Notes on what it would take to understand how one adverb works*

George Lakoff
Department of Linguistics
University of California,
Berkeley

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A natural language is a unified and integrated system, and the serious study of one part of the system inevitably involves one in the study of many other parts, if not the system as a whole. For this reason, the study of small, isolated fragments of a language--however necessary, valuable and difficult this may be -- will often make us think that we understand more than we really do. The fact is that you can't really study one phenomenon adequately without studying a great many other related phenomena, and the way they fit together in terms of the linguistic system as a whole. This is the sort of thing a linguist learns very early in his career. Experience in descriptive linguistics, even at an elementary level will force a linguist to come to grips with a wide range of complex data in some language, perhaps even English, and the truism soon emerges. But, due to the vagaries of our educational institutions, few philosophers or logicians receive training in linguistic description. Consequently much of the discussion of natural language in the philosophical and logical literature is based on a very small sampling of data which is skewed in nontrivial ways. True, one has to start somewhere, and a great deal has been learned by ordinary language philosophers who have looked at only a handful of relatively simple examples and by logicians who have studied what by natural language standards are only miniscule fragments (e.g., first-order predicate calculus, the various modal logics, etc.). But now that philosophers and logicians are turning to more detailed studies of natural language phenomena, it is perhaps the right time to suggest that philosophical and logical training be expanded to include the study of natural languages as entire systems. I don't mean to suggest, for example, that logicians should stop their systematic study of small fragments, but rather that a knowledge of the kinds of phenomena outside of those fragments can enrich the study of fragments and give one a more realistic picture of what one does and does not know about natural language.

The study of adverbs is a good case in point. Reichenbach,[1] in his analysis of conversational language, made a brave attempt to study a number of natural language phenomena that had previously been ignored

by logicians, including adverbs of a limited sort. No further work in this area was done for nearly two decades until Davidson[2] came up with an analysis of adverbs in support of his view of the analysis of events, and at about the same time Montague and Parsons[3], working independently, came to the conclusion that the logical properties of certain adverbs could be accounted for using the techniques of modal logic. This work has since been elaborated upon by Clark, Thomason, Stalnaker, Harman, Grandy, and others.[4] Two traditions have emerged, one (the Davidson tradition) treating adverbs as predicates of event variables and the other (the Montague tradition) treating them as predicate modifiers, that is, functions mapping propositional functions into propositional functions. Let us look at some phenomena relevant to both of these traditions.

One of the adverbs most frequently cited in both of the above-mentioned traditions is slowly. Perhaps the first thing a linguist would notice about slowly is that it is derivationally related to the adjective slow. Among the first questions a linguist would ask would be: Is the relation between slow and slowly systematic, and if so, what is the relation? Of those who have discussed slowly in recent philosophical work, only Harman[5] has tried to account for the relation between slowly and slow. Harman, noting that slow, like large, is a relative modifier, offers a Davidsonian analysis of slow as a relation between an event and a class of events, namely, that given in (1), which is Harman's proposed logical form for John walked slowly.[6]

(1) (Ee) (John walks in e and e is past and e is slow for (the class each member of which is) a walk by a man of John's age).

But now suppose one asks further questions of the sort a linguist would be likely to ask -- for example: What kinds of sentences can slow and slowly occur in? What do they mean in those sentences? Are there sentences with slow that are paraphrases or near-paraphrases of sentences with slowly? Consider examples like the following:

(2) a. This country changes slowly.

b. The rate at which this country changes is slow.

c. Change in this country is slow.

(2b) is a paraphrase of both (2a) and (2c). In (2b), slow is overtly predicated of a rate of change over time. Since (2b) means the same thing as (2a) and (2c), they too would seem to involve a predication of a rate of change over time. But rates of change over time are not events, and so it would seem that Harman's analysis is incorrect in this respect. This is supported by the nonsensicalness of sentences like (3).

(3) *That event was slow.

Slow may be predicated of rates of change over time, but not of events.

Such examples not only show the inadequacy of the Harman and Davidson analyses in terms of events, but they also show that the Montague style analysis is also inadequate. Simply saying that slowly maps predicates into predicates does not account for the relation between slowly and slow, nor does it account for the fact that sentences with slowly involve a rate of change over time.[7] In the mappings from predicates to predicates, such rates are not specified in any way. Moreover, the Montague-style analysis, which treats slowly as an operator, cannot handle cases which require quantification over action processes. The following syllogism is an example.

(4) a. John does everything that requires effort slowly.

b. Running requires effort.

c. Therefore, John runs slowly.

This would seem to suggest that a Harman-Davidson style analysis using quantification over rates rather than events might be made to work. Thus one might try to restate Harman's analysis in (1) in terms of rates rather than events.

(1') (Er) (John walks at rate r and r is slow relative to the class of rates at which a man of John's age walks)

But this too has problems. First, if one worries about the kind of ontological problems that usually concern Davidson and Harman, one might feel a little queasy about quantifying over rates, since if one accepts Quine's views about being commited to the real-world existence of values of variables (which I do not accept), (1') would commit one to the existence of rates as entities. I somehow doubt that Davidson and Harman, who accept Quine's views in this matter, would like having rates as

entities. But that is the least of the difficulties with (1'). Note that (1') does not mention times; it contains no occurrence of a variable over times or of any time descriptions. Yet slow and slowly involve rate change with respect to time, and any adequate analysis must show how time is involved. This is required by syllogisms of the following sort.

(5) a. Men of John's age typically walk a mile in twenty minutes.
b. John walked that mile in ten minutes.
c. Therefore, for a man of his age, John did not walk slowly in walking that mile.

Given an analysis of sentences with slowly that does not mention times, there will be no way to account for inferences like those in (5). And even if one were to find a way to include a specification of a time variable, say, in a function d/dt specifying change with respect to time, there would still be the question of just what it is that is changing. In a sentence like John walks slowly, what is changing with respect to time is John's location. But in logical forms like (1) and (1'), there is no variable over locations, which is what is needed if one is to express that location is changing with respect to time. The problem here is less with the analysis of slowly than in the analysis of walks. To understand how slowly modifies walks, one must realize that slowly indicates a change of something with respect to time, and in the case of walking, that something is location.

But even this much is problematical. Consider running instead of walking. It is possible to run in place, that is, to move one's legs in a running fashion but to stay in the same location. One can run in place either quickly or slowly without changing location. What is changing then is not location but the position of one's legs. Now suppose that John is running, moving his legs very quickly, but taking very small steps. John may be moving slowly, but I don't think it would be appropriate to say that John is running slowly. Nor do I think it would be appropriate to say that he is running fast. Slowly, when modifying run seems to require both a relatively slow rate of location change and a relatively slow rate of leg movement. I

find this judgment somewhat subtle; but I think it is correct, though I would not be surprised to find more complexities.

When one asks the question of what is changing when slowly modifies other verbs, still more problems arise. Take the following cases:

(6) a. The stew is cooking slowly.
b. Sam thinks slowly.
c. Teddy arrived at that decision slowly.
d. The earth developed slowly.

What is changing with respect to time in (6a) is the degree to which the stew is 'done' (in cooking parlance). Letting 'c' stand for degree of doneness, it would seem that the logical form of (6a) would have to contain something like a function d/dt(c) indicating rate of change of degree of doneness with respect to time. If 'c' is a variable over degrees of doneness, and if 'c' occurs in the logical form of (6a), then according to the Quinean view, one would be committed to having degrees of doneness for stew as entities in one's ontology. I doubt that even Davidson would be happy about such entities. My guess is that in order to account for what is changing in (6a), one would probably need to have variables over degrees of doneness, and since there is no such real-world entity as a degree of doneness, so much the worse for Quine's views on ontological commitment.[8]

The question of what changes with respect to time is thornier in (6b). Thinking is a process -- an action process carried on by an individual. The rate understood in (6b) is the rate at which this process is carried on. If one thinks of processes as sequences of states, then the change going on in (6b) is a change from one state of mind to another state of mind, if one can speak sensibly of states of mind. If processes are not taken to be sequences of states, then the question arises as to just what processes are. (6c) is even trickier than (6b) since it involves that rate at which the process of reaching a decision proceeds. Reaching a decision presumably involves going through some sequence of mental operations, which might be viewed as changes of mental state, where a mental state must be either associated with or defined in terms of thoughts that one has in one's mind. An understanding of

what slowly modifies in (6c) requires an understanding of just what it is that is changing when one is reaching a decision. In (6d), the physical state of the earth is changing over a very long period of time. There is no conceivable way of analyzing (6d) in terms of a single event.

The examples in (6) are not untypical of sentences containing the adverb slowly. There are as many as there are sentences in English describing processes. To understand how slowly works requires an understanding of how it works in all such sentences. And that in turn requires an understanding of what is taken to be changing in all sentences describing processes, since the logical form of each sentence with slowly should, I think, reveal exactly what it is that is changing. In short, the study of sentences containing slow or slowly requires the simultaneous study of processes and expressions of rate.

The problems with slow and slowly do not, of course, end there.

(7) a. John left the party slowly.

b. The guests left the party slowly.

(7a) would be appropriate if there was some procedure involved in leaving the party (getting one's coat, saying goodbye to the host, etc.) and John went through that procedure slowly. (7b) however is ambiguous. There is a rather forced reading parallel to that of (7a), in which each guest went through the leaving procedure slowly. The more normal reading however, is that the number of people at the party decreased at a slow rate, though each individual guest may have left quickly. The thing changing over time in (7b) can be either the location of each of the guests with respect to the location of the party or the number of guests at the party. To account for the ambiguity of (7b), one must provide two different logical forms specifying just what it is that is changing in each case.

Certain sentences with slowly reveal a different sort of ambiguity,

(8) John answers questions slowly.

(8) has a forced reading in which the rate at which John speaks while answering questions is slow. But the more normal reading of (8) is that it takes an inordinate length of time for John to deliberate before he starts to answer a question. This is the reading one finds in (9).

(9) John is slow to answer questions.

(10) provides a similar example where slowly can mean slow to.

(10) a. John reacts slowly when you throw something at him.

b. John is slow to react when you throw something at him.

This seems to suggest that there are two senses of slow.

(11) a. normal slow -- involves rate of change over time during process

b. slow to -- involves amount of time before process starts

Obviously, these two senses of slow are not unrelated and one would like to understand the relation between them. I don't like the idea of saying that there are two different slowly's in John reacts slowly and John runs slowly, just as I would not want to say that there are two different concepts slow in John was slow to leave the party and John was slow in leaving the party, though these sentences mean rather different things. Hopefully there should be a single concept slow that will cover both cases, but I have no idea at present what it might be.

So far we have looked exclusively at the sort of problems with slowly that philosophers and logicians have not looked at. Let us now turn to matters that they have investigated. Of primary concern has been the inference from (12a) to (12b).

(12) a. John ran slowly.

b. John ran.

It is this that motivates the proposals of both Reichenbach and Davidson, and writers in the Montague tradition have paid special attention to this mode of inference. Since writers in both traditions have confined themselves to two-valued logic, none of them has noticed that (12a) not only entails (12b), but also bears to (12b) the even stronger relation of presupposition.

(12) a. John didn't run slowly.

b. Did John run slowly?

c. John may have run slowly.

(12b) is entailed by each of the sentences in (13). This suggests a further inadequacy of the Davidsonian proposal, which accounts for the inference from (12a) to (12b), by conjunction simplification as shown in (14)

(14) a. (Ee) (John runs in e and e is past and e is slow)

b. (Ee) (John runs in e and e is past)

The inference from (14a) to (14b) follows from first-order logic. Unfortunately for this proposal, the inferences from the sentences of (13) to (12b) cannot be accounted for in this way. Indeed the presuppositional nature of the relationship between (12a) and (12b) is inconsistent with a Davidson-style analysis, since presuppositions cannot be accounted for by conjunctions. However, if we adopted an analysis along the lines tentatively and rather vaguely outlined above, we can at least reduce the problem of why John ran slowly presupposes John ran to the problem of why the rate at which John ran was slow presupposes John ran. Note that the definite description The rate at which John ran itself presupposes that John ran. Thus, given an analysis of slow as a two-place relation between a rate and a class of rates we could presumably account automatically for the presupposition relation between (12a) and (12b) -- with the proviso, of course, that we had some adequate analysis of sentences and definite descriptions involving rates, which we do not have at the moment. Incidentally, it should be noted that if we look at the paraphrases of (13) where the rate descriptions are made explicit, it becomes somewhat clearer why those sentences presuppose (12b):

(14) a. The rate at which John ran was not slow.

b. Was the rate at which John ran slow?

c. The rate at which John ran may have been slow.

Such an analysis in terms of rates can also account for an observation made by Thomason to the effect that adverbs like slowly cannot modify negatives. Thus, sentences like (15) are nonsense.

(15) John slowly didn't run.

Now, negatives are stative in nature. Negated sentences do not describe changes or processes or activities. But rates make sense only when one is talking about changes or processes. Thus one cannot say the rate at which John didn't run for the same reason as one cannot say the rate at which John was tall or the rate at which John believed that Sam was a fink. (15) doesn't make sense because it is nonsensical to talk of the rate of a state.

Parsons has shown that there are additional reasons for not adopting a Davidson-style analysis. For example, the following sentences

should, on the Davidson analysis, have the same logical form, though they mean very different things and have different entailments.

(16) a. John wrote painstakingly and slowly.

b. John painstakingly wrote slowly.

(16a) entails that John wrote painstakingly while (16b) does not. The Davidson style analysis can handle cases like (16a) but not cases like (16b). For the latter, Parsons, like Montague, suggests that adverbs operate as predicate modifiers, mapping propositional functions into propositional functions. As we have already seen, there are problems with such an analysis of slowly. If we look at paraphrases, or near-paraphrases of (16a) and (16b), we can see more clearly why they differ.

(17) a. John took pains in writing and the rate at which he wrote was slow

b. John took pains so that the rate at which he wrote was slow.

Thomason, adopting the Montague-Parsons view of adverbs as predicate modifiers, claims that they have the following logical property:

(18) $f(g(p)) = g(f(p))$

As examples, he considers such sentences as (19).

(19) a. John hit Harry in the yard with a hammer.

b. John hit Harry with a hammer in the yard.

Though (18) works for adverbials like those in (19), it does not hold in general. In fact, given $f(g(p))$, $g(f(p))$ may not even make sense.

(20) a. John wisely wrote slowly.

b. *John slowly wrote wisely.

Again, one can get an idea of what is going on by looking at paraphrases.

(21) a. John was wise in that the rate at which he wrote was slow.

b. *The rate at which John was wise in writing was slow.

(21b) doesn't make sense because one cannot be wise at a rate, since being wise is not a process. (20b) fails to make sense for the same reason. Simply saying that adverbs are predicate modifiers doesn't account for such facts. Moreover, since some adverbs do have the property of (18) and others don't, one needs to account for why some do and others don't.

Since different adverbs have different logical properties, one has to study their logical properties adverb-by-adverb. Thomason, looking at adverbs like locatives, came to the conclusion that predicate modi-

fiers in general had the following logical properties:

(22) a. $f(p \wedge q) = f(p) \wedge f(q)$

b. $f(p \vee q) = f(p) \vee f(q)$

This works fine for locatives.

(23) a. John kissed Sally and pinched Mary in the yard.

b. John kissed Sally in the yard and John pinched Mary in the yard.

(24) a. John kissed Sally or pinched Mary in the yard.

b. John kissed Sally in the yard or John kissed Mary in the yard.

In each case the (a) and (b) sentences are logically equivalent. But this is not true of adverbs like slowly.

(25) a. Sam ran a mile slowly and climbed a mountain slowly and did ten pushups slowly.

b. John ran a mile, climbed a mountain, and did ten pushups slowly.

(25a) does not entail (25b). He may have done each of the tasks slowly, but may have done the combined task quickly. The point here is that slowly is a relative modifier. Thus, Sam's running of a mile may be slow for a mile run in itself, but fast for the mile run part of a combined task of running a mile, climbing a mountain, and doing ten pushups. Similarly, (25b) does not entail (25a). Sam may have done the combined task slowly, though he may have done one part of it, say running a mile, very quickly and the other parts especially slowly. Because of the relative nature of slowly, logical relations like those in (22) do not hold at all.

Opacity is another matter that is discussed in the literature on Montague-style analyses of adverbs. It is usually assumed, as Parsons and Thomason explicitly claim, that predicate modifiers are referentially transparent. Though this is true for adverbs like in the yard and slowly, it is not true for adverbs like willingly, readily, reluctantly, enthusiastically, etc.

(26) a. Oedipus married Jocasta willingly.

b. Oedipus married his mother willingly.

These sentences have opaque readings in which (26a) is true and (26b) false.

The general question of opacity with adverbs is a complex matter that I have discussed elsewhere, and there is no space to go into it here in detail. The point to be made is that there is no single simple generalization for all adverbs, or even for those that act as predicate modifiers. In the case of slowly, the lack of opacity follows from the fact that descriptions of rates are opaque. Thus, in sentences of the form "The rate at which _____ drove was slow", the substitution of identicals for the blank will not change truth values.

If slow is a two-place predicate that takes a rate and a set of rates as its arguments, that would seem to be incompatible with the Montague-Parsons idea that it is a predicate-modifier. But there is something right about that idea. Consider a sentence like:

(27) Everyone ran slowly.

The logical form of (27) contains a universal quantifier and a rate description. Is the quantifier inside or outside of the rate description, or both?

(28) a. For every person x (in the specified group), the rate r such that x ran at r was slow.

b. The rate r such that, for every person x (in the specified group) x ran at r, was slow.

(28a) seems to convey the correct reading of (27), while (28b) does not. The reason is that in (28b) every person in the group under discussion is described as running at the same rate r, and (27) by itself does not have that reading, at least not in my speech. (Note that (27), like (28a) does not preclude that possibility; the point is that there is no ambiguity such as one finds in sentences like "Every man in the office dated a pretty secretary" where there are two distinct readings.) Even a sentence like (29) has, at least in my speech, only the (28a) reading and not the (28b) reading.

(29) The rate at which everyone ran was slow.

Again the "every" must have wide scope. If my intuitions on this matter are not deceiving me, and if they hold in general, then it would seem that slow is predicated not simply of a rate description, but of a rate description with a variable-place open! Strange, but maybe true.

As Harman noted, slow, like large, is a relative concept. Just as many logicians have suggested that large be analyzed as large for a ..., Harman suggested that slow be analyzed as slow for a ... . Harman is assuming, as others have before him, that the "for a" construction is well-understood. But as usual, there are complications. As Fillmore[9] noted, certain occurrences of the "for a" construction entail (at least pragmatically) a negative expectation. For example, the following sentences would not be compliments:

(30) a. McGovern is honest for a politician.

b. Kate is smart for a woman.

These sentences respectively entail that politicians are not expected to be particularly honest and that women are not expected to be particularly smart.

Hence the oddness of (31).

(31) Nelson is rich for a Rockefeller.

which implies that one would not expect a Rockefeller to be rich.

As Fillmore observed, in the case of positive expectations one has to use "even for a".

(32) Nelson is rich even for a Rockefeller.

In addition to entailments of negative expectation, there is the matter of paraphrases. The following seems to me to be paraphrases, or near-paraphrases, and seem to be at least logically equivalent.

(33) a. Kate is smart for a woman.

b. Kate is smart considering that she's a woman.

c. Kate is smart if you consider that she's a woman.

d. Kate is smart if you take into consideration the fact that she's a woman.

Do all these sentences have the same logical form? If so what is it? Does it contain the verb consider? Does it contain an if-then connective? Is smart a two-place relation in all of these sentences? (Note the impossibility of "Kate is smart for a woman considering that she's a woman!")

If they are not all of the same logical form, what accounts for their similarity (if not identity) of meaning?

Then there is the matter of what the "for a" construction can occur with.

(34) a. Harvey knows a lot of facts for a philosopher.

b. *Harvey knows that fact for a philosopher.

Both these sentences contain the same verb "know". But, as (34b) shows, the "for a" construction cannot in general modify the verb "know", even though that is what it appears to be doing in (34a). Actually, the "for a" construction in (34a) is modifying "a lot", as the near-paraphrase in (35) makes clear.

(35) The number of facts that Harvey knows is a lot for a philosopher to know.

In general, it would seem that "for a" constructions can only occur with 'predicates of extent'. However, the notion 'predicate of extent' is not completely clear either. One would tend to think of a verb such as "like" as a predicate of extent, since it can be modified by "very much", "a great deal", etc. However, "for a" constructions cannot in general occur with "like".

(36) *Sam likes {Sheila / every girl} for a philosopher

However, it can occur with certain genericly quantified objects.

(37) Sam likes girls for a philosopher.

In short, "for a" phrases present a great many problems and cannot be used in the analysis of relative attributes as though they were well-understood. One cannot safely say that relative attributes are relations between an individual and a class, where the class is represented in surface structure by the for-phrase. The matter is not that simple. Nor can one simply claim that relative attributes are two-place relations between an individual and a class; there are complications there too. First, the individual must be a member of the class. Thus, we don't have sentences like:

(38) a. Philosophers are smart for jockeys.

b. My wife is smart for a man.

c. That dog is small for an elephant.

d. Gil walks slowly for a turtle.

Secondly, there is the problem of what class is understood when none is overtly expressed. Harman suggested that one possible way of understanding "John runs slowly" might be "John runs slowly for a person of his age". Of course, the understood class can vary tremendously from context to context. If John races, even against people of a different age, one can understand "John runs slowly" as being relative to those he competes against. Or if one is mentioning an individual occurrence of running as in "John ran slowly yesterday", one might be comparing that occurence of John's running to other occurrences of John's running. But any individual is a member of an infinite number of classes and only a relatively small number of those can be understood when the class is not overtly mentioned. For example, John may be a member of the class of people born on April 16, 1944. I can't imagine any context in which "John runs slowly" could conceivably be understood as "John runs slowly for a person born on April 16, 1944." An understanding of relative attributes requires an understanding of which classes can be understood in which contexts (or understood at all) when none is overtly mentioned.

Finally there is the fact that all relative attributes are fuzzy concepts in the sense of Zadeh, Ross, and Lakoff[10]. They can be modified by hedges such as rather, sort of, very, etc., which indicates that they cannot be dealt with in two-valued logic. Rather they require a continuous-valued logic and a semantics with distribution functions.

I think it should be clear by now that none of the proposed analyses of adverbs, in either the philosophical or linguistic literature, has come close to a full understanding of an adverb like slowly, which I chose only because it had been widely discussed. My guess is that any other arbitrarily chosen adverb would lead one into the study of at least as many other, seemingly unrelated phenomena -- and I feel that I have only begun to investigate the complexities of slowly. This experience should make one wary of studying small fragments or just looking at a handful of sentences, as many philosophers have a tendency to do. Studying a single phenomenon, or even a single word, in a natural language is like fooling with a giant delicately balanced mobile. Touch one piece and the whole thing moves.

FOOTNOTES

[1]Reichenbach, Hans. Elements of symbolic logic. Macmillan. 1947, pp. 247-354.

[2]Davidson, Donald. "The logical form of action sentences", in Rescher, N., The logic of decision and action, U. of Pittsburgh Press, 1967.

[3]Montague, Richard. "English as a formal language" in Linguaggi Nelle Societe e nella Tecnica, Milan, Editione di Comunita, 1970.
Parsons, Terence. "Some problems concerning the logic of grammatical modifiers", in Synthese 21, 1970.

[4]Clark, Romane. "Concerning the logic of predicate modifiers", Nous, 4, 1970.

Thomason, Richmond. "A semantic theory of adverbs", Yale mimeo, 1970

Stalnaker, Robert. "Notes on adverbs in response to Thomason and Lakoff", mimeo, 1970.

Grandy, Richard, "On semantics for modifiers", Princeton mimeo, 1972.

[5]Harman, Gilbert. "Logical form". Princeton mimeo. 1971.

[6]Actually, Harman intended this to be the logical form for the "more explicit sentence" "John walked slowly for a man his age". Harman recognizes that "John walked slowly" is elliptical, but he does not worry about what it can and cannot be elliptical for. This is discussed briefly below.

[7]Sentences like (i) indicate that there is an underlying rate description in "John drove slowly" which the namely-expression is understood as modifying.
(i) John drove slowly -- namely, 25 miles per hour.

[8]I would prefer a semantics for natural language where the elements of the universe of discourse were not taken to be real-world entities or even possible real-world entities. The only ontological commitments would be to those entities in the extension of the predicate EXIST.

[9]Fillmore, Charles. "Entailment rules in a semantic theory", in Readings in the philosophy of language, ed. by J. F. Rosenberg and C. Travis, Prentice Hall, 1971.

[10]Lakoff, George. "Hedges", in Papers from the eighth regional meeting of the Chicago Linguistics Society, U. of Chicago Linguistics Dept., 1972

Ross, J. R. "A note on implicit comparatives", Linguistic Inquiry, 1970, I, 3, pp. 363-6.

Zadeh, Lofti. "Fuzzy sets", Information and Control, 1965, 8, pp. 338-53.