local causality. If a baby elephant or a baby antelope does not stand up immediately after their birth, they will die. Several events which we observe may be connected by causal chains. The amazing migration patterns and courtship rituals of birds and butterflies are encoded in their genes.

Our brains, evolved over millions of years, allow us to understand that the external physical reality should be governed by natural laws which we can try to discover. We succeeded in explaining observable properties of macroscopic objects assuming the existence of invisible atoms and molecules. Later, we discovered electrons, nuclei, elementary particles, resonances, and various fields that play an important role in the Standard Model. Various conservation laws are obeyed in macroscopic and in quantum phenomena.

Information about the invisible world is indirect and relative to how we probe it. Invisible charged elementary particles leave traces of their passage in photographic emulsion or in different chambers (sparks, bubble, multi-layer, etc.). They also produce clicks on detectors.

We accelerate electrons, protons, and ions and by projecting them on various targets we probe more deeply into the structure of the matter over smaller and smaller distances. We succeeded in trapping electrons and ions. We constructed atomic clocks and ion chips for quantum computing.

It is therefore surprising that the violation of various Bell-type inequalities [1–5] by some correlations between clicks on the detectors observed in spin polarization correlation experiments (SPCE) [6–11] may lead to the conclusion that that there is no objective physical reality, that the electron may be both here and a meter away at the same time, that a measurement performed by Alice in a distant location may change instantaneously an outcome of Bob's measurement or that apparently random choices of experimental settings in SPCE are predetermined due to super-determinism.

The fact that such conclusions are unfounded has been pointed out by several authors [12–83]. The violation of the inequalities confirms only that "*unperformed experiments have no outcomes*" [84], that one may not neglect the interaction of a measuring instrument with a physical system and that the "*non-invasive measurability*" assumption is not valid. It confirms the existence of quantum observables which can only be measured in incompatible experimental contexts.

It also proves that entangled photon pairs, produced in SPCE, may not be described as pairs of socks (local realistic hidden variable models- LRHVM) or as pairs of fair dice (stochastic hidden variable models-SHVM) [1–4].

We are unable to create any consistent *mental picture* of a "photon." We have the same problem with many other elementary particles, but the lack of *mental pictures* does not mean that they do not exist. These invisible particles are building blocks of the visible world around us, including ourselves.

A completely new approach is needed in order to reconcile the quantum theory with the theory of general relativity, and it is not certain whether we are smart enough to find it. We will surely not discover it, however, if we accept *quantum magic* as the explanation of phenomena which we do not understand.

The question in the title of this article was first asked by Einstein during his promenade with Pauli, after it was rephrased in different contexts by Leggett and Garg [85] and Mermin [86]. In this paper, we defend Einstein's position [87–89] as we believe that the moon continues to exist if nobody looks at it.

The paper is organized as follows:

In section Experimental Spreadsheets and Bell-Type Inequalities we show that Bell-CHSH, Leggett-Garg, and Boole inequalities [34, 70, 78, 90] are trivial arithmetic properties of some Nx3 or Nx4 spreadsheets containing ± 1 entries.

In section Local Realistic Models for EPR-Bohm Experiment we define LRHVM and explain why these models cannot reproduce quantum predictions for ideal EPRB experiments which are impossible to implement.

In section Contextual Description of Spin Polarization Correlation Experiments we show how, by incorporating in an LRHVM setting dependent parameters describing measuring instruments, we may explain in a locally causal way correlations between distant outcomes observed in SPCE.

In section Subtle Relationship of Probabilistic Models With Experimental Protocols we explain why Bell-1971 model [2, 91] and Clauser-Horne model [4] are inconsistent with experimental protocols used in SPCE.

In section Quantum Mechanics and CHSH Inequalities we define *quantum CHSH inequality* [92, 93] and Tsirelson bound [92] and we reproduce Khrennikov's recent arguments [43] that the violation of quantum CHSH inequality confirms the local incompatibility of some quantum observables.

In section The Roots of Quantum Non-locality we show that speculations about quantum non-locality are in fact rooted in the incorrect interpretation of von Neumann/Lüders projection postulates [94, 95].

In section Apparent Violations of Bell-Boole Inequalities in Elastic Collision Experiments we discuss simple experiments with elastically colliding metal balls [54] and we explain an apparent violation of Bell-Boole inequalities in these experiments. These experiments allow us to better understand LRHVM and why they fail to describe SPCE.

Section Conclusions contains some conclusions.

EXPERIMENTAL SPREADSHEETS AND BELL-TYPE INEQUALITIES

Let us examine properties of a spreadsheet with four columns each containing N entries ± 1. We may have N-identical rows or 16 different rows permuted in an arbitrary order. The entries may be coded values representing outcomes of some random experiment (e.g., flipping of four fair coins). They may display the results of some population survey or represent daily variations of some stocks. They also may be created by an artist as a particular visual display. Thus, the columns in the spreadsheet may be finite samples of a particular discrete time-series of data or they can be devoid of any statistical meaning.

If each line of the spreadsheet contains measured values (a. a, b, b′) of jointly distributed random variables (A, A′, B, B′) taking the values ±1 then b = b′ or b = –b′and then:

$$|s| = |ab - ab' + a'b + a'b'| \\ = |a(b - b')| + |a'(b + b')| \leq 2. \quad (1)$$

From (1) we immediately obtain CHSH inequality:

$$|s| \leq \sum_{a,a',b,b'} |ab - ab' + a'b + a'b'| \mathrm{p}(\mathrm{a}, \mathrm{a}', \mathrm{b}, \mathrm{b}') \\ \leq |\mathrm{E(AB)} - \mathrm{E(AB')}| + |\mathrm{E(A'B)} + \mathrm{E(A'B')}| < 2 \quad (2)$$

where p(a, a, b, b′) is a joint probability distribution of (A, A′, B, B′) and $E(AB) = \sum_{a,b} abp(a,b)$ is a pairwise expectation of A and B obtained using a marginal probability distribution $p(a,b) = \sum_{a',b'} p(a,a',b,b')$.

If A′ = B and B′ = C then E(BB) = 1 and we obtain from (2) Boule and Leggett-Garg inequalities satisfied by three jointly distributed variables (A,B,B′):

$$|\mathrm{E(AB)} - \mathrm{E(AC)}| + 1 + \mathrm{E(BC)} \leq 2 \Rightarrow |\mathrm{E(AB)} \\ - \mathrm{E(AC)}| \leq 1 - \mathrm{E(BC)} \quad (3)$$

The Bell (64) inequality $|P(\vec{a},\vec{b}) - P(\vec{a},\vec{c})| \leq 1 + P(\vec{b},\vec{c})$ is a Boole inequality (3) for $P(\vec{a},\vec{b}) = -E(AB)$, $P(\vec{a},\vec{c}) = -E(AC)$ and $P(\vec{b},\vec{c}) = -E(BC)$.

All these inequalities are deduced using the inequality (1) obeyed by any four numbers equal to ±1. The inequalities (2) and (3) are in fact necessary and sufficient conditions for the existence of a joint probability distribution of only pairwise measurable ±1-valued random variables [18, 19].

The inequalities (2) and (3) are of course also valid if |A|≤1, |A′|≤1|, |B|≤1, and |B′|≤1.

LOCAL REALISTIC MODELS FOR EPR-BOHM EXPERIMENT

In physics, Bell-CHSH inequalities [2] were derived in an attempt to reproduce quantum predictions for impossible to implement ideal EPRB experiments [96].

In EPRB experiments a source produces a steady flow of electron- or photon- pairs [60] prepared in a quantum spin-singlet state. One photon is sent to Alice and another to Bob in distant laboratories where they measure photons' spin projections in directions **a** and **b** (**||a||=||b||=1**) and the outcomes "spin up" or "spin down" are coded ±1. There are no losses and for any pair of experimental settings Alice's and Bob's measuring stations output correlated pairs of outcomes.

If Alice and Bob perform their experiments using four pairs of settings [(**a**, **b**); (**a′**, **b**); (**a**, **b′**); and (**a′**, **b′**)], then outcomes ±1 are the values of corresponding 4 binary random variables A_a, $A_{a'}$, B_b, and $B_{b'}$. In [1, 2] these values are determined by some ontic parameters λ (hidden variables) describing pairs of photons when they arrive at Alice's and Bob's measuring stations. Pairwise expectations of measured random variables, in different settings, are all expressed in terms of a unique probability distribution p(λ) defined on an unspecified probability space Λ:

$$E(A_a B_b) = \sum_{\lambda \in \Lambda} A_a(\lambda) B_b(\lambda) p(\lambda) \\ = \sum_{\lambda} A(\vec{a},\lambda) B(\vec{b},\lambda) p(\lambda) \quad (4)$$

$$E(A_a B_{b'}) = \sum_{\lambda \in \Lambda} A_a(\lambda) B_{b'}(\lambda) p(\lambda) \\ = \sum_{\lambda} A(\vec{a},\lambda) B(\vec{b}',\lambda) p(\lambda) \quad (5)$$

$$E(A_{a'} B_b) = \sum_{\lambda \in \Lambda} A_{a'}(\lambda) B_b(\lambda) p(\lambda) \\ = \sum_{\lambda} A(\vec{a}',\lambda) B(\vec{b},\lambda) p(\lambda) \quad (6)$$

$$E(A_{a'} B_{b'}) = \sum_{\lambda \in \Lambda} A_{a'}(\lambda) B_{b'}(\lambda) p(\lambda) \\ = \sum_{\lambda} A(\vec{a}',\lambda) B(\vec{b}',\lambda) p(\lambda) \quad (7)$$

If in (1) we replace a = A_a (λ) = A(**a**, λ), a′ = $A_{a'}$ (λ) = A(**a′**, λ), b = B_b (λ) = B(**b**, λ), and b′ = $B_{b'}$ (λ) = B(**b′**, λ) we obtain:

$$|S| = \sum_{\lambda} |A(\vec{a},\lambda)B(\vec{b},\lambda) - A(\vec{a},\lambda)B(\vec{b}',\lambda) + A(\vec{a},\lambda)B(\vec{b},\lambda) \\ + \mathrm{A}(\vec{\mathrm{a}},\lambda)B(\vec{\mathrm{b}}',\lambda)| p(\lambda) \leq 2 \quad (8)$$

Therefore, the expectations (4–7) obey the inequality (2).

Bell used the integration over hidden variables instead of the summation. In agreement with QM, he insisted that one cannot measure simultaneously or in a sequence different spin projections of the same photon, thus the expectations E(A_a $A_{a'}$ B_b $B_{b'}$) have no physical meaning. Nevertheless, the existence of those counterfactual non-vanishing expectations is necessary in order to prove (8). Namely there exists a mapping:

$$\lambda \to (A_a(\lambda), A_{a'}(\lambda), B_b(\lambda), B_{b'}(\lambda)) = (\mathrm{a}, \mathrm{a}', \mathrm{b}, \mathrm{b}') \quad (9)$$

which defines a joint probability distribution p(a, a′, b, b′) and a non-vanishing counterfactual expectation E(A_a $A_{a'}$ B_b $B_{b'}$) [56, 97].

If a joint probability distribution p (a, a′, b, b′) does not exist, the inequalities (2) and (8) cannot be derived. According to QM, such joint probability distributions do not exist in EPRB, thus, for some settings, quantum predictions violate CHSH inequalities.

For an ideal EPRB experiment, QM predicts: E(A_a B_b)**=** **– a · b=** – cos θ and E(A_a) = E(B_b) = 0. If **b** and **b′** are arbitrary orthogonal unit vectors (**b·b′ = 0), a = (b′-b)/$\sqrt{2}$** and **a′ = (b + b′)/$\sqrt{2}$** then S**=[(b′-b)·(b′-b)+(b′ + b)·(b′ + b]/$\sqrt{2}$ =** $4/\sqrt{2}$ **=** $2\sqrt{2}$. This value significantly violates CHSH and saturates the

Tsirelson's bound [92], which we discuss in section Quantum Mechanics and CHSH Inequalities.

According to QM: $E(A_aB_a) = -1$ and $E(A_aB_{-a}) = 1$ for any vector **a.** Thus, Alice and Bob when measuring spin projections using the settings (**a**, **a**) and (**a**, **–a**) should obtain perfectly anti-correlated or correlated outcomes, respectively. At the same time, these outcomes are believed to be produced in an *irreducible random way*, thus one encounters an impossible to resolve paradox:

> *"a pair of dice showing always perfectly correlated outcomes."*

In order to reproduce perfect correlations in LRHVM, one abandons the irreducible randomness and assumes that Alice's and Bob's outcomes are predetermined before measurements are done. Therefore, there exists a counterfactual joint probability distribution of all these predetermined outcomes and CHSH inequalities may not be violated [86, 97–99].

Fortunately, this paradox exists only on paper because an ideal EPRB experiment does not exist and in SPCE we neither observe strict correlations nor anti-correlations between clicks.

In the next section we show how imperfect correlations between clicks in SPCE may be explained in a locally causal way without evoking quantum magic.

CONTEXTUAL DESCRIPTION OF SPIN POLARIZATION CORRELATION EXPERIMENTS

In SPCE, correlated signals/photons, sent by some sources, arrive at distant measuring stations and produce clicks on the detectors. There are black counts, laser intensity drifts, photon registration time delays, etc. Detected clicks have time tags which are different for Alice and Bob. One has to identify clicks corresponding to photons that are members of the same entangled "pair of photons" which is a setting- dependent complicated task. Correlated clicks are rare events and estimated correlations depend on the photon-identification procedure used. A detailed discussion regarding how data is gathered and coincidences determined may be found, for example in Hess and Philipp [22], De Raedt et al. [80, 82], Adenier and Khrennikov [100, 101], and Larsen [102].

Even if all the above-mentioned difficulties had not existed, QM would not have predicted perfect correlations for real experiments. Settings of realistic polarizers may not be treated as mathematical vectors [47], but rather as small spherical angles; therefore instead of $E(A_a\,B_b) = -\mathbf{a}\cdot\mathbf{b} = -\cos\theta$ we obtain:

$$E(A_aB_b = \eta(\vec{a})\eta\vec{b}\int_{Oa}\int_{Ob}-\vec{u}\cdot\vec{v}d\vec{u}d\vec{v} \tag{10}$$

where $O_a = \{\vec{u} \in S^{(2)};\ |1-\vec{u}\cdot\vec{a}| \le \varepsilon\}$ and $O_b = \{\vec{v} \in S^{(2)};\ |1-\vec{v}\cdot\vec{b}| \le \varepsilon\}$

In order to estimate correlations, Alice and Bob have to choose correlated time windows. They retain only pairs of windows containing two types of events: "a click on a detector 1 and a click on a detector 2" or "a click on only one of the detectors." Therefore, in SPCE, random variables describing outcomes of these experiments have three possible values coded as ± 1 or 0.

To make a comparison with the notation used in [60] easier, where more details may be found, we denote different pairs of settings by (x, y),..., (x′, y′) and $E(A_xB_y) = E(AB|x, y)$.

Imperfect correlations estimated in SPCE may be reproduced by the following locally causal contextual hidden variable model [59, 60]:

$$E(A_xB_y) = \sum_{\lambda\in\Lambda_{xy}} A_x(\lambda_1,\lambda_x)B_y(\lambda_2,\lambda_y)p_x(\lambda_x)p_y(\lambda_y)p(\lambda_1,\lambda_2) \tag{11}$$

$$E(A_xB_{y'}) = \sum_{\lambda\in\Lambda_{xy'}} A_x(\lambda_1,\lambda_x)B_{y'}(\lambda_2,\lambda_{y'})p_x(\lambda_x)p_{y'}(\lambda_{y'})p(\lambda_1,\lambda_2) \tag{12}$$

$$E(A_{x'}B_y) = \sum_{\lambda\in\Lambda_{x'y}} A_{x'}(\lambda_1,\lambda_{x'})B_y(\lambda_2,\lambda_y)p_{x'}(\lambda_{x'})p_y(\lambda_y)p(\lambda_1,\lambda_2) \tag{13}$$

$$E(A_{x'}B_{y'}) = \sum_{\lambda\in\Lambda_{x'y'}} A_{x'}(\lambda_1,\lambda_{x'})B_{y'}(\lambda_2,\lambda_{y'})p_{x'}(\lambda_{x'})p_{y'}(\lambda_{y'})p(\lambda_1,\lambda_2) \tag{14}$$

$$E(A_x) = \sum_{\lambda\in\Lambda_{xy}} A_x(\lambda_1,\lambda_x)p_x(\lambda_x)p_y(\lambda_y)p(\lambda_1,\lambda_2) \tag{15}$$

$$E(B_y) = \sum_{\lambda\in\Lambda_{xy}} B_y(\lambda_2,\lambda_y)p_x(\lambda_x)p_y(\lambda_y)p(\lambda_1,\lambda_2) \tag{16}$$

where $A_x(\lambda_1, \lambda_x) = 0,\pm 1$, $A_{x'}(\lambda_1, \lambda_{x'}) = 0,\pm 1$, $B_y(\lambda_2, \lambda_y) = 0,\pm 1$, and $B_{y'}(\lambda_2, \lambda_{y'}) = 0,\pm 1$. Please note that $A_x(\lambda_1, \lambda_{x'})$, $A_{x'}(\lambda_1, \lambda_x)$, $B_y(\lambda_2, \lambda_{y'})$, and $B_{y'}(\lambda_2, \lambda_y)$ are undefined. The experiments performed in incompatible settings are described by dedicated probability distributions defined on 4 disjoint hidden variable spaces:

$$\begin{aligned}\Lambda_{xy} &= \Lambda_{12}\times\Lambda_x\times\Lambda_y\,;\ \Lambda_{x'y} = \Lambda_{12}\times\Lambda_{x'}\times\Lambda_y\,;\ \Lambda_{xy'}\\ &= \Lambda_{12}\times\Lambda_x\times\Lambda_{y'}\,;\ \Lambda_{x'y'} = \Lambda_{12}\times\Lambda_{x'}\times\Lambda_{y'}\end{aligned} \tag{17}$$

where $\Lambda_x \bigcap \Lambda_{x'} = \Lambda_y \bigcap \Lambda_{y'} = \varnothing$. Therefore, counterfactual expectations $E(A_x A_{x'})$, $E(B_y B_{y'})$, $E(A_x A_{x'} B_y B_{y'})$ do not exist and Bell and CHSH inequalities may not be derived.

The efficiency of detectors is not 100% and it is difficult to establish correct coincidences between distant clicks because of time delays. These two problems, called efficiency and coincidence-time loopholes, were discussed in detail by Larsen and Gill [103] in terms of the sub-domains of hidden variables corresponding to four experimental settings. They found that CHSH inequality has to be modified:

$$\begin{aligned}|E(A_xB_y|\Lambda_{xy}) \quad &-E(A_xB_{y'}|\Lambda_{xy'})| + |E(A_{x'}B_y|\Lambda_{x'y})\\ &+E(A_{x'}B_{y'}|\Lambda_{x'y'})| \le 4-2\delta\end{aligned} \tag{18}$$

where $\delta \propto p(\Lambda_{xy}\bigcap\Lambda_{xy'}\bigcap\Lambda_{x'y}\bigcap\Lambda_{x'y'})$. In our model $p(\varnothing) = 0$, thus the only constraint for S in our model is a no-signaling bound: $|S|\le 4$.

Our model contains enough free parameters to fit any estimated correlations. For example, if we start with k values of λ_1, k values of λ_2, and m values for each λ_x, $\lambda_{x'}$, λ_y, and $\lambda_{y'}$ we have km pairs of (λ_1, λ_x), 3^{km} functions $A_x(\lambda_1, \lambda_x)$, and 3^{km} functions $B_y(\lambda_2, \lambda_y)$. We also have m-1 free parameters for each $p_x(\lambda_x)$, $p_{x'}(\lambda_{x'})$, $p_y(\lambda_y)$, and $p_{y'}(\lambda_{y'})$ and $\left(\frac{k(k+1)}{2}-1\right)$ free parameters for $p(\lambda_1, \lambda_2)$. Thus, we have 4×3^{km} functions to choose and $4(m-1) + k(k-1)/2$ free parameters to fit 32 probabilities or eight expectations estimated in experiments performed using four pairs of settings. If instead of four pairs of settings Alice and Bob use nine pairs of settings, then we may increase m and k as needed to fit 72 probabilities or 12 expectation values, etc.

In mathematical statistics we concentrate on observable events: outcomes of random experiments or results of a population survey. Joint probability distributions are used only to describe random experiments producing several outcomes in each trial e.g., rolling several dice or various data items describing the same individual drawn from some statistical population. Probabilistic models describe a scatter of these outcomes without entering into the details of how outcomes are created.

Hidden variable probabilistic models introduce some invisible "hidden events" which determine subsequent real outcomes of random experiments. In Bell model (4–7), pairs of photons ("beables") are described by λ before measurements take place. Because clicks are predetermined by the values of λ there exists the mapping (9) and the probability distribution of "hidden events" described by $p(\lambda)$ which may be replaced by a joint distribution $p(a, a', b, b')$.

In contextual model (11–17), an outcome of "a click" or "no-click" is not predetermined and is created in a locally causal way in function of a hidden parameter describing a signal ("photon") arriving at the measuring station and a hidden parameter describing a measuring instrument in the moment of their interaction. The model (11–17) gives an insight into how apparently random outcomes are created in SPCE.

In model (4–7) there exists a joint probability distribution of all hidden events labeled by λ. In the model (14–17), hidden events form 4 disjoint probability spaces and there exist only four distinct joint probability distributions ($p_{xy}(\lambda_x, \lambda_1, \lambda_y, \lambda_2)$ on $\Lambda_{xy},\ldots$, $p_{x'y'}(\lambda_{x'}, \lambda_1, \lambda_{y'}, \lambda_2)$ on $\Lambda_{x'y'}$). A joint probability distribution of all possible hidden events ($\lambda_x, \lambda_1, \lambda_y, \lambda_2, \lambda_{x'}, \lambda_{y'}, \lambda_2$) does not exist because hidden events ($\lambda_x, \lambda_{x'}$) and ($\lambda_y, \lambda_{y'}$) may never occur together. This is why one may not prove CHSH assuming the existence of such probability distribution and a non-vanishing $E(A_x A_{x'} B_y B_{y'})$ used to prove (2–3, 8) does not exist.

SUBTLE RELATIONSHIP OF PROBABILISTIC MODELS WITH EXPERIMENTAL PROTOCOLS

In 1971, Bell [91] pointed out that whilst one may incorporate into his model additional hidden variables describing measuring instruments, it does not invalidate his conclusions because after the averaging over instrument variables the pairwise expectations still have to obey CHSH inequalities. We reproduce his reasoning in the notation consistent with (11–17).

If we average over the variables λ_x and λ_y we obtain:

$$E(A_xB_y) = \sum_{\lambda_1,\lambda_2} \overline{A}_x(\lambda_1)\overline{B}_y(\lambda_2)p(\lambda_1,\lambda_2) \tag{19}$$

$$E(A_xB_{y'}) = \sum_{\lambda_1,\lambda_2} \overline{A}_x(\lambda_1)\overline{B}_{y'}(\lambda_2)p(\lambda_1,\lambda_2) \tag{20}$$

$$E(A_{x'}B_y) = \sum_{\lambda_1,\lambda_2} \overline{A}_{x'}(\lambda_1)\overline{B}_y(\lambda_2)p(\lambda_1,\lambda_2) \tag{21}$$

$$E(A_{x'}B_{y'}) = \sum_{\lambda_1,\lambda_2} \overline{A}_{x'}(\lambda_1)\overline{B}_{y'}(\lambda_2)p(\lambda_1,\lambda_2) \tag{22}$$

where

$$\begin{aligned}\overline{A}_x(\lambda_1) &= \sum_{\lambda_x} A_x(\lambda_1,\lambda_x)p_x(\lambda_x)\ ;\ \overline{B}_y(\lambda_2)\\ &= \sum_{\lambda_y} B_y(\lambda_2,\lambda_y)p_y(\lambda_y)\end{aligned} \tag{23}$$

$$\begin{aligned}\overline{A}_{x'}(\lambda_1) &= \sum_{\lambda_{x'}} A_{x'}(\lambda_1,\lambda_{x'})p_{x'}(\lambda_{x'})\ ;\ \overline{B}_{y'}(\lambda_2)\\ &= \sum_{\lambda_{y'}} B_y(\lambda_1,\lambda_{y'})p_{y'}(\lambda_{y'})\end{aligned} \tag{24}$$

Since $|A_x(\lambda_1, \lambda_x)| \le 1$, $|A_{x'}(\lambda_1, \lambda_{x'})| \le 1$, $|B_y(\lambda_2, \lambda_y)| \le 1$, $|B_{y'}(\lambda_2, \lambda_{y'}) = | \le 1$ thus $|\overline{A}_x(\lambda_1)| \le 1$, $|\overline{A}_{x'}(\lambda_1)| \le 1$, $|\overline{B}_y(\lambda_2)| \le 1$, $|\overline{B}_{y'}(\lambda_2)| \le 1$ and:

$$\begin{aligned}&|\bar{A}_x(\lambda_1)||\overline{B}_y(\lambda_2) - \overline{B}_{y'}(\lambda_2)| + |\bar{A}_{x'}(\lambda_1)||\overline{B}_y(\lambda_2)\\ &\quad + \bar{B}_{y'}(\lambda_2)| \le 2\end{aligned} \tag{25}$$

Although the expectations calculated using the Equations (11–14) and (19–22) have the same values, the two sets of formulas describe different experiments. In the experiment described by the Equations (11–14), pairs of photons arrive sequentially to measuring instruments which produce in a locally causal way "a click" or "no-click," and a counterfactual Nx4 spreadsheet of all possible outcomes does not exist and may not be used to prove CHSH inequalities. Thus, the estimated pairwise expectations may significantly violate (8), which they do.

The Equations (19–22) describe an experiment, impossible to implement, which uses the following two-step experimental protocol:

1. For each arriving pair of photons estimate the averages (23–24).
2. Display estimated values $|\overline{A}_x(\lambda_1)| \le 1$, $|\overline{A}_{x'}(\lambda_1)| \le 1$, $|\overline{B}_y(\lambda_2)| \le 1$,and $|\overline{B}_{y'}(\lambda_2)| \le 1$ in four columns of a Nx4 spreadsheet.
3. Use all entries of this spreadsheet to estimate expectations (19–22).

Because the entries of each line of this spreadsheet obey the inequality (1), if we could implement this protocol the estimated expectations would obey CHSH for any finite sample.

There is a significant difference between a probabilistic model and a hidden variable model. If we average out some variables in a probabilistic model, we always obtain a marginal probability distribution describing some feasible experiment. If we average out some hidden variables in a hidden variable model, we may obtain a new hidden variable model which does not correspond to any feasible experiment.

For a similar reason, the experimental protocol of SHVM is inconsistent with the protocol used in SPCE. A much more detailed discussion of a subtle relationship of probabilistic models with experimental protocols may be found in [56].

As we demonstrated with Hans De Raedt [104], different experimental protocols, based on the same probabilistic model, may generate significantly different estimates of various population parameters.

If we want to compare the data obtained in SPCE with quantum predictions, we have to post- select only pairs of ± 1 outcomes which correspond to invisible entangled pairs of photons. Thus, instead of the Equations (11, 15–16) we obtain:

$$E(A_xB_y|A_x \neq 0, B_y \neq 0) = \sum_{\lambda \in \Lambda'_{xy}} A_x(\lambda_1, \lambda_x)B_y(\lambda_2, \lambda_y)p_x(\lambda_x)p_y(\lambda_y)p(\lambda_1, \lambda_2) \tag{26}$$

$$E(A_x|A_x \neq 0, B_y \neq 0) = \sum_{\lambda \in \Lambda'_{xy}} A_x(\lambda_1, \lambda_x)p_x(\lambda_x)p_y(\lambda_y)p(\lambda_1, \lambda_2) \tag{27}$$

$$E(B_y|A_x \neq 0, B_y \neq 0) = \sum_{\lambda \in \Lambda'_{xy}} B_y(\lambda_2, \lambda_y)p_x(\lambda_x)p_y(\lambda_y)p(\lambda_1, \lambda_2) \tag{28}$$

where $\Lambda'_{xy} = \{\lambda \epsilon\ \Lambda_{xy} | A_x\ (\lambda_1, \lambda_x) \neq 0 \text{ and } B_y\ (\lambda_2, \lambda_y) \neq 0\}$. In a similar way, we transform the expectations (12–14) into conditional expectations. Using these conditional expectations, we may not derive CHSH; thus our model does not exclude their violations in SPCE. It may also explain in a rational way an apparent violation of no- signaling reported in [79, 80, 100, 101, 105–108]:

$$\begin{aligned} E(A_x|A_x &\neq 0, B_y \neq 0) \neq E(A_x|A_x \neq 0, B_{y'} \neq 0); \\ E(B_y|A_x &\neq 0, B_y \neq 0) \neq E(B_y|A_{x'} \neq 0, B_y \neq 0) \end{aligned} \tag{29}$$

The setting-dependence of these marginal expectations does not prove no-signaling because E (A_x) and E (B_y) defined by (15–16) do not depend on the distant measurement settings.

Please note that the expectations (26) may not be transformed into a factorized form (21).

Naïve quantum predictions for a singlet state cannot explain the correlations observed in SPCE. One has to use much more complicated density matrices [109] containing free parameters, and still some discrepancies between the theoretical predictions and the data persist. A more detailed discussion of how the data are analyzed in SPCE and how the apparent violation of no-signaling may be explained may be found in [60].

Since our description of real data is causally local, all speculations about *quantum non-locality* are unfounded.

In the next section we explain that, contrary to what is believed, probabilistic predictions of QM are not in conflict with local causality.

QUANTUM MECHANICS AND CHSH INEQUALITIES

According to the statistical contextual interpretation [29, 52, 57, 89, 110, 111], QM provides probabilistic predictions for experiments performed in well-defined experimental contexts. In these experiments, identical preparations of physical systems are followed by measurements of physical observables. A class of identical preparations is described by a state vector $|\psi\rangle$ or by a density matrix ρ and a class of equivalent measurements of an observable A is represented by a Hermitian/self-adjoint operator$\hat{A}$. Outcomes of measurements are eigenvalues of these operators. In general, outcomes are not pre-determined and they are created as a result of the interaction of measuring instruments with physical systems. In the same experimental context, only the values of compatible physical observables, represented by commuting operators, give sharp values when measured jointly.

In SPCE, "photon pairs," prepared by a source, are described by a density matrix ρ and physical observables A and B by Hermitian operators $\hat{A}_1 = \hat{A} \otimes I$ and $\hat{B}_1 = I \otimes \hat{B}$ defined on a Hilbert space $H = H_1 \otimes H_2$. The correlations between measured values of these observables are evaluated using a conditional covariance between A and B [56, 58]:

$$\text{cov}(A, B|\rho) = E(AB|\rho) - E(A|\rho)E(B|\rho) \tag{30}$$

where, $E(A|\rho) = Tr\rho\hat{A}_1, E(B|\rho) = Tr\rho A\hat{B}_1$ and $E(AB|\rho) = Tr\rho\hat{A}_1\hat{B}_1$. If ρ is an arbitrary mixture of separable states then quantum correlations have to obey CHSH:

$$|E(AB|\rho) - E(AB'|\rho)| + |E(A'B|\rho) + E(A'B'|\rho)| \leq 2 \tag{31}$$

As we saw in section Experimental Spreadsheets and Bell-Type Inequalities, the inequality (31) may be significantly violated for entangled quantum states if specific incompatible pairs of settings are chosen.

The quantum description is contextual because a triplet $\{\rho, \hat{A}_1, \hat{B}_1\}$ depends explicitly on a preparation of "photon pairs" and on observables (A,B) measured using specific experimental settings. Different incompatible experimental settings are therefore described in QM by different specific Kolmogorov models.

In particular, Cetto et al. [73] have recently demonstrated that expectations E(AB | ψ), for a singlet state $|\psi\rangle \in H$, may be expressed in terms of the eigenvalues of operators $\hat{A} = \vec{\sigma} \cdot \vec{a}$ and $\hat{B} = \vec{\sigma} \cdot \vec{b}$ using specific dedicated probability distributions. We reproduce below their results in our notation:

$$E(AB|\psi) = -\vec{a} \cdot \vec{b} = \sum_{\alpha\beta} \alpha\beta p_{ab}(\alpha, \beta) = E(A_aB_b) \tag{32}$$

where $\hat{A} \otimes \hat{B}|\alpha\beta\rangle_{ab} = \alpha\beta|\alpha\beta\rangle_{ab}$, $p_{ab}(\alpha,\beta) = |\langle\psi|\alpha\beta\rangle_{ab}|^2$ and $\alpha = \pm 1$ and $\beta = \pm 1$. For the remaining settings we obtain:

$$E(AB'|\psi) = -\vec{a}\cdot\vec{b}' = \sum_{\alpha\beta'} \alpha\beta' p_{ab'}(\alpha,\beta') = E(A_{a'}B_b) \quad (33)$$

$$E(A'B|\psi) = -\vec{a}'\cdot\vec{b} = \sum_{\alpha'\beta} \alpha'\beta p_{a'b}(\alpha',\beta) = E(A_{a'}B_b) \quad (34)$$

$$E(A'B'|\psi) = -\vec{a}'\cdot\vec{b}' = \sum_{\alpha'\beta'} \alpha'\beta' p_{a'b'}(\alpha',\beta') = E(A_{a'}B_{b'}) \quad (35)$$

If 4 experiments are performed in incompatible (complementary) contexts then a joint probability distribution $p(\alpha\alpha'\beta\beta')$ and the expectation values $E(A_aA_{a'}B_bB_{b'})$ do not exist in agreement with the contextual model (11–14).

In 1982, Fine [18, 19] demonstrated that Bell-CHSH inequalities are necessary and sufficient conditions for the existence of a joint probability distribution of ± 1-valued observables (A,A′,B,B′).

As we saw in section Local Realistic Models for EPR-Bohm Experiment, QM predicts a significant violation of CHSH inequality: $S = 2\sqrt{2}$.

In 1980, Tsirelson [92] proved that $2\sqrt{2}$ is the greatest value of S allowed by QM:

$$|S| = |\left\langle\psi|\hat{S}|\psi\right\rangle| = |\left\langle\psi|\hat{A}\hat{B} - \hat{A}\hat{B}' + \hat{A}'\hat{B} + \hat{A}'\hat{B}'|\psi\right\rangle| \leq 2\sqrt{2} \quad (36)$$

where $|\psi\rangle \in H$ is an arbitrary pure state and all Hermitian operators on the left hand side are arbitrary elements of C* algebra having their norms ($\left\|\hat{A}\right\| = \sup_{\|\phi\|\leq 1}\left\langle\phi|\grave{A}|\phi\right\rangle$) smaller or equal to 1. In order to prove (36), Tsirelson used a following operator inequality:

$$\hat{S}^2 = \left(\hat{A}\hat{B} - \hat{A}\hat{B}' + \hat{A}'\hat{B} + \hat{A}'\hat{B}'\right)^2 \leq 4I + \left[\hat{A},\hat{A}'\right]\left[\hat{B},\hat{B}'\right] \quad (37)$$

From (37) he deduced immediately that $\left\|\hat{S}^2\right\| \leq 4 + \left\|\left[\hat{A},\hat{A}'\right]\right\|\left\|\left[\hat{B},\hat{B}'\right]\right\| \leq 4 + 2 \times 2 = 8$, thus $\left\|\hat{S}\right\| \leq 2\sqrt{2}$ proves *quantum CHSH inequality* (36). Landau [93] defined an operator $\hat{C} = \frac{1}{2}\hat{S}$ and noticed that if A, A′. B and B′ are ± 1-valued observables ($\hat{A}^2 = I$), then the inequality (37) becomes the equality $\hat{C}^2 = I + \frac{1}{4}\left[\hat{A}_1,\hat{A}_2\right]\otimes\left[\hat{B}_1,\hat{B}_2\right]$ and $\left\|\hat{C}\right\| \leq 1$.

Recently, Khrennikov discussed various implications of (37). CHSH inequality may be violated only if both $\left[\hat{A}_1,\hat{A}_2\right] \neq 0$ and $\left[\hat{B}_1,\hat{B}_2\right] \neq 0$. Therefore, the violation of CHSH proves the *local incompatibility* of Alice and Bob's specific physical observables [43] which has nothing to do with *quantum non-locality*.

The *local incompatibility* of some observables allows neither doubt over the *local causality* in nature nor the "*objective*" existence of elementary particles and atoms.

THE ROOTS OF QUANTUM NON-LOCALITY

Mathematical models provide abstract idealized descriptions of physical phenomena and in general are unable to explain, by detailed causal chains, why such a description is successful. For example, in Newton's equations describing the motion of planets, a small change in the position of one planet at time t seems to instantaneously change gravitational forces acting on distant planets. Newton admitted that no intuitive explanation of this mystery existed, but it did not diminish the value of his gravitation theory.

According to the special theory of relativity, the physical influences may not propagate faster than the speed of light c, thus it became clear that Newton's theory of gravitation should be modified. Einstein, by constructing the general theory of relativity, succeeded in reconciling the special theory of relativity with Newton's theory of gravitation which is still used with success by NASA.

Similarly, in a non-relativistic QM, relativistic effects are not important. The theory provides algorithms which allow probabilistic predictions to be made regarding outcomes of experiments performed in well-defined macroscopic contexts. A time-dependent Schrodinger equation describes only a time evolution of a complex valued function (probability amplitude), which, together with Hermitian/self-adjoint operators, is used to provide probabilistic predictions for a scatter of experimental outcomes.

Quantum predictions are consistent with Einsteinian no-signaling. Quantum field theory (QFT) is explicitly relativistic and field operators in space-like regions commute.

The speculations about *quantum non-locality* are only rooted in incorrect "individual interpretations" of QM according to which:

1. a pure state vector/wave function $|\psi\rangle$ is an attribute of an individual physical system;
2. a measurement of a physical observable A instantaneously changes/collapses the initial state vector onto an eigenvector vector $|a_i\rangle$ of the corresponding operator $\hat{A}$ with a probability $p = \langle a_i|\psi\rangle^2$;
3. a measurement outcome is an eigenvalue a_i corresponding to the vector $|a_i\rangle$;
4. if two physical systems, S_1 and S_2, interacted in the past and separated, a measurement of the observable A performed on the system S_1 and yielding a result $A=a_i$ determines instantaneously a state vector $|\phi\rangle_{A=a_i}$ of the system S_2 in a distant location.

Using (1–4) one concludes that measurements of observables A and B performed on systems S_1 and S_2 create in an "*irreducible random way*" perfectly correlated outcomes at distant space-like locations, thus we encounter the same paradox: "*a pair of dice showing perfectly correlated outcomes.*"

The statistical contextual interpretation of QM (SCI) [52, 57, 89] is free of paradoxes. According to this interpretation, a quantum state vector represents only an ensemble of identically prepared physical systems and, after a von Neumann/Lüders projection, a new state describes a different ensemble of physical systems. Namely: $|\phi\rangle_{A=a_i}$ describes all the systems S_2 such that measurements of the observable A on their entangled partners (systems S_1) gave the same outcome $A = a_i$.

The statistical interpretation does not claim that QM provides the complete description of individual physical systems and the question of whether quantum probabilities may be deduced from some more detailed description of quantum phenomena is left open [46, 52, 59, 61, 87–89, 112, 113].

Lüders projection and its interpretation have been discussed recently in detail by Khrennikov [44]. We reproduce below a few statements from the abstract of his article:

> "If probabilities are considered to be objective properties of random experiments, we show that the Lüders projection corresponds to the passage from joint probabilities describing all sets of data to some marginal conditional probabilities describing some particular subsets of data. If one adopts a subjective interpretation of probabilities, such as Qbism, then the Lüders projection corresponds to standard Bayesian updating of the probabilities. The latter represents degrees of beliefs of local agents about outcomes of individual measurements which are placed or which will be placed at distant locations. In both approaches, probability-transformation does not happen in the physical space, but only in the information space. Thus, all speculations about spooky interactions or spooky predictions at a distance are simply misleading."

In 1998, Ballentine explained in his book that "individual interpretation" of QM is incorrect: "Once acquired, the habit of considering an individual particle to have its own wave function is hard to break. Even though it has been demonstrated strictly incorrect." Therefore, talking about "passion at the distance," "predictions at the distance," and "steering at the distance" may only lead to incorrect mental pictures and create unnecessary confusion.

In QM, measuring devices always play an active role. Allahverdyan et al. [110, 111] recently solved the dynamics of a particular realistic quantum measurement and discussed what this implies for the interpretation of QM. On page 6 in [110] they wrote:

> "A measurement is the only means through which information may be gained about a physical system. Both in classical and in quantum physics, it is a dynamical process which couples this system S to another system, the apparatus A. Some correlations are thereby generated between the initial (and possibly final) state of S and the final state of A."

Claims that QM is a non-local theory are also based on an incorrect interpretation of a two-slit experiment. In this experiment, a wave function (representing an ensemble of identically prepared electrons) "passes" by two slits, but this does not mean that a single electron may be in two distinct places at the same time. If two detectors are placed in front of the slits, they never click at the same time, thus an electron (but not the electromagnetic field created by an electron) passes by only one slit. According to SCI, a wave function is only a mathematical entity and QM does not provide a detailed space-time description of how the interference pattern on a screen is formed by the impacts of individual electrons.

Another root of *quantum non-locality* is Bell's insistence that the violation of Bell-type inequalities in SPCE would mean that a locally causal description of these experiments is impossible [1]:

> "In a theory in which parameters are added to quantum mechanics to determine the results of *individual measurements, without changing the statistical predictions, there must be a* mechanism whereby the setting of one measuring device can influence the reading of another instrument, however remote. Moreover, the signal involved must propagate instantaneously, so that such a theory could not be Lorentz invariant."

Consider Alice and Bob, both doing a realistic EPRB-type experiment. Theo Nieuwenhuizen brought to my attention that the already nonsensical idea of faster-than-light communication (i.e., non-locality) becomes even more "mind-boggling" when the experiments have different durations.

Bell's statement is correct only if one is talking about an ideal EPRB which does not exist. The violations of various Bell-type inequalities in real SPCE prove only that these experiments may not be described by oversimplified hidden variable models. In SHVM, the outcomes, registered in distant measuring stations, are produced in an irreducible random way, thus the correlations between such outcomes are very limited. In LRHVM and in Eberhard model [5], a fate of a photon/electron is predetermined before the experiment is performed.

As we explained in section Contextual Description of Spin Polarization Correlation Experiments, imperfect correlations in SPCE may be explained in a locally causal way if instrument parameters are correctly included in a probabilistic model, closing the so-called Nieuwenhuizen's *contextuality loophole* [65–67].

Bell-CHSH inequalities may also be violated in social sciences by expectations of ± 1–valued random variables, which can only be measured pairwise but not all together. The violation of these inequalities in social sciences has nothing to say about the physical reality and the locality of nature [16, 37, 38, 114–116]. This is why we agree with Khrennikov [43], that we should get rid of *quantum non-locality* as it is a misleading notion.

In the next section we discuss simple experiments with colliding elastically metal balls in which the experimental outcomes are predetermined but an apparent violation of Bell and Boole inequalities may be proven [54]. We also discuss the violation of inequalities by the estimates obtained using finite samples.

APPARENT VIOLATIONS OF BELL-BOOLE INEQUALITIES IN ELASTIC COLLISION EXPERIMENTS

Let us consider a simple experiment with metal balls colliding elastically:

1. A 4 kg metal ball and a 1 kg metal ball are placed in some fixed positions, P_1 and P_2, on a horizontal perfectly smooth surface.
2. A device D, with a built in random numbers generator, is imparting on a lighter ball a constant rectilinear velocity with a speed described by a random variable V taking values v and distributed according to a probability density $f_V(v) = 1/10$ for $0 < v \leq 10$ and the ball is sliding without friction and without rotating toward the heavier ball.
3. After an elastic head-on collision, the heavier ball starts moving forward with the speed $V_1 = 2v/5$ and the lighter ball rebounds backwards with the speed $V_2 = 3v/5$. It is easy to check that the total linear momentum and energy are conserved: $1v = 4(2v/5)-1(3v/5)$ and $1v^2 = 4(2v/5)^2 + 1(3v/5)^2$.
4. After the collision, both balls arrive at two distant measuring stations, S_1 and S_2 (treated as black boxes), which for 4 different selected pairs of settings output values (±1) of only pairwise measurable observables (A, B), (A, C), (B, C), and (B, B).
5. Before each repetition of the experiment, Alice and Bob systematically or randomly choose a pair of settings, simply by pushing appropriate switches on their measuring stations.
6. We assume that boxes function in a locally causal way: the speed of a ball is measured and setting dependent coded values ±1 are outputted. Thus, A, B, and C denote physical observables, which are measured, which means that in the setting (B, B) the same physical observables are measured by Alice and Bob.

The observables A, B, and C are functions of hidden random variables, V_1 and V_2, which are distributed according to probability distributions $f_{V_1}(v_1) = 1/4$ and $f_{V_2}(v_2) = 1/6$ on the intervals [0, 4] and [0, 6], respectively.

Let us now define the specific functions of A(y), B(y), and C(y), where $y = v_1$ (if Alice is using a setting A) or $y = v_2$ (if (Bob is using a setting A). We have chosen that, after the collision, Alice measures the speed of the heavier ball, but it does change pairwise expectations.

- $A(y) = -1$ if $0 < y \leq 2$ and $A(y) = 1$ if $2 < y$,
- $B(y) = -1$ if $0 < y \leq 3$ and $B(y) = 1$ if $3<y$,
- $C(y) = 1$ if $0 < y \leq 3$ and $C(y) = -1$ if $3<y$.

If $V_1 = v_1$ then $V_2 = 3v_1/2$ and the pairwise expectation $E(AB) = \int_0^4 A(v_1)B(3v_1/2)f_{V_1}(v_1)dv_1$. We see immediately, that $E(AB) = \frac{1}{4}\left(\int_0^2 (-1)(-1)dv_1 + \int_2^4 (1)(1)dv_1\right) = 1$ and E (AC) = -E (AB) =-1. In a similar way we evaluate E(BC).

- If $v_1 \leq 2$ then $v_2 < 3$: $B(v_1)C(v_2) = (-1)(1) = -1$.
- If $2 < v_1 \leq 3$ then $3 < v_2 \leq 4.5$: $B(v_1)C(v_2) = (-1)(-1) = 1$.
- If $3 < v_1$ then $4.5 < V_2$: $B(v_1)C(v_2) = (1)(-1) = -1$.

Thus:

$$E(BC) = -\int_0^2 f_{V_1}(v_1)dv_1 + \int_2^3 f_{V_1}(v_1)dv_1 - \int_3^4 f_{V_1}(v_1)dv_1 \quad (38)$$

and $E(BC) = -2/4+1/4-1/4 = -1/2$ and $E(BB) = -E(BC) = 1/2$.

We see that Bell (+sign) and Boule (-sign) inequalities (3) seem to be violated:

$$|E(AB) - E(AC)| \leq 1 \pm E(BC) \quad (39)$$

because $|1-(-1)| > 1 \pm 1/2$.

The violation of (39) is surprising because the outcomes of our experiments are predetermined.

However, one has to pay attention before checking Bell-Boole-inequalities. Despite the fact that in the settings (A,B) and (B,C) Alice and Bob measure the same physical observable B, the output values ±1 are the values of 2 different random variables $B(V_1) \neq B(V_2)$. Therefore, the inequalities which are violated are not (39), but inequalities:

$$|E(A(V_1)B(V_2)) - E(A(V_1)C(V_2))| \leq 1 \pm E(B(V_1)C(V_2)) \quad (40)$$

Since for each trial, values of random variables [$A(V_1)$, $B(V_1)$, $B(V_2)$, $C(V_2)$] are predetermined by a value of the initial speed V imparted on the lighter ball, there exists an "invisible" joint probability distribution of these random variables and CHSH inequalities may not be violated:

$$|S| = |E(A(V_1)B(V_2)) - E(A(V_1)C(V_2)) + E(B(V_1)B(V_2)) + E(B(V_1)C(V_2))| = 1 + 1 + \frac{1}{2} - \frac{1}{2} \leq 2 \quad (41)$$

By treating measuring stations as black boxes, Alice and Bob do not know whether this *invisible* joint probability exists and that for each trial the values of measured observables are predetermined. Therefore they display the data obtained in different settings using four Mx2 spreadsheets and they estimate measurable pairwise expectations $E(A(V_1)B(V_2))$, $E(A(V_1)C(V_2))$, $E(B(V_1)C(V_2))$, and $E(B(V_1)B(V_2))$.

These estimates may violate the inequality (41) because, as we demonstrated in section Introduction, only the estimates obtained using all ±1 entries of Nx4 spreadsheets strictly obey CHSH inequality for any finite sample. Alice and Bob do not know that their outcomes are in fact extracted from specific lines of invisible Nx4 spreadsheet and that the columns of Mx2 spreadsheets are simple random samples drawn from the corresponding complete columns of Nx4 spreadsheet. This is why, if M and N are large, the estimated pairwise expectations may not violate the inequality (41) more significantly than is permitted by sampling errors.

In collision experiments, outcomes are predetermined and the correlations exist due to the energy and momentum conservation. In SPCE, the correlations between signals are created at the source.

There is a big difference between metal balls and photons in SPCE. In collision experiments, metal balls are distinct macroscopic objects with well-defined linear momenta. Measurements of speeds are, with a good approximation, noninvasive, thus measuring stations in fact register passively their preexisting values and output specific coded values ±1.

In SPCE we cannot observe and follow pairs of photons moving from the source to the measuring stations. By no means can the passage of a photon through a polarization beam splitter (PBS) be considered as a passive registration of a preexisting "spin up" or spin down" value. Clicks on the detectors are also the results of dynamical processes.

In collision experiments all observables are compatible, therefore Alice's modified measuring station might output in each trial values of (A (V_1), B (V_1)) and Bob's modified station values of (B(V_2), C(V_2)) which might have been displayed using a Nx4 spreadsheet. In SPCE it is impossible because the observables (A, A′) and (B, B′) are not compatible and their joint probability distribution and Nx4 spreadsheet do not exist.

The problem of how significantly finite samples, extracted from a counterfactual spreadsheet Nx4, may violate CHSH inequalities was studied by Gill [117]. Each pair of arriving photons are described by a line (±1,±1,±1,±1) from a counterfactual Nx4 spreadsheet containing predetermined values of observables (A,A′,B,B′). By randomly assigning setting labels to the lines and extracting corresponding pairs of outcomes from these lines, one obtains four simple random samples drawn from the corresponding pairs of complete columns of Nx4 spreadsheet. If these simple random samples are used to estimate pairwise expectations E(AB),E(AB′), E(A′B),E(A′B′) then:

$$\Pr\left(\langle AB\rangle_{obs} + \langle AB'\rangle_{obs} + \langle A'B\rangle_{obs} - \langle A'B'\rangle_{obs} \ \geq 2\right) \leq \frac{1}{2} \quad (42)$$

where $\langle AB\rangle_{obs}$ is an estimate of E(AB) etc. A more detailed discussion of various finite sample proofs of Bell-type inequalities may be found in [57, 117].

Let us see what happens if we display all experimental data (containing N data items for each pair of settings) in a 4Nx4 spreadsheet and randomly fill the remaining empty spaces by ±1. Pairwise expectations estimated using complete columns of this spreadsheet strictly obey CHSH inequality. One may ask a question: why can real data, being subsets of these columns, violate CHSH more significantly than it is permitted by (42)? The answer is simple: the outcomes obtained in SPCE for each pair of incompatible settings are not **simple random samples** extracted from corresponding columns of the completed 4Nx4 counterfactual spreadsheet.

In [104] we studied the impact of a sample inhomogeneity on statistical inference. In particular we generated two large samples (which were not simple random samples) from some statistical population and we estimated some population parameters. The obtained estimates were dramatically different.

De Raedt et al. [82] generated in a computer experiment quadruplets of raw data (±1,±1,±1,±1). Subsequent setting -dependent photon identification procedures, mimicking procedures used in real experiments, allowed the creation of new data samples containing only pairs (±1,±1) for each experimental settings. Because these new data sets were not simple random samples extracted from the raw data, the estimated values of pairwise expectations, obtained using these setting- dependent samples, could violate CHSH as significantly as it was observed in SPCE.

We personally do not believe that the fate of the photons is predetermined only by the preparation at the source and that the violation of Bell-CHSH inequalities is the effect of unfair sampling during a post selection.

For us, clicks registered by distant measuring stations in SPCE and coded by ±1 are of a completely different nature than the colors and sizes of socks or the positions and linear momenta of balls and electrons. Spin projections and clicks do not exist before the measurements are done. Thus, one may not describe incoming "pairs of photons" by lines of non-existing Nx4 spreadsheet containing ±1 counterfactual outcomes of impossible to perform experiments.

CONCLUSIONS

In this article we explained why the speculations about quantum non-locality and quantum magic are rooted in incorrect interpretations of QM and/or in incorrect "mental pictures" and models trying to explain invisible details of quantum phenomena.

For example, a "mental picture" of an ideal EPRB experiment in which twin photon pairs produce, in an irreducible random way, strictly correlated or anti-correlated clicks on distant detectors creates the impossible to resolve paradox:

"a pair of dice showing always perfectly correlated outcomes."

As we explained in section Local Realistic Models for EPR-Bohm Experiment, we do not need to worry because the ideal EPRB experiment does not exist.

In SPCE, setting directions are not mathematical vectors but only small spherical angles and we neither see nor follow pairs of entangled photons which produce "click" or "no- click" results on Alice's and Bob's detectors. There are black counts, laser intensity drifts, etc. Detected clicks have time tags and correlated time-windows are used to identify and select pairs of clicks created by the photons belonging to the same entangled pair.

Since various photon- identification procedures are setting – dependent, final post-selected data may not be described by the quantum model used to describe the non-existing ideal EPRB. In SPCE, not only do we not have strict correlations or anti-correlations between Alice and Bob's outcomes but marginal single counts distributions also depend on the distant settings that seems to violate Einsteinian no- signaling. This violation is only apparent because single count distributions estimated using raw data do not depend on the distant settings [60].

Raw and post- selected data in SPCE may be described in a locally causal way using a contextual model [59, 60] in which "a click" or "a no-click" are determined using setting dependent parameters describing a measuring instrument and parameters describing a signal arriving at the measuring station at the moment of the measurement. Still, a detailed description of how "Nature gets this done" is the real mystery underlying quantum correlations.

In contrast to LRHVM and SHVM, in the contextual model (11–17) and in QM the outcomes of four incompatible experiments performed in different settings are described by dedicated probability distributions defined on disjoint probability spaces. Only if all the physical observables measured in SPCE were compatible could these dedicated probability distributions be deduced as marginal probability distributions from a joint probability distribution defined on a unique probability space.

Khrennikov recently explained in [43, 44] that *quantum non-locality* is also rooted in incorrect individual interpretation of QM and in incorrect interpretation of Lüders projection postulate.

Plotnitsky pointed out in [118] that in QM there is no place for *spooky action at a distance*, however his insistence on *spooky predictions at a distance* contributes to general confusion [44].

Other convincing arguments against *quantum non-locality* have recently been given by Jang [119, 120], Bough [121], Wilsch et al. [122], and De Raedt et al. [123].

We want also to mention a recent paper of Griffiths [124] in which he arrives also to the conclusion, that quantum mechanics is consistent with Einstein's locality principle and that the notions of *quantum nonlocality* and of *quantum steering* are misleading and should be abandoned or renamed.

As we mentioned in the introduction, it would be surprising if the violation of Bell-CHSH inequalities, which are proven using simple algebraic inequalities satisfied by any quadruplet of 4 integer numbers equal to ± 1, might have deep metaphysical implications. In fact, such metaphysical implications are quite limited and may be summarized in a few words: "*unperformed experiments have no results*" [84].

Therefore, the violation of various Bell-type inequalities may neither justify the existence of non-local influences nor justify doubts that atoms, electrons, and the Moon are not there when nobody looks.

AUTHOR CONTRIBUTIONS

The author confirms being the sole contributor of this work and has approved it for publication.

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