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1

Russell's Version of the Theory of Definite Descriptions

1. introduction

It is mildly ironic that the title of this chapter is an unfulfilled (or improper) definite description because Russell really had two versions of the theory of definite descriptions. The two versions differ in primary goals, character and philosophical strength.

The first version of Russell's theory of definite descriptions was developed in his famous essay of 1905, 'On Denoting'.[1] Its primary goal was to ascertain the logical form of natural language statements containing denoting phrases. The class of such statements included statements with definite descriptions, a species of denoting phrase,[2] such as 'The Prime Minister of England in 1904 favored retaliation' and 'The gold mountain is gold'. So the theory of definite descriptions contained in what Russell himself regarded as his finest philosophical essay is a theory about how to *paraphrase* natural language statements containing definite descriptions into an incompletely specified

[1] Bertrand Russell, 'On Denoting', *Mind*, New Series: XIV (1905), pp. 479–493; Alfred North Whitehead and Bertrand Russell, *Principia Mathematica*, (Second Printing), Cambridge, At the University Press (1910), Volume 1. 'On Denoting' is reprinted in *Bertrand Russell: Logic and Knowledge* (editor, Robert C. Marsh), George Allen and Unwin, Ltd., London (1956), pp. 41–56. All references here to 'On Denoting' are to the reprinted version in the Marsh collection.

[2] Dismissed by G. F. Stout as rubbish, 'On Denoting' was praised by F. P. Ramsey as a paradigm of philosophical analysis. Russell's own opinion of the quality of his famous essay is reported on page 39 in the Marsh collection cited in the previous note.

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formal language about propositional functions. Russell used this version of his theory to disarm arguments such as Meinong's arguments for beingless objects. Such reasoning, he said, is the product of a mistaken view about the logical form of statements containing definite descriptions.

The second and later version is presented in that epic work of 1910, *Principia Mathematica* (hereafter usually *Principia*). Its primary goal, in contrast to the first version, was to provide a foundation for mathematics, indeed, to reduce all of mathematics to logic. In chapter *14 Russell introduces a special symbol, the inverted iota, and uses it to make singular term-like expressions out of quasi-statements. They serve as the formal counterpart of definite descriptions, and the expression 'definite description' is extended to cover the formal counterparts themselves, not an uncommon procedure in logic. Then contextual definitions are offered which are said to "define" definite descriptions in all the possible statements in which they can occur. Definite descriptions are regarded not as a referring kind of expression but as a certain variety of "incomplete symbol". So, in *Principia*, Russell's theory of definite descriptions is a theory about how to treat the logical counterpart of natural language expressions of the form 'the so and so' where 'the' is used in the singular. As such it is a *definitional extension* of a formal language, the first order fragment of which is similar to the predicate logic found in most contemporary textbooks of symbolic logic, minus names. Russell uses definite descriptions in *Principia* for all sorts of purposes; for example, he uses them to define descriptive functions.

The chronological order of the two versions will be reversed and the second version will be discussed first. It is the most complicated of the two versions, is more prone to technical complaint, and mainly because of these same complaints, it is weaker in philosophical strength than the first version of the theory. In fact, the first version is a very natural antidote to many of the problems besetting the second version.

2. RUSSELL'S THEORY IN *PRINCIPIA MATHEMATICA*

In what follows Russell's inverted iota is replaced by a smaller case '**i**', the dot notation is replaced by parentheses, '**&**' replaces '**.**', his sign for conjunction, and the higher case English letters '**P**' and '**Q**' replace his Greek symbols 'Φ' and 'Ψ'.

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Russell's formal theory is captured in the following pair of *contextual* definitions along with an explanation of their distinctive character.[3]

CD1 $\mathbf{[ix(Px)]\ Qix(Px) = (\exists y)((x)(Px \equiv x = y)\ \&\ Qy)\ Df}$
CD2 $\mathbf{E!ix(Px) = (\exists y)((x)(Px \equiv x = y))\ Df}$[4]

In contrast to the contextual definitions of identity (in chapter *13), and of the conditional (in section A of part I), which, Russell says, define the introduced expressions, **CD1** and **CD2** define neither '**i**' nor '**ix(Px)**' even though the signs for identity, the conditional and the sign '**i**' are not primitive signs. According to Russell, **CD1** and **CD2** merely "define" any "proposition in which [the phrase '**ix(Px)**'] occurs".[5]

The contextual definitions of definite descriptions in *Principia* are very complex. First, they introduce not one but two symbols along with a symbol pair; '**i**', '**E!**' and the left and right hand brackets '**[**' and '**]**'. Second, they introduce the important notion of the scope of a definite description; this is the function of the brackets around an expression of the form '**ix(Px)**' in the definienda of **CD1** and **CD2**. Third, they show that '**E!**', in contrast to predicates such as '**P**' and '**Q**', only appears next to definite descriptions. Fourth, they reveal that definite descriptions occur in positions in statements often occupied by (logically proper) names or variables, and that '**E!**' occurs in positions in statements often occupied by predicates. For instance, in

$$\mathbf{a = ix(Px),}$$

'**ix(Px)**' occupies a position often occupied by a name or variable, and in

$$\mathbf{E!ix(Px),}$$

'**E!**' occupies a position often occupied by predicates, for instance, the predicate '**Q**'. Despite this fact, '**E!**' is not treated as a predicate

[3] Russell actually gives a third definition in *Principia* for ordering the occurrence of definite descriptions in statements. That definition is neither important nor essential to the current discussion.
[4] 'CD' abbreviates 'contextual definition'. Neither the expression nor the abbreviation appear in Russell's statement of the two definitions in chapter *14 of *Principia*.
[5] Ibid., p. 175.

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(primitive or defined), and definite descriptions are not treated as names or referring expressions (primitive or defined).

In this version of Russell's theory, why it would be a disaster to treat definite descriptions as names, as symbols "directly representing... object[s]", is easily answered. If they were so treated, then

$$\mathbf{ix(Px \mathbin{\&} \sim Px) = ix(Px \mathbin{\&} \sim Px)}$$

would be a substitution instance of the valid *Principia* principle

$$\mathbf{x = x}$$

and hence would be true. But the statement in question is false when evaluated via **CD1** because it is false that

$$\mathbf{(\exists x)(Px \mathbin{\&} \sim Px).}$$

On page 72 of *Principia* Russell claims to have *proved* on pages 67 and 68 that definite descriptions are not (logically proper) names, hence that they are "incomplete symbols" and do not stand for "constituents" of "facts". The relevant passages are these:

> Suppose we say: 'The round square does not exist.' It seems plain that this is a true proposition, yet we cannot regard it as denying the existence of a certain object called the 'the round square'. For if there were such an object, it would exist: we cannot first assume that there is a certain object, and then proceed to deny that there is such an object. Whenever the grammatical subject of a proposition can be supposed not to exist without rendering the proposition meaningless, it is plane that the grammatical subject is not a proper name, that is, not a name directly representing some object.
>
> ...By an extension of the above argument, it can easily be shown that [**ix(Px)**] is *always* an incomplete symbol. Take, for example, the following proposition: 'Scott is the author of Waverley'. [Here "the author of Waverley" is [**ix**(**x** wrote Waverley)].] This proposition expresses an identity; thus if "the author of Waverley" could be taken as a proper name, and supposed to stand for some object *c*, the proposition would be 'Scott is c'. But if c is any one except Scott, this proposition is false; while if *c* is Scott, the proposition is "Scott is Scott," which is trivial, and plainly different from "Scott is the author of Waverley." Generalizing, we see that the proposition **[a = ix(Px)]** is one which may be true or may be false, but is never merely trivial, like [**a** = **a**], whereas, if [**ix(Px)**] were a proper name [**a** = **ix(Px)**] would necessarily be either false or the same as the trivial proposition [**a** = **a**]. We may express this by saying that [**a** = **ix(Px)**] is not a value of the propositional function [**a** = **y**], from which it follows that [**ix(Px)**] is not a value of [**y**]. But since [**y**]

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may be anything, it follows that [**ix(Px)**] is nothing. Hence, since in use it has meaning, it must be an incomplete symbol. . . .

Certain peculiarities of, and complaints about, this version of Russell's arise immediately. Among the more striking are the following. First, consider Russell's treatment of singular existence statements. That treatment seems arbitrary and, in a certain sense, appears to reduce the expressive power of this version of his theory. On the one hand,

$$\mathbf{E!ix(Px)}$$

and

$$(\exists \mathbf{y})(\mathbf{y} = \mathbf{ixP(x)})$$

are logically equivalent in the second version, where '**P**' is any one-place predicate. The second of these statements, in fact, is another way of asserting existence. So, on the other hand, one would suppose,

$$\mathbf{E!a}$$

and

$$(\exists \mathbf{y})(\mathbf{y} = \mathbf{a})$$

would represent alternative ways of asserting the existence of the object named by '**a**'. But this is not so because though the latter statement in the immediately preceding pair is logically true, the former, according to Russell, is "meaningless". If, to protect this doctrine, '**E!**' is accorded privileged status as the means of expressing singular existence, the decision seems to be simply an arbitrary syntactical choice with no substantive explanatory power. Moreover, in Spinoza's philosophy, Substance is the one and only one thing that exists. But the natural paraphrase of this descriptive phrase is not well formed in Russell's *Principia* theory because it would juxtapose '**E!**' to a variable as in

$$\mathbf{ix((y)(E!y \equiv y = x))},$$

a juxtaposition which Russell regards as meaningless. So the second version lacks a certain expressive power.[6]

[6] Russell's view of the meaninglessness of '**E!x**' is stated on pages 174–175 of *Principia Mathematica*.

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Second, Russell's use of definite descriptions in *Principia* is anomalous. Consider, for instance, the definition of a descriptive function in Chapter 30. That definition is expressed as follows:

R'y = ıx(xRy) Df.[7]

However, according to Russell, normally a definition like this one is "a declaration that a certain newly-introduced symbol or combination of symbols is to mean the same as a certain other combination of symbols of which the meaning is already known".[8] As such it serves as justification for the replacement of the definiendum by the definiens in formulae of the formal language. In the preceding definition the "other combination of symbols of which the meaning is already known" is the definite description

ıx(xRy).

But, as noted earlier, Russell claims that definite descriptions in the second version do not have meaning in isolation, that they themselves are never defined but only the propositions in which they occur. Because definite descriptions have no meaning in and of themselves, they have no meaning that is already known, as is presupposed in the definition above of a descriptive function. So the use of the definite description in the definiens of the above definition is anomalous since evidently it is not treated there as an incomplete symbol.

Russell is aware of the problem as a look at the bottom of page 232 of *Principia* makes clear. His solution is to say that the definition above of a descriptive function is a definition in a very special sense; it is, he says, "more purely symbolic than other definitions". But a dilemma looms. If the definition of a descriptive function is not a definition in Russell's standard sense, then it is hard to see how it can be used to *justify* the replacement of the definiens by the definiendum in the formulae of *Principia*, as Russell clearly intends. Indeed, in the "purely symbolic sense", the definition amounts simply to an unjustified declaration that the definiens and the definiendum are interchangeable. If, on the other hand, the above definition is taken in Russell's usual sense, then the definiens is *not* being treated as an incomplete symbol. This is dramatic evidence of Russell's vacillation in the second version over

[7] Ibid., p. 232.
[8] Ibid., p. 11.

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the status of definite descriptions. On some occasions, he treats them as logical subjects and, on others as incomplete symbols, and hence *not* as logical subjects.[9]

Third, Russell's "proof" that definite descriptions in the second version are incomplete symbols is highly questionable. The structure of the proof contained in the previously quoted passages is hard to discern. In particular, it is not clear what Russell means by the phrase "By an extension of the above argument . . . ". Does this phrase suggest an argument by cases in which, first, unfulfilled definite descriptions are shown to be incomplete symbols, and, then, fulfilled definite descriptions are so shown? Or does it suggest merely that there is a basic kind of context, other than the context of non-existence, namely, the context of identity, in which definite descriptions, fulfilled or unfulfilled, can be shown not to be (logically proper) names? Whatever the exact superstructure of Russell's proof, several of its evident premises are open to serious question.

In the first place, the demonstration that the statement 'The round square does not exist' leads to contradiction if its constituent definite description is treated as a name, as a phrase "directly representing some object", rests on the assumption that being an object entails being an existent. Here Russell, apparently, is making tacit appeal to his earlier 1905 "demolition" of Meinong's view that there are nonexistent objects. But, as current discussion has shown, the most Russell's famous argument in 'On Denoting' established is that the principle,

The so and so is (a) so and so,

is, on its most common construal, false. This principle was indeed espoused by Meinong, but it is modifiable or expungible without damaging Meinong's belief in nonexistent objects.[10] In fact, current philosophical logic abounds in provably consistent treatments of nonexistent objects.[11]

[9] It should be observed that the definition of a descriptive function on page 132 does not provide a context – namely, an identity context – in which the definite description occurs. Contexts of the form ' . . . = _ Df' are not identity contexts as Russell points out on page 11 of *Principia*.

[10] See Karel Lambert, *Meinong and the Principle of Independence*, Cambridge University Press (1983), pp. 33–34.

[11] See, for example, Terence Parsons, *Nonexistent Objects*, Yale University Press, New Haven (1980).

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In the second place, Russell's assertion that a statement of the form

a = ix(Px)'

where '**a**' is a logically proper name, is "never merely trivial" is itself false; it suffices to let '**ixP(x)**' be '**(x = a)**'. The resulting statement is just as "trivial" as

a = a.

In the third place, it is a controversial matter whether the principle of the *substitutivity of identity* holds in contexts of the form

. . . is trivial,

a principle apparently exploited by Russell when, in effect, he substitutes the expression 'the proposition that Scott is the author of Waverley' for the expression 'the proposition that Scott is Scott' in the statement 'The proposition that Scott is Scott is trivial'. Moreover, even assuming that the the *substitutivity of identity* holds in contexts of the form

. . . is trivial,

there is the Fregeian position with which to contend. Such contexts are indirect (*ungerade*) from Frege's point of view and, thus, the expressions replacing '. . . ' in '. . . is trivial' will refer not to their ordinary references, propositions (qua statements) for Russell, but to their senses. But in contexts of the form

. . . is identical with _

the expressions in question to refer to their ordinary references, and, hence, 'The proposition that Scott is the author of Waverley is trivial' cannot be derived by the *substitutivity of identity* from 'The proposition that Scott is Scott'. Russell's implicit discontent with this solution again apparently relies on his 1905 argument in 'On Denoting' – the perplexing Gray's *Elegy* argument – that Frege's doctrine of sense and reference is incoherent. This latter argument, however, is at worst dubious and at least very controversial, even on the most sympathetic interpretation.[12] It seems appropriate to conclude that Russell's boast

[12] Op cit., 'On Denoting', pp. 48–51. See Alonzo Church, 'Carnap's *Introduction to Semantics*', *Philosophical Review*, III (1943), pp. 256–272; John R. Searle, 'Russell's

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in *Principia Mathematica* to have *proved* that definite descriptions are incomplete symbols is exaggerated.

Fourth, and perhaps most importantly, the second version violates a condition on formal languages that Russell himself seems to acknowledge. The condition is that a formal language should not be ambiguous with respect to logical form.[13] One can think of logical form as the way a statement is evaluated for truth-value.[14] For example,

This is duplicitous,

in contrast to

Someone is duplicitous,

is a predication because, in standard semantics, it is evaluated by locating the referent of 'this' and ascertaining whether or not it is a member of the set of things associated with the predicate 'is duplicitous'. Because the word 'someone' doesn't even purport to have a referent, the contrast statement can't be a predication. Of course, this way of talking about logical form is at the very most only implicit in *Principia* because its semantics is never formally specified. Nevertheless, there is little, if any, distortion of Russell's view that formal languages should be, and are easily made to be, unambiguous with respect to logical form. So consider

Qa,

and

Qix(Px),

objection to Frege's theory of sense and reference', *Analysis*, XVIII, (1958), pp. 137–143; and Simon Blackburn and Alan Code, 'The power of Russell's criticism of Frege: 'On Denoting', pp. 48–50', *Analysis*, 38 (1978), pp. 65–77.

13 Op cit., 'On Denoting', pp. 52–53.

14 The rough and ready account of logical form adopted here is due to David Kaplan. See his 'What is Russell's theory of definite descriptions?' in *Physics, History and Logic* (editors, W. Yourgrau and A. Breck), Plenum: New York (1967), pp. 277–295. Actually Kaplan's account does raise questions (not especially troublesome in the current discussion). For example, positive and negative free logicians both count 'Vulcan is Vulcan' as a predication, but the former evaluates the statement true while the latter evaluates the statement false. On Kaplan's account these adversaries have different conceptions of the logical form called 'predication'.

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where '**a**' is a logically proper name – perhaps, the word 'this'. These two statements appear to have the same logical form; their syntactic structure would lead one to think that they would be evaluated in the same way. But they are not. Were names to be added to *Principia*,

Qa

would certainly be a predication, but not

Qix(Px).

Indeed the latter statement gets evaluated via **CD1,** and, as is evident, is much more complicated in that regard than the former statement. So Russell's second version violates the condition that formal languages not be misleading in their syntax with respect to logical form.

A similar situation arises *vis-à- vis* the pair of statements

E!ix(Px)

and

Qix(Px)

because in the Russellian scheme of things '**E!**' cannot go in all places where the predicate '**Q**' can. For example, it cannot occupy the place of '**Q**' in

Qx

or in

Qa,

where '**a**' is a logically proper name. In *Principia,* Russell says that contexts of the form

E!a,

where '**a**' is a logically proper name, and contexts of the form

E!x

are not meaningful, but contexts of the form

Qa

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