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Why "is at"?—On Quine's Objection to Carnap's *Aufbau* in "Two Dogmas of Empiricism"

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In "Two Dogmas", Quine indicates that Carnap's *Aufbau* fails "in principle" to reduce our knowledge of the external world to sense data. This is because in projecting the sensory material to reconstruct the physical world, Carnap gives up the use of operating rules and switches to a procedure informed by general principles. This procedure falls short of providing an eliminative translation for the connective "is at", which is necessary for the reduction. In dissecting Quine's objection, I argue that Quine has at best proven the claim that the use of general principles essentially fails the task of radical reductionism. However, in order to establish the conclusion that the *Aufbau* fails *in principle*, Quine needs to further vindicate two other claims. They are: first, a switch from operating rules to general principles is necessary; second, the set of general principles Carnap adopts is the best alternative. By disambiguating the notion of "explicit definition" and examining the concept of definability in the *Aufbau*, I explore the possibility of justifying these two claims that Quine overlooks in his objection. The result suggests that Quine's objection stands in tension with his radical reductionist reading of the *Aufbau*.

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1. Introduction

The early reception of Rudolf Carnap's *Der logische Aufbau der Welt* (*The Logical Structure of the World*, henceforth *Aufbau*) in the English-speaking world as a radical reductionist project can be traced back to two of its influential critiques that long predated the publication of the English translation of the book in 1967. These two critiques, both published in 1951, are Nelson Goodman's *The Structure of Appearance* and W. V. Quine's "Two Dogmas of Empiricism" (henceforth "Two Dogmas"). While both of them take the *Aufbau* as an attempt to answer the problem of the external world in the same vein as Russell—through reconstructing our knowledge of it from sense data—they pose different challenges to this radical reductionist enterprise. In *The Structure of Appearance*, Goodman argues that unless doubtful, extra-systematic assumptions are introduced, the *Aufbau* is doomed to failure. This is because the method of "quasi-analysis" Carnap employs in constructing *quality classes* is unable to fence itself off from two unfavorable conditions, namely those of "companionship" and "imperfect community". If these unfavorable conditions arise, the construction will yield either an incomplete or distorted picture of the structure of our elementary experience. Unfortunately, the *Aufbau* is not equipped with the machinery to even detect these unfavorable conditions. Nevertheless, Goodman remains positive towards the reductionist approach of the *Aufbau*. Since the current problem is due to the application of "quasi-analysis" motivated by a Gestaltist basis of elementary experience, the most straightforward solution is either to refine the technique of "quasi-analysis" or simply to adopt a different basis. Goodman opts for the latter in his own constructional system.[1]

On the other hand, in "Two Dogmas", Quine singles out the *Aufbau* as the paradigm example of radical reductionism. According to Quine, the *Aufbau* embodies what he calls "the second dogma" of logical empiricism: in assuming that every meaningful statement "is held to be translatable into a statement (true

[1] In fact, before *The Structure of Appearance*, Nelson Goodman and Henry Leonard already raise and tackle the problem of "imperfect community" by using a "calculus of individuals" they develop in their joint paper "The Calculus of Individuals and Its Uses" (1940). As a footnote of the paper indicates, the paper was read before a joint meeting of the Association for Symbolic Logic and the American Philosophical Association in December 1936. Goodman and Leonard believe that this calculus will allow us to "deal efficaciously with certain relational properties which are often ignored or misunderstood, sometimes to the detriment of constructional undertakings like Carnap's *Logischer Aufbau der Welt*" (1940, 50). A full-scale application of this calculus to deal with both the problems of "imperfect community" and "companionship" can be found in Goodman's doctoral thesis *A Study of Qualities* (1941) (which was first published in revised form as *The Structure of Appearance*). Interestingly enough, as Cohnitz and Rossberg (2006) suggest, it is unlikely that this technical innovation in Goodman's own constructional system was originally motivated by the *Aufbau*. This is because the idea of a calculus of individuals first appeared in Leonard's doctoral thesis *Singular Terms* (1930)—a time when both Leonard and Goodman, like the majority of the philosophical community in the English-speaking world back then, were probably not aware of the *Aufbau*. I am indebted to an editor of this journal for pointing this out to me.

Another attempt to refine the constructional technique can be found in Lewis (1969).

According to Carnap, the reason for adopting a solipsistic, phenomenal—"autopsychological"—basis in the *Aufbau* is to capture the "epistemic order" of objects in the external world (§64). Carnap alludes to the possibility of carrying out the construction on different bases, for instance, a physical one (§59) or a psychological one (§60). He indeed took the physical option seriously, as we can see in his correspondence with Schlick concerning the title of the *Aufbau*. For Carnap's correspondence with Schlick, see Coffa (1991, 403), Friedman (2007, 135) and Neuber (2016, 100).

or false) about immediate experience" (Quine 1951, 36), the *Aufbau* at the same time suggests that statements about the external world can be verified or confirmed in isolation from each other. Unlike Goodman's challenge, which focuses on the construction *within* the autopsychological domain, Quine's objection is directed at the assembling of the physical world from objects of the autopsychological domain, viz., cross-domains construction. Quine writes:

> Carnap did not seem to recognize, however, that his treatment of physical objects fell short of reduction not merely through sketchiness, but *in principle*. Statements of the form 'Quality q is at point-instant $x; y; z; t$' were, according to his canons, to be apportioned truth values in such a way as to maximize and minimize certain over-all features, and with growth of experience the truth values were to be progressively revised in the same spirit. I think this is a good schematization (deliberately oversimplified, to be sure) of what science really does; but it provides no indication, not even the sketchiest, of how a statement of the form 'Quality q is at $x; y; z; t$' could ever be translated into Carnap's initial language of sense data and logic. *The connective 'is at' remains an added undefined connective*; the canons counsel us in its *use* but not in its *elimination*. (Quine 1951, 37–38; emphasis added)

I am going to examine and evaluate Quine's objection in this paper. Two questions immediately come to the fore. First, an *interpretative* one: is radical reductionism an accurate characterization of the *Aufbau*? Second, an *argumentative* one: does Quine's objection successfully disprove the *Aufbau*, understood as a radical reductionist project? Critics have been predominantly occupied with the *interpretative* question, namely, whether or not the Quine-Goodman radical reductionist reading does full justice to the *Aufbau*. In the past few decades, the growing interest in the *Aufbau* has been revolving around the possibility of endowing it with a non-reductionist appraisal. The *argumentative* question is largely neglected.

This neglect, to a certain extent, is understandable. First, the argumentative question comes after the interpretative question. If radical reductionism is a misinterpretation of the *Aufbau*, then the argument directed against its "radical reductionism" should perhaps be discarded as irrelevant. Second, it is unclear how seriously we should take Quine's objection, given that he retracted it 40 years later in "Two Dogmas in Retrospect". There he says that radical reductionism is merely "a straw man" in "Two Dogmas";[2] the real target is moderate reductionism. This comment dims the importance of this objection in understanding Quine's own position in "Two Dogmas". Furthermore, critics in general agree that should the *Aufbau* be accepted as a radical reductionist project, Quine's objection is spot on. There does not seem to be room for dispute concerning the argumentative question. As Carnap eventually acknowledges, Quine is correct to point out that the method he employed to construct the physical world from individual's sense data "is different from the methods of concept formation used elsewhere in my book" (Carnap 1963, 19). In the Preface to the Second Edition of the *Aufbau*, Carnap writes:

> One of the most important changes is the realization that the reduction of higher level concepts to lower level ones cannot always take the form of *explicit definitions*; generally more liberal forms of concept introduction must be used. Actually, without clearly realizing it, I already went beyond the limits of *explicit definitions* in the construction of the physical world. For example, for the correlation of colors with space-time points, only *general principles*, but no clear *operating rules* were given (§127)[3]. (Carnap 1928/1969, viii; emphasis added)

Yet, the focus of this paper is the *argumentative question*. Although Carnap, in retrospect, came to terms with Quine's criticism, I believe that Quine's criticism misses the target—even if

[2]Notice that in claiming that radical reductionism is a straw man, Quine is not saying that radical reductionism is a misrepresentation of the *Aufbau*'s position, but a misrepresentation of his own target in "Two Dogmas".

[3]The number preceded by the symbol "§" in the bracket refers to the section number in the *Aufbau*. For instance (§127) refers to section 127 of the *Aufbau*.

we agree with Quine that radical reductionism is at the core of the *Aufbau*. Quine has at most established the claim that *those particular canons* adopted in the *Aufbau* fail "in principle" to provide the requisite translation for the physical connective "is at". But in order to refute radical reductionism and clear the ground for his confirmation holism, Quine needs to establish the claim that the *Aufbau itself* fails "in principle" to provide the translation. Quine does not distinguish these two claims.

Moreover, despite the fact that the *Aufbau* might merely be a scapegoat of radical reductionism in "Two Dogmas", it is mistaken to say that Quine did not take the objection seriously. Although he subsequently revised his views towards the analytic-synthetic distinction, as well as the holism he proposed in "Two Dogmas", Quine maintained the same objection to the *Aufbau* throughout his intellectual career. We can find similar complaints in his 1969 "Epistemology Naturalized",[4] his 1995 *From Stimulus to Science*,[5] and even in a short response to Keith Lehrer in 1999,[6] to name a few. Even if Quine's objection does not succeed in disproving the radical reductionism of the *Aufbau*, it highlights the important fact that Carnap's treatment of physical objects in the *Aufbau* deviates from the mechanics set out for the whole construction. This deviation needs an explanation. As we will see below, an adequate explanation requires us to disambiguate the notion of "explicit definition" as well as to clarify the concept of definability in the *Aufbau*. (I will pick up these tasks in Sections 4 and 5 below.) These are key concepts in understanding the lifelong philosophical disputes between the two philosophers.

Last but not least, addressing the *argumentative question* and dissecting Quine's argument will shed new light on the *interpretative question*. As I will show, Quine's objection stands in serious tension with his radical reductionist interpretation of the *Aufbau*. Additionally, by drawing attention to Carnap's later responses to Quine's objection, I want to point out that not only Quine, but also Carnap himself, is puzzled about the objective of the *Aufbau*.

I am going to proceed as follows. In Section 2 I will reformulate Quine's objection into an argument consisting of three different claims. In Section 3 I will briefly describe the objective of the *Aufbau*. Then I will examine Quine's criticism of the *Aufbau* in "Two Dogmas". I will reconstruct Quine's argument as to why the canons or general principles Carnap substitutes for operating rules fail to provide the reductive translation for the connective "is at", which is the last of the three claims I put forward in Section 2. This involves two tasks. First, in Section 4,

[4] In "Epistemology Naturalized", Quine writes (1969, 76–77; original emphasis):

> The fact is, though, that the construction which Carnap outlined in *Der logische Aufbau der Welt* does not give translational reduction either. It would not even if the outline were filled in. the crucial point comes where Carnap is explaining how to assign sense qualities to positions in physical space and time. These assignments are to be made in such a way as to fulfill, as well as possible, certain desiderata which he states, and with growth of experience the assignments are to be revised to suit. This plan, however illuminating, does not offer any key to *translating* the sentences of science into terms of observation, logic, and set theory.

[5] Quine writes (1995, 13):

> Carnap's construction of the sensory domain was strictly by definition, as far as it went, and he gave reason to believe that that job could be completed by further definition on the same basis. For the subsequent construction of the physical world, however, one could not hope to proceed purely by definition; for minimization requires us to go back and reconsider past spatial allocations of qualities in the light of later ones. We are given a canon of procedure, and a brilliant one, but not one that makes the theory of the external world translatable into the language of sense experience. That is too much to ask.

[6] Quine writes (1999, 261):

> Sense data were the traditional epistemological proving ground for natural science. Translation of science in to a language of sense data was accordingly seen as the way to justify science. Hence Carnap's *Logischer Aufbau der Welt*. It provided translation up to a point and then proceeded with counsels for rational reconstruction short of translation. But the reduction of science to sense data fails if the translation cannot be completed: and one sees clearly from the *Aufbau* that it cannot.

I will disambiguate the notion of "explicit definition" (*explizite Definitionen*) in the *Aufbau*; then in Section 5, I will scrutinize the criteria of "explicit definition" in question. In Section 6 I will examine the role of the canons or general principles in the *Aufbau*, with an attempt to vindicate the first and second of the three claims I put forward in Section 2, which I believe Quine has overlooked in his objection. In Section 7 I will conclude with a few remarks on the interpretative question.

2. Breakdown of Quine's Objection: Three Claims

According to Quine, logical empiricism has two untenable assumptions—what he calls the "two dogmas". The first assumption is the belief in the synthetic-analytic distinction. It suggests that while the truths of synthetic statements depend on the states of affairs in the world, the truths of analytic statements are grounded exclusively in their meanings, which are independent of matters of fact. The second assumption is reductionism. It is the conviction that every cognitively significant statement can be reduced to statement about individual's sense experience, and therefore be independently verified or confirmed.

The repudiations of these two dogmas result in respectively two new insights: first, "no statement is immune to revision" (Quine 1951, 40); and second, "our statements about the external world face the tribunal of sense experience not individually but only as a corporate body" (1951, 38). Together the two dogmas entail a more holistic as well as pragmatic epistemology—one that does not posit any unchangeable, sharp boundary between speculative metaphysics and natural science.

Unless one has already taken for granted the truth of the holistic epistemology, which is far from self-evident, Quine's denunciations of the two dogmas should be seen as attempts to justify his proposal of holism, instead of the other way round. The attack on the first dogma is directed at the fact that the synthetic-analytic distinction is explanatorily circular. The notion of "analyticity" has to be expounded in terms of "synonymity", "necessity" or "linguistic rules" which are equally in need of clarification by "analyticity" itself.

Quine attacks the second dogma via attacking Carnap's *Aufbau*. As we can see from the very first passage quoted above (p. 2), Quine believes that Carnap mistakenly entrusts the reduction of physical objects to certain "canons". These "canons" fail "in principle" to deliver the constructional definition necessary for the task (more on constructional definitions in the next section). Quine concludes that since the radical reductionism of the *Aufbau* fails, radical reductionism *per se* also fails.

The assumption in Quine's attack of the second dogma is clear: the *Aufbau* is not only a radical reductionist system, but also the best possible incarnation. This assumption is by no means trivial. It has two parts. First, compared to other radical reductionist systems, the *Aufbau* is the most promising; second, the current *Aufbau* is the best possible version—it cannot be further improved as a radical reductionist system. Since my focus is the *Aufbau* instead of radical reductionism, I will mainly focus on the second part of the assumption. Now if we put together Quine's objection and Carnap's retrospective comments I quoted above (p. 2), then it becomes clear that what has gone wrong in the *Aufbau* is a substitution of "general principles" (or "canons") for "operating rules" in handling the reduction of physical objects.[7]

[7]By "operating rules" here I am referring to operational definitions that allow one to define or translate a scientific predicate to "observable criteria" (Quine 1984, 124). We should pay attention to two points. (1) The "observable criteria" here should be understood as direct experiential or instrumental terms. Any indirect connection between theoretical terms and observational language is not accepted as a satisfactory "operating rule". This is to draw a sharp contrast between "operating rules" and "general principles". This reading complies with Bridgman's Operationism, which has a close connection with logical positivism. The crucial point is whether non-observational, theoretical terms can be fully defined by observational language. (2) These "operating rules" should also be carefully distinguished from the "operational rules" Carnap refers to when he suggests that the constructions in the *Aufbau*

Thus we can reformulate Quine's argument as follows:

(P1) A switch from the use of operating rules to the use of general principles is necessary for the reduction of physical objects.
(P2) The set of general principles adopted by the *Aufbau* is the best possible set of general principles for the reduction of physical objects.
(P3) The set of general principles adopted by the *Aufbau* fails *in principle* for the reduction of physical objects.

(C) The *Aufbau* fails *in principle* (for the reduction of physical objects).

The reformulation incorporates the second part of the assumption I have just mentioned—since Quine is not just claiming that the *Aufbau* fails, but that it fails *in principle*.

And in order to show that the *Aufbau* fails *in principle*, it is necessary for Quine to demonstrate that, within the boundary of radical reductionism, no revision of the *Aufbau* can redeem its fault. Quine has drawn attention to the fact that the particular set of canons Carnap adopts for the *Aufbau* is problematic, i.e., (P3); but this is certainly not sufficient to establish the conclusion that the *Aufbau* fails *in principle*. What Quine needs to prove in addition is that the adoption of *this* particular set of canons is inevitable, and this is what (P1) and (P2) are supposed to justify. All three claims are indispensable.[8]

In what follows, I will first reconstruct Quine's argument for (P3), which is anything but clear in "Two Dogmas". Then, by examining (P1) and (P2), I will show that there are two readings of why a switch from the use of operating rules to general principles is inevitable. However, one way or the other, Quine's objection is uncharitable to the *Aufbau*.

Two disclaimers. First, my rebuttal of Quine's objection does not gamble on the hope that the *Aufbau* may not be the best possible version of radical reductionism, thus a better version may survive Quine's criticism. Again, it is doubtful whether or not the *Aufbau* should be understood as a radical reductionist project (Friedman 1987; Richardson 1998; Creath 1982). My focus is on the *Aufbau* itself, not radical reductionism. By breaking down Quine's objection into three different claims, I attempt to demonstrate that Quine's objection does not give the *Aufbau* a fatal blow. This is because proving that *those* canons adopted in the *Aufbau* necessarily fail the reduction of physical objects does not amount to proving that the *Aufbau itself* necessarily fails the reduction of physical objects, unless we take *those* canons as essential to the *Aufbau*. But if this is the case, then perhaps it would be more reasonable for us to give up the radical reductionist reading, for the switch from "operating rules" to "general principles" is exactly what defies radical reductionism. More will be said on this *interpretative* issue when I discuss (P1) and (P2) in Section 6.

Second, in demonstrating the deficiency of Quine's objection, I do not attempt to rescue the *Aufbau* as a radical reductionist project (or radical reductionism itself). Upon realizing the flaw of radical reductionism, Carnap has already shifted towards a more liberal and holistic approach towards scientific knowledge in the 1930s, not long after the publication of the *Aufbau*. As he points out in the above passage (p. 2), this is because "reduction cannot

are given in four different languages (§§8, 95). The latter, "operational rules", belong to "language of fictitious constructive operations", in which "construction is envisaged as a rule for a constructive operation." These "operational rules" are merely one of the three translations of the proper language of the system, namely, the language of logistics. Friedman (2007, 143–44) points out that such operating rules are the idealistic counterpart of the realistic translation of the constructional rules. They are therefore irrelevant to my discussion of "operating rules" understood as *explicit definitions* (understood narrowly) in this paper. I will give a detailed account of explicit definitions in Section 4.

[8] As we will see later, Quine's argument can actually do without (P2), given that it is the use of general principles *per se* instead of any particular set of general principles that is responsible for the shortcoming of the *Aufbau*. However, as I will also explain later, the reason for Carnap choosing *that* particular set of general principles in the *Aufbau* is crucial for us in understanding why Quine's objection is problematic.

always take the form of explicit definition". Thus it does not seem appropriate to ask, whether Carnap would have worked out the solution within the "limits", if Carnap has tried hard enough. As Quine would say, "[t]hat is too much to ask" (Quine 1995, 13).

Yet, if I am correct that Quine's objection does not fully disprove the *Aufbau*'s radical reductionism, then Carnap's response to the objection may be mistaken as well. As Carnap confesses, he did not realize at the time of the *Aufbau* that he had trespassed the limit of "explicit definition" in the treatment of physical objects. Clearly here Carnap is alluding to the switch from operating rules to general principles. This raises an intriguing question: what did Carnap not realize? Is it the fact that there is a switch of procedures? Or, Carnap actually knew that there is a switch of procedures, but he failed to realize that such a switch is illegitimate in the context of the *Aufbau*, i.e., that it goes beyond the limit of explicit definition? Both Quine's objection and Carnap's response seem to suggest that the switch itself is overlooked; but as I will show towards the end of this paper, textual evidence in the *Aufbau* suggests otherwise. Answering this question will allow us to get a better picture of Carnap's own understanding of the *Aufbau's* objective.

3. The *Aufbau*'s Reductionism

To understand the reductionism of the *Aufbau*, we need to understand its notion of "construction". As Quine lauded on one occasion:[9]

> Russell had talked of deriving the world from experience by logical *construction*, but his *constructions* were sketchy and slight. Carnap, in *Der logische Aufbau der Welt* (1928), set himself to the task in earnest. (Quoted in Creath 1990, 456)

As Carnap admits, the new logic Russell and Whitehead developed allows him to tackle the "questions of epistemology"—"questions of the *reduction* of cognitions to one another" (1928/1969, xvi; emphasis added). The *Aufbau* thus upholds "the supreme maxim in scientific philosophizing" Russell puts forward in "The Relation of Sense-Data to Physics" (1914) and "Logical Atomism" (1924): "Wherever possible, logical constructions are to be substituted for inferred entities".[10] These "inferred entities", according to Carnap, are by and large existing concepts resulting from "unreflected and spontaneous development" (1928/1969, v). By re-constructing them with materials with which we are better acquainted, namely our sense data, we can gain better insight into our cognition and secure a firm ground for scientific knowledge. Thus the epistemic task of constructing the external world aims at its possible "reduction" to our sense data.[11]

While Russell speaks of those "inferred entities" as "logical fictions", we should be careful that the constructional enterprise of the *Aufbau* carries no ontological connotation: what is being reduced is neither unreal nor less "real" than what it is being re-

[9]This line appears in a letter dated 21 January 1963, in which Quine replied to Ernest Moody's request for "a letter giving your estimate of Carnap's life achievement as a philosopher, and of the significance of his work as an influence on the work done in philosophy during the past thirty years", with an eye to have UCLA honor Carnap (Creath 1990, 456).

[10]To what extent Carnap is attempting in the *Aufbau* to answer Russell's problem of the external world is a controversial issue. On one hand, critics such as Friedman (1987) and Richardson (1998) argues that the *Aufbau* should be understood as grounded in neo-Kantian philosophy. In a recent paper, Richardson (2016) suggests that the *Aufbau* is embedded in the German epistemological context of that time, which is best represented by the philosopher and psychologist Karl Gerhards' "Der mathematische Kern der Außenweltshypothese". On the other hand, Pincock (2002; 2009) also rejects the "received view" of Quine and Goodman; but maintains at the same time that the *Aufbau* is a genuine Russellian project—except that we have to be careful about which Russellian project we are talking about. Since my target is Quine's objection to the *Aufbau*, my argument will proceed along the lines of the "received view" of Quine and Goodman.

[11]This is not to suggest that construction is merely analysis or reduction in reverse, or vice versa. For a detailed discussion, see Linsky (2016).

duced to. The reduction is conceptual in nature. When Carnap speaks about the reducibility of "objects", he is focusing on their epistemic aspect. The conviction is that every cognitively significant statement can be reduced to a statement about one's immediate experience, namely the verification condition of the original statement. What cannot be reduced as such, with the exception of analytic statements, is unverifiable and therefore meaningless. This is because although analytic statements are devoid of empirical content, their truth-values are determined exclusively by the meanings of their words. The legitimacy of a concept, object, or statement thus depends on its reducibility. This spells out Carnap's anti-metaphysical stance: debates in metaphysics, for example, between realists and idealists, are meaningless for they essentially boil down to "metaphysical statements" that are neither analytic nor reducible to empirically verifiable statements.

According to Carnap, to say that a can be *constructed* from b and c is to say that a is in principle *reducible* to b and c. And to say that an object a is *reducible* to b and c is to say that all statements about a can be translated into statements exclusively about b and c (§35). Thus to *construct* a from b and c, what we need is a *constructional definition* that ordains how statements in which a occurs should be translated into statements in which b and c have completely replaced a—statements where a is eliminated. In this regard, the *Aufbau* can be viewed as "a system of definitions" (Creath 1982, 386). It translates every linguistic expression standing for higher-order objects to expressions referring only to objects at the most basic level, whether directly or indirectly via a chain of definitions.

In the *Aufbau*, lexicons at the basic level, viz., the autopsychological domain, are not only confined to terms of sense data but also include notations of elementary logic and set theory. Carnap further reduces all the objects of this domain into one single relation, namely *recollection of similarity* (Rs) between *elementary experiences* (*Erlebnisbestandteil* or *erlebs*), which are the primary, "given" "experiences themselves in their totality and undivided unity" taken at an instance (§67). In short, the goal of the *Aufbau* is to rewrite every empirical concept ultimately in terms of Rs.

For Quine, this initial operation of the *Aufbau*, which deals with the construction of objects within the autopsychological domain, is promising. From §108 to §121, Carnap proceeds step by step from Rs to provide constructional definitions for various relations, classes, and concepts in terms of Rs, notations of elementary logic and simple set theory. These constructions include: *part identity*, *similarity circles*, *quality classes*, *senses*, etc.

Once the construction of these objects at the basic level is completed, Carnap proceeds to "project" them outwards to reconstruct the physical world (§§126–27). Quine finds that Carnap, at this "especially important step in the constructional system" (§94), departs from his preceding procedure. Instead of providing any constructional definition as he does for the autopsychological objects, Carnap instructs the construction of the physical world with twelve general principles, or canons. Quine's worry is that these canons are unable to fulfill *Aufbau*'s goal of radical reductionism. More specifically, at this stage Carnap has to assign the two-dimensional sensory quality to the four-dimensional "world points". The corresponding reduction requires Carnap to translate the statement "quality q is at $x; y; z; t$", which is supposed to stand for the state of affairs in the physical world, into statement about one's immediate experience. Now "quality q", as an autopsychological object, has been defined previously;[12] and the quadruple "$x; y; z; t$" is merely basic logistic notation. What Carnap needs to take care of is the connective "is at". But the canons only offer us a general outline of how to assign "quality q" to "$x; y; z; t$"; no definition for the connective "is at" is given. Without the definition, the connective "is at" cannot be eliminated or translated "away"; the whole project of reduction is halted at this step.

[12] *Color*, having been defined in §112, is used for an illustration in the *Aufbau*.

4. "Explicit Definition" vs. "Definition In Use"

In this section I am going to examine (P3), the claim that those canons adopted in the *Aufbau* fail "in principle" to provide an adequate definition for the physical connective "is at". We have just seen that Quine is dissatisfied with the fact that the physical connective "is at" in the statement "quality q is at x, y, z, t" remains undefined. The *Aufbau* provides no "operating rules" (to borrow Carnap's phrase) that inform us how this connective can be translated into the language of sense data and logic.

Here it would be helpful to separate two closely connected questions. First, in what sense the *Aufbau* fails to provide a definition of the connective? And second, in what sense the *Aufbau* fails *in principle* to provide a definition of the connective? The first question requires us to find out what *kind* of definition is missing here. The second question asks us to examine what *criteria* this kind of definition has to satisfy.

To answer the first question, let us again look at the passage from the Preface to the Second Edition of the *Aufbau* I quoted earlier on (p. 2). In confessing, "without clearly realizing it, I already went beyond the limits of *explicit definitions* in the construction of the physical world", Carnap suggests that what we need here is an "explicit definition" for the connective "is at". If we take "explicit definition" as the abbreviations of already understood terms, or in Quine's words "the introduction of novel notations for purposes of sheer abbreviation" (1951, 26), then it is undeniable that the *Aufbau* provides no such term-to-term abbreviation for the connective "is at". Unlike those autopsychological objects such as *The Basic Elements* (*elex*), *Part Similarity* (*Ps*), etc. of which constructional definitions are provided one by one from §108 to §121, there is no "explicit definition" for the connective given in the same manner, viz., in the form "Construction: *is at* $=_{\text{Df}}$ . . . . . ". We find no phenomenal term-to-term translation for the physical connective "is at" in the *Aufbau*.

However, it would be reckless to ask for such an "explicit definition" for the physical connective "is at" here. First, in discussing the second dogma (reductionism), Quine is targeting the presumption that each *statement* can be individually verified or confirmed in isolation. The focus has shifted from term-to-term to statement-to-statement translation. And the latter does not require a term-to-term translation of each meaningful component in the statement. Thus an "explicit definition" in the sense of term-to-term translation is not necessary here. Quine may only be complaining that Carnap does not provide any adequate translation scheme for reducing statements containing the physical connective "is at" into statements containing only phenomenal vocabularies.

Second, and more importantly, a closer look suggests that the notion of "explicit definition" is indeed ambiguous in the *Aufbau*. Clarifying the two different senses of "explicit definitions" as well as their respective roles in the *Aufbau* will allow us to understand in what sense an adequate definition of the connective "is at" is missing.

According to Carnap, two different types of definition are responsible for the task of logical construction in the *Aufbau*. They are *explicit definitions* (*explizite Definitionen*) and *definitions in use* (*Gebrauchsdefinition*). The former define objects by *explicitly indicating* them; the latter deal with "quasi-objects" that cannot be so indicated—they have no meaning in isolation but only *in use* (§38).

Nonetheless, we should be careful that whether an object can be indicated explicitly is not an absolute matter, it depends on what semantic resources are available. In fact, these two different kinds of definition can be given to the same object simultaneously in most cases.[13] The illustrations in the *Aufbau* suggest that whether a definition is *explicit* or *in use* depends ultimately

[13] The exceptions are objects in the most basic or lowest domain. They cannot be given any definition in use because such a definition, as we will soon see, necessarily reduces objects from one domain to objects in a lower domain.

on whether or not the objects being referred to by the defined and defining terms are of the same order. It is only when we use objects of the same order to provide a definition that we can *explicitly* indicate what we try to define. For instance, defining a cardinal number by other numbers and basic mathematical operations is counted as an explicit definition (§39); on the other hand, defining a cardinal number in terms of equinumerosity results in a definition in use (§40).

Consider the cardinal number 5. An explicit definition would be given in the form of "$5 =_{df} 4 + 1$" (together with "$4 =_{df} 3 + 1$", "$3 =_{df} 2 + 1$", "$2 =_{df} 1 + 1$"). This is because objects being referred to in the definition, i.e., objects being referred to on both flanks of "$=_{df}$", are all of the same order—they are numbers and an operation that takes numbers as arguments. At the same time, the cardinal number 5 can also be given a *definition in use* when it is being defined in terms of equinumerous sets. The objects that are presumed to define the number 5 are sets, which occupy a lower order compared to the object being defined, viz., the number 5. In this regard, the number 5 is being seen as a *class* of equinumerous sets, occupying a higher order. This example suggests that in the *Aufbau*, *explicit definitions* are responsible for intra-domain translations, as they aim to identify an object by other objects in the same domain; on the contrary, *definitions in use* are responsible for inter-domain translations, as they intend to map objects in one domain to objects in another domain. In the case of definitions in use, the thing being defined is a "quasi-object" *relative to* the defining objects (§§27, 33, 34). This is because the former is not an object in the strict sense, *from the standpoint of the latter*.

However, as Carnap advises, we should be careful that the distinction between these two types of definition is not always clear.[14] For example, defining prime numbers with numbers and multiplication appears only to be *explicit* due to the grammatical form of the definition. Indeed, it is a case of definition in use (§39). Unlike defining a cardinal number in terms of addition and other numbers as we have seen in the previous paragraph, the term "prime numbers" denotes a *class* of numbers instead of an actual number. Thus it is in the same vein of defining a cardinal number in terms of equinumerosity, where the cardinal number is being regarded as a *class* of equinumerous sets.

It will be helpful to consider Russell's account of definite descriptions as an example of *definition in use*, in order to gain a better understanding of how a definition in use or contextual definition is *syntactically* different from an *explicit definition*.[15] Given its definite article and the grammatical rules, a definite description appears as a genuine logico-grammatical object. However, in Russell's analysis, a definite description "*the F*" is taken as an incomplete symbol. Unlike a logically proper name such as "I" or "this", a definite description does not stand for any object by itself. The term acquires its semantic value only when it is being *used* in the *context* of a complete sentence. Such a sentence, in the form of "*the F is G*", is analyzed as a propositional function:

(f) $\exists x(Fx \,\&\, \forall y(Fy \rightarrow x = y) \,\&\, Gx)$

[14] Coffa avers that Carnap's treatment of contextual definition is "hopelessly confused" (Coffa 1991, 221). He maintains that Carnap's application of "definition in use" has no connection with Russell's conception of "contextual definition"; on the contrary, it aligns with what Russell and we would call "explicit definition". Coffa's reason seems to focus on Carnap's elucidation of the case of understanding the definition of a prime number as "a natural number [that] has only 1 and [itself] as divisors" as a definition in use, despite its explicit definition outlook. However, Coffa overlooks the fact that Carnap indeed intends his notion of "definition in use" to be understood as a kind of "explicit definition"—contra to Russell's understanding of implicit definition (§39). Similarly, Leitgeb (2011) seems to have confused the two types of definition when he suggests that the use of "definition in use" will be allowed in his new *Aufbau*.

[15] Carnap also points out in the *Aufbau* that another case involving *definition in use* would be *relations*—propositional functions having more than one argument place. A relation is thus a quasi-object relative to its relata. And the symbol of a relation will stand for its extension, i.e., a set of ordered pairs (triples, quadruples, quintuples. . .) (§34).

Under this analysis, the definite description becomes a quasi-object. It stands for a class instead of any particular individual when it is being used in a statement, even though in many cases there is only one member in the class. No definitional correspondence of the original term "*the F*" can be singled out in (f), the analysans. In other words, there is no "object" "*the F*" stands for that we can *explicitly indicate*.

Let me introduce some terminologies to illustrate the syntactical difference between explicit definitions and definitions in use. Following Gupta (2015), I analyze a definition in normal form "$X : \ldots X \ldots =_{\mathrm{Df}} -----$" into three different components:

1. The *defined term*: the term being defined, i.e., "X";
2. The *definiendum*: an expression containing the defined term, i.e., "$\ldots X \ldots$";
3. The *definiens*: another expression that is equated by the definition with the definiendum, i.e., "$-----$".

Going back to Russell's theory of definite descriptions, "*the F*" will then be the defined term "X"; "*the F is G*" will be the definiendum "$\ldots X \ldots$"; and its analysans (f), i.e., "$\exists x(Fx \ \& \ \forall y(Fy \rightarrow x = y) \ \& \ Gx)$", will be the definiens "$-----$". Notice that both the definiendum and definiens are unspecified expressions. They belong to the same syntactic category; but what this category is varies from case to case. They may both be sub-sentential expressions or they may both be statements. Moreover, the definiendum may or may not be identical with the defined term "X". In the case of *definitions in use*, both the definiendum and definiens will be statements; and the defined term is embedded in the definiendum, which serves as its context of *use*. In the case of *explicit definitions*, the defined term will be identical with the definiendum.

Let me quickly summarize what I have said so far about the difference between the two types of constructional definition in the *Aufbau*. From a semantic point of view, the difference is whether the object being defined is an object proper, or a quasi-object with reference to the defining objects. From a syntactic point of view, the difference depends on whether the defined term is identical to the definiendum—i.e., whether a synonymous term or a statement of the same meaning is given as definens (precisely what "of the same meaning" means will be discussed later). To reiterate, on the one hand, for *explicit definition proper*, "the new symbol [i.e., the defined term, which is equivalent to the definiendum in this case] is declared to have the same meaning as the compound one. [i.e., the definiens]" (§38). On the other hand, every *definition in use* "indicates that a propositional function which is expressed with the aid of a new symbol means the same as a propositional function which is expressed only with the older symbols" (§40). Both definitions are equally necessary and irreplaceable for the *Aufbau*, since both intra- and inter-domains constructions are needed.

It should now be clear that since autopsychological objects and physical objects essentially belong to two different domains, defining the latter in terms of the former requires *definitions in use* instead of *explicit definitions*. Thus if Quine's complaint with regard to the missing translation of the physical connective "is at" is to be legitimate at all, he can only be read as complaining that Carnap provides no *definition in use* or contextual definition for the connective, rather than *explicit definition*. This is because it is *definition in use* that is required to reduce a higher-order object to lower-order objects (or by the same token, to construct the former from the latter). As Carnap suggests, "*the ascension to a new constructional level takes place always through a definition in use*" (§40; original emphasis).

Furthermore, we should expect that the definition would come in the format of statement-to-statement, instead of a term-to-term phenomenal correspondence that is substitutable for the physical connective in all occurrences without altering the grammaticality of the original statements. *Definition in use* allows the translation to be carried out in the form of "constructional transformation". As Carnap puts it:

> If a is reducible to b, c, then the propositional functions $K, L, \ldots$ about a are coextensive with the propositional functions K', L', which are exclusively about b, c. The *constructional transformation* (i.e., the elimination of the object a with the aid of its constructional definition) consists in the transformation of the propositional functions $K, L, \ldots$ into $K', L' \ldots$. (§50)

However, in the Preface to the Second Edition of the *Aufbau* and a similar passage from his "Intellectual Autobiography" in the Schilpp volume,[16] Carnap seems to suggest that the construction of the physical world from autopsychological objects falls short of the standard of the *Aufbau* precisely because the general principles he adopts fail to provide the requisite "explicit definitions". This goes against the above conclusion that what is crucial at this stage should be *definitions in use* instead of *explicit definitions*.

Here we should be aware that the notion of "explicit definitions" is in fact ambiguous in the *Aufbau*. To save Carnap from a blatant self-inconsistency, here "explicit definitions" should be understood as "explicit definitions in the wider sense" (as contra "implicit definition": §39) instead of "explicit definition proper" (as contra "definition in use") we have been discussing under the guise of "explicit definition" so far.

The ambiguity of "explicit definitions" in the *Aufbau* is puzzling. In fact, the ambiguity survives in his 1934 *The Logical Syntax of Language*.[17] And it is unclear when exactly Carnap began to use the term "contextual definition" to refer to *definition in use* and reserve the term "explicit definition" exclusively for *explicit definition proper*. In his later writings such as *Meaning and Necessity* (1947) and "Meaning Postulates" (1952), Carnap sharply distinguishes the two notions—long before the Preface to the Second Edition as well as the "Intellectual Autobiography" in the Schilpp volume, which were written in 1961 and 1963 respectively.[18]

After all, it should be clear now that "explicit definitions" Carnap refers to retrospectively in these two writings can only be *explicit definitions in the wider sense*, and in particular, *definitions in use*, since he is concerned with defining objects in the physical domain by objects in the autopsychological domain. Moreover, the syntactical difference between these two types of definition also suggests that Quine's objection in "Two Dogmas" is directed correctly at *definitions in use*, for what he asks for is a statement-to-statement instead of term-to-term translation. Quine writes:

> Radical reductionism, conceived now with *statements* as units, sets itself the task of specifying a sense-datum language and showing how to translate the rest of significant discourse, statement by statement, into it. Carnap embarked on this project in the *Aufbau*. (Quine 1951, 36; emphasis added)

5. Definability: Co-extensiveness

Once it is clear that *definition in use* is responsible for the translation of the physical connective "is at", we can proceed to answer the second question, namely, in what sense the *Aufbau* fails *in*

[16] Carnap writes (1963, 19; emphasis added), "As Quine has pointed out correctly, this procedure is different from the methods of concept formation used elsewhere in my book. In general, I introduced concepts by *explicit definitions*, but here the physical concepts were introduced instead on the basis of general principles of correspondence, simplicity, and analogy."

[17] He writes (1934/2001, 24; original emphasis), "To the *explicit definitions*, in the wider sense in which the word is used here, belong both the explicit definitions in the narrower sense—that is to say those where the definiendum consists only of the new symbol . . . —and the so-called definitions *in usu* [sic]—those where the definiendum contains other symbols besides the new symbol . . . "

[18] In his correspondence with Quine, Carnap first used the term "contextual definition" in a letter dated 25 November 1943, replying to Quine's comments on his manuscript of *Meaning and Necessity* in two earlier letters (Creath 1990, 360). My conjecture of Carnap's "officially" adopting the term "contextual definition" has to do with the debate concerning the method of extension and method of intension, with which his correspondences with Quine at that period were occupied. However, a full address of the issue is beyond the scope of this paper.

principle to provide a definition of the connective? This leads us to scrutinize the criteria for successfully defining a term by definition in use. Carnap suggests:

> . . . every *definition in use* indicates that a propositional function which is expressed with the aid of a new symbol means the same as a propositional function which is expressed only with the older symbols. By "same meaning", we mean that both propositional functions are satisfied by the same objects. (§40; emphasis added)

The criterion, broadly speaking, is trivial. Of course, the definiendum and the definiens have to have the same meaning in order for the latter to replace the former. However, there are two components in this criterion.

First, there is an *equivalence requirement*, i.e., that the definiendum and the definiens have to have the same meaning. The sameness of meaning here is to be understood as *co-extensiveness*—both the definiendum and the definiens, as propositional functions, are to be "satisfied by the same objects". Second, there is an *eliminability requirement*, i.e., the defined term (i.e., new symbol, the connective "is at" here) has to be *eliminated* in the definiens, given that the definiens is expressed *only* with older symbols.

Although they are lumped together in the *Aufbau*, it is important to separate these two requirements in assessing the adequacy of a definition (in use). As Belnap (1993, 121) points out, we should not ask for elimination indiscriminately in all contexts, for example, co-extensiveness will allow elimination in an extensional context but it may not allow elimination in an intentional or explanatory ("becausal") context. Thus depending on how the equivalence requirement is understood, we should not expect eliminability outside contexts guaranteed by the equivalence requirement.

Moreover, one can have definability without eliminability, for example, in implicit definitions. If we accept that a logical connective is being defined by its rule of introduction and rule of elimination in a natural deduction system, then we may agree that we have successfully "defined" the logical connective without at the same time knowing how to eliminate it in all occurrences. Therefore, the two requirements are not necessarily tied together.

Going back to Quine's complaint that the canons "counsel us in its *use* but not in its *elimination*", we can conclude that he is particularly concerned with the *eliminability requirement*.[19] However, given that the *Aufbau* is a "system of *definitions*", it seems more appropriate for us to focus on the general notion of *definability* instead of the specific requirement of *eliminability*, since the latter is not necessary for the former. Simply put, even if we can prove that the canons fail the eliminability requirement in their treatment of physical objects, it still does not show that they do not deliver the requisite constructional definition—unless we can demonstrate that eliminability is necessary for a constructional definition. A comprehensive investigation of the relation between eliminability and definability will go beyond the scope of this paper. But in what follows, I will focus on the equivalence requirement instead of the eliminability requirement; such a move is justified in the current discussion for two reasons.

First, for Quine, eliminability is both a necessary and sufficient condition for definability. As he points out on different occasions, that "[t]o define is to eliminate, to excuse, to exonerate. Statements containing the defined term import no content, no risk of error, not already present in statements lacking the term" (Quine 1984, 124). For him, "definitions properly so-called" are "definitions with eliminable definienda" (Quine 1964, 71). Since the equivalence requirement is a necessary condition for definability, showing that the canons' failure in satisfying the latter via a detour to the former will at the same time vindicate Quine's objection against eliminability. Indeed, the two requirements

[19] It should be clear now, by "use" here Quine cannot be referring to the "use" in "*definition in use*", for a definition in use should also satisfy the requirement of eliminability.

are not clearly separated in the *Aufbau* (Carnap 1936, 464). Thus shifting our focus from eliminability to equivalence would not bring us too far from the original intention of both philosophers.

Second, the equivalence requirement is more fundamental than the eliminability requirement. The latter presupposes the former, but not vice versa. Thus if we can successfully prove that the canons are unable to satisfy the equivalence requirement, we will at the same time provide the reason why they are unable to satisfy the eliminability requirement.

Indeed, what is at stake here is equivalence, not eliminability. Richardson in a recent paper suggests that "[i]f we think less linguistically", we can simply read the statement "quality q is at $x; y; z; t$" as a quintuple $\langle q, x, y, z, t\rangle$. In this regard the issue of eliminability of "is at" simply disappears for there is nothing needed to be eliminated in the new notation, as the quintuple "is a set that does not take us beyond the resources of type theory and the autopsychological to form" (Richardson 2016, 5). But this obviously does not resolve the present problem, since what we want to know is whether or not the value assignment to this quintuple ordained by the canons is correct. What we really care is *under what condition* a term can be eliminated in a purported translation, and this has to do with the equivalence requirement.

To understand why the canons fail to meet the equivalence requirement of *co-extensiveness* and fall short of performing the job of an adequate definition in use, let us look at how these canons are supposed to instruct the assignment of the two-dimensional sensory quality to four-dimensional "world points".

As Carnap suggests, "assignments should be in accordance with the experiences as far as possible, there should be a minimum of change in the course of time, and a maximum of regularity" (Carnap 1963, 19). Yet, these rules are at best "principles of correspondence, simplicity and analogy" (Carnap 1963, 19). Unlike any operating rule that provides a definite translation, these canons focus on the maximization/minimization of various features in guiding how sense quality should be assigned to four-dimensional "world points". More specifically, they bring in pragmatic concerns such as state-identity and process-identity (§132). As Quine recaps on another occasion:

> . . . minimization requires us to go back and reconsider past spatial allocations of qualities in the light of later ones. We are given a canon of procedure, and a brilliant one, but not one that makes the theory of the external world translatable into the language of sense experience. (Quine 1995, 13)

Quine's complaint is that this procedure can at most provide us a translation that is open for revision, as he points out, "with growth of experience the truth values were to be progressively revised in the same spirit" (Quine 1951, 37).

However, I do not think that the open-endedness of such a procedure is a real problem here. For why cannot such a procedure lead us eventually to an adequate translation? In saying so of course I am not appealing to a slim hope that perhaps after a sufficiently large number of revisions we may be lucky enough to arrive at the correct translation. Can't we just stipulate one? Indeed the *Aufbau* is "explicative" in spirit. As Carnap affirms, the motivation behind the whole constructional project is to come up with new definitions for the sake of scientific formulation. And these new definitions "should be superior to the old in clarity and exactness, and above all, should fit into a systematic structure of concepts" (Carnap 1928/1969, v).

But let's not allow ourselves to be carried away by the *interpretative* issue at this moment, as we are still dealing with the *argumentative* question, namely whether Quine's original argument does successfully expose the shortcoming of the *radical reductionism* of the *Aufbau*. The real problem here is that appealing to those canons does not even provide us with a single translation that we can subsequently revise and narrow down. In other words, whether we believe that there exists an objective translation to be discovered or we should decide at a certain point that a particular translation would be adequate for the sake

of scientific knowledge, the *Aufbau*'s procedure just gives us too many options to start with. Why is this so?

The answer is clear: given that the overarching guideline in working out the translation is to satisfy various factors as much as possible, then we may have more than one way of balancing these factors when these factors come into conflict. These different translation schemes are equally legitimate, insofar as they all balance these factors to the same extent. Nonetheless, there is no warranty that the translations provided by these different schemes will be co-extensive. Given the equivalence requirement, at most one of these translations is correct—but we have no way to tell which. In other words, these canons, which require us to coordinate various features, do not guarantee a *unique* output. The problem of this procedure is not merely that the resulting translation is subject to future revision, but each time it is likely to produce multiple legitimate but conflicting translations.

Of course, one may point out that positing co-extensiveness as an equivalence requirement is far from satisfactory. Intuitively, co-extensiveness does not guarantee the sameness of "meaning", or more accurately, the sameness of verification condition that is at stake here. Indeed, in the Preface to the Second Edition of the *Aufbau*, Carnap has already suggested that we should strengthen this criterion: the co-extensiveness between the definiendum and the definiens should not be accidental but necessary—either in a logical or nomological sense (1928/1969, ix).

But the problem here has nothing to do with the mismatch between extension and sense. By adopting an extensional method, Carnap does allow statements of different senses to be equally legitimate translations for another propositional function, insofar as they are co-extensive. Carnap has made it clear that what the *Aufbau* aims to preserve is logical significance but not epistemic sense.[20] The fundamental problem in consulting these canons in question is that they cannot even give a unique *co-extensive* translation. Thus strengthening the equivalence requirement will not solve the problem, for it will only result in a standard further unreachable for these canons, which are already too weak to satisfy the original requirement of co-extensiveness.

But what if we weaken the requirement? Can we safeguard the use of these general principles or canons? Goodman argues that co-extensiveness is in fact too strong a requirement for the system of *Aufbau*. As he points out, the *Aufbau* should be understood as "a 'structural description' rather than as a book of synonyms or as a full-color portrait of reality" (Goodman 1963, 555). The equivalence relation in a constructional definition, i.e.. "$=_{Df}$", is therefore to be understood as "is here to be mapped as" (1963, 554) instead of a straight identity that "claim[s] a literal and exclusive truth" between the definiendum and the definiens. Goodman suggests that a more pertinent requirement would be isomorphic equivalence "in a certain specific way" (1963, 556). In other words, perhaps we can stipulate an equivalence requirement that is weaker than co-extensiveness. This suggestion, I believe, concurs with Carnap's intention. And it is conducive to the goal of explication I briefly mentioned above.

However, it is doubtful whether the use of general principles can satisfy even a weaker equivalence requirement. Suppose we only require that a legitimate translation retain a certain semantic value s of the original physical statement. By appealing to principles that aim at maximizing/minimizing certain pragmatic factors, we will once again run into the same situation where different schemes of maximization/minimization are equally legitimate, given that they satisfy the pragmatic consideration to the same degree. Nonetheless, they may deliver translations with different values of s. What is at stake here is whether our translational scheme will always give us a translation of unam-

[20] "Since the construction of an object in the constructional system has always to do only with logical value and not with epistemic value (§50), a constructional definition which employs the indicator of an object and which thus produces a logical translation achieves exactly what we demand of it." (§51)

biguous value. But general principles or canons, which take into account different pragmatic concerns, do not guarantee that this basic requirement can be met.

Yet, there remains the possibility that by adopting a weaker equivalence requirement, Carnap may not need to switch to the use of general principles. Here I cannot undertake a thorough examination of such possibility, but it should be clear now why a leap from operating rules to general principles is an intriguing issue for the *Aufbau*. It is to this question I now turn.

6. General Principles vs. Operating Rules

The discussions in the last two sections show that the use of canons or general principles does not live up to the standard of the *Aufbau*'s radical reductionism. This is because instead of providing a strict statement-to-statement translation, these canons ask us to work out the translation by harmonizing different pragmatic factors. Such a strategy is likely to provide multiple translations, and these translations are not necessarily co-extensive. Given that co-extensiveness is what the *Aufbau* posits as the equivalence requirement for definition in use, the canons are unable to deliver the requisite translation. Hence, these canons fail "in principle" to translate statements about the physical world into statements about one's immediate experience. (P3) is therefore vindicated.[21]

In this section I will examine (P1) and (P2). As I have pointed out earlier, they are claims that Quine overlooks in his objection. (P1) suggests that a switch from operating rules to general principles is necessary; (P2) suggests that the general principles Carnap adopts for the *Aufbau* are the best possible general principles. Without them, (P3) itself is not sufficient to prove that the *Aufbau*, as a radical reductionist project, necessarily fails.

Let me begin with (P2). If we can prove that (P2) is false, i.e., that the general principles Carnap employs in handling the reduction of physical objects are not the best possible options, then maybe we can rescue the *Aufbau* by equipping it with a better set of general principles. However, a closer look at the argument in the previous sections will soon dismiss this optimism.

In fact, the truth of (P3) has nothing to do with the *particular* set of canons Carnap adopts in the *Aufbau*; rather, the failure of the *Aufbau* arises merely from its reliance on a strategy of harmonizing different pragmatic factors to achieve the most satisfactory outcome. Such strategy is simply a common feature associated with the use of general principles, regardless of whether the world one aims to reconstruct is "the laziest" or one with other attributes. Unlike operating rules that lay down definite, strict translation, general principles are concerned with balancing different non-semantic factors. The output of it, as we have seen, may not always be unique and conclusive.

If this is the case, then (P2) becomes redundant, for even if we adopt other general principles, the same problem persists—for formally speaking we will still be engaging in the same procedure of harmonizing various factors, even though what these factors are differ. The problem of Carnap's treatment of physical objects in the *Aufbau* results from a procedure relying on general principles *per se*, rather than any particular canons he adopts for this procedure. In this regard, Quine's objection can be trimmed down to (P1) and (P3) only. What Quine needs to prove is simply that first, a switch from the use of operating rules to the use of general principles is necessary in the *Aufbau* ((P1)); and second, that the use of general principles necessarily fails to deliver the requisite translation ((P3)).

[21] The argument for (P3) should be seen as an independent argument against the second dogma. Quine's own argument against the *Aufbau*'s radical reductionism in "Two Dogmas" is far from substantial. Although Quine suggests that the two dogmas are "at root, identical", the argument I put forward to demonstrate the shortcomings of the general principles in the previous section has nothing to do with the circularity of dubious intensional notions such as "synonymity", "necessity" or "linguistic rules" that Quine relies on in attacking the first dogma. As it is clear that what is at stake here is the incapability of these general principles to satisfy the extensional criterion of the *Aufbau*.

Nevertheless, examining (P2) is crucial for addressing Quine's objection. For without fully appreciating the role of those canons that Carnap actually adopts in the *Aufbau*, it is hard to see whether a switch from operating rules to general principles is justified in the *Aufbau*. This issue, as we will see, is also closely connected with the *interpretative* question of Quine's objection.

In what follows I am going to examine why Carnap prefers the use of this particular set of canons to other alternatives in constructing the physical world from sense data. But this is not to offer a comprehensive comparison of Carnap's own strategy with other possible options. Rather, by drawing our attention to the fact that Carnap is indeed aware that these canons are inferior to the operating rules he offers for the constructions of autopsychological objects, I aim to provide a plausible explanation why Carnap still decides to switch from the operating rules to these canons in the *Aufbau*.

Recall that at this point Carnap is dealing with the assignment of sensory qualities to four-dimensional world points. As Carnap points out, there are two factors affecting the choice of general principles here (§92). First, how the two-dimensional order of the visual field is constructed; second, how, from the two-dimensional order of the visual field, the three-dimensional order of the space of visual things (or the four-dimensional space-time world) is constructed. Let me discuss them in turn.

First, Carnap points out that there is more than one method in deriving the order of the visual field places. This is because the real process of cognition is overdetermined by rational reconstruction, i.e., more than one determination is legitimate in the sense that each would be sufficient by itself. These various determinations also differ in what they assume to be the basic relation for the spatial order of the visual field. The construction Carnap adopts in the *Aufbau* uses only the similarity of the location sign of proximate visual field places (*Propxl*). However, Carnap admits that different assumptions of cognitive psychology will result in different constructional strategies. For instance, one may think that these location signs are not comparable hence no similarity is exhibited; or one may assume that the recognition of such similarity is not completely visual but depends on small motion of the eyes, or kinesthetic sensations of the eye muscles; or one may even take a one-dimensional visual focus as fundamental (§92).

Presumably, these different models of visual field construction will result in different ways of constructing the three-dimensional spatial order of physical reality. For example, the last strategy mentioned in the previous paragraph would allow us to skip the construction of the visual organ, as this construction has practical value only in facilitating the two-dimensional order, which is being constructed rather than taken as elementary in this strategy. The important point here is that once physical reality is completely constructed, the two-dimensionality of the visual field can be interpreted and explained by different aspects of physical reality. All these different strategies are legitimate as they all lead to the construction of the same physical reality, albeit via different routes.

Carnap does compare and contrast his own strategy with other ways of constructing the three-dimensional order of the space of visual things from the two-dimensional order of the visual field (§124). Carnap's own method is to construct the entire four-dimensional space-time world at once. This differs from Gerhards' method which proceeds from the construction of an unchangeable world based on individual aspects; it also departs from Russell's method which first constructs visual things individually based on unperceived *possible aspects*, i.e., *sensibilia*; and it rejects Whitehead's method which constructs space and time after the construction of things. According to Carnap, although "our kind of construction of physical points and of the physical space is by no means a fully satisfactory solution" (§124), his method has the advantage of complying with the Russellian maxim, namely "whenever it is possible, inferred entities should be replaced by construction". This is because instead of

"inferring" the "unperceived points and states of a thing" that is required by the other methods, the *Aufbau* proceeds to actually construct them. As Carnap puts it, "we assign quality classes, as well as components of qualities and more complex structures formed from them, to intermediate time points as well, even though no point of view and no elementary experience corresponds to them" (§132).

So for Carnap, there is more than one way to construct the external world, and they differ in what fundamental assumptions they begin with. Now it is unclear whether these other strategies promise us operating rules or similarly rely on general principles. But let us not go into details here, for my aim is neither to evaluate the pros and cons of Carnap's strategy, nor to judge whether his is better than the others. What we should pay attention to is the fact that Carnap is not ignorant about all these other possible strategies. More importantly, he is certain that his strategy of constructing the external world all at once—both space and objects, instead of step by step—has one decisive advantage: it is the one most faithful to the Russellian maxim of logical construction. And this is why he consults that particular set of canons. For example, with regards to analogy, since the central tasks of constructing objects of the external world include accounting for not only their "real, experienced" aspects but also the possible ones, the projection of phenomenal objects to the physical, external world therefore has to appeal to analogy (of what has been previously constructed) in order to vindicate facts about physical objects when they are not featured in an immediate experience. Carnap also points out in the last canon, that even if we have fulfilled the other eleven canons to the largest possible extent, "[w]e shall later on supplement and correct the assignment" (§126). Thus we have a partial explanation of why Carnap switches from operating rules to general principles: he believes that this strategy is most conducive to the original Russellian spirit behind the whole project.

But this is not the whole story, for one may ask, why does Carnap allow the *Aufbau*'s original plan of providing operating rules to be overridden by this particular strategy that requires him to defect to general principles? To put the question somewhat differently: why does Carnap believe that this thorough construction has to be instructed by general principles, but not operating rules instead?

Here I want to draw attention to the fact that Carnap is aware of the deficiency of this strategy. As he admits, "our kind of construction of physical points and of the physical space *is by no means a fully satisfactory solution*" (§124; emphasis added). In fact, Carnap believes that the current stage of knowledge is not mature enough for us to formulate the operating rules or constructional definitions in the language of logic. Nonetheless, he remains hopeful that this can eventually be accomplished, as he writes:

> However, in the following outline we shall give only the construction of the lower levels in this language. The reason for this does not lie in the fact that the objects of higher type offer particular difficulties of expression for this language, but in the fact that the problem of constructing the higher objects has itself not been solved with precision and that these constructions therefore can be given only in *bold outline*. As soon as the content of the construction of any object is precisely known, *there are no difficulties in the way of a logistic formulation*. (§95; emphasis added)

Thus Carnap clearly knows that the assignments provided by those canons can only be an approximation of the "logistic formulation" he is after in the *Aufbau*. Moreover, he thinks that the other strategies are equally impotent in formulating the operating rules. If so, then we should see the switch from operating rules to general principles as merely a methodological compromise, or a provisional concession, given the imperfection of our current state of knowledge. In other words, Carnap only intends to use these canons for a preparatory proceeding in figuring out the "content of the construction", but not a wholesale replacement of the operating rules. These canons are not constructional

definitions and they are never meant to be. Thus the outcomes of these canons are open for revisions. It would be more reasonable to see these general principles as preliminary measures of constructing physical reality instead of the constructions themselves.

So we have the remaining half of the explanation as to why a switch from operating rules to general principles is necessary: since the problem of the external world has not been solved "with precision", we can only defer the final construction to the future when we have a better insight of how it should be done. At this stage we can only opt for a sketch with reference to some general considerations. Thus it is *necessary* for us to take a *detour* from operating rules and rely for the time being upon these general principles.

If the above interpretation is correct, then Quine's objection is wide of the mark. Granted that Carnap remains faithful to the Russellian goal throughout the *Aufbau*, and given that at the same time he makes explicit that the general principles are not up to the standard of providing the required construction, it is more reasonable to see the switch to these general principles as merely a temporary concession. Let's call this an *instrumental* reading of the general principles—they are intended to sort out the constructions so as to prepare for more rigid, logistic formulations.

Under the *instrumental* reading, it would be unfair to accuse Carnap of providing general principles instead of operating rules. This is because Carnap by no means tries to replace the latter with the former, which are below the standard of radical reductionism. Quine's objection thus becomes a straw man—not in the sense that radical reductionism is not the real agenda here (as Quine later admits in "Two Dogmas in Retrospect"), but precisely because it *is*, for the objection is legitimate only if Carnap really intends to replace *completely* the operating rules with general principles. Yet the instrumental reading suggests otherwise.

But is it possible for us to justify Quine's objection by taking a non-instrumental, or *unconditional* reading of these canons, i.e., that the canons are posited as wholesale substitutes for the operating rules? Given that Carnap fully realizes that these canons are inferior to the operating rules in delivering the requisite task of construction, this would amount to accusing Carnap of deliberately smuggling a faulty procedure into the system. I think it would be equally uncharitable for us to allege that this is what Carnap does—unless we admit at the same time that he is departing from radical reductionism. If Carnap has prepared to replace the operating rules by general principles once and for all, it would be more reasonable for us to see that Carnap is not fully engaging in a radical reductionist project in the *Aufbau*—but an eclectic one ready to embrace at least a moderately holistic epistemology.[22]

[22]Quine does in fact give a fairer account of the *Aufbau* in "Carnap's Postivistic Travail", a paper that is commonly overlooked. In it, Quine suggests that Carnap's construction of physical objects has in fact incorporated Duhem's holistic thesis (though Duhem is nowhere mentioned in the *Aufbau*)—a thesis that he admits he was ignorant of at the time when "Two Dogmas" was first published in 1951, as he admits later in "Two Dogmas in Retrospect" with respect to an added footnote in the 1963's republication of "Two Dogmas". The passage from "Carnap's Positivistic Travail" is worth quoting in length:

> It was a strange one, too, not only because Duhem's point seems so evident, but because Carnap himself recognized it and failed to appreciate its significance. Already in his *Scheinprobleme*, 1928, there is a hint of the Duhemian holism, and that same year in the *Aufbau* the very mechanism of the Duhem effect is strikingly and imaginatively depicted. What I think of in the *Aufbau* is Carnap's account of the assigning of perceived colors to positions in physical space. Direction from the eyes is determined directly by the position of the color in the visual field, but distance from the eye is assigned in the light only of systematic considerations affecting all the assignments together. The guiding principle is the principle of least action: so choose the distances as to minimize the differences of color within short intervals of space and time. This is a very perceptive caricature of the role of simplicity considerations in scientific theory, and it is holistic. It is one of Carnap's deepest insights, and we can only regret that it did not play a fundamental role in his subsequent philosophy. (Quine 1984, 125–26)

7. Conclusion

I have shown above that Quine's objection has at most proven (P3). (P3) indicates no more than the fact that the *particular set of canons* adopted by the *Aufbau* fails to translate statements about the physical world to statements about one's immediate experience. It does not go all the way to show that the *Aufbau itself* fails "in principle" to deliver such statement-to-statement translations. In other words, Quine's objection at most invalidates the "general principles" in fully performing the role of constructional definitions, but it does not show that the missing constructional definitions cannot be eventually worked out. Quine perhaps correctly identifies the source of the problem but he only reveals the limit of *those* "canons", and not the limit of "explicit definitions" as Carnap comes to acknowledge later. This is the result of my investigation into the *argumentative question* of Quine's objection.

In order for Quine's objection to go through, I suggest that we have to address two other claims, namely (P1) and (P2). I argue that in order to see whether the switch to general principles in the *Aufbau* is necessary ((P1)), we need to understand the role Carnap appoints to these principles in the assignment of two-dimensional sensory qualities to four-dimensional "world points" ((P2)).

My examination of (P2) shows that the use of general principles in the *Aufbau* is deeply connected with the Russellian maxim of logical construction, which is at the heart of radical reductionism. On the one hand, the specific way of constructing the external world that Carnap adopts, namely to construct it all at once, is motivated by a wish to comply with the Russellian maxim. On the other hand, logical construction requires rigid formulation, which, according to Carnap, is a remote goal for the present stage of knowledge. It is therefore necessary for him to seek refuge in general principles.

This opens up two different readings of (P1), namely an instrumental one and an unconditional one. The former suggests that a *detour* from operating rules to general principles is necessary to work out the raw materials for the constructional definitions; the latter, on the contrary, claims that a *replacement* of operating rules by general principles is necessary.

If we take the unconditional reading and see the general principles simply as reckless substitutes for constructional definitions, then the *Aufbau* does of course deviate from the goal of radical reductionism in its transition from the autopsychological to the physical domain. However, given that Carnap at the same time realizes that these general principles can at best give us a "bold outline" of the constructions, such a reading is uncharitable—for we should in fact take the switch as a decisive statement that renounces the whole radical reductionist project. But then Quine's objection would be incoherent with his reductionist reading of the *Aufbau*.

On the contrary, if we take these general rules as merely instrumental to formulating the operating rules, Quine's objection is no less uncharitable—since the general principles are not meant to replace the operating rules. Of course one may argue that perhaps the rigid logistic formulations cannot be worked out at last, or appealing to general principles is not the best strategy, but then Quine would owe us an explanation as to why these are the case.

The above considerations regarding the switch from operating rules to general principles in the *Aufbau*, of course, do not settle the *interpretative question* of Quine's objection. But they at least show that Quine's objection against the *Aufbau*'s radical reductionism can only be based on uncharitable readings of the *Aufbau*, one way or the other. The lesson to learn is that the argumentative strength of Quine's objection comes in significant conflict with its interpretative correctness.[23] On the one hand, if

[23] A more thorough critique along the same lines can be found in "Epis-

the *Aufbau* is truly a radical reductionist project relying on constructional definitions, then the use of general principles is not to be blamed—for it is merely a preliminary measure. On the other hand, if the use of general principles is to be blamed, then we have to admit that these general principles play a central role in the whole system—after all, should we still see the *Aufbau* as a radical reductionist project?

This is not to dismiss the insight of Quine's objection. It is undeniable that Quine correctly points out the anomaly in the *Aufbau*. Furthermore, if the Russellian goal is what Carnap really aims at in the *Aufbau*, then the strategy of proceeding from general principles to operating rules never really works in Carnap's favor. As Carnap has shown subsequently in "Testability and Meaning" (1936), "Meaning Postulates" (1952) and "The Methodological Character of Theoretical Concepts" (1956), it is preferable to introduce theoretical terms by "reduction sentences" rather than by explicit definitions. This weaker form of constructional definition provides a more flexible way to accommodate dispositional properties and law-like statements, which call for replacing the standard of verifiability with one of confirmability. This suggests that radical reductionism is no longer appropriate to characterize our empirical science. While Carnap gives philosophical reasons why operating rules should give way to general principles in the enterprise of science, or more precisely, why in addition to explicit definitions, reduction sentences are also needed, the switch here is no longer instrumental—but unconditional.[24]

My investigation leaves several questions unanswered. First, can constructional definitions for physical objects be worked out directly if we adopt Goodman's suggestion and replace the equivalence requirement with one even weaker than co-extensiveness? Second, if the *Aufbau* is really a "system of definitions", then why can we not simply accept these general principles as providing together an implicit definition of the physical connective "is at"? All these questions hinge on the separation of eliminability from definability, and I can only leave them for another occasion.[25]

To end this paper, I want to return to an earlier question: so precisely what Carnap did not realize when he was writing the *Aufbau*? What is he referring to when he confesses in the Preface to the Second Edition that "without realizing it, I have already gone beyond the limit of explicit definition"? Is it the *switch* from operating rules to general principles *itself*, or the *illegitimacy* of such a switch?

temology Naturalized". There Quine contends that while the move from term-to-term explicit definition to statement-to-statement contextual definition is "unassailable" (Quine 1969, 73) within the boundary of reductionism, the switch from operating rules to more liberal measures, such as "reduction sentences" or general principles, betrays the original intention of reduction in the enterprise of epistemology. He further argues that if reduction were not possible, then it would be more reasonable for philosophers to engage in psychological study of actual concept formations instead of the rational reconstruction of concepts that relies on these liberal measures that fail to provide eliminative definitions. This critique is relevant only to the unconditional reading here, for under such reading we take the *Aufbau* as non-reductive.

[24] As Carnap writes in his "Intellectual Autobiography" (1963, 57):

> In addition to the requirement of complete verifiability we must abandon the earlier view that the concepts of science are explicitly definable on the basis of observation concepts; more indirect methods of reduction must be used. For this purpose I proposed a particular form of reduction sentences. In the course of further investigations it became clear that a schema of this simple form cannot suffice to introduce concepts of theoretical science. Still, the proposed simple form of reduction sentences was useful because it exhibited clearly the open character of the scientific concepts, i.e., the fact that their meanings are not completely fixed.

[25] With regard to Goodman's suggestion, my quick reply is that if we give up co-extensiveness as the criterion for the equivalence requirement, it means that we give up at the same time eliminability in extensional contexts. But if eliminability is given up in extensional contexts, it is unclear whether eliminability should still be maintained as a requirement anymore.

Suppose we understand a logical connective as being *defined* by its respective introduction and elimination rules in a system. These rules, being meaning-conferring, do not render the logical connective being defined eliminable.

Let's consider (P1) again, namely the claim that a switch from operating rules to general principles is necessary in the *Aufbau* in constructing the physical world from immediate sensory experience. If we take the instrumental reading of (P1), then it is clear that Carnap is aware of the switch of strategies. More importantly, he does it intentionally and with good methodological reason. Given that he does not intend to replace operating rules with general principles, the switch is therefore not illegitimate at all.

On the contrary, if we take a non-instrumental, unconditional reading of (P1), the move becomes illegitimate from the perspective of radical reductionism. However, Carnap cannot be ignorant about the switch, and more importantly, about the fact that such a switch is deemed to be illegitimate if it is unconditional—for he knows perfectly well that the general principles are only inferior surrogates for operating rules.

Thus it seems that Carnap himself is also unclear about the real objective of the *Aufbau*. To worsen the case, I want to point out that there are passages in the *Aufbau* suggesting that these general principles or canons will ultimately give us a complete construction.[26] Perhaps we should follow Richardson's advice: "we must grant the mismatch between what Carnap says he is doing and what he does at this crucial step" (2016, 7).

[26]For example, in §136, Carnap wrote:

> Even if we consider the color spots alone, the application of this procedure brings the assignments very considerably closer to completion. Further supplementations result from the mutual support of the various senses. Through such supplementations, new things and regularities become known, or old ones become better known; with the aid of this information, further supplementations become possible. Thus, we find mutual advancement between the recognition of general laws which hold for things and processes on one hand, and the supplementation of the assignment of qualities to points in the perceptual world on the other.

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