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## INDICATIVE CONDITIONALS


#### Abstract

In this paper a semantics and logic of conditional necessity is developed as the basis of a logic of indicative and subjunctive conditionals and of causal sentences. It is argued, against E. Adams and D. Lewis, that these three types of statements differ only in presupposition.


In "Counterfactuals" (1973) David Lewis has developed a logical system VC for counterfactuals. For the normal cases of counterfactuals, in which the antecedent is false, VC can be replaced by VW, a system based on weak instead of strong centering. In this paper I shall try to show that this system can also be applied to indicative conditionals and can generally be used for a comprehensive and unified treatment of the logic of conditionals. The main obstacles in generalizing Lewis' analysis of counterfactuals to include indicative conditionals are, first, that the intuitive background he provides for his semantics favours strong centering, and, second, an argument by Ernest W. Adams in (1970) to the effect that "subjunctive and indicative conditionals are... logically distinct species" so that the truth-conditions for the former cannot be derived from those for the latter by adding suitable presuppositions. Our first step will be a reconsideration of that argument.

## 1. Adams'argument

If counterfactuals derive from indicative conditionals or both from a basic type of conditionals then it should be true that:
(1) an indicative conditional 'If it is the case that $A$, then it is the case that $B$ ' (shortly: 'If $A$, then $B$ ') has for non-A the same truth conditions as the counterfactual 'If it were the case that A , then it would be the case that B '.

According to Adams the two following sentences form a counter-example to (1), since we consider (2) true and its antecedent false, but (3) false:
(2) If Oswald didn't shoot Kennedy, then someone else did.
(3) If Oswald badn't shot Kennedy, then someone else would have.

Now Lewis and before him N. Goodman, N. Rescher and R. Stalnaker, have analyzed the truth conditions of conditionals as dependent upon ceteris-paribu;conditions C, not explicitly mentioned in the conditional, that are compatible with the antecedent. If we change our assumptions as to the truth or compatability of C with A, then we also change our assessment of the truth of the conditional. In the example we are only prepared to accept (2) as true if we know that Kennedy was indeed shot in Dallas, and if we consider that compatible with Oswald's innocence, although this may be very unlikely for us as things stand. If, on the other hand, we consider (3) to be false, we take it that Oswald did indeed shoot Kennedy and that there was no one else around with intentions or means to assassinate the president. We therefore do not consider Kennedy's being shot in Dallas compatible with Oswald's innocence. ${ }^{1}$ So we have changed our assessment of the ways things might have been, if Oswald hadn't shot Kennedy. But (1) presupposes that this assessment remain the same for the indicative and the counterfactual conditional.

The difference in our assessment of the truth of (2) and (3) seems to be a consequence of the fact that, while (2) speaks about the author of the killing in Dallas, (3) implies that Kennedy was somehow fated to be killed anyhow, which is not implied in the logical representation of (3). There are indeed quite a lot of differences of meaning in natural language, for instance by a change in topic and comment as in Goodmans example of Georgia and New York ${ }^{2}$, that are not accounted for in the usual straight-forward logical representation. So Adams' example is not a conclusive argument against (1).

## 2. Types of conditionals

In traditional grammar three types of conditionals are distinguished: Those using the indicative in the antecedent and consequent (indicative conditionals) and two forms of subjunctive conditionals. These two forms can be distinguished morphologically in some languages, as in Latin, by their use of present or past tense (Si boc credas, erres vs. Si boc crederes, errares), but generally they have to be determined by the fact that one type (the counterfactual) carries the presupposition that the antecedent (and also normally the succedent) is false, while the other type (in Latin potentialis) carries no such presupposition but expresses the speaker's opinion that the antecedent is improbable or uncertain. We shall also include causal statements of the form "Since it is the case that A, it is the case that B" in our investigation of conditionals. Such sentences presuppose that A (and hence B) is true.

The grammatical subdivision of conditionals is of little logical interest since it mixes syntactical criteria (mood) with semantical (presupposition) and pragmatical

[^0]ones (beliefs of the speaker). As we are here only after truth conditions and not after expressive meaning components the difference between indicative conditional and potentialis is not relevant for us. And since we shall not consider partial interpretations ${ }^{3}$ we shall take no account of presuppositions. We want to argue that we can get along then with only one type of conditional which we write $A \Rightarrow B$. We say that $A \Rightarrow B$ is used as an indicative conditional if it is undecided (for the speaker) whether the antecedent $A$ holds or not. $A \Rightarrow B$ is used as a counterfactual if (for the speaker) it is a fact that $\urcorner \mathrm{A}$. And it is used as a causal statement "Since it is the case that $A$, it is the case that $B$ " if (for the speaker) it is a fact that $A .{ }^{4}$

We think, therefore, that the difference between the indicative, counterfactual, and causal conditional is not a difference of truth-conditions but only a difference in presupposition.

If we assert for instance

## (1) If Jack believes that John is married, then be is wrong,

and are told that Jack does not believe John to be married, then we are committed to the statement.

## (2) If Jack were to believe that Jobn is married, he would be wrong.

The reason for asserting (1), viz. that John is not married, is the same as that for asserting (2). And if we assert (2) we are committed to (1) if we hear that it is really uncertain whether Jack believes John to be married or not.

And if I assert (1) then if I learn that Jack really does believe John to be married, then I am committed to the statement

## (3) Since Jack believes that Jobn is married, be is wrong.

And conversely, if I assert (3) and then learn that it is not sure that Jack believes John to be married, I shall say that (1) is true.

One or a few examples are not conclusive evidence for our thesis of course. They just serve to give it a certain intuitive plausibility. Our main argument has to be that the semantic analysis of $A \Rightarrow B$ is such that the thesis is intuitively adequate.

## 3. Similarity of worlds

D. Lewis gives several types of semantics for the language of conditionals. The intuitively fundamental one is that of comparative similarity systems. Such systems are based on relations $j \leq i k$ on the set $I$ of possible worlds for all $i \varepsilon I . j \leq i k$

3 Cf. for instance Kutschera (1974a).
$4 \quad$ We shall not discuss the difference between subjective and objective presuppositions here.
5 Cf. Lewis (1973), pp. 48 seq.
says that the world $k$ is at least as similar to $i$ as $j$ is. $j \leq_{i} k$ is to be a weak ordering for which several conditions hold, among them
(1) $\mathrm{j} \ll_{\mathrm{i}} \mathrm{i}$ for all $\mathrm{i}, \mathrm{j} \varepsilon \mathrm{I}$ and $\mathrm{j} \neq \mathrm{i}$.

This is the condition of strong centering, which says that every world is more similar to itself than any other world.

Lewis' truth condition for $A \Rightarrow B$ is
(2) $A \Rightarrow B$ is true in $i$ iff $A$ is impossible or there is an $A$-world $j$ so that all A -worlds that are at least as similar to i as j are B -worlds.
From (1) we obtain then
$A \wedge B \supset(A \Rightarrow B)$.
This is harmless for counterfactuals which normally are used only under the presupposition that $\neg A$. It is unacceptable, however, if we want to interpret $A \Rightarrow B$ as the basic form of conditionals, since every causal conditional would then be true.

If we replace (1) by the condition for weak centering
(1') $\mathrm{i} \leq{ }_{\mathrm{i}} \mathrm{i}$ for all $\mathrm{i}, \mathrm{j} \varepsilon \mathrm{I}$,
then (3) is not valid anymore, but the assumption that there is a world, different from $i$, which is to $i$ just as similar as i itself, is counterintuitive. Similarity of $j$ and $i$, according to Lewis, is to be overall-similarity, so that $j$ is the more similar to i the more details they have in common and the more important these common details are. Since for $j \neq i j$ must in some details, however few and unimportant, be different from $i$, $i$ itself must certainly be more similar to $i$ than $j$. To obtain an adequate semantics for our $A \Rightarrow B$ there remain then only two possibilities: Change (2) or change the whole intuitive background of the semantics.

We might, for instance, change (2) to
$A \Rightarrow B$ is true in $i$ iff $A$ is impossible or $A \wedge B$ necessary or there is an A-world $\mathfrak{j}$ and a $\neg A$-world $k$ so that all $A$-worlds at least as similar to $i$ as $j$ or $k$ are $B$-worlds.
In case $A$ is false in $i$ this coincides with (2), i.e. nothing is changed for counterfactuals. (2') expresses the fact that $A \Rightarrow B$ holds if we can infer $B$ from $A$ together with a suitable ceteris-paribus-condition compatible both with A and $\neg \mathrm{A}$, and looks, therefore, like a good candidate for indicative conditionals. The trouble with $\left(2^{\prime}\right)$, however, is that the logic we obtain from this condition is too weak. Conditionals are a type of inference-relation and though many of the fundamental principles valid, for instance, for logical entailment or material or strict implication (like the laws of contraposition, strengthening of the premiss or transitivity) are not valid for conditionals ${ }^{6}$ they are valid in the normal cases. From ( $2^{\prime}$ ), however, we do not obtain sufficiently strong restrictions of these laws.
$6 \quad$ Cf. Lewis (1973), 1.8.

We shall therefore follow the second course and abandon the use of worldsimilarities for the interpretation of $A \Rightarrow B$ altogether.

## 4. Conditional necessity

We shall interpret $A \Rightarrow B$ as a statement about conditional necessity and read it as "On condition that A, it is necessary that B". The notion of conditional necessity is a generalization of the usual notion of (unconditional) necessity, as conditional probability or conditional obligation are generalisations of the notions of (unconditional) probability and obligation. Under different conditions different propositions may be necessary. From conditional necessity we obtain two concepts of unconditional necessity: proposition p is weakly necessary if it is necessary on a tautologous condition, and p is strongly necessary if it is necessary under all conditions. $p$ is weakly necessary if under the given circumstances $p$ is normally the case. Therefore $A \Rightarrow B$ expresses a notion of weak necessity: on condition that $A$, it is normally the case that $B$.

Conditional possibility can then be defined by
D4.1. $A \Rightarrow B:=\neg(A \Rightarrow \neg B)$
We read $A \Rightarrow B$ as "On condition that $A$, it is (weakly) possible that $B$ ". A proposition p is unconditionally weakly possible if under the given circumstances it would not be abnormal if p were the case. And p is strongly possible if there is a condition under which $p$ is (weakly) possible.

Before we discuss these intuitive concepts further let me give the formal definitions:
Let $\mathbb{C}$ be the language obtained from that of Predicate Logic by stipulating that $(A \Rightarrow B)$ be a sentence if $A$ and $B$ are. To economize on brackets $\neg, \wedge, \vee$ are to bind stronger and $\supset, \equiv$ weaker than $\Rightarrow$, so that we may write

$$
A \wedge B \Rightarrow B \vee C \supset C \Rightarrow \neg A \text { instead of }((A \wedge B) \Rightarrow(B \vee C)) \supset(C \Rightarrow \neg A)
$$

D4.2. An interpretation of $\mathbb{C}$ is a quadruple $\langle\mathrm{U}, \mathrm{I}, \mathrm{f}, \Phi\rangle$ so that:
(1) U is a non-empty set of (possible) objects.
(2) I is a non-empty set of (possible) worlds.
(3) $\quad f(i, X)$ is a function on $I \times P(I)(P(I)$ being the power set of $I)$ so that for all $\mathrm{i} \varepsilon \mathrm{I}$ and $\mathrm{X} \supset \mathrm{I}$
(a) $f(i, X) \subset X$
(b) $X \subset Y \wedge f(i, X) \neq \Lambda \supset f(i, Y) \neq \Lambda^{\top}$
(c) $\quad X \subset Y \wedge f(i, Y) \cap X \neq \Lambda \supset f(i, X)=f(i, Y) \cap X$
(d) $\quad i \varepsilon f(\mathrm{i}, \mathrm{I})$

[^1](4) For all i $\varepsilon$ I $\Phi_{\mathrm{i}}$ is a function from the set of sentences of $\mathbb{C}$ into the set $\{t, f\}$ of truth values so that
(a) $\quad \Phi_{\mathrm{i}}(\mathrm{a})=\Phi_{\mathrm{j}}(\mathrm{a})$ for all $\mathrm{j} \varepsilon \mathrm{I}$ and all individual constants a.
(b) $\Phi_{i}$ satisfies the conditions for interpretations of the language of Predicate Logic over U.
(c) $\quad \Phi_{i}(A \Rightarrow B)=t$ iff $f(i, A) \subset[B]$, where $[B]=\left\{j \varepsilon I: \Phi_{j}(B)=t\right\}$.

In modal logic we set $\Phi_{i}(N A)=t$ iff $S_{i} \subset[A]$, where for all $i \varepsilon I S_{i}$ is a non-empty subset of $I$. $S_{i}$ is the set of worlds possible from the standpoint of $i$. (4c) is the straight-forward generalization for conditional necessity: $f(i, A)$ is the set of worlds (weakly) possible under condition that A from the standpoint of $i$.

If we set $S_{i}=\bigcup_{X} f(i, X)$, then $S_{i}$ is the set of worlds strongly possible from the standpoint of i, i.e. the proposition $X$ is strongly necessary iff $S_{i} \subset X$, and $X$ is strongly impossible iff $S_{i} \subset \overline{\mathrm{X}}$. We construe f so that
( $\alpha$ ) $\mathrm{f}(\mathrm{i}, \mathrm{X})=\Lambda \equiv \mathrm{S}_{\mathrm{i}} \subset \overline{\mathrm{X}}$,
i.e. $f(i, X)$ is empty iff $X$ is strongly impossible. This follows from the conditions of $D 4.2(3)$, and the definition of $S_{i}$. If $S_{i} \subset \bar{X}$ then $f(i, X)=\Lambda$ according to (a). If $S_{i} \cap X \neq \Lambda$ then there is a $Y$ with $f(i, Y) \cap X \neq \Lambda$, so according to (c) $f(i, X \cap Y)$ $=f(i, Y) \cap X \neq \Lambda$, and according to (b) $f(i, X) \neq \Lambda$.

From the definition of $S_{i}$ we obtain
(ß) $\quad S_{i} \subset X$ iff for all $Y \subset I f(i, Y) \subset X$,
And ( $\alpha$ ) together with (a) implies
$(\gamma) \quad S_{i} \subset X \equiv f(i, \bar{X}) \subset X$.
We can, therefore, define strong necessity and possibility by
D4.3. (a) $N A:=\neg A \Rightarrow A$
(b) MA: $=\neg \mathrm{N}\urcorner \mathrm{A}$,
while weak necessity and weak possibility are defined by
D4.4. (a) $L A:=T \Rightarrow A$, where $T$ is a tautology.
(b) $\mathrm{PA}:=\neg \mathrm{L} \neg \mathrm{A}$.

Now condition D4.2(3a) says that all worlds (weakly) possible on condition that X are X -worlds.

Condition (b)-always of $\mathrm{D} 4.2(3)$-says that if X is strongly possible and $\mathrm{X} \subset \mathrm{Y}$, then Y is strongly possible. This is the law $\mathrm{A} \supset \mathrm{B} \vdash \mathrm{MA} \supset \mathrm{MB}$ of modal logic.

Condition (c) (in view of (a) and (b)) is equivalent to $f(i, Y) \cap X \neq \Lambda \supset$ $f(i, X \cap Y)=f(i, Y) \cap X$; i.e. if among the worlds (weakly) possible on condition that Y there are some X -worlds, then these are the worlds (weakly) possible on condition that X and Y . This implies the law of im- and exportation of premisses $A \Rightarrow B \supset(A \wedge B \Rightarrow C \equiv A \Rightarrow B \supset C)$.

Condition (d) finally says that $i$ is weakly possible from the standpoint of $i$. (d) together with (c) implies the law of modus ponens for conditional necessity: $A \wedge(A \Rightarrow B) \supset B$. A word, perhaps, is also in order on condition D4.2 (4a): All individual constants are interpreted as standard names. S. Kripke has given good reasons for such a procedure in (1972). Since we are not interested in existence here we have not introduced sets $U_{i}$ of objects existing in $i$. If $E$ is a one-place predicate constant of $\mathbb{C}$ we could set $\Phi_{i}(E)=U_{i}$ and define quantification over existing instead of possible objects by $\Lambda . \mathrm{xA}[\mathrm{x}]:=\Lambda \mathrm{x}(\mathrm{Ex} \supset \mathrm{A}[\mathrm{x}])$.
D4.5. An interpretation $\mathfrak{M}=\langle\mathrm{U}, \mathrm{I}, \mathrm{f}, \Phi\rangle$ satisfies a sentence A in i iff $\boldsymbol{\Phi}_{\mathrm{i}}(\mathrm{A})=\mathrm{t}$. A is valid in $\mathfrak{M}$ iff $\mathfrak{M}$ satisfies A for all i $\varepsilon \mathrm{I}$. And A is $C$-valid iff A is valid in all interpretations of $\mathbb{C}$.

Our concept of interpretation appears in Lewis (1973), 2.7 as that of a model based on a weakly centered selection function. His selection functions, however, are introduced on the basis of comparative similarity concepts for which only weak centering is counterintuitive, as we have seen. To arrive at such functions- $f(i, A)$ being interpreted as the set of A-worlds most similar to $i$-the Limit-Assumption has to be assumed, that for all i and A there is an A -world most similar to A . Though this makes no difference for the resulting logical system it is intuitively not well-founded as Lewis points out, since the similarity of worlds may depend on the values of real valued parameters like places, times, masses etc. in them. Our approach avoids these difficulties in giving another interpretation to the selection functions.

If we want to consider iterated applications of modal operators the principles
(ס) NA $\supset$ NNA of C.I. Lewis' system S4, and
( $\varepsilon) ~ \neg \mathrm{NA} \supset \mathrm{N}\urcorner \mathrm{NA}$ of S 5
suggest themselves. As $\mathbf{S 5}$ seems to be intuitively most adequate, we may incorporate the condition
(e) $j \varepsilon S_{i} \supset S_{j}=S_{i} \quad$ for all $j \varepsilon I$
into D4.2(3a).
If we want to obtain principles for iterated applications of $\Rightarrow$ it seems best to generalize $(\delta)$ and $(\varepsilon)$. The following two conditions are the likeliest candidates:
(弓) $A \Rightarrow B \supset L(A \Rightarrow B)$
( $\boldsymbol{\prime}) \quad \neg(A \Rightarrow B) \supset L \neg(A \Rightarrow B)$.
These two conditions are equivalent with postulating in $\mathrm{D} 4.2(3)$ also
(f) $\quad j \varepsilon f(i, I) \supset f(j, X)=f(i, X) \quad$ for all $j \varepsilon I$ and $X \subset I$.

If we assume
(弓') $\mathrm{A} \Rightarrow \mathrm{B} \supset \mathrm{N}(\mathrm{A} \Rightarrow \mathrm{B})$ and
$\left.\left(\eta^{\prime}\right) \quad \neg(A \Rightarrow B) \supset N\right\urcorner(A \Rightarrow B)$,
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instead of $(\zeta)$ and $(\eta), A \Rightarrow B$ holds iff it holds on all conditions, and does not hold iff it holds on no conditions, all conditionals would be necessarily true or false, and we would have $L A \equiv N A$ and $N(A \supset B) \equiv A \Rightarrow B$, i.e. conditional necessity would coincide with strict implication.

This is not adequate since it may be true that If my barometer goes $u p$, then the atmospheric pressure rises but, since my barometer does not necessarily function correctly, it is false that The going up of my barometer strictly implies that the atmospheric pressure rises.

If, on the other hand, we postulate
( (') $\quad A \Rightarrow B \supset A \Rightarrow(A \Rightarrow B)$ and
$\left(\eta^{\prime \prime}\right) \quad \neg(A \Rightarrow B) \supset A \Rightarrow \neg(A \Rightarrow B)$
this would not be intuitively correct, since, if $A \Rightarrow B$ holds, then $A$ is a reason for $B$, but not a reason for $A \Rightarrow B$.
5. The logic of conditional necessity

Let $\boldsymbol{C}_{0}$ be Predicate Logic plus the following rule and axioms
CR: AトNA
C1: $A \Rightarrow A$
C2: $\quad N A \supset B \Rightarrow A$
C3: $\quad N(A \supset B) \wedge(C \Rightarrow A) \supset C \Rightarrow B$
C4: $\quad(A \Rightarrow B) \wedge(A \Rightarrow C) \supset A \Rightarrow B \wedge C$
C5: $\quad A \Rightarrow B \supset(A \wedge B \Rightarrow C \equiv A \Rightarrow B \supset C)$
C6: $A \Rightarrow B \supset(A \supset B)$
C7: $\quad \Lambda x(A \Rightarrow B[x]) \supset A \Rightarrow \Lambda x B[x]$
$\boldsymbol{C}_{1}$ is to be $\boldsymbol{C}_{0}$ plus
C8: NA $\supset$ NNA
C9: $\mathrm{NA}^{\mathrm{N}} \supset \mathrm{N} \rightarrow \mathrm{NA}$
$\boldsymbol{C}_{2}$ is to be $\boldsymbol{C}_{1}$ plus
C10: $A \Rightarrow B \supset L(A \Rightarrow B)$
C11: $\neg(A \Rightarrow B) \supset L \neg(A \Rightarrow B)$.
The propositional part of $\boldsymbol{C}_{0}$, i.e. $\boldsymbol{C}_{0}$ minus C 5 , is equivalent with D . Lewis' system VW in (1973), pp. 132 seq.
$\boldsymbol{C}_{0}$ contains the basic modal system M of von Wright (or T of R. Feys) together with the Barcan formula $\Lambda \times N A[x] \supset N \Lambda x A[x]$, and therefore $\mathbb{C}_{1}$ contains S 5 .

For inference relations $>$ like logical entailment and material or strict implication the following principles are fundamental:
(1) $\mathrm{A}>\mathrm{A}$
(2) $(\mathrm{A}>\mathrm{B}) \wedge(\mathrm{B}>\mathrm{C}) \supset(\mathrm{A}>\mathrm{C})$
(3) $(A>B) \supset(A \wedge C>B)$
(4) $(\mathrm{A}>\mathrm{B}) \supset(\mathrm{A}>\mathrm{B} \vee \mathrm{C})$
(5) $(\mathrm{A}>\mathrm{B} \wedge \mathrm{C}) \equiv(\mathrm{A}>\mathrm{B}) \wedge(\mathrm{A}>\mathrm{C})$
(6) $(\mathrm{A} \vee \mathrm{B}>\mathrm{C}) \equiv(\mathrm{A}>\mathrm{C}) \wedge(\mathrm{B}>\mathrm{C})$
(7) $\quad(\mathrm{A}>\mathrm{B}) \supset(\neg \mathrm{B}>\neg \mathrm{A})$
(8) $\mathrm{A} \wedge(\mathrm{A}>\mathrm{B}) \supset \mathrm{B}$
(9) $(\mathrm{A} \wedge \mathrm{B}>\mathrm{C}) \equiv(\mathrm{A}>\mathrm{B} \supset \mathrm{C})$

For $\Rightarrow$ in place of $>$ only (1), (4), (5) and (8) hold. In place of (2) we have $(B \Rightarrow A) \wedge(A \Rightarrow B) \wedge(B \Rightarrow C) \supset A \Rightarrow C$,
in place of (3)
$(A \Rightarrow C) \wedge(A \Rightarrow B) \supset A \wedge C \Rightarrow B$,
in place of (6)
$(A \vee B \Rightarrow A) \wedge(A \vee B \Rightarrow B) \supset(A \vee B \Rightarrow C \equiv(A \Rightarrow C) \wedge(B \Rightarrow C))$,
in place of (7)
$\neg L B \wedge(A \Rightarrow B) \supset \neg B \Rightarrow \neg A$
and in place of (9) we have C5.

## 6. Conditional necessity and conditionals

We have to show now that conditionals can be adequately analyzed in terms of statements $A \Rightarrow B$ about conditional necessity.

## (A) Indicative Conditionals

$A$ statement $A \Rightarrow B$ may be used as an indicative conditional if $P A \wedge P \neg A$, i.e. if under the given circumstances $A$ and $\neg A$ are both weakly possible (it may very well be the case that A, but also that $\neg$ A). N. Goodman's analysis of counterfactuals in (1965) can in part be carried over to indicative conditionals. Then a sentence (1) "If A, then B" is not only true if $N(A \supset B)$ but also if there is a relevant condition $C$, not mentioned in "If $A$, then $B$ " so that $N(A \wedge C \supset B)$. C cannot be a free parameter for then (1) would have no definite truth value. C cannot be the conjunction of all true statements, for if $A$ is false (1) would always be true since $N(A \wedge \neg A \supset B)$. $C$ has to be at least consistent with $A$. C cannot always be true since, on condition that $\mathrm{A}, \mathrm{C}$ might be an implausible assumption if $\neg \mathrm{A}$. As Goodman has shown, the truth condition "If A , then B " is true iff there is a $C$ so that $\neg N(A \supset \neg C)$ and $N(A \wedge C \supset B)$ violates the principle
$M B \wedge(A \Rightarrow B) \supset \neg(A \Rightarrow \neg B)$.
So we will have to choose a stronger relation than $\neg \mathrm{N}(\mathrm{A} \supset\urcorner \mathrm{C})$ between A and $C$ which Goodman calls cotenability. It seems natural to take $A \Rightarrow C$ in place of such a relation. If $A \Rightarrow C$ then $C$ is (weakly) necessary or the normal case on condition that A , so that assuming A C goes without saying. This accounts
for $C$ not being mentioned in (1). We then have:
"If $A$, then $B$ " iff there is a condition $C$ such that $A \Rightarrow C$ and $N(A \wedge C \supset B)$. From this it follows that "If $A$, then $B$ " holds iff $A \Rightarrow B$ does. For if $A \Rightarrow B$ is true we can take $A \supset B$ as our $C$; then $N(A \wedge C \supset B$ ) and (in view of $C 3$ ) $A \Rightarrow C$. And if we have $A \Rightarrow C$ and $N(A \wedge C \supset B)$, then we have $A \Rightarrow A \wedge C$ (C1 and C4) and therefore $A \Rightarrow B$, in view of C3.

This equivalence

$$
A \Rightarrow B \equiv C(A \Rightarrow C \wedge N(A \wedge C \supset B))
$$

also holds for counterfactuals and causal conditionals and so is a valuable argument for the correctness of our analysis.

In the normal case of an indicative conditional "If A, then B" B is not weakly necessary, i.e. we have $\mathrm{\imath LB}$. We would, for instance, not normally say "If Nixon is still president next year, then he will be over sixty."
(1) normally expresses that there is a connection between the facts expressed by $A$ and $B$ so that it may very well be that $\neg B$ if $\urcorner A$. The case $L B$ can be excluded by using the strong conditional defined by
D6.1. $A \underset{\mathrm{~s}}{\Rightarrow} B:=(A \Rightarrow B) \wedge\urcorner(\neg A \Rightarrow B)$,
which implies that $\neg B$ is (weakly) possible under condition that $\neg A$. For $\neg L B$, $A \underset{\mathrm{~s}}{\Rightarrow} B$, and $A \Rightarrow B$ are equivalent.
$\mathrm{A} \Rightarrow \mathrm{B}$ can be read as "If it is the case that A , then it may be the case that B". As Lewis points out, such "may"-conditionals (speaking of counterfactuals he had his eye on "might"-conditionals) also play a role in ordinary discourse. They come quite naturally from the standpoint of conditional necessity.

## (B) Counterfactuals

$A \Rightarrow B$ may be used as counterfactual if $\neg A$. Then we also have $P \neg A$. We shall not stipulate $\neg \mathrm{PA}$, i.e. $\mathrm{L} \neg \mathrm{A}$ however, since under the given circumstances, though $A$ is false, A might very well be possible. So on our definition for PA $A \Rightarrow B$ may be used both as an indicative and a counterfactual conditional, depending on the speakers knowledge, of which we have taken no account in our semantics.

The logic of counterfactuals then coincides with that given by D. Lewis. In the normal case of a counterfactual we again have $ᄀ \mathrm{LB}$. If we want to imply that B is in fact true, we say "If it were the case that A, then it would still be the case that $B$ ". By use of $\Rightarrow$ instead of $\Rightarrow$ we can exclude this case LB, while for $\neg \mathrm{LB} A \Rightarrow B$ is equivalent again to $A \Rightarrow B$.

## (C) Causal conditionals

$\mathrm{A} \Rightarrow \mathrm{B}$ may be used as a causal statement (2) "Since it is the case that A, it is the case that $B$ ", if $A$ is true. Then we have PA, but we don't stipulate that $\neg P \neg A$, i.e. that $A$ is normally the case. In fact from $A \Rightarrow B$ and $L A$ we obtain LB,
so that $B$ is normally the case too; from LA (or PA) and LB $A \Rightarrow B$ already follows, so that this statement is quite uninformative in case of LA. In ordinary discourse, we usually only give reasons for phenomena that are unexpected, strange or unusual as for instance J. König, H. Hart and H. Honore, J. Passmore and E. Scheibe have pointed out in their discussions of the notion of explanation. This is not always so, but $\neg \mathrm{LB}$ (and therefore $\urcorner \mathrm{LA}$ ), is certainly an important case in the use of causal statements. Taking $A \Rightarrow B$ instead of $A \Rightarrow B$ (equivalent with $A \Rightarrow B$ again in case of $\neg L B$ ) we do not exclude the case $L B$, but only $\neg L A \wedge L B$ and give an informative sense to a causal statement in case of LA $\wedge L B$ since $\neg(\neg A \Rightarrow B)$ does not follow from LA and LB. $A \vec{s} B$ then states that $A$ is necessary for the weak necessity of $B$.

It should finally be emphazised first that a causal conditional "Since A, B" does not state that A is a cause of B . As in

Since my barometer is going $u p$, the atmospheric pressure rises
or
Since the period of the pendulum is $t$, its length is $\mathrm{g}\left(\frac{\mathrm{t}}{2 \pi}\right)^{2}$
A may be an effect or a symptom for $B$ that is a reason for believing that $B$. Moreover, A may not be the only possible or actual reason for $B$. But in $A \Rightarrow B$ if $A$ were not to be the case then at least $B$ might be false.

The question, wether C10 and C11 are adequate, is very hard to decide, since we lack reliable truth criteria for ordinary language sentences with iterated "if-then's".

Take the following examples of sentences of the form $A \Rightarrow(B \Rightarrow C),(A \Rightarrow B) \Rightarrow C$, and $A \wedge B \Rightarrow C$ :
(3) If Jobn will come, then if Jack will come too, it will be a nice party.
(4) If Jack will come in case Jobn comes, it will be a nice party.
(5) If Jobn and Jack will come, it will be a nice party.

If under the given circumstances it is possible that John will come (PA), then (3) according to C 10 and C 11 is equivalent to
(6) If Jack will come, then it will be a nice party.

And if under the given circumstances it is possible that Jack will come in case John comes $(P(A \Rightarrow B))$, (4) is equivalent to
(7) Under the given circumstance it will be a nice party.

Under condition that $\mathrm{P}(\mathrm{A} \Rightarrow \mathrm{B})(5)$ follows from (3), but is not equivalent to (3).

All this is hardly convincing. But I doubt that we can really spell out the difference of meaning between (3), (4), and (5). Such constructions are very rare so that we have only a narrow basis for a test for the adequacy of C 10 and C 11 .

If we do not want to exclude such iterated "if-then"'s altogether-and we wouldn't lose much for ordinary language analyses thereby - then it seems best to adopt strong principles that permit us to reduce many such sentences to simple "if-then"'s, as C8 and C9 do in Modal Logic.

## 7. Conclusion

An explication of a concept is adequate if the explicatum is coextensional with the explicandum for the great mass of normal instances, and if the explicatum is simple and fruitful. I have tried to show that our explication of conditionals captures the main ideas that we express by them. The simplicity of the explicatum is achieved only by leaving open the question of how to give a precise sense to the notion of relative necessity ${ }^{8}$ and by passing over a lot of problems connected with natural language analyses.

As for fruitfulness just one example: Many conditional obligations have to be analyzed in the form 'If it is the case that A , then it is obligatory that B ', for which a rule of detachment holds, so that we can infer from $A$ that $B$ is obligatory. ${ }^{9}$

If we take the 'if-then' here as a material or a strict implication, this conditional obligation takes no exceptions. As we can think, with a little imagination, for almost every current conditional obligation of situations, in which it would not hold, i.e. of conditions $C$ so that, if $A \wedge C$, then not $O(B)$, we would have inconsistency in almost all our normative systems. ${ }^{10}$ But if we analyze 'If $A$, then $O(B)^{\prime}$ as $A \Rightarrow O(B)$, then $O(B)$ is only said to be the normal case on condition that $A$, not that $O(B)$ holds in A-worlds in which extraordinary circumstances and strange coincidences obtain. It may then very well be the case that $A \Rightarrow O(B)$, but $\rightarrow(A \wedge C \Rightarrow O(B))$. Such restrictions to normal cases are implied, I think, in most every-day statements of conditional obligation.

## References

Adams, E. W. (1970), Subjunctive and Indicative Conditionals. Foundations of Language 6, pp. 89-94.
Framsen, B. van (1973), Values and the Heart's Command. The Journal of Philosophy 70, pp. 5-19.

[^2]Goodmar, N. (1965), Fact, Fiction, Forecast. 2nd ed. Indianapolis 1965.
Kripke, S. (1972), Naming and Necessity. pp. 253-355, 763-769 in G. Harman and D. Davidson (ed.): Semantics of Natural Language, Dordrecht: Reidel.

Kutschera, F. von (1974a), Partial Interpretations. To appear in E. Keenan (ed.): Formal Semantics for Natural Language, Cambridge.
Kutschera, F. von (1974b), Normative Präferenzen und bedingte Obligationen. In H. Lenk (ed.): Normlogik. München-Pullach.
Lewis, D. (1973), Counterfactuals, Cambridge, Mass.: Harvard Univ. Press.
Stalnaker, R. C. (1968), A Theory of Conditionals. pp. 98-112 in N. Rescher (ed.): Studies in Logical Theory, Oxford.


[^0]:    1 Cf. also Lewis (1973), p. 71.
    2 Cf. Goodman (1965), pp. 14 seq.

[^1]:    7 For the sake of brevity we use the logical operators of $\mathbb{C}$ also as metatheoretical symbols.

[^2]:    8 In another paper I define an epistemic interpretation of conditional necessity and try to show that a purely "objective" interpretation is, as in the cases of unconditional necessity or the similarity of worlds, impossible.
    9 For other types of conditional obligations cf. Lewis (1973), 5.1, and Kutschera (1974b).
    10 For a way out of this problem that comes close to deontic suicide cf. B. van Fraassen (1973).

