ON REPRESENTING RELATIONS BETWEEN PHYSICAL CONCEPTS[1]

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Abstract

The paper has three objectives: to expound a set-theoretical triplet model of concepts; to introduce some triplet relations (symbolic, logical, and mathematical formalization; equivalence, intersection, disjointness) between object concepts, and to instantiate them by relations between certain physical object concepts.

Keywords

Concept, modelling, triplet, base, representing, part, linkage, relations, formalizations, equivalence, intersection, disjointness.

Introduction

Currently explorations in cognitive science, cognitive psychology, and artificial intelligence have led to dramatic changes in the standard understanding of common concepts. They have turned out to be more intriguing objects of study (Margolis et al. 1999) than proponents of their logical analysis presupposed. Researchers have intensively used quantitative and experimental methods in studies of concept roles in knowledge attainment and object recognition. As consequence of this, most experts have now modeled common concepts as the complex structures. Only a few researchers have simulated concepts as the unstructured entities (Fodor 1998; Peacocke 1992).

The diversity of tasks generated by recent concept studies creates a situation when each model proposed is suitable to resolving specific problem(s) and none answers all reasonable and hot questions about concepts. As result,

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many models of common concepts have been put forward (Barsalou 1993; Ganter et al. 1996; Goldstone 1996; Kangassalo 1992; Loocke 1999; Palomäki 1994; Prinz 2002; Rosch 1999; Smith 1990; Wille 1982). Practically, all of them can be described set-theoretically. It means that models regard a concept as a special set-theoretical construction.

There are many kinds of set-theoretical models of concepts. Their peculiar features, coverage and heuristics depend on three main factors.

The first factor is the set-theoretical construction that imitates a concept. Traditionally, this construction is a certain set or an ordered pair/triad of sets.

The second factor is a model of properties used in a concept model. As a rule, researchers have simulated a property of entities covered by a concept as a logical dichotomous predicate. Alternatively, a property is regarded in terms of its yes-no possession by entities in question, i.e. set-theoretically a property is modeled as a set of entities that possess it. However, there is the more promising modeling of a property in terms of 1) the set of entities that can possess a property, 2) property values, and 3) procedures for determining these values (Burgin et. al. 1993). Such a modeling, on the one hand, does not identify a concept of the entity and a property of the entity, and, on the other hand, differs in kind a concept of properties and a property of concepts.

The third factor is the set theory describing the set-theoretical construction that models a concept. Now there are many options: standard, non-standard fuzzy (Zadeh 1975), named (Burgin 1990), rough (Pawlak 2002), etc. set theory in any of its numerous informal and formal versions. Typically, researchers have operated under a standard naive set theory and sporadically a certain version of naive fuzzy set theory.

Let us illustrate these points.

According to the well-known intuition, the evidence that a human person possesses a concept of sensible entities of any kind is equivalent to their effective recognizing by the person. Note, that even such a (perceptible) concept is not sensible, but ideational. The recognition is a procedure that decides whether an entity generated a given percept belongs to the set of entities covered by the concept in question. It is universally accepted, that recognizing is a procedure of processing information about not, 'bare' entities, but their properties. Now there are only more or less elaborated hypotheses about the mechanisms and stages of this processing. The hypotheses on the processing of information about sensible properties are most well grounded experimentally and theoretically, but even they are a subject to permanent changes. Practically nothing is known about 'recognizing' non-sensible entities and processing of non-sensible information. That is a reason why most researchers prefer to simulate concepts leaving aside the specifics of the appropriate information processing. They have taken for granted the information about entities and their properties and have modeled concepts in terms of the entities that fall under concepts and properties of those entities.

Thus, modeling concepts, one starts from considering together entities and their properties. Alternatively, entities are treated as carriers of their properties. This leads to the next standard set-theoretical model of concepts.

Its first component is a set of entities that fall under a concept. Interchangeable terms 'volume', 'extension', 'category' and 'reference' denote this set.

The second component is a set of (separately necessary and jointly sufficient) properties of entities falling under a concept. Mostly the terms 'content' and sometimes 'intension' denote this set.

Note, that the 'extensional' concept models take into account only the first component, while 'intensional' ones – only the second component of the standard model of concepts.

Cognitive scientists and cognitive psychologists have criticized this model and proposed various prototype, exemplar and theory-theory alternatives (Smith et al. 1981; Cohen et al. 1984; Komatsu 1992). Many of them associate with a concept another pair of sets. For example, the prototype theoreticians model a concept by means of at least two sets. The elements (prototypes) of the first set are entities that most people 'automatically, undoubtedly' subsume under a concept. One has selected the elements of the second set due to a great degree of their similarity to prototypes. Thus, a concept has been modeled as an open set-theoretical structure: the fixed prototypical set and the generated variable set whose elements are being included in it on the basis of their similarity to prototypes.

Contrary to all distinctions between models proposed, their proponents have shared, at least, two presuppositions. The first is the idea that all concepts are of the same kind. Taking common sense concepts as typical objects of studies, researchers have often overlooked the peculiarities of scientific concepts. One of a few exceptions is a work of Meir Buzaglo devoted to mathematical concepts (Buzaglo 2002).

The second presupposition is a concentration upon the composition and features of *separate* concepts. In consequence, the relations between concepts have received a little attention. There have been only sporadic attempts at studying such relations. To the best of author's knowledge, Stephan Körner studied relations between concepts about 50 years ago. In the frame of an extensional logical model of ostensive concepts, he analyzed their inclusion, intersection and exclusion (Körner 1959, 24-35). Recently, Buzaglo studied systematically such relations between mathematical concepts as forced expansions. It should be also mentioned the experimentally based research of concept relations induced by influence of one concept on another (Goldstone 1996).

Thus, one of the important and interesting areas of the set-theoretical concept modeling is its application for analyzing and cataloguing relations between concepts. Evidently, that the concept cataloguing as well the cataloguing of relations between concepts depends on the model of concepts.

To a first approximation, one can speak of about external and internal approaches to the cataloguing task. External approach associates with a concept the list of its holistic features. Accordingly, that or this combination of features generates a certain class of concepts. Internal approach connects with a concept its hypothetical internal composition of concepts. Correspondingly, possessing of that or this part of the composition generates a definite class of concept.

In either case of concept modeling, one can develop a scheme of cataloguing relations between concepts.

All studies known to the author introduce and describe relations between concepts in terms of relations between the sets associated with related concepts. Usually, only one set has been corresponded to each concept in question. However, as prototype modeling illustrates, one may use two (and more) sets for imitating a concept. In this framework, it would appear natural to describe concept relations in terms of relations between various set-theoretical constructions that correspond to concepts. Apparently, the 'richer' construction in question, the 'richer' the spectrum of relations between concepts being imitated by the construction.

The simplest extensional set-theoretical modeling of concepts allows one to introduce such relations between concepts as extensional subordination, compatibility and disjunctivity. The first is induced by inclusion, the second — by intersection, the third – by non-intersection of sets of entities falling under concepts in question (i.e. volumes/extensions of concepts). Put otherwise, the concept **C*** is subordinate (compatible/disjunctive) to the concept **C** if and only if the volume of the concept **C*** is a subset of the volume of the concept **C** (intersection of the volume of the concept **C*** and the volume of the concept **C** is a nonempty/empty set).

However, many relations between scientific concepts are 'invisible' under such modeling. Examples are such relations between physical concepts as symbolization, quantification, mathematization, theorization, etc. In the framework of extensional modeling, these relations are indiscernible as concepts in question in many cases have the same set of entities falling under them.

In the intensional model one can consider relations between synonymous (denoted by the same term/name) coextensive (with the same extension) qualitative and quantitative concepts. In the simplest case, relations between a qualitative property from content of a qualitative concept and a synonymous quantitative property from content of an appropriate quantitative concept induce these relations. The chief drawback of standard modeling of this situation is an identification of property with a logical predicate that essentially restricts its applicability to real physical concepts. Physical magnitudes or properties of physical objects have more complicated structure than predicates and need more adequate modeling.

In any case, studies of physical concepts are deficient in the area of concept analysis. Few researchers have conducted such studies (Boniolo 2001; Diez 2002) and they have not touched systematically the issue of relations between concepts.

In what follows we will consider a triplet model of concepts and apply it to analysis of the internal structure of theoretical physical concepts and some relations between them. This model regards a theoretical physical concept as a complicated set-theoretical construction characterized by means of structures of physical theories. Henceforward we use the term "concept" to mean "theoretical physical concept".

Compared to common concepts, physical concepts have discriminative marks that any realistic model of them ought to take into account. Firstly, it is an essential role of mathematical and theoretical structures in their constructing, change, functioning, and application. Secondly, they associate not only with properties of entities falling under them, but also with quantitative values of properties. Various procedures of experimental measurement, theoretical reasoning, and numerical computation help in determining these values.

The triplet approach has modeled a physical concept as a triple consisting of three set-theoretical informational structures: a concept base, a concept linkage and a concept representing part. These structures are supposed to be carriers of information by means of which one constructs, distinguishes, identifies, connects, and applies concepts.

For terminological clarity, we shall use capital bold symbols, letters, words, and word combinations for denoting concepts. We construct the names (terms) of concepts from these denotations by putting them into double quotes. For instance, the capital bold letter **C** denotes a concept and "**C**" denotes its name.

There might be many various denotations of the same concept and, correspondingly, many its synonymous names. Against the obvious (at least contextually) difference between a concept and its name, many researchers without reservation have interchangeably used their denotations. This creates additional difficulties in concept studies. Among other things, this leads some researchers also to the identification of a concept with its name (see, critique of this in Rosser 1953).

1. The Essentials of Triplet Modeling

1.1. Presuppositions of the triplet modeling of concepts

Concepts are pertinent and important objects of philosophical, logical, and psychological analysis over many hundreds of years. Their studies also abound in contemporary linguistics, cognitive psychology, artificial intelligence, informational science, cognitive science, pedagogy, etc. Paradoxically, only few researchers are ready to confess explicitly that theoretically they study concepts by means of constructing and analyzing their models. Many researchers from traditional areas (first of all, philosophy, logic and classical psychology) have presented outcomes of their studies as if concepts were immediately given to their consciousness as simple and unstructured (formless, structureless) objects. Metaphorically speaking, researchers mainly consider concepts as indivisible, but 'seeable by our mental capabilities' atoms whether it be of our consciousness, cognition, knowledge, language, mind, thinking, thought, and the like. It is widely believed that there are complex concepts, but one has practically always connected their complexity with their acquisition, recognition, application, and practically never with their inner structure.

Contrary to the prevailing view of concepts, the triplet approach has taken as an initial point of departure the following statements.

According to the triplet view, concepts are not Platonic ideas. In a broad sense, their reality is produced by human thinking and processes involved in it. Objectivity of concepts is connected with a realization of certain of their cognitive functions. Among them are recognition and classification of objects, collecting, deepening, summarizing, abstracting, instantiating, extending, ordering, circumstantiating information about objects and evaluating its adequacy.

In light of the contemporary understanding of the physical world and its cognition, it would be highly conjectural to follow G.Frege in associating objectivity of concepts with their hypothetical immutability and belonging to the outness. In particular, he stated that "the concept is something objective that we do not form and is not formed in us" (Frege 1984, 113).

It seems, that physicists have constructed the physical concept as a holistic mental entity that fulfills specific cognitive functions. Concept's possessors assembled a concept from some available structures that are not a concept itself. Ironically, one can reduce a concept to any of these structures. Among them are names of a concept and names of entities falling under it; definitions and descriptions of a concept; entities falling under a concept; properties of entities covered by a concept; images and other perceptual representations of these entities, names and values of properties of those entities, etc. Only after some conscious and unconscious, native and guided, goal-directed and time-consuming many-staged processing these structures may create (not always) a concept. The proper end of concept creation is performing certain cognitive task(s). The change of a task, growing knowledge and cognitive skills of concept's possessor have resulted in assembling a new concept even with the old name and extension. Fundamentally, any nontrivial use of a concept by its possessor will cause its appropriate transformation. Contemporary cognitive psychology and cognitive science do not tell us much about the details of processes mentioned above and physiological, neurological, psychological, and cognitive mechanisms of their realization. However, hypothesis about the existence of these processes and mechanisms seems sound.

At some general level of analysis, all concepts reveal the same universal structure. Some concept models imitate its specific substructures. Examples are the above-mentioned extensional and intensional models. However, there are also structures appertained only to some concepts. For instance, mathematized concepts of theoretical physics have associated, at least, with mathematical symbols and structures that 'are lacking' in their synonymous common, if available, counterparts. Thus, it would be wrong to expand automatically to all concepts the outcomes of analysis of a certain type of concept.

The multitude of concept models witnesses the internal complexity of concepts. Among other factors, it explains intricacy of concept acquisition, etc. What we usually call the same concept reveals its various aspects in various situations of its use.

Practically, all concept models proposed are adequate and effective under stated cognitive tasks, certain presuppositions of analysis of concepts and specific conditions of their application. However, most concept models concentrate on some more or less simple aspect of a concept and reduce it to this aspect. In a sense, such models give partial pictures of concepts. In many cases, there is no practical need to put together these partial pictures.

Any model is not a universal frame for resolution of all problems generated by concepts, their origin, compositions, functions, roles, development and so on. The triplet model constitutes no exception. Up to now it has proved to be effective in resolving problems of concept internal structure and classification. Its applicability beyond this area is an open question.

1.2. Concept as information carrier

There are many concept classes (Kuznetsov 1997; Medin et al. 2000). One may construct (introduce, invent, suggest, put forward, etc.) a concept of anything in the world. Some examples are the concepts **GOD**, **UNIVERSE**, **METAL**, **SPACE**, **DURATION**, **WORD**, **SIGN**, **LOVE**, **DISEASE**, **HUMANITY**, **WAR**, **SET**, **NUMBER**. We restrict our attention only to so called physical concepts, i.e. concepts used by physicists in their study of the material world. It is widely accepted that the extension (volume) of physical concepts includes various forms of matter differentiation. Discriminating entities covered by physical concepts, one can draw a distinction between object concepts (e.g., **PHYSICAL OBJECT**, **PLANET, STAR, ATOM**), attributive concepts (e.g., **PHYSICAL PROPERTY, MASS, ELECTRIC CHARGE, DENSITY**), and relational concepts (e.g., **PHYSICAL RELATION, DISTANCE, FORCE, VELOCITY**). Sometimes, one has united attributive and relational concepts as **PHYSICAL MAGNITUDES**. The triplet modeling of any physical concept is specific to its class.

To avoid confusion of objects and concepts of objects as well as names of objects and names of concepts of objects, we will use the following conventions. We will denote (refer to) physical objects by means of small bold symbols, letters, words and word combinations. We construct the names of physical objects by putting denotations of physical objects into double quotes. Thus, we will distinguish **particle** and **PARTICLE** as the object and the concept of this object. Correspondingly, we will distinguish "**particle**" and "**PARTICLE**" as the name of **particle** and the name of **PARTICLE**.

If necessary, small italic characters will denote physical magnitudes or properties and relations of physical objects. An example is the magnitude *mass.* One of names of this magnitude is "*mass*". Physicists use the symbol *m* simultaneously for denotation of the magnitude *mass* and for naming this magnitude. The distinction between these two uses of the symbol *m* commonly is evident from the context.

For simplicity's sake, we will consider only object concepts, i.e., concepts of isolated and supposedly independent physical objects like **stars, planets, macroscopic bodies, molecules, atoms, nuclei, sub-nuclear constituents.** As a main example we will take the various versions of the concept **ELEMENTARY PARTICLE** (in short, **PARTICLE**) and related concepts. Today there is no final and completed version of this concept. Physicists are in process of constructing its various and numerous variants. Rapidly alterable experimental data and dramatically reformative theories have caused transitions from one variant to another.

The triplet modeling treats an object concept **C** as a (mental) construction assembled from three kinds of interrelated information: the concept base, the concept representing part and the concept linkage. Any separate piece of these informative kinds is not a concept itself, but only its triplet component. Such components can play the role of the concept representer acting as a substitute for a concept. This concept is something objective when a physicist has been thinking of the object of her study by means of concept representers relevant to a given situation. Typically, the concept's holders as well as researchers of concepts have identified a concept with some of its representers.

The concept's possessor may assemble different concepts with the same name from the information available. It depends on many cognitive and psychological factors. Among them there are the cognitive tasks that concept's possessor wishes to resolve and requirements to the task solutions as well as the state of her mind and memory. The astronomer has used various concepts **STAR** when she thinks about internal structure of **red dwarfs** and about the role of **giants** in evolution of our Galaxy.

1.3. The concept base

The first kind of information associated with a concept **C** concerns objects falling under it. This is information about such objects considered as independent and separable entities. It includes pieces of knowledge (with some measure of conclusiveness) about the supposed existence of these objects, about their attributes (properties, relations, functions, quantity, composition, physical states, regularities, etc.) One can arrange this information according to hypotheses about the ontological structuring of the world. Today the most-used hypothesis assumes understanding the physical world in terms of ontological structure: objects – properties (and their values) of objects – relations (and their values) between objects (see Figure 1) or objects – their magnitudes – values of magnitudes (see Figure 2). It is worth noting that there are many connections between components of this structure. These connections have generated such its substructures as *relations between properties*, *properties of a relation between objects,* and *a relation between numeric values of the same relation*. The information ordered in such a way comprises the base of the concept **C** or the concept base ***B***(**C**).

The extensional model of concepts operates only with a part of the concept base. This is the information about the set of objects falling under a concept and, factually, about some shared property of these objects that simultaneously functions as a characteristic property of this set. The intensional model operates with another part of the concept base. This is the information about so-called first order properties of objects, i.e. properties directly connected with objects as independent and separable entities. Note, that *n+1-*

order properties of objects are determined as the properties of the *n*-order properties of objects.

property values
➚
the set of properties of various orders
➚
objects
➘
the set of relations of various orders
➘
relation values

Figure 1

objects ➔ *the set of magnitudes of various orders* ➔ magnitude values

Figure 2

In addition to this information, the base of any sophisticated physical concept includes also the following pieces of knowledge. They are about: properties of these properties; relations between objects; relations between properties; properties of relations; qualitative and quantitative values of properties and relations just listed; values of physical constants, etc.

For instance, the base of the concept **PARTICLE** contains pieces of knowledge that deal with facts and hypotheses concerning: the place of particles in the structure of the physical world; classes of particles (leptons, mesons, hadrons, etc.); types of particle interactions (gravitational, weak, electromagnetic, strong, electroweak, etc.); external and internal properties of particles (charge, mass, spin, isotopic spin, strangeness, stability, beauty, color, etc.); numeric values of properties; physical constants and their values (e – electron charge, $\hbar$ – Planck's constant, c – speed of light in a vacuum, etc.) It is significant that the base of the concept **PARTICLE** is a subject of permanent changes (elaborating, supplementing, rejecting, and restructuring) during the last seventy years. To see this, one need only compare overviews of particles data published in different years.

The concept base contains, as it were, the outcomes of the first stage of the preprocessing of information connected with a concept. Many concept models consider only a particular base substructure and identify it with a concept.

Physical concepts associate with such specialized, abstract, and highly organized systems as physical theories. One says practically nothing about the contemporary concept **PARTICLE** without the profound and extensive use of a certain physical theory, i.e. its languages, models, problems, methods, estimations, etc. Moreover, a particular theory proposes a specific way of representing and processing available information about particles. It means that, at least in the course of modeling physical object concepts, one should not miss the second stage of preprocessing information about objects falling under concepts. The input of this preprocessing stage is the concept base. The output constitutes the concept representing part. Mechanisms of preprocessing are beyond the scope of this paper.

1.4. The concept representing part

Thus, the second kind of information is an outcome of second stage of preprocessing information associated with a concept. The main aim of such a preprocessing is to prepare conditions for the effective use of concepts in cognitive processes (application and constructing hypotheses and models, formulating and resolving problems, deducing and justifying statements, etc.).

From the formal point of view, the second kind of information or the concept representing part has been expressed by general linguistic and specific theoretical structures from physical theories relevant for a concept in question.

There are many kinds of such structure. Their examples are expressive forms of languages. In the case of assertive languages these are letters of alphabets, words, word combinations, sentences, texts; in the case of programming languages – constants, variables, commands, operators, programs, data, bases and banks of knowledge, etc. Various structures of physical theories are vitally important for constituting and processing physical concepts. Among them are laws, potential, partial and full models (Balzer et al. 1987; Sneed, 1971), abstract properties, measuring, computing models, principles of symmetry and supersymmetry, problems, hypotheses, estimations, heuristics, procedures (Burgin et al. 1994).

In what follows we consider only formal linguistic and theoretical structures of expressing the concept representing part. Thus, we reduce this part to its formal structural aspect. It means that we consider so-called formal structural representing part ***R***(**C**) of the physical concept **C**.

In this paper, we leave aside semantic questions of meaning and sense. It is a significant limitation, but, happily, one may analyze many important relations between concepts without delving into semantic issues.

From the reference point of view, components of ***R***(**C**) may function as names, descriptions, definitions, pictures, images, diagrams, tables, models, specific theories (theory of spin, theory of internal quantum numbers, etc.) of components from the concept base.

The extensional model of concepts contains implicitly only such representing structures as general and singular names of objects falling under a physical concept. However, the representing parts of physical versions of the concept **PARTICLE** include expressive forms of many physical theories: quantum mechanics, relativity theory, quantum field theory, theories of symmetries and supersymmetries, etc. It would be an unjustified and fruitless simplification to suggest that ***R*****(PARTICLE)** contains only the names of different degrees of generality like "**particle**", "**electron**", "**proton**", "**neutron**", or even their symbols "**e**", "**p**", "**n**". It is interesting to note that ***R*****(PARTICLE)** does not include singular (individual) names of particles, possibly, due to absolute identity of particles of the same class.

1.5. The concept linkage

The third kind of information associated with a concept concerns the nature and ways of establishing the correlation between components of its base (i.e., pieces of the information of objects and their attributes) and structures that depict them in its representing part. This information comprises the linkage ***L***(**C**) of a concept **C**. In the case of physical concepts this correlation is not equal to the trivial and simple juxtaposition of two sets of elements. The components of this correlation are constituted by performing complex operations of the denotation, abstraction, idealization, modeling, computer simulation, argumentation, problem formulation and resolution, interpretation, observation, experimentation, measurement, calculation, computation, etc.

Let us illustrate this by considering briefly a case of using sophisticated mathematical apparatus in contemporary theories of particles. Physicists have provided the quantitative description and explanation of some details of particle interaction at high energies by topological constructions of fiber bundle and section (Bernstein and Phillips 1981). It means that these constructions are components of the representing part of the some contemporary version of the concept **PARTICLE**. The linkage between them and experimentally measured values of properties and relations of particles, i.e. between some components of the representing part and the base, is a result of physicist's creative activity. It includes all operations above mentioned except direct observation of elementary particles. In a sense, all versions of this concept are not ostensive ones.

1.6. Fluidity of concepts

These three kinds of information are vitally important for construction and uses of theoretical physical concepts. Without either of these, it would be difficult to say that a physicist possesses a certain concept. None of these informative kinds exists in a final and finished form.

Indeed, any nontrivial utilization of a concept results in the growth of information about its base as well as in elaborating and restructuring its representing part and linkage. The permanent improvement of measuring technique, the development of new experimental and mathematical methods cause changes in the linkage of most scientific concepts. The physicist's tendency to generalize information at hand, to explain the refined as well as principally new information about the concept base leads to using new mathematical structures in the concept representing part. Examples are the application of matrix calculus by Heisenberg for the development of many concepts of quantum mechanics or using group-theoretical methods by theoretical physicists for constituting many concepts of contemporary elementary particles physics. In some instances physicists are even forced to invent new mathematical structures and use these in the representing parts of their concepts. Newton's invention of differential and integral calculus has played a principal role in building such concepts of classical mechanics as **MOTION, FORCE**, **VELOCITY**, **ACCELERATION**, etc.

It would appear reasonable to suggest that these augmentations of information should be included by one or another way into a physical concept. The point here is that the nontrivial use of a concept has resulted in changes of all kinds of information associated with a concept. However, these changes are not total because some essential pieces of information remain stable. Thus, the standard presupposition of concept analysis that concepts are finished and unchanged entities (similar to Plato's ideas) should be taken with reservations.

It is notable that the most stable informative structure connected with a concept is its name, i.e., the term for a concept. However, constancy of the concept name is not identical with invariability of three kinds of information associated with a concept. Formalizing concepts, many experts factually assume such invariability. It is true only for a momentary static picture of concepts.

1.7. Set-theoretical descriptions of triplet structure of concepts

From the above discussion, it appears that there are some sound grounds for modeling a concept **C** as a triple (***B***(**C**)**,** ***L***(**C**)**,** ***R***(**C**)). One may describe this triple by means of different mathematical theories: category theory, theory of "ordinary" sets, theories of fuzzy sets, theory of named sets, some topological and algebraic theories, etc. In what follows we will use only some elementary notions of informal naive theory of ordinary sets. However, one may naturally continue the line of consideration proposed for the case of other set theories (see, for example (Kuznetsov et al. 1998)).

1.8. Holistic and local relations between concepts

It is possible to introduce the notions of holistic and local relations between two concepts.

Let **C** and **C*** be concepts and (***B***(**C**)**,** ***L***(**C**)**,** ***R***(**C**)) and (***B***(**C***)**,** ***L***(**C***)**,** ***R***(**C***)) be, correspondingly, their triplet models.

The holistic kind is associated with relations between triplet structures (***B***(**C**)**,** ***L***(**C**)**,** ***R***(**C**)) and (***B***(**C***)**,** ***L***(**C***)**,** ***R***(**C***)) each of which being considered as one aggregate.

One may schematically picture holistic concept relations by means of the diagram 1.

The pair (α**,** β) characterize holistic relations between **C** and **C***. Here α is a relation between ***R***(**C**) and ***R***(**C***) and β is a relation between ***B***(**C**) and ***B***(**C***).

$$
\begin{array}{ccccc}
 & & \alpha & & \\
 & \boldsymbol{R}(\mathbf{C}) & \rightarrow & \boldsymbol{R}(\mathbf{C^*}) & \\
\boldsymbol{L}(\mathbf{C}) & \uparrow & & \uparrow & \boldsymbol{L}(\mathbf{C^*}) \\
 & \boldsymbol{B}(\mathbf{C}) & \rightarrow & \boldsymbol{B}(\mathbf{C^*}) & \\
 & & \beta & &
\end{array}
$$

Diagram 1.

There are also some natural restrictions on the bases, linkages and representing parts of meaningfully related concepts. Commutivity of the diagram ($\alpha\boldsymbol{L}(\mathbf{C}) = \boldsymbol{L}(\mathbf{C^*})\beta$) is one of these restrictions. We illustrate this with concepts **C** and **C*** that are various temporal states of the 'same' concept. As the criterion of sameness we take the condition $\boldsymbol{B}(\mathbf{C}) \subseteq \boldsymbol{B}(\mathbf{C^*})$.

Let us suppose that we have the concept **PARTICLE** = **C**, as it exists at some moment *t* of the history of particle physics development. The simplest triplet model of this concept can be described as follows. The base associates with particle classes known at *t*. The representing part includes only names of these particle classes. The linkage contains the relations of naming particle classes by their names and inverse relations. In this case, it is naturally to assume the fulfillment of the next three conditions.

1) For any class of elementary particles from the base, there are its names in the representing part, i.e. there are no nameless particle classes. 2) There are no empty names in the representing part, i.e. names for which there are no known particle classes. 3) The linkage nomenclates (tags, titles) certain names to the appropriate particle classes and juxtaposes particle classes and their names.

Concepts for which these conditions are valid will be called self-consistent with respect to the procedure of naming and the inverse procedure, or, in short, self-consistent.

The discovery of a new particle class causes changes of the base, the representing part and the linkage of **C**. These changes result in transforming the concept **C** in the new concept **C***. The change of the base is associated with its extension by the new class of particles and is exemplified by the relation β that transforms $\boldsymbol{B}(\mathbf{C})$ into $\boldsymbol{B}(\mathbf{C^*})$. The change of the representing part is associated with its extension by new names for the new class of particles and is exemplified by the relation α that transforms ***R***(**C**) to ***R***(**C***). The changes of the base and the representing part generate the change of the linkage.

The holistic relation between concepts **C** and **C*** will be 'good' if both concepts are self-consistent and the diagram 1 is commutative. The latter means that the path from ***B***(**C**) via ***R***(**C**) to ***R***(**C***) ends with the same result as the path from ***B***(**C**) via ***B***(**C***) to ***R***(**C***). Informally, on one hand, this means that transforming ***R***(**C**) should be such that there will be no empty names in ***R***(**C***) and new names will be names of the particle class discovered. On the other hand, the transformation of ***B***(**C**) should be such that ***B***(**C***) includes only such new particle class for that there are names. To put it differently, composition α***L*(C)** of relations α and ***L*(C)** should be identical to composition ***L*(C*)**β of relations ***L*(C*)** and β.

Relations between substructures of triplet structures of concepts induce the local concept relations. Examples are relations between ***R***(**C**) and ***R***(**C***) or between ***B***(**C**) and ***B***(**C***).

Evidently, in the extensional model framework relations between the concept extensions induce relations between concepts. This means that 'extensional' relations between concepts are special cases of local relations between concepts in the framework of triplet model.

In this paper, we concern only local relations between concepts induced by relations between substructures of their representing parts. These will be called RR*-relations or, in short, R-relations. The local relations induced by relations between concept bases considered in (Kuznetsov 2003).

1.9. Formal structural components of the concept representing part

Analyzing R-relations, we need a detailed set-theoretical description of the concept representing part.

Let us consider a situation in which a physicist possesses some object concepts. Usually one supposes that a physicist operates only with the information about properties of objects covered by concepts. However, more realistically to admit that she associates with concepts certain ontological hypotheses about inner structure, properties, and relations of objects falling under these concepts. According to the triplet modeling, such information is contained in the bases of concepts and characterizes objects and their attributes as they supposed to be. For example, a physicist can describe some physical object as orbiting in an ellipse in a definite spatial region, as having spherical form and huge mass. Here special emphasis is placed on the hypothetical ontological content, but not on forms of its expression.

In principle, a physicist expresses the content of the accessible information by means of linguistic and specific theoretical structures. A physicist may use all of these in the concept representing part. However, a rather limited set of expressive structures is associated with a certain concept at any moment of its history. Modeling concepts, researchers, as a rule, have taken explicitly into account such linguistic structures as separate words and word combinations that function as object names.

Let us return to our favorite example. The simplest extensional model of the concept **PARTICLE** refers explicitly only to the set of physical objects falling under it. As this takes place, names of various degrees of generality and abstractness have been implicitly used for description both of this set, its subsets, and elements. Their examples are words and word combinations that in plural refer to some sets and in singular – to its elements: "**microobject(s)**", "**constituent(s) of atoms**", "**elementary particle(s)**", "**hadron(s)**", "**nucleon(s)**", "**proton(s)**", etc.

Considering scientific practice, one can notice the following. Not only words and word combinations, but also letters of various alphabets, sentences of informal and formal languages, combinations of sentences, and special structures of physical theories (structuralist models, see Diez 2002) are among components of concept representing parts. Examples are the symbol *e* denoting electrons, the sentences 'The mass of an electron is not equal to zero', 'Charged leptons interact through electromagnetic forces', 'Electrons are spinor fields', etc.

Usually, sentences and their sets function as definitions and descriptions of components from the concept base. However, at least the representing parts of physical concepts contain some sentences that function as true statements. These statements are consequences of abstract assertions, mathematical formulas, sophisticated hypotheses, complex theoretical models, etc. In turn, theoretical models are specific combinations of letters and symbols, words and statements that may be considered and analyzed as some wholeness (see, for example, the structuralist reconstruction of theoretical models (Sneed 1971; Balzer et al. 1987)).

In the light of this, we describe a concept representing part in the following terms.

Let L^n be some language with the alphabet A^n, the vocabulary V^n, the set of expressions (sentences) E^n and the set of texts T^n. Here and in what follows $n = 1, 2, 3, \ldots$. All these sets are supposed to be finite. These sets will be called the language constitutive sets or, in short, constitutive sets. Practically, only the alphabets are more or less stable, other constitutive sets are subjects of permanent change. It should be noted that important characteristics of language are also its rules for constructing, evaluating and transforming its constitutive sets. In this paper, we will not touch these rules.

The point of this paper is that one can formally and structurally characterizes the representing part of a concept by means of certain combinations of the subsets of language constitutive sets. It is well to bear in mind that the representing part of physical concepts includes, as a rule, subsets of constitutive sets from several languages. For simplicity' sake, we consider only two languages L^1 and L^2. From this standpoint, the representing part of the concept **C** includes some subsets of unions of 'cognominal' constitutive sets of two languages:

$\boldsymbol{R}(\mathbf{C}) = <A(\mathbf{C}), V(\mathbf{C}), E(\mathbf{C}), T(\mathbf{C})>$, where $A(\mathbf{C}) \subseteq A^1 \cup A^2$, $V(\mathbf{C}) \subseteq V^1 \cup V^2$, $E(\mathbf{C}) \subseteq E^1 \cup E^2$, $T(\mathbf{C}) \subseteq T^1 \cup T^2$.

In principle, it is necessary also to take into account 'hybrid' constitutive sets generated by the simultaneous utilizing of two languages. The most natural medium for these is the set of expressions and the set of texts. Indeed, in the case of two languages the concept representing part includes meaningful expressions and texts built from both alphabets. It contains also bilingual meaningful expressions contained components from two vocabularies. The same is true for the set of texts. From this viewpoint, it would be more precise to operate with $E(\mathbf{C}) \subseteq E^1 \cup E^2 \cup E^{12}$, $T(\mathbf{C}) \subseteq T^1 \cup T^2 \cup T^{12}$. Here E^{12} is a set of meaningful expressions consisting of letters and words from both languages and T^{12} is a set of texts constructed from those expressions.

2. R-relations between concepts

Let us consider two concepts **C** and **C*** with representing parts ***R***(**C**) = <*A*(**C**), *V*(**C**), *E*(**C**), *T*(**C**)> and ***R***(**C***) = <*A*(**C***), *V*(**C***), *E*(**C***), *T*(**C***)>. In the framework of such modeling, it is possible to introduce many R-relations. We will consider only some of these.

2.1. Formalizations

There are many meaningful relations of formalization of concepts.

Definition 1. A concept $^{L1,L2,\ldots,Ln}$**C** is *n*-lingual if its representing part includes elements and subsets of constitutive sets from *n* languages.

As a rule, common concepts are monolingual. Let us consider the following example. The English translation of the Ukrainian word ‘школа' (and the Russian word ‘школа’) is the word ‘school’. The representing parts of concepts **ШКОЛА** and **SCHOOL** are expressed, correspondingly, by means of Ukrainian and English languages. Taking into account the differences of teaching systems, kinds and contents of textbooks, terms of study, one might conclude that these concepts are not identical. Note that, after twelve years of the Ukrainian independence and various transformations of the Ukrainian and Russian educational systems, the Ukrainian concept **ШКОЛА** is not identical to the Russian concept **ШКОЛА** and neither is identical to the Soviet concept **ШКОЛА**.

Physical concepts are, at least, bilingual. Representing parts of many general physical concepts (even in the case of its informal exposition) include letters of Greek and Latin alphabets. Representing parts of theoretical concepts include expressive tools of many mathematical languages. Thus, these concepts are multilingual. For example, the representing part of the classical mechanics concept **FORCE** includes constructions from vector algebra, theory of functions, differential calculus, theory of differential equations, etc. Thus, it would be unjustified simplification to suggest that theoretical physicists have built the representing parts of their concepts by means of only one mathematical language.

The use of expressive means from many languages in representing parts of physical concepts is not an end in itself. Often this is a unique way of obtaining nontrivial information about the concept bases. From this point of view, the grasping of many physical concepts presupposes the profound learning, possessing and understanding of a great deal of mathematics and its numerous languages.

For example, according to contemporary physics, objects falling under the quantum mechanical concept **WAVE FUNCTION** are not spatially localized and visualized objects of classical physics. Only physicists with deep knowledge of theory of Hilbert spaces and theory of differential equations in partial derivatives are able to describe internal and external properties of quantum-mechanical objects, predict and compute their experimentally measurable values.

In principle, multilingualism of the concept representing part is justifiable when each of the languages used possesses unique expressive and transformational capabilities and could not be eliminated without essential loss of concept effectiveness.

For example, the use of languages of probability theory in the representing part of the concept **WAVE FUNCTION** allows physicists to predict the probability of the location of a quantum object at a spatial point. The additional use of the languages of differential and integral calculus allows one to compute the probability of detecting a quantum object in some spatial region.

Let us consider two concepts ${}^{L1}\mathbf{C}$ and ${}^{L1,L2}\mathbf{C}$ with the same base $\boldsymbol{B}$. Constitutive components from the representing part of the former concept are constructed by language L^1, and those of the latter – by languages L^1 and L^2. As this takes place, some constitutive components from $\boldsymbol{R}({}^{L1}\mathbf{C})$ enter into $\boldsymbol{R}({}^{L1,L2}\mathbf{C})$ without changes and others – after their translation into L_2.

Definition 2. The bilingual concept ${}^{L1,L2}\mathbf{C}$ is a nontrivial complete (representing; linkage; base) L^2-lingual extension of the concept ${}^{L1}\mathbf{C}$, if and only if the $\boldsymbol{R}({}^{L1,L2}\mathbf{C})$ contains such constitutive components constructed by language L^2 that their processing has resulted in obtaining more effectively or otherwise unknown information about the concept **C** (its representing part; linkage, base).

For example, let us take the concept **INTEGER** as a monolingual concept ${}^{L1}\mathbf{C}$ with a representing part expressed in ordinary language L^1. In particular, names of counting numbers are expressed by special English words – numerals, relations between integers are expressed by words ‘greater’, ‘lesser’, ‘equal’, and operations over integers are expressed by words ‘summation’, ‘addition’, ‘subtraction’, etc. However, even the simplest arithmetic operations with large numbers we perform better when corresponding combinations of ciphers substitute for these words. Thus, strictly speaking, the new concept **INTEGER** with such a representing part will be bilingual one because the Arabic numerals, strictly speaking, are not elements of the English alphabet. From this follows, that the second concept is a nontrivial base L^2-lingual extension of the first concept.

Definition 3. The relation of L^2-lingual extension between concepts ${}^{L1}\mathbf{C}$ and ${}^{L1,L2}\mathbf{C}$ with the same bases is:

— local (full) symbolic L^2-formalization (L^2-symbolization), if letters of alphabet of the language L^2 are used as symbols that substitute for some (all) constitutive components of the concept ${}^{L1}\mathbf{C}$;
— local (full) logical L^2-formalization (L^2-logicalization), if L^2 is a logical language in terms of which some (all) constitutive components of the concept ${}^{L1}\mathbf{C}$ are expressed. (A specific logical language determines its own type of formalization. For example, the first order predicate language determines the first order predicate formalization of concepts);
— local (full) mathematical L^2-formalization (L^2-mathematization), if L^2 is a language of certain mathematical theory in terms of which some (all) constitutive components of the concept ${}^{L1}\mathbf{C}$ are expressed. (Depending on the kind of mathematical theory, to a first approximation, one can introduce such specific kinds of mathematization as algebraization, arithmetization, categorization, functionalization, geometrization, metrization, set-theoretization, topologization etc.)

Contrary to a widespread opinion, the logical formalization of a concept is not a prerequisite of its effective mathematical formalization. Historically, many physical concepts were given mathematical expression without any attempt of their logical formalization .

In turn, it should be noted that symbolization of physical concepts is only a prelude to their effective mathematization.

Definition 4. The component type of the concept **C** is *L*-symbolic (verbal, sentential, textual) if its representing part includes elements from the alphabet (the vocabulary, the set of sentences, the set of texts) of the language *L*.

The component type of common concept is simultaneously verbal, sentential and textual. In addition to this, the component type of many physical concepts is symbolic. For example, representing parts of concepts of many elementary particle classes include their symbolic names that are letters from the Greek alphabet with superscripts and subscripts.

Early in its "development" a new physical concept may have any component type. For example, the representing part can contain only some fuzzy set of texts. Later the set of sentences characteristic of the concept had been formulated and then the set of characteristic words has been associated with it. However, upon becoming familiar with an existent concept, a person usually learns one of its names (i.e., a word or a word combination). Then she learns a sentence or sentences containing this name and finally text(s) associated with the concept.

According to the extensional modeling, one regards a concept as the set of objects falling under it and denotes this set by the same general name with the same spelling as the name of this concept. It is implicitly assumed, that the concept representing part includes only such name. Based on it, one can say that the component type of a concept is verbal. However, on closer inspection one should take into account the properties of objects falling under a concept. In this case, the concept representing part includes not only names of objects, but also names of properties of these objects and values (qualitative and quantitative names) of properties in question. It also includes sentences such as 'The object with a name "*general name of objects in question*" possesses the property with a name "*general name of property*"'. What this means is that the component kind of the concept in question is sentential.

Intuitively, some physical concepts have the status of symbolic concepts that processed only as symbols of a formal system. For example, in the case of the concept **VELOCITY AS CONTINUOUSLY DIFFERENTIABLE FUNCTION** the immediate subject of derivation or integration is the symbol *f* from its representing part, but not the word 'function'.

This intuition may be explicated as follows.

Let σ be structure that transforms according to the rules of a formal system Σ built in language *L*[1].

Definition 5. The concept **C*** = (***B***(**C***)**,** ***L***(**C***)**,** ***R***(**C***) = <σ>) is a constructive *B*-conservative σ-formalization of the concept **C =** (***B***(**C**) = ***B***(**C***)**,** ***L***(**C**)**,** ***R***(**C**) = <*A*(**C**), *V*(**C**), *E*(**C**), *T*(**C**)>) if structure σ substitutes for any component from *A*(**C**), *V*(**C**), *E*(**C**), *T*(**C**) of the representing part ***R***(**C**) of the concept **C** and transformation of σ has resulted in new information about the concept base ***B***(**C***) = ***B***(**C**).

Definition 6. The constructive *B*-conservative σ-formalization is:

— mono-symbolic if σ is an elementary symbol of system Σ (and language L^1);
— poly-symbolic if σ is a combination of elementary symbols of system Σ (and language L^1);
— structure-symbolic if σ is a relational structure constructed from symbols of system Σ (and language L^1);

Let us consider some details of classical and quantum-theoretical (non-relativistic) mathematical formalizations of the informal concept **PARTICLE**. Its representing part contains words and word combinations 'an elementary particle', 'energy', 'momentum', 'spatial localization', 'equation of motion', etc.

According to the classical mathematical formalization, one should replace such components of the representing part by mathematical constructions from arithmetic, vector algebra and theory of continuous functions. In particular, the word 'mass' is replaced by the symbol *m* that denotes scalar mathematical function with positive numeric values. The words 'force' and 'acceleration' are substituted, correspondingly, by symbols ***f*** and ***a*** that denote specific finite-dimensional vector continuous functions of spatial and temporal coordinates. These substitutions are examples of mono-symbolic formalization. The expression '$\boldsymbol{f} = m\boldsymbol{a}$' constructed from these symbols substitutes for the word combination 'equation of motion'. This substitution is an example of structure-symbolic formalization. The classical mechanics holds that the solution of equation of motion permits one to describe classical properties of elementary particles and predict (under knowing some initial conditions) quantitative values of these properties. Notice that the solution of equation of motion is a result of its specific transformation according to the theory of differential equations.

In the early twentieth century, it was experimentally demonstrated, that the classical mathematical formalization of the concept **PARTICLE** was limited in applicability. The non-relativistic quantum formalization is more effective, but is not universally applicable, too.

According to the last formalization, the symbol ψ denoting a vector from infinite-dimensional functional space replaces the phrase 'an elementary particle'. Physicists have replaced the classical equation of motion by the

Schrödinger equation or some other fundamental quantum mechanical equation for the wave function ψ. It is assumed that solutions of this equation predict the distribution of measured values of some quantum mechanical properties of particles at low energies. In particular, the phrase 'a spatial localization of a particle' is replaced by the phrase 'a probability of finding a particle in a spatial point *x*', which, in turn, is replaced by a certain poly-symbolic combination known as $|\psi(x)|^2$.

It is interesting to note that, in the former case, formalizations of some particle properties (*mass,* etc.) take place. However, in the latter case physicists replace the name 'an elementary particle' by the symbol ψ that enters quantum mechanical equation. In both cases, formalizations are constructive because they have resulted in new information about the concept base.

Definition 7. Let **C** and **C*** be concepts and ***R***(**C***) in comparison with ***R***(**C**) contains additionally the symbol (correspondingly, word, phrase, sentence, text). The concept **C*** is a nontrivial one-element symbolization (verbalization, phrasalization, sententialization, textualization) of the concept **C** if and only if the processing of ***R***(**C***) has resulted in new information in comparison with the processing of ***R***(**C**).

One may also characterize constitutive components of the concept representing part in terms of their reference to constituents of the base.

Definition 8. The concept **C** is *b*-named (*b*-definitional, *b*-modeling, *b*-theoretical) if its representing part contains language structure that functions as a name (correspondingly, a definition, a model, a theory) of the component $b \in$ ***B***(**C**).

Notice that many components of ***R***(**C**) do not refer immediately to constituents of ***B***(**C**).

2.2. Representing equivalencies of concepts

Let us consider some other local R-relations between concepts.

Definition 9. The concept **C** is completely R-equivalent to the concept **C*** if and only if *A*(**C**) = *A*(**C***), *V*(**C**) = *V*(**C***), *E*(**C**) = *E*(**C***), *T*(**C**) = *T*(**C***).

It should be emphasized, that the identity of concepts does not follow from complete R-equivalency of concepts because these may have various bases.

Proposition 1. The concept **C** is completely R-equivalent to itself.

Proposition 2. The relation of complete R-equivalence is transitive, symmetric and reflexive.

Definition 10. The concept **C** is $a(v, e, t)$-locally R-equivalent to the concept **C*** if and only if $a \in A(\mathbf{C})$ and $a \in A(\mathbf{C^*})$ (correspondingly, $v \in V(\mathbf{C})$ and $v \in V(\mathbf{C^*})$; $e \in E(\mathbf{C})$ and $e \in E(\mathbf{C^*})$; $t \in T(\mathbf{C})$ and $t \in T(\mathbf{C^*})$).

The representing part of any object concept contains the word 'object'. What this means is that all object concepts are 'object'-locally R-equivalent.

According to the non-relativistic quantum mechanics, the representing parts of concepts of various particle classes (electrons, protons, neutrons and others) include many constructions. Among these are the wave function (ψ is an element of the alphabet of quantum mechanical language) and the quantum mechanical equation of motion (the Schrödinger equation $\partial\psi = 0$ is an expression from the set of expressions of quantum mechanics). By this is meant that concepts in question are ψ ($\partial\psi = 0$)–locally R-equivalent.

Proposition 3. If the concept **C** is t-locally R-equivalent to the concept **C*** then there are such e and a, that $e \in t$ and $a \in e \in t$ and for which **C** is e-locally and a-locally R-equivalent to **C***.

Proposition 4. If the concept **C** is e-locally R-equivalent to the concept **C*** then there is such a, that $a \in e$ and for which **C** is a-locally R-equivalent to **C***.

Definition 11. The concept **C** is $a\{v, e, t\}$-coherent if and only if $a \in A(\mathbf{C})$, then there are such $v \in V(\mathbf{C})$, $e \in E(\mathbf{C})$ and $t \in T(\mathbf{C})$, that $a \in v \in e \in t$.

2.3. Representing disjointnesses of concepts

Let us consider some concept relations induced by set-theoretical disjointness.

Definition 12. The concepts **C** and **C*** are completely R-disjoint if and only if $A(\mathbf{C}) \cap A(\mathbf{C}^*) = \varnothing$, $V(\mathbf{C}) \cap V(\mathbf{C}^*) = \varnothing$, $E(\mathbf{C}) \cap E(\mathbf{C}^*) = \varnothing$, $T(\mathbf{C}) \cap T(\mathbf{C}^*) = \varnothing$.

The English translation of the Ukrainian word ‘частка’ is the word ‘particle’. In the light of this, informal concepts **ЧАСТКА** and **PARTICLE** with representing parts expressed, correspondingly, in terms of Ukrainian and English languages are completely R-disjoint concepts. However, mathematized versions of these concepts are not completely R-disjoint because these use standard Latin and Greek symbols and their combinations, in particular, motion equation.

Proposition 5. The relation of complete R-disjointness is not transitive on the set of all object concepts, i.e., for arbitrary **C**, **C*** and **C****, complete R-disjointness of **C** and **C**** does not follow from complete R-disjointness of **C** and **C*** and complete R-disjointness of **C*** and **C****.

Definition 13. The concepts **C** and **C*** are $a(v, e, t)$-locally R-disjoint if and only if $A(\mathbf{C}) \cap A(\mathbf{C}^*) = \varnothing$ (correspondingly, $V(\mathbf{C}) \cap V(\mathbf{C}^*) = \varnothing$; $E(\mathbf{C}) \cap E(\mathbf{C}^*) = \varnothing$; $T(\mathbf{C}) \cap T(\mathbf{C}^*) = \varnothing$).

Proposition 6. The relation of $a(v, e, t)$-partial R-disjointness is not transitive on the set of all object concepts.

Proposition 7. The completely R-disjoint concepts are $a(v, e, t)$-locally R-disjoint.

2.4. Representing intersections of concepts

Let us consider concept relations induced by various set-theoretical intersections.

Definition 14. Concepts **C** and **C*** are completely R-intersecting if and only if $A(\mathbf{C}) \cap A(\mathbf{C}^*) \neq \varnothing$, $V(\mathbf{C}) \cap V(\mathbf{C}^*) \neq \varnothing$, $E(\mathbf{C}) \cap E(\mathbf{C}^*) \neq \varnothing$, $T(\mathbf{C}) \cap T(\mathbf{C}^*) \neq \varnothing$.

Proposition 8. The relation of complete R-intersection is reflexive and symmetric.

Proposition 9. The relation of complete R-intersection is not transitive on the set of all object concepts. It means that for arbitrary **C**, **C*** and **C**,** complete R-intersection of **C** and **C**** does not follow from complete R-intersection of **C** and **C*** and complete R-intersection of **C*** and **C****.

Definition 15. The subset of the set of all object concepts is a non-transitive R-subset if it includes only concepts related by the non-transitive complete R-intersection.

Proposition 10. The relation of complete R-intersection is tolerant on the non-transitive R-set.

Informally, tolerance of complete R-intersection of concepts means the following. Complete R-intersection of concepts could be interpreted as an expression of their similarity in respect to their representing parts. In this case tolerance of the relation between completely R-intersecting concepts means that concepts **C** and **C**** that are similar to the concept **C*** are not necessarily similar to each other.

There are many propositions about the connections among the introduced above types of concept relation. One example is the next proposition.

Proposition 11. Completely R-equivalent concepts are completely R-intersecting.

3. Perspectives

All representing relations between concepts introduced above are homogeneous in a sense that these are induced by single-type relations between homonymous constitutive components of the representing parts of concepts. For example, the relation of *a*-local R-intersection is induced by intersection of sets consisting from elements of the alphabet(s).

There are also inhomogeneous concept relations that are induced by heterogeneous set-theoretical relations between homonymous constitutive components of the representing parts of concepts. An example is the relation between the concept representing parts that is induced simultaneously by intersection of their sets of letters and the relation 'to be a subset' between their sets of sentences.

There are clusters of closely connected physical concepts. An example is the cluster of classical mechanics concepts. It includes concepts **MASS**, **SPATIAL COORDINATE**, **TEMPORAL COORDINATE**, **TRAJECTORY, VELOCITY**, **ACCELERATION**, **FORCE**, **TIME**, **SPACE**, **ENERGY**, etc. Nontrivial relations between these concepts are induced not by set-theoretical, but by specific mathematical relations between constitutive components of their representing parts. For instance, as physicists say, momentary velocity at a point is equal to derivative of spatial coordinate with respect to temporal coordinate.

In the framework of triplet modeling, one can explicate these concept relations in terms of composition of constitutive components of concept representing parts. In turn, various kinds of composition are useful in analysis of problems of concept combinations.

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