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THE TRIPLET MODELING OF CONCEPT CONNECTIONS[1]

1. INTRODUCTION

In the last three decades, concepts have been the focus of intensive attention of cognitive scientists and psychologists, philosophers of science, experts in computer science, terminology, and artificial intelligence. The results of their studies may be summarized as follows.

With a few exceptions, researchers have treated concepts as complicated and multifaceted entities studied by means of their models. There are now at least two classes of concept models. The first class deals with isolated concepts as well as with processes of their construction, recognition, and comprehension. Models of this class depict conjecturable aspects of concepts in a form of their internal structures. Experts (Komatsu, *Recent Views*) identify many model types: the classical, the family resemblance, the exemplar, the explanation-based views, etc. The second class deals with collections of concepts in a form of connectionist systems. Here various connections between concepts have expressed concept complexity (Goschke and Koppelberg, *The Concept*).

Up to this point, these classes of models have been developed independently of one another. Nevertheless, there is a possibility of mutual enrichment of one class in relation to another class. Indeed, internal structures of isolated concepts to some extent presuppose certain limitations on possible connections between concepts. Correspondingly, connections between concepts determine the internal structures that concepts should have.

Let us illustrate this intuition by a very simple example. Some researchers define a concept as a set of objects falling under it. In these terms, connections between concepts are modeled as set-theoretical relations between sets corresponding to concepts considered. In turn, starting from set-theoretical relations between concepts, we inevitably associate some sets with our model of concepts.

This paper has two objectives: to introduce informally a triplet model (TM) of concepts and to show the kinds of concept connections that are established as a result of TM.

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[1] The draft. The final version, see: **The Triplet Modeling of Concept Connections. *Philosophical Dimensions of Logic and Science // Selected Contributed Papers from the 11th International Congress of Logic, Methodology, and Philosophy and Science.* Ed. by A. Rojszczak, J. Cachro and G. Kurczewski, Dordrecht: Kluwer, 2003, 317-330.**

2. THE TRIPLET MODELING OF CONCEPTS

For the sake of simplicity, we consider only so-called object concepts like **GENE**, **HORSE, STAR**, and **ATOM**, i.e. concepts about **genes**, **horses**, **stars**, and **atoms**. As an illustration, we will use the concept **ELEMENTARY PARTICLE** (in short **PARTICLE**) and concepts associated with it.

TM (Kuznetsov) characterizes an object concept **C** by three interrelated kinds of information. The first deals with objects falling under a concept **C**. This information is organized around the ontological hypotheses on what a concept is about. It is a so-called concept base ***B***(**C**). The base includes also the information on attributes of objects, attributes of attributes of objects, connections between attributes, etc. Examples of attributes are those of the isolated objects (their internal structures, qualitative and quantitative properties, functions, processes, etc.), binary connections between objects and so on.

The second kind of information has focused on representing structures that depict objects and their attributes in some intelligent or information storage system. This kind has organized according to the rules, resources and history of representative and communicative systems, primarily natural and artificial languages, and knowledge systems. It is a so-called concept representing part ***R***(**C**) that includes names, descriptions, definitions, images, models, etc. of elements from ***B***(**C**).

The third kind of information concerns on the correspondences between objects (and their attributes) and appropriate representing structures. This information – a so-called concept link ***L***(**C**) – has expressed the relationships between the concept base ***B***(**C**) and the concept representing part ***R***(**C**). Typically, these relationships are not simple one-to-one correspondences between the elements of the former and the structures of the latter. Many peculiar operations and processes (of naming, modeling, interpretation, observation, measurement, etc.) contributed to generating these correspondences. All this information is vitally important for formation and use of any concept.

The hypothetical triplet structure (***B***(**C**)**,** ***L***(**C**)**,** ***R***(**C**)) and its substructures can be described by means of various set-theoretical constructions. In these terms, it is possible to speak of partial and holistic kinds of connections between two concepts. Let **C** and **C*** be concepts, (***B***(**C**)**,** ***L***(**C**)**,** ***R***(**C**)) and (***B***(**C***)**,** ***L***(**C***)**,** ***R***(**C***)), correspondingly, their triplet models.

The partial kind of concept connection associates with relations between substructures of triplet structures. Examples are relations between ***B***(**C**) and ***B***(**C***) or between ***R***(**C**) and ***B***(**C***).

The holistic kind associates with relations between triplet structures (***B***(**C**)**,** ***L***(**C**)**,** ***R***(**C**)) and (***B***(**C***)**,** ***L***(**C***)**,** ***R***(**C***)).

Concept holistic connections may be depicted in terms of diagrams like the following one.

$$\begin{array}{ccc} & \alpha & \\ \boldsymbol{R}(\mathbf{C}) & \rightarrow & \boldsymbol{R}(\mathbf{C^*}) \\ \boldsymbol{L}(\mathbf{C})\uparrow & & \uparrow\boldsymbol{L}(\mathbf{C^*}) \\ \boldsymbol{B}(\mathbf{C}) & \rightarrow & \boldsymbol{B}(\mathbf{C^*}) \end{array}$$

β

Various relations between substructures of triplet structures of **C** and **C*** induce many holistic concept connections (**α, β**). Experts have investigated only a few of them. There are some restrictions on the bases, links and representing parts of meaningfully connected concepts. Communicativity **(α*L*(C) = *L*(C*)β)** of the diagram induces some of these restrictions.

In this paper, we restrict our attention to partial connections between concepts induced by relations between their bases. These will be called BB*-connections or, in short, B-connections.

3. OBJECTS, ATTRIBUTES AND THE CONCEPT BASE

To characterize B-connections, we need a detailed description of the concept base. We will consider a situation in which an intelligent system possesses object concepts. It means that such a system has certain ontological hypotheses containing information on some discriminating mark(s) (*differentia)* of objects in question and the collection of other attributes of these objects. Assume that at present knowledge this information describes objects and their attributes as they are. This intuition may be formalized in the following way.

We introduce the ground set ***G***(**C**) of objects falling under a concept **C** and consider it as constituted by discriminating mark(s) of these objects. Additionally, we introduce the collection ***A***(**C**) of their other attributes. In these terms, the concept base ***B***(**C**) has a form < ***G***(**C**), ***A***(**C**) >.

Let us relate these constructions and terminology to the familiar modeling of a concept in terms of its extension and content. According to the extension-content model, extension is identified with the set of objects falling under a concept and content is identified with attributes of objects from this set. At first glance, there is no difference between the extension-content model and the concept base.

The difference appears if we take into account the existence of, first, attributes of various orders in relation to ***G***(**C**) and, second, relevant connections between objects and between attributes. The necessity of considering the appropriate information is evident for any nontrivial scientific concept. Obviously, a physicist lacks the "contemporary" concept **PARTICLE** if he or she is not able to characterize physical properties of particles, relationships between properties, the "nature" of properties and relationships, etc. With the progress of high-energy physics the concept **PARTICLE** has been a subject of substantial shifts.

The set-theoretical construction of abstract property (Burgin, *Abstract Theory*; Burgin and Kuznetsov, *Properties*) is useful in explicating the notion of an attribute. We will consider only constructions needed for the simplest triplet modeling of a concept base.

Informally, an (one-place) attribute $\boldsymbol{a}^{1}(\boldsymbol{G})$ of the first order in relation to the set $\boldsymbol{G}$ is a triple $(\boldsymbol{G}, p^{1}, L^{1})$. Here L^{1} is the scale of $\boldsymbol{a}^{1}$, i.e. set of attribute values, and p^{1} is a procedure of corresponding (measurement, calculation, etc.) elements from $\boldsymbol{G}$ to elements from L^{1}.

Let us illustrate this for the concept **PARTICLE.** Its ground set ***G*****(PARTICLE)** consists of tiny pieces of matter that (according contemporary physics) possess

mass, ***electric charge***, ***isotopic spin***, ***lepton charge***, among other attributes. All of these are attributes of the first order in relation to particles.

A (one-place) attribute $\boldsymbol{a}^{\mathbf{2}}(\mathbf{C})$ of the second order in relation to the set $\boldsymbol{G}$ is a (one-place) attribute of the first order in relation to (one-place) attribute of the first order in relation to the set $\boldsymbol{G}$. One of the second order attributes in relation to the set of particles and one of the first order attributes in relation to ***mass*** is its ***positiveness***. Commonly, but not always, the second order a^2-attributes for scientific concepts have more abstract "nature’, than the first order a^1-attributes.

An object connection (or a two-place attribute) $\boldsymbol{con}^{\mathbf{1}}(\mathbf{C})$ of the first order in relation to the set $\boldsymbol{G}$ is a triple ($\boldsymbol{G} \otimes \boldsymbol{G}$, $con^{\mathbf{1}}$, $M^{\mathbf{1}}$). Here $\otimes$ is the direct product of sets, $M^{\mathbf{1}}$ is the scale of $\boldsymbol{con}^{\mathbf{1}}$, i.e. set of connection values, and $con^{\mathbf{1}}$ is a procedure of corresponding elements from $\boldsymbol{G} \otimes \boldsymbol{G}$ to elements from $M^{\mathbf{1}}$.

One of the object connections of the first order in relation to the set of particles is gravitational force between particles.

A m-place attributive connection $\boldsymbol{att}^{\mathbf{1}}(\mathbf{C})$ of the first order in relation to the set $\boldsymbol{A}^{\mathbf{1}}$ of the attributes of the first order in relation to $\boldsymbol{G}$ is a triple ($\boldsymbol{A}^{\mathbf{1}} \otimes_1 \dots \otimes_m \boldsymbol{A}^{\mathbf{1}}$, $att^{\mathbf{1}}$, $Q^{\mathbf{1}}$). Here $Q^{\mathbf{1}}$ is the scale of $\boldsymbol{att}^{\mathbf{1}}$, i.e. set of attributive connection values, and $att^{\mathbf{1}}$ is a procedure of corresponding m-tuples from $\boldsymbol{A}^{\mathbf{1}} \otimes_1 \dots \otimes_m \boldsymbol{A}^{\mathbf{1}}$ to elements from $Q^{\mathbf{1}}$. Here and in what follows $\boldsymbol{A}$ symbolizes a set $\{\boldsymbol{a}_{\mathbf{1}}, \boldsymbol{a}_{\mathbf{2}}, \dots \}$

A typical attributive connection for the set of elementary particles is a pattern (regularity) connecting some their attributes. Examples are the relationship between such first order attributes of hadrons (subsets of particles) as ***hypercharge***, ***charm***, ***beauty***, ***baryon number*** and ***strangeness***, and the famous second law of Newton establishing the fixed link between ***mass***, ***force***, and ***acceleration***.

It is possible to introduce compositions of various kinds for any constructions mentioned.

In the light of these considerations, the concept base $\boldsymbol{B}(\mathbf{C})$ has a form <$\boldsymbol{G}(\mathbf{C})$, $\boldsymbol{A}^{\mathbf{1}}(\mathbf{C})$, $\boldsymbol{A}^{\mathbf{2}}(\mathbf{C})$, $\boldsymbol{Con}^{\mathbf{1}}(\mathbf{C})$, $\boldsymbol{Att}^{\mathbf{1}}(\mathbf{C})$>.

As its partial cases the base includes the pieces of information on concept extension (or $\boldsymbol{G}(\mathbf{C})$) and content (usually some subset of the set $\boldsymbol{A}^{\mathbf{1}}(\mathbf{C}) \cup \boldsymbol{A}^{\mathbf{2}}(\mathbf{C})$).

As a rule, researchers restrict their consideration of concepts to attributes of the first order, i.e. attributes of the objects from $\boldsymbol{G}(\mathbf{C})$. However, for most scientific concepts such a restriction is too narrow. Their treatment has to take into account attributes of these attributes of the first order that have the second order in relation to objects. Examples are ***positiveness of mass*** and ***discreteness of electric charge***. Particles interact and an important attribute of their interaction (particle connection) is its strength. Some theories of microphysics operate with four values of interaction strength: gravitational, weak, electromagnetic, and strong. Many object and attributive connections of various orders should be explained by future theories of particles.

It seems that considerations introduced demonstrate the relevance of the construction of the concept base. Moreover, these show also that the modeling of concepts has always a relative character. It is dependent on our ontological hypotheses, on our knowledge and understanding of these hypotheses, on depth and detail of model we wish to build, on the problem we intend to solve with the help of the model and so forth. In the end, to have a working model, we should decide which constructions associated with the concept base must be included in this model.

In a sum, the information about higher than first order attributes is an essential and integral part of our information about any scientific concept.

4. B-CONNECTIONS BETWEEN CONCEPTS

4.1. Two Interpretations of B-Connections

Let us consider two concepts **C** and **C*** with the bases $\boldsymbol{B}(\mathbf{C}) = <\boldsymbol{G}(\mathbf{C}), \boldsymbol{A}^{1}(\mathbf{C}), \boldsymbol{A}^{2}(\mathbf{C}), \boldsymbol{Con}^{1}(\mathbf{C}), \boldsymbol{Att}^{1}(\mathbf{C})>$ and $\boldsymbol{B}(\mathbf{C^*}) = <\boldsymbol{G}(\mathbf{C^*}), \boldsymbol{A}^{1}(\mathbf{C^*}), \boldsymbol{A}^{2}(\mathbf{C^*}), \boldsymbol{Con}^{1}(\mathbf{C^*}), \boldsymbol{Att}^{1}(\mathbf{C^*})>$.

B-connections between concepts may be interpreted in two ways. A so-called god's eye-view assumes that modeling a concept we possess full, perfect, final and unchanged knowledge about its bases. In triplet terms, this means that we are able to fix once and for all set-theoretical structures from the concept base. The god's eye-view goes back to Plato with his world of ideas. Advocates of logical models of concepts have *de facto* accepted this view.

However, humans do not possess absolute knowledge about almost all concepts. Trivially scientific knowledge is a subject of change. A so-called human's eye-view has given proper weight to this fact. In triplet terms, this means that we allow changes in set-theoretical structures from the concept base. For example, the number of elements in sets may increase or elements may transform their nature. In what follows, we will use the human's eye-view of concepts.

4.2. Qualitative and Quantitative Concepts

We need the two definitions.

Definition 1. The concept **C** is an a-dichotomous qualitative (in short, a-qualitative) if the information associated with the attribute ***a*** fixes only its presence or absence.

The concept **PARTICLE** is strong interaction-dichotomous qualitative if the information on such a two-place attribute as interaction between particles fixes only existence or nonexistence of this interaction type between particles.

The ground set of the hadron-qualitative concept **HADRON** includes only particles that take part in strong interaction.

Definition 2. The concept **C** is an a-dichotomous quantitative (in short, a-quantitative) if the information associated with the attribute ***a*** fixes not only its presence or absence, but also values (quantities) of the attribute in the case of its presence.

Particles with half-integer values of spin constitute the ground set of the spin-quantitative concept **FERMION** and particles with integer values constitute the ground set of the spin-quantitative concept **BOSON**. The concept **PARTICLE** is spin-dichotomous quantitative one if the information associated with such a one-place attribute as spin of particle fixes not only that any particle has spin, but also spin values (quantities) for each particle.

Many concepts at every stage of their growth are simultaneously a^1-qualitative and a^2-quantitative where $\mathbf{a^1}$ and $\mathbf{a^2}$ are some of their attributes.

4.3. Types of B-identity

Let us start from types of such a B-connection as a B-identity.

Definition 3. The concept **C** is totally B-equal to the concept **C*** if and only if $\boldsymbol{G}(\mathbf{C}) = \boldsymbol{G}(\mathbf{C^*})$, $\boldsymbol{A}^1(\mathbf{C}) = \boldsymbol{A}^1(\mathbf{C^*})$, $\boldsymbol{A}^2(\mathbf{C}) = \boldsymbol{A}^2(\mathbf{C^*})$, $\boldsymbol{Con}^1(\mathbf{C}) = \boldsymbol{Con}^1(\mathbf{C^*})$, $\boldsymbol{Att}^1(\mathbf{C}) = \boldsymbol{Att}^1(\mathbf{C^*})$.

Proposition 1. The concept **C** is totally B-equal to itself.

Another example of totally B-equal concepts relies on the use of different natural languages for naming a concept and consequently triplet structures associated with it. Holistically (for instance, when their representing parts have been taken into account) these B-equal concepts are not equal.

In the case of B-changes a concept at some stage is not totally B-equal to "the same" concept at other stage. The reason is that there should be changes, at least, in one of the components of **B**. For instance, quantification of first order attributes is usually the outcome of complex and time-consuming theoretical and experimental activity. As the result of this, the qualitative concept **C** is not totally B-equal to the corresponding quantitative concept **C*** because, at least, $\boldsymbol{A}^1(\mathbf{C}) \neq \boldsymbol{A}^1(\mathbf{C^*})$.

Definition 4. The concept **C** is G-partially B-equal to the concept **C*** if and only if $\boldsymbol{G}(\mathbf{C}) = \boldsymbol{G}(\mathbf{C^*})$.

Proposition 2. The concept **C** is G-partially B-equal to itself.

Proposition 3. If the concept **C** is totally B-equal to the concept **C***, then **C** is G-partially B-equal to **C***.

Proposition 4. The quantitative concept is G-partially B-equal to the corresponding qualitative concept if there is no difference in their ground sets.

Definition 5. The concept **C** is A^1-partially B-equal to the concept **C*** if and only if $\boldsymbol{A}^1(\mathbf{C}) = \boldsymbol{A}^1(\mathbf{C^*})$.

Proposition 6. The concept **C** is A^1-partially B-equal to itself.

Proposition 7. If the concept **C** is totally B-equal to the concept **C***, then **C** is A^1-partially B-equal to **C***.

From G-partially B-equality of concepts does not follow their A^1-partially B-equality. It is typical for an ordinary (common) concept and its scientific counterparts. For example, for many educated people the ground set of the concept **PLANET OF SOLAR SYSTEM** is equal to the ground set of corresponding concept of experts in planetary physics. However, their concepts are not A^1-partially B-equal, because experts operate with many additional attributes of planets that, as a rule, are unknown for laymen.

In many cases, A^1-partially B-equality of concepts may lead after some transformations of their bases to their unification in some new concept.

For example, before classical physics scientists operated with two different concepts: **CELESTIAL BODY** and **EARTHLY BODY**. The ground set of the former included the Sun, planets, stars and comets, while the ground set of the latter contained objects on the Earth. Establishing the A^1-partially B-equality of their mechanical attributes (***mass***, ***position***, ***force***, ***acceleration***, etc.) led to an appearance of the concept **MACROBODY**. Its ground set is the union of the former and the latter ground sets and its attributes are mechanical by nature. Moreover, celestial and earthy bodies obey the same mechanical laws.

Definition 6. The concept **C** is A^2-partially B-equal to the concept **C*** if and only if $\boldsymbol{A^2}(\mathbf{C}) = \boldsymbol{A^2}(\mathbf{C^*})$.

Proposition 8. The concept **C** is A^2-partially B-equal to itself.

Proposition 9. If the concept **C** is totally B-equal to the concept **C***, then **C** is A^2-partially B-equal to **C***.

Definition 7. The concept **C** is Con^1-partially B-equal to the concept **C*** if and only if $\boldsymbol{Con^1}(\mathbf{C}) = \boldsymbol{Con^1}(\mathbf{C^*})$.

Proposition 10. The concept **C** is Con^1-partially B-equal to itself.

Proposition 11. If the concept **C** is totally B-equal to the concept **C***, then **C** is Con^1-partially B-equal to **C***.

Contrary to all differences between such classes of elementary particles as leptons and baryons, these take part in gravitational interaction. Owing to this, the concepts **LEPTON** and **BARYON** are gravitation-partially B-equal.

Definition 8. The concept **C** is Att^1-partially B-equal to the concept **C*** if and only if $\boldsymbol{Att^1}(\mathbf{C}) = \boldsymbol{Att^1}(\mathbf{C^*})$.

Proposition 12. The concept **C** is Att^1-partially B-equal to itself.

Proposition 13. If the concept **C** is totally B-equal to the concept **C***, then **C** is Att^1-partially B-equal to **C***.

According to the contemporary physical worldview, all states of an elementary particle are subject of restrictions established by various principles of symmetry and supersymmetry. Some of these principles establish identity of certain attributes of particles before and after admissible particle transformations. Such principles are partial cases of two-place attributive connections. In this sense, concepts **LEPTON** and **BARYON** are Att^1-partially B-equal where Att^1, for example, may be substituted by principle of electrical charge conservation.

4.4. Types of B-disjointness

Let us consider concept connections induced by the relation of mutual disjointness between sets. In some sense, these connections are opposite to B-identity connections.

Two sets A and B are mutually disjoint if and only if $A \cap B = \varnothing$ where $\cap$ is a class intersection.

Definition 9. The concepts **C** and **C*** are totally B-disjoint if and only if $\boldsymbol{G}(\mathbf{C}) \cap \boldsymbol{G}(\mathbf{C^*}) = \varnothing$, $\boldsymbol{A^1}(\mathbf{C}) \cap \boldsymbol{A^1}(\mathbf{C^*}) = \varnothing$, $\boldsymbol{A^2}(\mathbf{C}) \cap \boldsymbol{A^2}(\mathbf{C^*}) = \varnothing$, $\boldsymbol{Con^1}(\mathbf{C}) \cap \boldsymbol{Con^1}(\mathbf{C^*}) = \varnothing$, $\boldsymbol{Att^1}(\mathbf{C}) \cap \boldsymbol{Att^1}(\mathbf{C^*}) = \varnothing$.

The concepts **PARTICLE** and **PLATONIC IDEA** are totally B-disjoint.

Definition 10. The concepts **C** and **C*** are G-partially B-disjoint if and only if $\boldsymbol{G}(\mathbf{C}) \cap \boldsymbol{G}(\mathbf{C^*}) = \varnothing$.

Proposition 14. If the concepts **C** and **C*** are totally B-disjoint, then they are G-partially B-disjoint.

It seems that common concept intuition prevents the situation in which G-partially B-disjoint concepts are A^1-partially B-equal. Informally, it means that two disjoint sets (which do not contain identical elements) have the same attributes. However, it is possible to give counterexample to this intuition.

Let ***U*** be a set of all elementary particles with the exception of the set of so-called true neutral elementary particles. The set ***U*** is the union of two disjoint sets: the set of particles and the set of antiparticles. Under these conditions, the concepts **PARTICLE** and **ANTIPARTICLE** are G-partially B-disjoint. Nevertheless, these concepts are A^1-partially B-equal if under their attributes $\boldsymbol{A^1}$ of the first order we take their so-called quantum numbers describing elementary particles interactions and do not take into account the difference in sign of appropriate quantum numbers values.

Definition 11. The concepts **C** and **C*** are A^1-partially B-disjoint if and only if $\boldsymbol{A^1}(\mathbf{C}) \cap \boldsymbol{A^1}(\mathbf{C^*}) = \varnothing$.

Proposition 15. If the concepts **C** and **C*** are totally B-disjoint, then they are A^1-partially B-disjoint.

At early developmental stage of quantum physics, the concepts **WAVE** and **CORPUSCLE** appeared to be A^1-partially B-disjoint. The reason was that some attributes of waves and corpuscles were understood as disjoint. Moreover, at this stage the ground sets of these concepts were interpreted as identical and equal to the set of quantum objects.

4.5. Types of B-intersection

Let us consider some concept connections induced by the relation of non-void class intersection between sets: $A \cap B \neq \varnothing$.

Definition 12. The concepts **C** and **C*** are relatively total B-joint if and only if $\boldsymbol{G}(\mathbf{C}) \cap \boldsymbol{G}(\mathbf{C^*}) \neq \varnothing$, $\boldsymbol{A^1}(\mathbf{C}) \cap \boldsymbol{A^1}(\mathbf{C^*}) \neq \varnothing$, $\boldsymbol{A^2}(\mathbf{C}) \cap \boldsymbol{A^2}(\mathbf{C^*}) \neq \varnothing$, $\boldsymbol{Con^1}(\mathbf{C}) \cap \boldsymbol{Con^1}(\mathbf{C^*}) \neq \varnothing$, $\boldsymbol{Att^1}(\mathbf{C}) \cap \boldsymbol{Att^1}(\mathbf{C^*}) \neq \varnothing$.

The concepts **CLASSICAL OBJECT** and **QUANTUM OBJECT** are relatively total B-joint. The first reason is that some objects belong simultaneously to the set of classical objects and to the set of quantum objects (for instance, superfluid liquid). The second reason is that classical and quantum objects share certain common attributes (for instance, mass, its positiveness, gravitational interaction and the famous relation between energy and mass).

Proposition 16. The concept **C** is relatively total B-joint with itself.

Proposition 17. The relation of relatively total B-jointness is not transitive.

Proposition 18. The relation of relatively total B-jointness is symmetrical.

Analogously, it is possible to consider relatively partial B-joint concepts for which only one of the conditions from *Definition 12* is valid.

4.6. Types of B-inclusion

Let us consider briefly B-connections between concepts induced by set-theoretical relation of class inclusion $\subseteq$ valid between the corresponding components of the concept bases. Inclusive B-connections may be called also B-inclusions between concepts.

Definition 13. The concept **C*** is a total B-inclusive generalization of the concept **C** (or the concept **C** is a total B-inclusive specification of the concept **C***) if and only if $\boldsymbol{G}(\mathbf{C}) \subseteq \boldsymbol{G}(\mathbf{C}^*)$, $\boldsymbol{A}^{\mathbf{1}}(\mathbf{C}) \subseteq \boldsymbol{A}^{\mathbf{1}}(\mathbf{C}^*)$, $\boldsymbol{A}^{2}(\mathbf{C}) \subseteq \boldsymbol{A}^{2}(\mathbf{C}^*)$, $\boldsymbol{Con}^{\mathbf{1}}(\mathbf{C}) \subseteq \boldsymbol{Con}^{\mathbf{1}}(\mathbf{C}^*)$, $\boldsymbol{Att}^{\mathbf{1}}(\mathbf{C}) \subseteq \boldsymbol{Att}^{\mathbf{1}}(\mathbf{C}^*)$.

Proposition 19. The concept **C** is a total B-inclusive generalization and a total B-inclusive specification of itself.

Proposition 20. The relation of a total B-inclusive generalization (specification) is transitive, i.e. if **C**** is a total B-inclusive generalization (specification) of **C*** and **C*** is a B-total generalization (specification) of **C**, then **C**** is a total B-inclusive generalization (specification) of **C**.

Proposition 21. If the concept **C*** is a total B-inclusive generalization of the concept **C**, then the concept **C*** and **C** are relatively total B-joint.

Many examples of the total B-inclusive generalization (specification) of concepts are connected with a situation of a so-called incremental growth of knowledge.

The construction of a total B-inclusive generalization offers an explication of the notion of the "sameness" of a concept. Indeed, speaking about progressive changes of the "same" concept, we suppose that at moment t_2 the concept should be total B-inclusive generalization of "this" concept at moment t_1 ($t_2 \geq t_1$). The "development" of the concepts **PARTICLE** and **NUMBER** provide many nontrivial examples of their total B-inclusive generalization.

There are many partial B-inclusive generalizations of concepts.

Definition 14. The concept **C*** is a G-partially B-inclusive generalization (in short, G-generalization) of the concept **C** if and only if $\boldsymbol{G}(\mathbf{C}) \subseteq \boldsymbol{G}(\mathbf{C}^*)$.

Proposition 22. The concept **C** is a G-generalization of itself.

Proposition 23. If the concept **C*** is a total B-inclusive generalization of the concept **C**, then the concept **C*** is a G-generalization of the concept **C**

Proposition 24. The relation of G-generalization is transitive, i.e. if **C**** is G-generalization of **C*** and **C*** is G-generalization of **C**, then **C**** is G-generalization of **C**.

In a similar manner, it is possible to introduce the notions of A^1-(A^2-, Con^1-, Att^1-) partial B-inclusive generalization.

4.7. Some types of "mixed" B-connections

It is interesting to employ some meaningful combinations of conceptual relations introduced above for the case of the concept growth.

Let $\mathbf{C_1}$ and $\mathbf{C_2}$ be the different stages of the "same" concept **C** at the moment t_1 and t_2 ($t_2 \geq t_1$).

Definition 15. The concept $\mathbf{C_2}$ is a G-conservative additive A^1-extension (A^1-restriction) of the concept $\mathbf{C_1}$ if and only if $\boldsymbol{G}(\mathbf{C_1}) = \boldsymbol{G}(\mathbf{C_2})$ and $\boldsymbol{A_1}^{\mathbf{1}} \subseteq \boldsymbol{A_2}^{\mathbf{1}}$ ($\boldsymbol{A_2}^{\mathbf{1}} \subseteq \boldsymbol{A_1}^{\mathbf{1}}$).

Informally, it means that in the course of concept growth at least one new attribute sets in to be associated (ceases to be associated) with the same concept ground set. For example, the G-conservative additive spin-extension of the concept **PAR-**

TICLE took place in 1925 when the notion about such a quantum attribute of particles as ***spin*** was introduced.

Definition 16. The concept $\mathbf{C_2}$ is a G-conservative A^1-incommensurable partner of the concept $\mathbf{C_1}$ if and only if $\boldsymbol{G}(\mathbf{C_1}) = \boldsymbol{G}(\mathbf{C_2})$ and $\boldsymbol{A_1^1} \cap \boldsymbol{A_2^1} = \varnothing$.

Contrary to the prevailing opinion followed from a so-called incommensurability thesis, real examples of G-conservative A^1-incommensurability of concepts are few in number. In the early stages of microphysics, the corpuscle-like attributes of particles were interpreted as incommensurable with the wave-like attributes. It means that physicists of that time operated with G-conservative attribute-incommensurable concepts of particles. This interpretation was abandoned after an appearance of the notion of quantum-mechanical dualism.

Proposition 25. If the concept $\mathbf{C_2}$ is a G-conservative A^1-incommensurable partner of the concept $\mathbf{C_1}$, then the concept $\mathbf{C_1}$ is a G-conservative A^2-incommensurable partner of the concept $\mathbf{C_2}$.

Definition 17. The concept $\mathbf{C_2}$ is an A-conservative G-incommensurable partner of the concept $\mathbf{C_1}$ if and only if $\boldsymbol{A_1^1} = \boldsymbol{A_2^1}$ and $\boldsymbol{G}(\mathbf{C_1}) \cap \boldsymbol{G}(\mathbf{C_2}) = \varnothing$.

According to many-worlds interpretation of quantum mechanics, many identical «parallel» physical worlds do not physically interact with each other. If one interprets this “fact” as disjointness of sets of particles from such universes, then the concepts of particles associated with these worlds are A-conservative G-incommensurable with each other.

Definition 18. The concept $\mathbf{C_2}$ is an A-conservative G-extension (G-restriction) of the concept $\mathbf{C_1}$ if and only if $\boldsymbol{G}(\mathbf{C_1}) \subseteq \boldsymbol{G}(\mathbf{C_2})$ ($\boldsymbol{G}(\mathbf{C_2}) \subseteq \boldsymbol{G}(\mathbf{C_1})$) and $\boldsymbol{A_1^1} = \boldsymbol{A_2^1}$.

By means of the notion of A-conservative progressive G-extension, it is possible to explicate changes in the concept **PARTICLE** associated with a new particle prediction and discovery. Sometimes, the new particle has been predicted on the ground of its hypothetical possession of the attributes of previously known particles.

It is possible to introduce other types of B-connections between concepts and find plausible interpretations of these types as well as to formulate propositions about links between these types.

5. SOME PERSPECTIVES

The triplet modeling of concept connections provides important insights into the study of concept change, concept elaboration, concept formation, etc. It significantly extends tools for concept analysis.

There are, at least, three directions of the further development of the triplet modeling of concept connections. The first one is an analysis of concept connections induced by relations of compositionality between the base substructures of concepts connected. The second one is a consideration of concept connections taking into account not only the concept base, but also the concept representing part and the concept link. The third one is a modeling of concept connections by means of various generalizations of ordinary set theory (theories of multisets, theories of fuzzy sets and theory of named sets (Burgin)).

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6. NOTE

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