

Pluralism and the Logical Basis of Metaphysics

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1. Introduction. Proof-Theory, negation and metaphysics

According to Dummett and Prawitz, the meanings of the logical constants may be given completely by their introduction and elimination rules in a system of natural deduction. Negation is the crucial constant when it comes to the question which the proof-theoretic justification of deduction has been purpose-built to decide: which of the two metaphysical positions realism and anti-realism is the correct one? Dummett reconstructs the realism/anti-realism debate as one about whether a certain logical principle holds: the principle of bivalence. Realism is equated with adopting classical logic, which keeps the principle, anti-realism with intuitionistic logic, which rejects it. The core idea is that the proof-theoretic justification of deduction enables us to solve the dispute from metaphysically as well as logically neutral grounds. It is independent of semantic assumptions, like the principle of bivalence, and thus independent of metaphysical assumptions, given the Dummettian reconstruction of the debate. Dummett argues that it is settled depending on which logic turns out to be the justified one: proof-theory is the logical basis of metaphysics. It is common knowledge that Dummett and Prawitz think that intuitionistic logic emerges as the proof-theoretically justified one and accordingly that anti-realism is the metaphysics to be favoured.¹

I have argued elsewhere² that the definition of the meaning of intuitionistic negation given by Dummett and Prawitz is not workable, because the rule *ex*

¹ This gloss of the debate skirts the question whether the dispute is rather one about the verification transcendence of truth and whether there could be an anti-realist justification of classical logic. I take it, however, that at least at an initial stage – in Dummett’s development of his ideas as well as in how he envisages the problem to be tackled – this equation is the moving force behind the project, as guaranteeing metaphysical as well as epistemological neutrality. Anti-realism sets off using only intuitionist logic, as the logic emerging from the proof-theoretic justification of deduction; to establish that classical logic is anti-realistically acceptable arguments at a further stage in the development of a comprehensive theory would be called for.

² In my Ph.D. thesis and an extract of it ‘Negation: A Problem for the Proof-Theoretic Justification of Deduction’, currently in preparation for publication. This paper is also an extract from my thesis.

falso quodlibet does not guarantee that \perp is always false. The symbol ' \sim ' defined in ' $\sim A =_{def.} A \supset \perp$ ' is not negation, or if it is, then only because non-proof-theoretic considerations have implicitly been appealed to. The meaning of negation cannot be defined proof-theoretically, but rather has to be presupposed as given together with the meanings of the atomic sentences.

The purpose of this paper is to investigate into the repercussions of this result for the logical basis of metaphysics. Essentially, it means that the proof-theoretic justification of deduction does not provide for a way of deciding the issue between intuitionist and classical logicians. I shall argue that both logics have to count as unobjectionable from the perspective of proof-theory, as both the intuitionistic as well as the classical treatment of negation constitute legitimate ways of formalising and regimenting our informal, pre-theoretical concept of negation. Negation is underspecified in the sense that 'considered judgements of logicity' do not speak decisively for or against one or other option when we consider the cases which are at issue between classicists and intuitionists. This logical pluralism I argue for raises the question whether accepting two logics is at all coherent. I argue that it is. What needs to be given up however is the idea that proof-theory could be a logical basis for metaphysics.

2. The proof-theoretic justification of deduction should not be rejected

Before going into any details it might be worth reflecting why one shouldn't take the stance that, as the programme of the proof-theoretic justification of deduction has failed to meet its main objective – i.e. to decide between classical and intuitionistic logic –, it should be rejected as being a failed approach to the justification of deduction. This response should be particularly attractive to philosophers – the majority, I presume – who hold that intuitionistic and classical logic are in some sense 'rivals' for the title of the correct logic. In this light, the outcome that the proof-theoretic justification of deduction leaves us with (at least) *two*³ acceptable logics rather than just one may be perceived as rather problematic. The reason why I should not recommend this way with the proof-theoretic justification of deduction is straightforward. There is much to be said in favour of Dummett's and Prawitz' programme. It is arguably the only workable systematic proposal for a justification of deduction. Semantic approaches presuppose a notion of truth and run the danger of circularity: the logical laws, like *tertium non datur*, that are to be established are implicitly assumed through properties of truth. So the choice is between living with a justification of de-

³ In fact, I argue that there is a whole range of acceptable logics in addition to classical and intuitionistic logic which are unobjectionable from the proof-theoretic justification of deduction, in particular relevance logic and some of its relatives.

duction which fails to decide between classical and intuitionistic logic, and an approach which hardly deserves this name.⁴ Proof-theory provides the most powerful method for justifying deduction ever proposed. If it fails to make a decision between classical and intuitionistic logic, then this is *prima facie* reason to accept them both as correct. Thus it is mandatory to investigate whether a logical pluralism is possible which accepts that classical and intuitionistic logic are equally good logics. To explore this is the purpose of this paper. But first, let's have a look at whether there might be some other way of deciding which logic to accepted. After all, if negation has to be presupposed as an undefined primitive in proof-theory then it might be thought that our previously given understanding of negation as used in ordinary discourse provides the means for deciding which negation rules to use, as it is this which informs our choice of them. I shall argue in the next section that this understanding is as indecisive when it comes to the question which of the two options for negation rules are the correct ones as is proof-theory. This consolidates the pluralist conclusion drawn earlier, as both, classical and intuitionistic negation rules may be backed up by reflection on the use of negation in ordinary discourse.

When in the following I talk about 'intuitions' and 'evidence' these are not to be understood as 'untutored', but rather as the basis of 'considered judgements of logicity' in the spirit of Mark Sainsbury and Michael Resnik: they are pre-theoretical logical insights on which formal logical theorising builds.⁵ I shall call the negation of ordinary discourse 'informal negation', in contrast to its formal analysis as classical or intuitionistic negation. The aim of the next section is to argue that informal negation can intelligibly be used in either classical or intuitionistic fashion.

3. Indecisive intuitions

Intuitions concerning our pre-theoretical, informal concept of negation and its use would appear to open up a way of deciding which rules for formalised negation are the correct ones if it was possible to single out by means of them which set of rules matches them best. This however is unlikely to succeed if the choice is between classical and intuitionistic logic. Both logics agree in a large class of cases in their treatment of negation and these cases provide for the core of the use of negation in ordinary discourse, namely where sentences are used which may with some right be called *decidable*. These are the only cases where we can expect to have strong and decisive intuitions concerning the correct use of

⁴ There are of course other approaches, but typically these are not *systematic* ones. Cf. the literature cited in footnote 5.

⁵ Sainsbury (2002), Resnik (1996).

negation, but they are precisely cases on which a decision between classical and intuitionistic negation cannot be built. Classical and intuitionistic logic diverge only in circumstances quite arcane relative to common discourse, namely where undecidable sentences are used, e.g. involving quantification over an infinite domain. It is unlikely that evidence is forthcoming which could be strong enough to decide which logic to use here. There are no paradigm cases of discourse which could be cited to back up a claim that negation behaves classically or intuitionistically when the domain of quantification is infinitely large. Quite to the contrary, the mere fact that intuitionistic mathematics has been developed seems to speak for the thesis that there are two reasonable ways of treating negation in such cases. Thus for mathematics at least, no decision is forthcoming. Surely there are other regions of discourse where Dummettian realists and anti-realists disagree whether negation satisfies *tertium non datur* $A \vee \sim A$ in particular discourse about the future and subjunctive conditionals. To substantiate the claim that a decision between classical and intuitionistic negation based on evidence from reflecting on ordinary discourse is not possible in a wider class of cases either, let's have a closer look at two examples.

First, the future. Consider the statement that next week, I'll drink that bottle of Sancerre that's been sitting on my shelf for days now and that I haven't managed to drink yet. Are we to say that *tertium non datur* holds for 'I'll drink that bottle of Sancerre next week' or not?⁶

Pro

During the course of the week either I drink the bottle at some point or I don't. These two cases exhaust the possible options there are, *tertium non datur*. Thus either I'll drink the bottle or I won't. That *tertium non datur* holds for the future tense may be based on the fact that *tertium non datur* undoubtedly holds for the corresponding present tense sentence 'I drink that bottle' at some time during the course of next week, as at some point during the next week either it or its negation is bound to be true.

Contra

There is not yet a moment in time lying in the next week which would make either of the present tense sentences 'I drink the bottle' and 'I don't drink the bottle' true. Whether or not I drink it next week also depends on factors which are unpredictable now: other social events might come up which force me to give up my hopes that I'll drink it. We cannot base its truth on present intention that I'll drink it. Thus it is not determinate whether I drink the bottle or not and thus *tertium non datur* should be rejected.

⁶ To see what is going to happen, we could just wait until the week has passed, so the example is reasonably far removed from cases of undecidable sentences of mathematics.

Both options constitute reasonable views on the behaviour of negation in future tense sentences, but neither argument is conclusive. Both views have their rationale in the light of the evidence. To accept *tertium non datur* for statements about the future focuses on the intuition that the two cases – either I drink the bottle or not – exhaust the possibilities and one of them has to materialise in the course of the next week. Rejecting *tertium non datur* is to do justice to the “openness” of the future. Both views focus on different *aspects* of informal negation, one might say. In the absence of a principled way of excluding one or other view, informal negation as used in discourse about the future has to count as underspecified.

Some philosophers may of course have views about the nature of the future that provide them with grounds for rejecting one or other option. Such a philosopher would have to show that the reasoning goes astray in one of the cases. But whatever reasons one could give to support such a claim, they would be of a rather different nature than the evidence appealed to above. They would be metaphysical reasons and thus we may exclude them from consideration, as the aim is to base metaphysics on logic rather than the other way round.

Similar considerations may be made in the case of counterfactuals. Suppose someone starts writing a Ph.D. and at some point during his course he drops out and takes to bee-keeping instead. Then we may ask ourselves whether the conditionalised instance of *tertium non datur* holds for ‘Had he continued working on it, he either would have written an excellent Ph.D. or not.’ is true.⁷ Again we can give two lines reasoning. First, *pro*: writing an excellent Ph.D. or not doing so exhaust the possible options, *tertium non datur*. Hence either had he continued working on it, he would have written an excellent Ph.D. or he wouldn’t. Secondly, *contra*: as in fact he hasn’t continued working on it, there is no fact of the matter whether his Ph.D. would have been excellent or not had he continued working on it. Hence the conditionalised instance of *tertium non datur* should be rejected. There are more robust cases of counterfactuals where a conditional *tertium non datur* may be beyond reasonable doubt: for instance, had he completed his thesis and handed it in, then either he would have passed or he wouldn’t (excluding unfortunate events that prevent preconditions for passing or failing to obtain). But one may doubt that all counterfactuals are of this kind, as the forgoing example shows. Thus counterfactuals provide further examples that show informal, pre-theoretical negation to be underspecified. I should argue that Dummett’s example ‘Jones was brave’ is another case where informal, pre-theoretical negation does not decide whether or not *tertium*

⁷ This may be considered to be a more genuine case of undecidability, as we do not have *scientia media*, but it still is notably different from mathematical examples: in the case of counterfactuals, there is not much we can do to decide what is the case, whereas in the case of undecided sentences of mathematics, at least we might hit on a proof one day.

non datur holds and so, I take it, is fictional discourse. But there's no space to go into any more details here.

This discussion underpins the unintended result of the proof-theoretic justification of deduction rehearsed in section 1 and provides independent support for the conclusion I draw from it. There is more than one option for formalising informal negation. Intuitionist and classical logic both have their rationale. Each logic captures different aspects of informal negation and focuses on different intuitions. Each regiments these aspects, but leaves out other aspects. If a metaphor may be allowed, formalising informal negation is like the straightening of a river: there are constraints on doing it properly, but there are several viable options of doing so, and you'll always leave some cut-off meanders. The discussion also gives independent support to the claim made in section 2 that the fact that the proof-theoretic justification of deduction fails to be decisive shouldn't lead us to reject it. That both classical and intuitionistic logic are proof-theoretically acceptable mirrors our pre-theoretical intuitions. Informal negation may thus be said to be underspecified relative to formalisation: it does not determine one of the two options of negation rules as the only correct ones. It is not determinate whether classical or intuitionistic principles should be applied. Informal negation is neither classical nor intuitionistic.

4. Is it incoherent to have two logics?

I have argued that we have two equally acceptable options of formalising negation. It might be objected that while it may very well be true that negation in natural language is neither quite classical nor quite intuitionistic, we'd better change this as it is questionable whether *both* logics could possibly be correct. In other words, it might be objected that this indecisiveness merely points to an inadequacy in our pre-theoretical, informal concept of negation. There is a simple argument employing reasoning acceptable to both, intuitionistic and classical logicians that purports to show that accepting two logics is inconsistent. If there are two logics, then it should be the case that there is a set of assumptions Γ and a conclusion A , such that according to one logic, A follows from Γ , but according to the other logic, it does not follow. But then A does and does not follow from Γ . Contradiction. So there cannot be two distinct correct logics. Assuming that there are some correct standards of logical reasoning, it follows that there can be only one correct logic.

Here is another problem one might find in pluralism. Assume all assumptions in Γ are accepted as true. Then either I am or I am not entitled to assert A , one is inclined to say, and the logic that tells me which is the case is the correct one. If there were two logics, in such a situation we would not know whether or not we can rely on the truth of A in our actions. Logic would fail to be a guide of thought. Pluralism is thus incoherent.

The first problem is a logical one, the second a pragmatic one. I'll discuss them in the next two sections and show that they are not really problems for pluralism.

a) Pluralism is not logically inconsistent

The logical argument against pluralism is easy to answer, strong and convincing as it looks at a first glance. A second glance shows that it is simply invalid. Although of course it is possible that a formula A follows from a set of formulas Γ according to classical logic, but not according to intuitionistic logic, no contradiction arises. It is true that, for certain Γ and A , $\Gamma \not\vdash_I A$ and $\Gamma \vdash_C A$, where \vdash_I and \vdash_C are the intuitionistic and classical consequence relations. But this is as much a contradiction as the one between aRb and $\sim aSb$. Thus no logical problem arises from accepting both logics as correct.

It might be objected that it nonetheless cannot be the case that both, classical and intuitionistic logic, are correct formalisations of our pre-theoretical notion of consequence, and thus although $\Gamma \not\vdash_I A$ and $\Gamma \vdash_C A$ do not formally contradict each other, they cannot both correctly capture this notion. This objection misses the point that if negation is underspecified, so is the pre-theoretic notion of consequence. If there is more than one way of giving rules for negation, it follows that there is more than one way of capturing our pre-theoretic notion of consequence by logical consequence as determined by what counts as a deduction. If negation can be formalised in two different ways, the same counts for our pre-theoretical notion of consequence. Now in any case of well-formed formulae Γ, A where $\Gamma \not\vdash_I A$ and $\Gamma \vdash_C A$, some of Γ, A must be undecidable. In other words, the cases where there is a real choice of logics are exactly those discussed in section 3, such that we have no grounds for favouring classical or intuitionistic logic. Hence this objection poses no further problem to what has already been discussed.

An opponent of logical pluralism might wish to strengthen her point: it may well be that no unique formal system captures all our intuitions about consequence, and that there are two formal systems which are equally adequate; nevertheless there *ought* to be only one logic, and hence we are under an obligation to make a decision which logic is the correct one and to declare some intuitions to be fallacious. If this course is taken we are back where we started: there is no reasonable means of making such a decision. Any decision would either beg the question – e.g. you chose the logic you assumed right from the start to be your favourite one – or it is based on grounds too feeble to support a choice as important as the choice of logic – e.g. you chose the one you've been trained to use in your undergraduate years of studying philosophy. A meaningful 'ought' should imply a 'can', should it not? Here we have a case where we can *not* do what allegedly we ought to do. If my arguments are correct, then the claim that there ought to be only one logic is pointless. There is no adequate rationale on which to base the decision which logic it would be.

I conclude that there is no logical problem with logical pluralism. So let's move on to the pragmatic problem.

b) Pluralism is not pragmatically incoherent

Here is again the pragmatic objection to logical pluralism. Given $\Gamma \not\vdash_I A$ and $\Gamma \vdash_C A$ and you accept the premises Γ , should you go on to assert A or not: which logic are you to apply? If there are two logics, then you seem to have a choice, but we are inclined to say that it is not upon us to make a decision. Given all premises Γ are true, A either is or isn't, and logic should tell you that: logic should guide your thought and tell you whether you are entitled to assert A or not. But this is possible only if there is just one logic.⁸

First note that the problem cannot raise a point against logical pluralism: there is no reason to believe that the question which logic to apply in a case of reasoning has a *general* solution with *one* answer that covers all cases. This practical issue does not force one to narrow down the range of acceptable logics to one system.

A monist might advance the following reasoning. Given $\Gamma \not\vdash_I A$ and $\Gamma \vdash_C A$ and we accept all of Γ , we could always go on asserting A : classical logic provides us with a sufficient justification for asserting the conclusion. Thus the problem which logic to use has a simple solution: always use classical logic, as it is the stronger logic. Now it may very well be true that one could always use classical logic. However, this does not address the question whether this is always the right way of looking at a given case. The proof-theoretic justification of deduction shows that the classical analysis of arguments is not all there is to logic. So although it may be possible to treat every argument classically, this does not show that this treatment is always adequate, let alone that it is the only possible treatment. That this is so should be obvious in the case of conditionals. What I am arguing here is that the same phenomenon extends to negation. The examples of statements about the future and counterfactual situations discussed earlier show that negation may intelligibly be treated in a classical as well as in a non-classical way.

Much of the force of the pragmatic problem stems from the way it has been stated. A closer look at how such a problematic case could arise shows that it

⁸ The situation is in some ways similar to a familiar one in mathematics. After the invention of Non-Euclidean geometries the question arose whether Euclidean or Non-Euclidean geometry should be used to describe the world. This is not a question for mathematics to decide; rather it depends on observations and experiments in physics. One might object that the case of alternative logics is inherently different from the case of alternative geometries, as there are no *experimenta cruceum* which could decide which logic is the correct one: logic has no subject matter; it is 'topic neutral'. But this is not quite right. There are such experiments: our pre-theoretical logical intuitions provide the relevant data. However, the problem with them, as argued earlier, is that they do single out a unique logic as the right one.

does not in fact introduce any new problems. Let's assume for simplicity's sake that the same formal language is used for intuitionistic and classical logic, and let's write Γ° and A° for the ordinary language sentences we are formalising by Γ and A and \Vdash for our intuitive notion of consequence. Applying the resources of formal logic, we discover that we have two ways of regimenting the informal argument for A° from Γ° , a classical and an intuitionistic case, and $\Gamma \vdash_{\text{C}} A$ and $\Gamma \vdash_{\text{I}} A$. Then, accepting all of Γ° , we ask ourselves: should we assert A° or not? In other words, should we take $\Gamma^\circ \Vdash A^\circ$ to hold or not? Well, under which conditions can this question arise? That both $\Gamma \vdash_{\text{C}} A$ and $\Gamma \vdash_{\text{I}} A$ happens only in very uncommon situations, namely if undecidable sentences are involved. Thus Γ° and A° will be sentences similar to the ones discussed in section 3. Thus we may recycle what has been said there. Our informal concept of negation is neither classical nor intuitionistic in the sense that neither of the two logics can claim to capture this concept either "entirely" or better than the other logic. Both logics give reasonable, well motivated ways of regimenting the informal concept. Extrapolating to the present case, whether or not you should consider $\Gamma^\circ \Vdash A^\circ$ to hold and go on asserting A° depends on whether you intend to focus on the classical or the intuitionistic aspect of informal negation. There is no absolute answer to the question, no answer, that is, which would be independent of the formalisations.

I conclude that there is no pragmatic problem for pluralism either. I'll say a little more connected to this in the conclusion.

5. Conclusion

The problem of how to formalise natural language sentences to some degree always arises. It is a problem that everyone faces who thinks that formal logic may serve in the analysis of informal arguments. It can hardly be denied that in formalising natural language for the purposes of logic there is a bunch of options one can choose from. For instance, shall I treat 'or' as a primitive, or shall I analyse it in terms of conjunction and negation? Shall I formalise a phrase 'the F' as a complete expression (a term) or as an incomplete one (a Russellian description)? Shall I formalise a conditional as a material one, a strict one, a variably strict one or a relevant one? What I am arguing for is more of this kind, only in a more radical case, as it does not seem to have been suggested very often in the case of negation. If, in analysing a natural language argument, natural language sentences are represented by formulas, a decision has to be made not only concerning how to represent the structure of the sentences in question, but also concerning which machinery the 'logical words' in them are subject to, this way making them precise. Formalisation, in other words, involves conceptual analysis. In the case of informal negation, the analysis involves making

a decision whether it is to be treated classically or intuitionistically. Due to the underspecification of informal negation, if you analyse an argument in formal logic, you need to make a decision which aspect of negation it is that you are focussing on, the classical one or the intuitionistic one. Informal negation is neutral between the two. Once we've realised that there are two options of analysing negation, we can make explicit which one we focus on in an argument. But neither is "nearer to the truth" or "more fundamental" than the other. We have to live with two options, no absolute decision between them being possible. But this is not incoherent. You just need to make clear which of them you are using. Formal logic helps us making these different aspects precise (or, indeed, helps us noting their existence).

I have argued that neither logical nor pragmatic problems arise from accepting that both, classical and intuitionistic logic are all right. But a problem remains. It can hardly be the case that both realism and anti-realism are correct! For while it is true that no decision needs to be made which of classical and intuitionistic logic is "the right logic", we cannot equally accept both metaphysics that each logic according to Dummett gives rise to. At least one of them has to go. But as we have no basis for deciding which one, given the proof-theoretic justification of deduction fails to decide between classical and intuitionistic logic, we should reject *both* metaphysics. We should give up the thought that proof-theory could provide a logical basis for metaphysics and that using one or other logic commits one to a certain metaphysics. Proof-theory is metaphysically neutral.

This leaves the question what to do about the notion of truth: does it or does it not satisfy the principle of bivalence? I take it that this question can adequately be dealt with by adopting a minimalist or pro-sentential theory of truth, but there is no space to go into this here.

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