

Colours from a logical point of view

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This paper presents a philosophical and logical investigation of colours, in contrast to other kinds of colour analysis, such as physical, physiological, chemical, psychological or cultural analyses. Neither philosophical nor logical analysis of colours concerns specific aspects of colours. Rather, these kinds of colour analysis concern what one might call “logical foundations of colour theory”. I will illustrate such a basal approach to color analysis first by completing a philosophical and then a logical analysis of colours.

1 Philosophical Analysis

Philosophical analysis of colours concerns the question: What are colours (i.e., to what category do colours belong)? A philosophical analysis intends to mitigate intellectual confusions, such as solipsism. The method of philosophical analysis consists in analysing the meaning of propositions rather than purporting or explaining the truth of certain propositions. Thus, a philosophical colour analysis does not intend to assert anything about colours or to provide a causal explanation of colours. Instead, it is confined to the analysis of color propositions, which is prior to or implied by colour theory.

Basically, the philosophical discussion offers three answers to the question of what colours are: sensations, dispositions, and properties of bodies.

1.1 Sensations

According to causal theories, such as physical or physiological theories, colours are not properties of bodies but sensations caused by light (or firing neurons). Colours are secondary qualities; they seem to be qualities of bodies, but, in fact, they are qualities of subjects (cf. figure 1). According to the tenets of causal theory, one might suggest the following analysis of colour propositions:

(A): “The table is red” = “Someone looking at the table has a red-sensation caused by light of long wavelengths reflected by the table.”

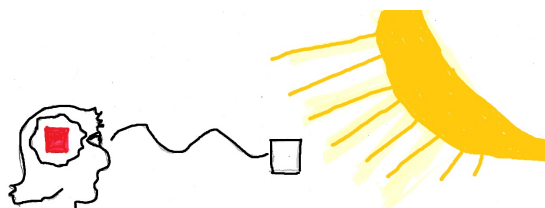


Figure 1: Colours as Sensations

The explication of colours as sensations is full of philosophical traps, ending with solipsism and scepticism. Clarifying the actual use of colour propositions is the primary step to resolve these intellectual confusions.

The main differences between sensations, such as pain, and colours are the following: (i) We do not identify our own pain; we express it, e.g., by saying, “I have pain” (or by screaming). Yet, we do not express colours; we identify them by looking at bodies. (ii) We might compare the colour of a body to colour paradigms if we are unclear about the exact colour attribution. Yet, there are no “pain paradigms” that might be used for comparison while doubting which kind of pain one has. (iii) We do not attribute pain to bodies. We say, “I have pain”, but not, “The needle has pain”. Yet, we attribute colours to bodies and not to ourselves. We say, “The table is red”, but not, “I am red” (unless I want to indicate the colour of my body or some part of it, e.g., due to a sunburn). (iv) We distinguish between colour-attributions and our perception of colours. Thus, it makes sense to state, “The table is red although I do not see it.” But we do not distinguish between pain and our sensation of it. It does not make sense to say, “I have pain although I do not feel it”. (v) It does not make sense to doubt or investigate one’s own pain. Yet, we can doubt the colour of a body, and we can methodically investigate it. (vi) We define colours by ostensive definition to colour paradigms. We might explain the meaning of the colour attribution in “The table is red” by saying, “The colour of the table is this ↗”, thereby referring to a red-paradigm. Yet, we do not define pain by pointing to pain paradigms. We cannot explain the meaning of “He has headache” by saying, “He has this ↗”, while we point to some pain-paradigm. Instead, we refer to the behaviour of beings to identify their sensations. This does not mean that we identify pains with such behaviour. A person suffers because of pain, not because of pain-behaviour.

Yet, we basically look at bodies to identify their colour, and the colour is nothing but what we see while looking at those bodies.

Thus, colour-words have a completely different logical grammar than sensation-words. According to pure analysis of the use of colour propositions, colours are not sensations but properties of bodies.

1.2 Dispositions

However, the analysis of the ordinary use of colour propositions seems to conflict with a common view of colour causation. According to the identity criteria of our ordinary use of colour words, colours are properties of bodies and not properties of the mind. Yet, according to common contemporary theory of colour causation, colours are the final links of a causal chain ending in our mind. The analysis of colours as dispositions offers a solution to this problem. Colours are said to be dispositions of bodies that cause certain sensations given standard conditions. On the one hand, one wants to do justice to the attribution of colours to bodies. On the other hand, one insists that this attribution is due to the fact that colours cause sensations of colours in our minds. According to this point of view, colour propositions are analysed as follows:

(B): “The table is red” = “The table has the disposition to cause a red-sensation” = “Given day light (more precisely, standard-conditions) and a non-colour blind person (more precisely, a standard-observer) the table causes a red-sensation in that person.”

However, nothing really changed compared to the sensualistic paradigm. The lack of change becomes evident by the fact that the dispositional theory of colours is in harmony with figure 1. As in the case of sensualistic analysis, providing a causal story analyses the meaning of colour propositions. The same applies for similar approaches that identify colours not with dispositions but with disjunctions of physical properties to cause colour-sensations (cf. e.g., Jackson 1996) or with something between physical properties and dispositions (cf. Campbell 1993).

To categorise colours as dispositions is a misleading attempt to resolve the mentioned conflict between the ordinary use of colour propositions and a theory of colour causation. Taken as an analysis of the meaning of colour propositions, the dispositional theory of colours is full of circularities and categorical confusions. First of all, the concepts of a standard-observer and of standard-conditions are usually defined by assuming colour attribution and not vice versa (compare Hacker

1987, p.127). A standard-observer is a person who perceives that body X has colour Y if body X has colour Y. Thus, to identify colour-blind persons, we may ask the person to distinguish red and green objects. If he or she is not able to do so, we identify the person as red-green blind. Standard-conditions are conditions in which a body X looks as having colour Y if body X has colour Y. Thus, darkness is not a standard condition because we are unable to identify the colours of objects at night. These ordinary definitions are not open to dispositionalism because dispositionalism becomes circular if it assumed those definitions. However, even if dispositionalism defines “standard-observer” and “standard-conditions” otherwise and non-circular, a dispositional analysis of colour propositions still refers to “colour-sensations”. Thus, for example, in the mentioned analysis of the proposition “The table is red”, dispositionalism must refer to “red-sensation”. This is obviously circular because the meaning of “red” is defined by using the word “red”. If the dispositionalist (as well as anyone explaining colours either as causes of colour-sensations or as colour-sensations themselves) tried to avoid circularity by defining a red-sensation as the effect of wavelengths (or, more generally, by some physical or physiological cause), he or she would not be permitted to refer to perception as a criterion for correlating certain colours with certain wavelengths (or certain physical or physiological causes). Yet, then the dispositionalist has no reason to correlate, for example, long wavelengths with a “red-sensation”. Furthermore, the dispositionalist is unable to justify that red is more similar to violet than to green or that the same colour can be caused by many different wavelengths (e.g., white). In short, the dispositionalist would lack the prior identity criteria to judge the correlation of physical (and physiological) causes and colour perception. Finally, classifying colours as dispositions is a category-mistake because it makes no sense to replace “I see a red table” with “I see the disposition of the table to cause my red-sensation.” Contrary to colours, dispositions are not visible, only their manifestations might be visible. However, unlike the circularity-problems, referring to physical properties, instead of dispositions, solves this problem.

From a philosophical point of view that takes into account an analysis of ordinary colour propositions, a dispositional theory of colours is as problematic as a sensualistic theory. Colours are neither sensations nor causes of sensations.

1.3 Properties of Bodies

Thus, if one refers to the ordinary use of colour propositions as a criterion to answer what colours are, the answer can only be that colours are, roughly, properties of bodies (see section 2.2 for more details). Colours are ostensibly defined and

perceived by us if we look at bodies. Analysis of the meaning of a colour attribution refers to a colour paradigm presented in standard-situations, i.e., at standard conditions (daylight) to a (non-colour blind) standard observer.

(C): “The table is red” = “The colour of the table is this ↗”.

This is not an explicit definition that defines colour words by other expressions, such as “standard-conditions”, “standard-observer” or “red-sensation”. We do not come to learn colour-words by explicit definitions (at least if we are not colour-blind or blind) but by ostensive definition, which define expressions implicitly.

Implicit definitions “define” primitive expressions without presuming the knowledge of the meaning of other expressions. Ostensive definitions only explain the meaning of expressions given certain conditions. To understand the meaning of colour-words by ostensive definition, one must, for example, be capable of distinguishing colours from other sorts of things, such as the shape or density of bodies, and one must be able to distinguish between different colours. This ability, in turn, presupposes certain objective conditions, such as sufficient brightness, and certain subjective conditions, such as the constitution of one’s eyes. Satisfaction of these conditions cannot be taken for granted, which becomes clear in the case of infants or colour-blind or blind persons. Infants first have to become familiar with basic differences in the world by experience and primitive education to correlate primitive expressions. A red-green blind person is not able to understand the meaning of “red” or “green” by ostensive definition. For such people, the colour words “red” and “green” are not defined implicitly but explicitly, e.g. by the following definition: “red is the colour of those things that non colour-blind persons call ‘red’ and not ‘green’” (and one might add “although I do not perceive any difference”). This definition is not an implicit, ostensive definition. Thus, the distinction between “red” and “green” is not primitive in the language of a colour-blind person.

However, one must not confuse conditions of a successful application of ostensive definitions with parts of the meaning of the so-explained expressions. The reference of an ostensive definition gives meaning to colour-words. Only coming to know this meaning by such a definition implies certain objective and subjective conditions. However, colour propositions, such as “The table is red”, refer to paradigms as criteria of colour attributions. For daily use, it suffices to refer to vague paradigms of typical objects (“clean snow is white”, “cloudless sky is blue”, “ripe tomatoes are red”, “ripe bananas are yellow”, etc.). If a more precise praxis of colour attributions is necessary, classification systems are introduced,

which make precise and ordered discriminations available, e.g., by a catalogue of colour paradigms. Such a catalogue can be used to identify colours of objects, placing the object and the catalogue side by side. Sensations or their invisible causes cannot serve as identity criteria in this or any other way.

The question is how to harmonise this understanding of the meaning of ordinary colour propositions with a causal explanation of colours that depends on our eyes and brains reacting to light waves (or photons). The key to answer this question is already apparent in the question itself: One must carefully distinguish between the *analysis of the meaning* of colour-words involving the explication of identity criteria of colour propositions from the *causation of colour perception and its explanation*. Standard conditions, standard observers, and standard causal settings condition possibilities of identifying and defining colours. Thus, one might say that the meaning of colour propositions and the possibility to identify colours implies certain causal regularities (whether one knows about them or not). However, the possibility of using meaningful propositions and the application of identity criteria are, in turn, prior to providing causal explanations. Only in consequence of identifying certain correlations between the perception of red objects and certain physical or physiological cause is one justified to say, “This table looks red *because* it reflects waves of a certain length falling on the retina of some standard observer in standard conditions.” However, from that statement it by no means follows that “This table is red” or even the proposition “This table looks red” *is identical in meaning* with a causal explanation of perceiving a red table. Figure 2 shows how to harmonise causal analysis and analysis of meaning in contrast to the attempt illustrated by figure 1.

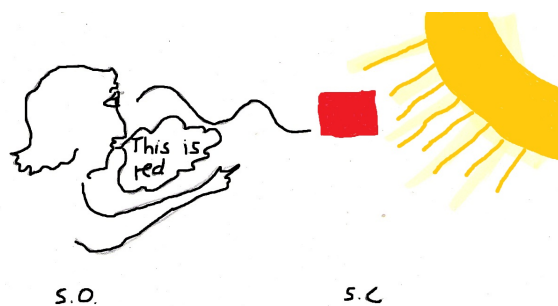


Figure 2: Colours properly defined and explained

Compare the following analogy to astronomy: To explain what we see in the night sky, we must be able to identify the position of the sun and other stars be-

fore we explain those positions. Thus, the proposition (S) “The sun rises” is prior to any geocentric or heliocentric world-views that attempt to explain the rising of the sun. However, the meaning of (S) presupposes certain regularities of the world, e.g., those assumed by a heliocentric world-view. Completely irregular movements and intensities of the stars would it make impossible to establish identity criteria for stars and their positions. Such irregularities would not only rule out the possibility of explaining the movements of stars but also the possibility of identifying and speaking meaningfully about stars.

Defining colours as sensations or in terms of dispositions or physical properties causing colour-sensations confuses causal explanation of colour perception with analysis of the meaning of colour propositions. The latter is prior to the former and must not be disregarded by answering what colours are.

1.4 The Fallacy of Solipsism

Insisting that colours are properties of bodies does not mean to deny colour illusions. A correct philosophical analysis of colour illusions clarifies the meaning of propositions, such as the following statement, which might be given in a situation of complementary after-images:

(D) “It looks like there is a green square in the middle of the white paper, although there is only a white sheet of paper”.

In case (D), we do not explain what it means “to look like a green square without being one” by ostensive definition. Instead, we are able to explain (D) by referring to other expressions, such as “standard conditions”, “standard observer” or “being a green square on a white sheet of paper”. The meaning of (D) implies the following statements: (i) what is seen is similar to what is seen when a standard observer looks at a green square on a white sheet of paper under standard conditions, (ii) the situation under consideration is not a standard situation and (iii) under standard conditions, a standard observer would perceive a white sheet of paper without a green square in the middle.

Thus, contrary to the analysis of an ordinary colour proposition in terms of (C), analysis of propositions about colour illusions, such as (D), implies a reference to standard conditions and standard observers. Propositions about colour illusions, such as (D), distinguish between standard and non-standard situations, and they already presume successful colour attributions. To identify an illusion means to identify some difference to a reliable perception of a colour. According to the

terminology of Wittgenstein, the possibility of distinguishing how colours appear and how they are and, thus, to identify illusions is a “secondary language game”, which is based upon the primary language game of attributing colours to bodies, cf. e.g., Wittgenstein 1972, p. 370f. This distinction refers back to the distinction between explicit and implicit definitions that is already prominent in Wittgenstein’s *Tractatus*, cf. Wittgenstein 1994, remark 3.263. Implicit definitions are the foundation of any understanding of language, which, in turn, presume certain correlations to the world that are not open to doubt if understanding language should be possible.

Solipsism or scepticism commits a fallacy by concluding from the possibility of illusions that *all* vision might be an illusion. Solipsism is not refuted by rejecting that colour vision causally depends on our eyes and brains or by insisting that we often seem to agree on colour attributions. Instead, the fallacy of solipsism is induced by disregarding that the possibility of identifying colours is prior to their causal explanation. The correct reaction to the position of solipsism is that it does not take into account the conditions of the possibility of meaningful colour propositions. The meaning of colour propositions presupposes the possibility to establish a standard for colour attributions. Such a standard is not open to doubt because it primarily establishes the possibility to doubt. The possibility of meaning of propositions, such as (D), and, therefore, the possibility to identify colour illusions, relies on the possibility of successful colour attribution, which is based on ostensive definition. Ostensive definitions, in turn, establish what a certain colour *is* by referring to a colour paradigm. Coming to understand the meaning of colour words by such definitions presupposes standard situations. Thus, the possibility of *some* colour illusions due to non-standard situations relies on the fact that *not all* vision is an illusion.

1.5 Conclusion

According to a philosophical analysis of colours, which relies on the analysis of colour propositions, colours are neither sensations nor (invisible) causes of colour-sensations. Instead, colours are visible properties of bodies. This analysis refers to the fact that our meaningful discussion about colours is based upon ostensive definitions. It follows from this analysis that solipsism ends with nonsense: Maintaining that the world, as we see it, is a complete illusion contradicts the conditions of the possibility to identify colour-illusions.

2 Logical Analysis

A logical analysis of colours considers the question of an adequate formal representation of colour propositions. The method of a logical analysis of colours constructs a proper formal language for colour representation. A logical analysis of colours intends to express necessary features of the meaning of colour propositions by syntactic features. Thus, for example, colour-exclusion, i.e., the impossibility of two colours occupying the same place at the same time, must follow from a proper formal representation of colour propositions. Likewise, it should follow from the formal representation of colour propositions (i) that colours relate internally to each other and (ii) that colours relate internally to the surfaces of bodies.

In the following, I argue that formalising colour propositions within first-order logic does not satisfy the aims of a logical analysis of colours. Instead, I will provide principle ideas of an alternative formal analysis. However, it should be noted that I abstain from all kinds of subtleties regarding (i) the concept of a body and the related concept of (visible) matter as well as (ii) colour analysis stemming from *theories* of colours, such as physical, chemical or physiological colour theories. Concerning the concept of a body, I use “body” in a broad, pre-theoretical, naive sense, meaning visible matter distinguishable from its surroundings. *Visible* matter does not imply properties as density, inertia or being compound of elements. Thus, for example, clouds and even blue sky are bodies in this sense, without implying any theory about the elements that make up those bodies. With regard to theories of colours, it makes, for example, good sense to state that two colours are at the same place at the same time in so far as simple and compound colours are distinguished. Similarly, one may speak of “red light” within a physical theory of colour, although the light itself is not meant to be red and in chemical theory, pigments, rather than surfaces of bodies, are coloured. However, this section only concerns principles of a logical analysis of an ordinary understanding of colour propositions, independent of any sophisticated theory of colours and matter. The means of a logical analysis are, of course, not “unsophisticated” or “pre-theoretic”. Yet, the meaning of the colour propositions to be analysed is that of a more or less “brute force” understanding of propositions, such as “This ball is red”.

2.1 Against first-order formalisation

Let me begin by considering how to formally represent the predicate “x is a colour”. One might suggest representing this ordinary language predicate by a propositional function within first-order logic. However, this results in ambiguous formalisations. Consider, the following two arguments.

| Argument 1 | Argument 2 |
|---------------------------|-------------------------------------|
| P1 This table is red. | P1 This table contains all colours. |
| P2 Red is a colour. | P2 Red is a colour. |
| C This table is coloured. | C This table is red. |

To prove the validity of both arguments within first-order logic, the second premise must be formalised differently. Thus, the following logically valid formalisations are suggested (cf. Brun 2004, p. 335):

| Argument 1 | Argument 2 |
|-----------------------------------|-----------------------------------|
| P1 Rt | P1 $\forall x(Cx \rightarrow Tx)$ |
| P2 $\forall x(Rx \rightarrow Cx)$ | P2 Cr |
| C Ct | C Tr |

The two formalisations refer to the following legends:

| Legend Argument 1 | Legend Argument 2 |
|-----------------------|-----------------------|
| t : this table, | r : red, |
| Rx : x is red, | Tx : x is a table, |
| Cx : x is coloured. | Cx : x is a colour. |

This ambiguity is avoided if the second premise of both arguments, “Red is a colour”, is analysed as a pseudo-proposition in terms of “Red is a value of the formal concept of colour”. It is impossible to represent such pseudo-propositions within first-order logic because no material property is attributed to an individual. A formal concept does not state anything about individuals but specifies values of a variable. Whereas material properties might be formalised by propositional functions within first-order logic, formal properties are depicted by variables. Failing to distinguish between formal and material concepts was one of the main objections of Wittgenstein against Frege’s and Russell’s use of first-order

formalism, cf. Wittgenstein 1984, remark 4.1271f.:

Every variable is the sign for a formal concept. [. . . The formal concepts] are represented in conceptual notation by variables, not by functions or classes (as Frege and Russell believed).

Analyzing “ x is a colour” as a formal concept that is to be represented by a variable results in the following formalisations of the two arguments:

| Argument 1 | Argument 2 |
|------------------|-------------------|
| P1 Rt | P1 $\forall C Ct$ |
| P2 $-$ | P2 $-$ |
| C $\exists C Ct$ | C Rt |

C is used as a variable of colours, whereas R is a value of this variable. t is a constant (name) for this table. Like R , t can be conceived as a value for a variable of bodies. However, I will consider a more detailed analysis of depictions of colours, bodies and their relation in the following section. For now, it suffices to simplify matters and indicate colours and bodies by constants that I shall not, for now, further analyse. The given formalisation is valid on the basis of existential introduction and universal quantifier elimination.

This kind of formalisation already departs from a common understanding of first-order logic because it does not distinguish a priori between individuals and properties. Instead, it simply distinguishes between the constant parts and the variable parts of a proposition. Thus, being red is not essentially a property and “this table” does not necessarily refer to an individual. The distinction between individuals and properties has no ontological basis. It is simply due to a distinction between constant and variable parts of a proposition (cf., Wittgenstein 1994, remarks 5.522f., Ramsey 1954, p. 271, for more details Lampert 2000, chapter 5). In consequence, when I described colours as “properties of bodies”, I did not mean to imply any ontological distinction between properties and individuals. Instead, it only meant that colour propositions combine colour-words with words referring to bodies (unlike, for example, expressions of sensations: “I have pain” is, roughly speaking, similar in meaning to “it hurts”). Classifying colours as properties of bodies does not make a claim about the specific ontological status of colours other than their necessary connection to physical bodies.

The common logical notation of first-order logic, with its distinction of function and argument and set-theoretical semantics, does not only lead to ambiguous

formalisations. Also, logical analyses in line with first-order logic neither solve the problem of colour-exclusion nor depict (i) the internal relation of colours and (ii) the internal relation of colours and bodies.

First-order logic does not reduce colour exclusion to a type of logical impossibility. Instead, even a straightforward formalisation, in terms of a conjunction of two atomic propositions, allows for stating the impossible within the language of first-order logic.

(E): “This spot s is red (R) and green (G) at the same time t .” = $R(s, t) \wedge G(s, t)$

$R(s, t) \wedge G(s, t)$ is no contradiction within first-order logic. According to the common set-theoretical interpretation of first-order logic, there is no syntactic criterion to identify colour exclusion.

Formalising “ x is a colour” in terms of a propositional function does allow for representing pseudo-propositions, such as “Red is a colour”, in terms of meaningful propositions. Classifying “Red is a colour” as a meaningful proposition also shows that such an analysis allows for meaningful colour propositions without attributing colours to physical bodies. Furthermore, formalising colour-predicates in terms of propositional functions and colour propositions in terms of $f(x, y)$ does not preclude syntactically the articulation of meaningless propositions, such as “1 is red at 1 o’clock.”

Finally, whereas propositional functions identify *sets* of arbitrary elements that are not internally related, formal concepts apply to values of systems. Colours are an example of internally related elements of a *system*. One prominent outstanding issue of a logical foundation of colour theory is a formal theory of possible colour systems. However, it would take me too far here to consider this issue. Instead, I only want to point out that any understanding of colours as sets represented by propositional functions does not do justice to the internal relations between colours. A proper formal representation of colours should depict these internal relations by syntactic properties of colour symbols. One should be able to identify the location of a colour in a system due to its symbol. One possibility for doing so is by representing colours by tuples of coordinates that identify a position in a colour space. I will come back to this possibility in the next section.

I conclude that first-order logic is not a proper logical notation for satisfying the aims of a logical analysis of colours. On the contrary, applying this formalism to colour propositions on the basis of its set-theoretical interpretation induces logical confusions. In the following, I will sketch an alternative formal representation of colour propositions that refers to the ideas of Wittgenstein in the period between

1927 to 1934 (for a different approach of the early Wittgenstein that still makes use of the language of first-order logic cf. Wittgenstein 1994, remark 6.3751 and Lampert (2000), chapter 4).

2.2 A Wittgensteinian Alternative

Colour, space and time form systems of internally related elements. Coordinates that reveal a position within the respective system identify internally related elements. That is why symbols, in terms of tuples of coordinates, identify the internal relation to other elements of the system based on syntactic properties. There is no need to refer to the meaning (reference) of symbols, such as $\langle 1, 1, 1 \rangle$, $\langle 2, 1, 1 \rangle$, $\langle 3, 1, 1 \rangle$, to derive that the second symbol identifies an element that is “in between” the elements symbolised by the first and third symbols. Names within first-order logic symbolise individuals that are not essentially related to each other. Propositional functions put together the isolated individuals. Thus, it can be asserted that individuals form the elements of a set. In contrast, symbols in terms of coordinates symbolise *positions* within ordered systems.

Colour propositions result from a combination of elements of different systems that together form a logical space, i.e., a space of possible states of affairs. Any combination of coordinates of different systems identifies a logical place, i.e., a possible state of affairs. Unlike symbolising a state of affairs by an atomic proposition of form $f(x)$, $f(x, y)$, ... within first-order logic, the combination of coordinates does not essentially distinguish function and argument; all coordinates are on the same logical level. There is no need for functions that put together certain individuals by asserting that they satisfy some property or relation. Instead, the combined coordinates identify a logical space. Any type of coordinate essentially connects with the other type of coordinate. A meaningful attribution of colour, thus, implies the combination of colour coordinates with some other type of coordinates. Colour propositions, which identify possible states of affairs, can be represented by such combinations of different coordinates.

Let us illustrate this kind of analysis in more detail. The logical space of colour propositions already presumes a naive physical space (= physical_n space), in which visual bodies are located at certain areas of space-time. Thus, one may conceive the physical_n space as a combination of systems of space, time and visual matter (= matter_v). Any point in the physical_n space depicts the possibility of matter_v occupying a space-time point. I call bounded areas of space-time occupied by matter_v “bodies” and the two-dimensional bounded area of bodies “surface”. I abbreviate bodies by $\langle M_v, |S|, |T| \rangle$, which symbolises a combination

of matter_v with intervals of space-points and time-points (= areas of space-time). I introduce the index _S to indicate those space-time points that mark the surface of a body: $\langle M_v, |S_S|, |T_S| \rangle$. Thus, referring to a surface of a body already implies complicated propositions about the boundaries of matter_v occupying space-time points.

Colour propositions can be formalised as combinations of coordinates that symbolise positions within the colour space with parts of surfaces. Let us, for short, assign tuples of colour coordinates as values of the variable C and let us conceive parts of surfaces as intervals within intervals. Thus, the form of colour propositions is $\langle C, \langle M_v, ||S_S||, ||T_S|| \rangle \rangle$. Contrary to any first-order analysis of the form of colour propositions in terms of atomic propositional functions, this analysis guaranties that any colour proposition identifies a possible state of affairs. Thus, it is impossible to construct meaningless propositions, such as “1 is red at 1 o’clock”, because the analysis ensures that coordinates of the colour system combine with coordinates of the system of physical_n space. Furthermore, this analysis reveals that colour propositions are complex, rather than atomic, propositions, implying propositions asserting that matter_v occupies points of space-time.

Of course, this analysis of colour propositions abstains from the details and anomalies of colour attributions. However, the purpose of this paper is to illuminate the basic principles of an alternative to first-order formalisations that does justice to the mentioned aims of logical analysis. For this purpose, it suffices to refer to the mentioned analysis as a paradigm of a logical analysis of colour propositions.

Finally, the representation of colour propositions as combinations of coordinates of a different type of systems solves the problem of colour-exclusion, too. For the sake of simplicity, let us (i) abstain from visual matter, (ii) symbolise colours by referring to a one-dimensional colour space (e.g., the spectrum) and by signifying colours with only one discrete coordinate, (iii) refer to a two-dimensional circle as a surface that is defined by a radius r and a centre, which is defined by the coordinates $\langle a, b \rangle$, (iv) combine the coordinates of the different dimensions in a two-dimensional representation that signifies a certain combination of coordinates of different scales by adjusting a pointer and (v) represent a combination of coordinates at a certain time by just one such adjustment. We then get the following logical analysis:

(F): “The spot with radius $r = 4.9$ at position $\langle a, b \rangle = \langle 2.7, 7.7 \rangle$ is at a certain time green (= colour 3.1).” = Figure 3

Within the syntax of the representation in figure 3, it is impossible to represent

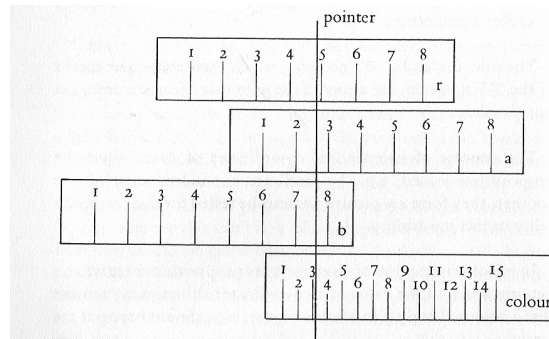


Figure 3: Logical Analysis of a colour proposition, taken from Wittgenstein 1975, p.112

a surface being green and red at the same time because the pointer can only signify *one* combination of colour coordinates with the coordinates identifying a surface. Thus, impossibility is not expressed by stating the impossible within a formalism but by the impossibility of representing the impossible within a proper notation: “[. . .] it is impossible to set one scale simultaneously at two graduation marks” (Wittgenstein 1975, p.112).

2.3 Conclusion

First-order logic, with its distinction between function and argument and its set-theoretical semantics, does not provide the means for an adequate formal representation of colour propositions. Instead, an adequate formal representation represents colour propositions in terms of a combination of coordinates of different systems that together denote a point in logical space. The syntax of such a representation solves the problem of colour exclusion and represents the internal relations between colours and between colours and parts of surfaces of bodies.

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