Philos Stud (2014) 168:619–628
DOI 10.1007/s11098-013-0146-2

The supertask argument against countable additivity

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Published online: 7 June 2013

Abstract This paper proves that certain supertasks constitute counterexamples to countable additivity even in the frame of an objective (not subjective, *à la* de Finetti) conception of probability. The argument requires taking conditional probability as a primitive notion.

Keywords Supertasks · Probability · Countable additivity

1 A first approach

Let us consider a random experiment consisting in tossing a coin (one not tampered with in any way) in the air. Let H be the "heads" event and T the "tails" event. We know that the probability P(H) = ½ = P(T). Let us suppose that we toss the coin an infinite numerable amount of times: in the instants $t_n = 1/n$ with n = 1, 2, 3, 4, ……. In other words, we follow the rule:

> R: coin tossing will take place at the instants $t_n = 1/n$ (with n positive integer) and only at these instants.

Let H_n be the "heads in the instant $t_n = 1/n$" event and T_n the "tails in the instant $t_n = 1/n$" event. Now let us move on to the T_n & H_{n+1} & H_{n+2} & H_{n+3} &… event. The common intuition that its probability is null may be rigorously justified. Given that, in a general manner, P(A&B) = P(A) · P(B/A) when P(A) ≠ 0, we have successively:

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$$P(T_n \,\&\, H_{n+1} \,\&\, H_{n+2} \,\&\, H_{n+3} \,\&\ldots\ldots) = P(T_n) \cdot P(H_{n+1} \,\&\, H_{n+2} \,\&\, H_{n+3} \,\&\ldots\ldots/T_n) \tag{1}$$

$$P(T_n \,\&\, H_{n+1} \,\&\, H_{n+2} \,\&\, H_{n+3} \,\&\ldots\ldots) = P(T_n \,\&\, H_{n+1}) \cdot P\big(H_{n+2} \,\&\, H_{n+3} \,\&\, H_{n+4} \,\&\ldots\ldots/T_n \,\&\, H_{n+1}\big) \tag{2}$$

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$$P(T_n \,\&\, H_{n+1} \,\&\, H_{n+2} \,\&\, H_{n+3} \,\&\ldots\ldots) = P(T_n \,\&\, H_{n+1} \,\&\, H_{n+2} \,\&\ldots\&\, H_{n+m}) \cdot P\big(H_{n+m+1} \,\&\, H_{n+m+2} \,\&\ldots\ldots/T_n \,\&\, H_{n+1} \,\&\, H_{n+2} \,\&\ldots\&\, H_{n+m}\big), \quad \text{with } m \geq 1, \tag{3}$$

as

$$P(T_n) = 1/2 \neq 0, \tag{4a}$$

$$P(H_n) = 1/2, \tag{4b}$$

and, furthermore:

$$P(T_n \,\&\, H_{n+1}) = 1/4 = 1/2^{1+1} \neq 0 \tag{5}$$

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$$P(T_n \,\&\, H_{n+1} \,\&\, H_{n+2} \,\&\ldots\&\, H_{n+m}) = 1/2^{1+m} \neq 0, \quad \text{whatever } m \geq 1 \text{ may be.} \tag{6}$$

But all the conditional probabilities P(B/A) mentioned above are certainly such that P(B/A) $\leq$ 1, which means that

$$P(T_n \,\&\, H_{n+1} \,\&\, H_{n+2} \,\&\, H_{n+3} \,\&\ldots\ldots) \leq 1/2^{1+m}, \quad \text{for all } m \geq 1.$$

As, given an arbitrary event A, P(A) $\geq$ 0, it follows that:

$$P(T_n \,\&\, H_{n+1} \,\&\, H_{n+2} \,\&\, H_{n+3} \,\&\ldots\ldots) = 0, \quad \text{with } n = 1, 2, 3, 4, \ldots\ldots.$$

By E_n I refer to the event T_n & H_{n+1} & H_{n+2} & H_{n+3} &……, so that $P(E_n) = 0$.

By using E_0 to refer to the event H_1 & H_2 & H_3 &……, reasoning in all ways like the one above proves that $P(E_0) = 0$ also.

2 The argument

Finally, and crucially, let us consider that the random experiment is conducted by tossing the coin at random in the instants $t_n = 1/n$ but only until tails comes up, when the coin tossing stops (if tails does not come up in any instant $t_n = 1/n$ the coin will be tossed for the last time in any case in $t_1 = 1/1 = 1$). We shall see below that the previous conclusions $P(E_n) = 0$ and $P(E_0) = 0$ are maintained but now $P(E_0 \vee E_1 \vee E_2 \vee E_3 \vee\ldots) = 1$, thus violating countable additivity.

The procedure for ensuring that the tossing of a coin leads to the violation of numerable additivity may be made more precise by making use of the following explicit rule:

> R*: coin tossing will only take place at the instants $t_n = 1/n$ (with n positive integer) and, furthermore, for any positive integer n, the coin is tossed at $t_n = 1/n$ if and only if no positive integer $m > n$ exists such that the coin has been tossed at $t_m = 1/m$ and has come up tails.

From R* it follows immediately that:

> (A1) If, in a given tossing of the coin, tails comes up, the coin shall not be tossed again. In particular, it is not possible for two tossings of the coin to give tails as a result. If at $t_{n+1} = 1/(n + 1)$ the coin is tossed and comes up heads the coin will be tossed again at $t_n = 1/n$.

> (A2) The coin cannot be tossed only a finite number of times (zero included).

If the coin is first tossed at $t_{n^*} = 1/n^*$ (or even if the coin is not tossed at all for times $t \leq 1/n^*$) then the coin is not tossed at $t_{n^*+m} = 1/(n^* + m)$, with m any positive integer. But then no positive integer $k > n^* + 1$ exists such that the coin has been tossed at $t_k = 1/k$ and, in particular, no positive integer $k > n^* + 1$ exists such that the coin has been tossed at $t_k = 1/k$ and has come up tails. Therefore, by R* it follows that the coin has been tossed at $t_{n^*+1} = 1/(n^* + 1)$, which contradicts the previous statement that the coin has not been tossed at $t_{n^*+m} = 1/(n^* + m)$, with m any positive integer.

> (A3) For any positive integer n, if the coin is not tossed at $t_{n+1} = 1/(n + 1)$ then it will not be tossed at $t_n = 1/n$ either.

If the coin is not tossed at $t_{n+1} = 1/(n + 1)$, by R* some positive integer $m > n + 1$ exists such that the coin has been tossed at $t_m = 1/m$ and has turned up tails. But then, obviously, some positive integer $m > n$ exists such that the coin has been tossed at $t_m = 1/m$ and has come up tails. Therefore, once again by R*, it follows that the coin is not tossed at $t_n = 1/n$.

From (A1), (A2) and (A3) it is immediately clear that the event $E_0 \vee E_1 \vee E_2 \vee E_3 \vee \ldots$ is sure and, therefore, has probability 1. Indeed, (A1) guarantees that if tails comes up, the coin shall not be tossed again (tails will not come up more than once), (A2) guarantees that there will be infinite coin tosses and, together with (A3), guarantees that a single positive integer N exists such that the coin is tossed at all the instants $t_n = 1/n$ with $n \geq N$ and only at the instants $t_n = 1/n$ with $n \geq N$ (as, by R*, coin tossing will only take place at the instants $t_n = 1/n$, with n positive integer). Having proved that $P(E_0 \vee E_1 \vee E_2 \vee E_3 \vee \ldots) = 1$, all that remains is to see that, under rule R*, the conclusions $P(E_n) = 0$ and $P(E_0) = 0$ can still be justified. The proof requires taking conditional probability as a primitive notion (it is possible to develop conditional probability first and then to define non-conditional probability in terms of it[1]) and is based on the following two premises (besides the common assumption that, given an arbitrary event A, $1 \geq P(A) \geq 0$):

[1] And remember that conditional probabilities may be defined for non-null events of zero probability. See, for example, Vickers (1988).

(I) Popper's axiom for primitive conditional probabilities

$$P(A\&B/C) = P(A/B\&C) \cdot P(B/C)$$

(II) For every n, the probability of the coin landing heads/tails at $t_n = 1/n$ (provided that the coin lands heads at $t_m = 1/m$ for all $m > n$) is 1/2. More formally: for every n,

$$P(T_n/H_{n+1} \& H_{n+2} \& H_{n+3} \& \ldots\ldots) = P(H_n/H_{n+1} \& H_{n+2} \& H_{n+3} \& \ldots\ldots) = 1/2$$

The argument is similar to the one given under rule R but, instead of $P(A\&B) = P(A) \cdot P(B/A)$ when $P(A) \neq 0$, we use $P(A\&B) = P(A/B) \cdot P(B)$ without the condition that $P(B) \neq 0$, which follows from (I) by taking the sure event as C (see Footnote 1).

Instead of (1), (2) and (3) we have respectively:

$$P(T_n \& H_{n+1} \& H_{n+2} \& H_{n+3} \& \ldots\ldots) = P(T_n/H_{n+1} \& H_{n+2} \& H_{n+3} \& \ldots\ldots) \cdot P(H_{n+1} \& H_{n+2} \& H_{n+3} \& \ldots\ldots) \quad (1*)$$

$$P(T_n \& H_{n+1} \& H_{n+2} \& H_{n+3} \& \ldots\ldots) = P(T_n \& H_{n+1}/H_{n+2} \& H_{n+3} \& H_{n+4} \& \ldots\ldots) \cdot P(H_{n+2} \& H_{n+3} \& H_{n+4} \& \ldots\ldots) \quad (2*)$$

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$$P(T_n \& H_{n+1} \& H_{n+2} \& H_{n+3} \& \ldots\ldots) = P\big(T_n \& H_{n+1} \& H_{n+2} \& \ldots \& H_{n+m}/H_{n+m+1} \& H_{n+m+2} \& \ldots\ldots\big) \cdot P\big(H_{n+m+1} \& H_{n+m+2} \& \ldots\ldots\big), \quad \text{with } m \geq 1 \quad (3*)$$

And instead of (4a), (4b), (5), (6):

$$P(T_n/H_{n+1} \& H_{n+2} \& H_{n+3} \& \ldots\ldots) = 1/2 \neq 0, \quad (4*a)$$

$$P(H_n/H_{n+1} \& H_{n+2} \& H_{n+3} \& \ldots\ldots) = 1/2, \quad (4*b)$$

(we cannot assume that $P(T_n) = P(H_n) = 1/2$ since there is no guarantee the coin will be flipped at t_n) and, furthermore:

$$P(T_n \& H_{n+1}/H_{n+2} \& H_{n+3} \& H_{n+4} \& \ldots\ldots) = 1/4 = 1/2^{1+1} \neq 0 \quad (5*)$$

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$$P(T_n \& H_{n+1} \& H_{n+2} \& \ldots \& H_{n+m}/H_{n+m+1} \& H_{n+m+2} \& \ldots\ldots) = 1/2^{1+m} \neq 0, \quad (6*)$$

whatever $m \geq 1$ may be.[2]

[2] This result may be deduced easily from (4*a) and (4*b) by using Popper's axiom

$$P(A \& B/C) = P(A/B \& C) \cdot P(B/C)$$

For instance, (5*) (which is (6*) with m = 1) is obtained thus:

But all the non-conditional probabilities P(B) mentioned in the right members of (1*), (2*) and (3*) are certainly such that $P(B) \leq 1$, which means that

$$P(T_n \,\&\, H_{n+1} \,\&\, H_{n+2} \,\&\, H_{n+3} \,\&\ldots\ldots) \leq 1/2^{1+m}, \quad \text{for all } m \geq 1.$$

As, given an arbitrary event A, $P(A) \geq 0$, it follows that:

$$P(T_n \,\&\, H_{n+1} \,\&\, H_{n+2} \,\&\, H_{n+3} \,\&\ldots\ldots) = 0, \quad \text{with } n = 1, 2, 3, 4, \ldots\ldots.$$

so that $P(E_n) = 0$. And reasoning in all ways as above proves that $P(E_0) = 0$ also.

The classic argument against countable additivity involves drawing a positive integer at random:

> …for if the probability of selecting, e.g., 1 is 0, then due to uniformity, the probability of choosing any other natural number is also 0. Thus sigma-[countable] additivity leads to a contradiction because the probability of choosing a natural number is 1 and not 0. On the other hand, if the probability of choosing 1 is positive then sigma-[countable] additivity leads again to a contradiction (the probability of the certain event would be infinite) (Székely 1986, p. 177).

However, Howson and Urbach (1993, p. 81) say about this:

> it is not at all clear what selecting an integer at random could possibly amount to: any actual process would inevitably be biased toward the "front end" of the sequence of positive integers.

Bartha (2004, pp. 311–312) puts it quite clearly:

> If, so far as all knowledge that could influence our assessment of likelihood goes, there is no basis for distinguishing between two sets of outcomes……, then we are inclined to regard them as equiprobable……… Obviously, we are appealing here to a version of the Principle of Indifference

In view of what has been said until now, the advantages of my proof of the failure of countable additivity are clear. To begin with, it is an example of physical chance and not of subjective probability, which helps to attenuate any discrepancy concerning the initial assignation of probabilities. Furthermore it does not depend on any selection made at random from an infinite population whose probability is founded on the principle of indifference. Evidently, an event is selected randomly from the numerable infinite set of events $\{E_0, E_1, E_2, E_3, \ldots\}$,[3] but, rather than being based on any principle of indifference, the equiprobability of these is based on the actual calculation of probabilities, as we have seen above. We may consider the

Footnote 2 continued

$$\begin{aligned} P(T_n \,\&\, H_{n+1}/H_{n+2} \,\&\, H_{n+3} \,\&\, H_{n+4} \,\&\ldots\ldots) &= P(T_n/H_{n+1} \,\&\, H_{n+2} \,\&\, H_{n+3} \,\&\, H_{n+4} \,\&\ldots\ldots) \\ &\quad \cdot P(H_{n+1}/H_{n+2} \,\&\, H_{n+3} \,\&\, H_{n+4} \,\&\ldots\ldots) \\ &= (1/2) \cdot (1/2) = 1/4. \end{aligned}$$

[3] This clearly implies selecting a non-negative integer at random and therefore provides God with a procedure to perform His lottery (see McCall and Armstrong, 1989).

assumed values $P(H) = ½ = P(T)$ as being based in turn on a prior available experience and, in any case, it is clear that my argument works just as well for any non-null values of P(H) and P(T) ($0 < P(H) < 1$, $0 < P(T) < 1$, $P(H) + P(T) = 1$).

The player who decides to toss the coin in the instants $t_n = 1/n$ (n = 1, 2, 3, 4,.......) with the condition of stopping tossing the coin the moment tails comes up may bet safely on obtaining an infinite, uninterrupted sequence of heads. He will win the bet despite that fact that $P(H) = ½$ and that, of course, every single tossing of the coin is independent from the others.[4]

My argument against countable additivity works only as a consequence of a supertask, but not any supertask will lead to my result. The random experiment consisting in tossing the coin randomly a numerable infinite amount of times in the instants $t_n = 1/n$ with n = 1, 2, 3, 4,....... (following rule R enunciated above) does not imply any violation whatsoever of countable additivity if no additional conditions are imposed.

Finally, it is interesting to note the parallels between the situation considered in my supertask (which violates numerable additivity) and the one that appears in the St. Petersburg Paradox: in both cases the random experiment is conducted by tossing the coin at random in the instants t_n (with n = 1, 2, 3, 4,.......) but only until tails comes up. The difference lies solely in the ordinal type of the set of instants t_n. In my case it is an infinite ordinal type ω* (as $t_n = 1/n$) while in the St Petersburg Paradox it is an infinite ordinal type ω (for example, $t_n = -1/n$). Thus, the situation studied in the St. Petersburg Paradox can be obtained by following a rule R** that comes directly from R* simply by changing n for -n and m for -m in all its mathematical expressions. Indeed:

> R**: coin tossing will only take place at the instants $t_n = -1/n$ (with n positive integer) and, furthermore, for any positive integer n, the coin is tossed at $t_n = -1/n$ if and only if no positive integer m exists, with $-m > -n$, such that the coin has been tossed at $t_m = -1/m$ and has come up tails.

In other words, bearing in mind that $-m > -n$ if and only if $m < n$:

> R**: coin tossing will only take place at the instants $t_n = -1/n$ (with n positive integer) and, furthermore, for any positive integer n, the coin is tossed at $t_n = -1/n$ if and only if no positive integer $m < n$ exists such that the coin has been tossed at $t_m = -1/m$ and has come up tails.

There are parallels too with the Benardete Paradox of the Gods and its variants (Benardete 1964), of which it may be considered a probabilistic version (although I shall not develop the point here, see below). This network of relations with problems well known in the philosophical literature today is, in my view at least, an additional argument in favour of the intrinsic interest of the supertask argument against countable additivity.

[4] This surprising result has a certain "family likeness" to the one presented in Bacon (2011). Both use crucially the peculiarities of ω* ordinal-type supertasks but Bacon's example is compatible with countable additivity.

3 Replies to possible criticisms

(1) Someone might say that, although following the rule R* is not logically impossible, it is probabilistically impossible because it implies an event occurring (some infinite and uninterrupted sequence of heads) that has probability 0, i.e. that it is probabilistically impossible. But this is not a good argument. Let us imagine that, instead of rule R*, rule R is used. Following R also implies the occurrence of an event (some infinite and uninterrupted sequence of heads and/or tails) which has probability 0, i.e. that it is probabilistically impossible. But from here nobody would deduce that following R is probabilistically impossible.[5]
(2) A second criticism might state that following R* is logically impossible because such a procedure makes certain the appearance of an event belonging to an infinite numerable set of events, each with null probability a priori. But this is tantamount to saying that following R* is logically impossible because it violates numerable additivity. This is clearly a circular argument in that it seeks to defend numerable additivity by assuming to begin with that numerable additivity is true.
(3) A third criticism would involve transferring to the supertask argument against countable additivity some of the standard criticisms made of Benardete's Paradox of the Gods, in which each god_n of a numerable infinity of gods controls with a barrier a point x_n (where $x_m < x_n$ if and only if $m > n$) such that god_n will immobilize a man at x_n if and only if no god_m (with $m > n$) does so before at x_m and the man arrives at x_n (see more details in Benardete (1964)). The criticism would be founded on the parallel, mentioned above, between my argument and the paradox referred to. Talking of the latter, Yablo (2000, p. 151) for instance says:

> If there's a paradox here, it lies in the difficulty of combining individually operational subsystems into an operational system. But is this any more puzzling than the fact that although I can pick a number larger than whatever number you pick, and vice versa, we can't be combined into a system producing two numbers each larger than the other?

To see how this diagnosis would apply to our case, let us rewrite rule R* supposing that if the coin is tossed at $t_n = 1/n$ it will be tossed by god_n:

> R*: coin tossing will only take place at the instants $t_n = 1/n$ (with n positive integer) and, furthermore, for any positive integer n, the coin is tossed by god_n at $t_n = 1/n$ if and only if no positive integer $m > n$ exists such that the coin has been tossed by god_m at $t_m = 1/m$ and has come up tails.

The difficulty Yablo mentions would now run thus: although a god_n may follow the "individually operational subsystem" consisting in "tossing the coin at $t_n = 1/n$ if

[5] One might attempt to justify that R is even logically impossible by arguing that the ω*-type supertask on which it is based is too. In this paper I assume from the beginning the logical possibility of ω*-type supertasks: my intention here is not to discuss the status of supertasks as such but rather to explore some of their consequences.

and only if no positive integer $m > n$ exists such that the coin has been tossed by god_m at $t_m = 1/m$ and has come up tails", the gods cannot combine such subsystems in the "operational system" defined by rule R*. But what reasons have we for saying that R* cannot be followed? In the Benardete Paradox, if the gods perform their plan then a force F (not exerted by any barrier) should immobilize the man. Yablo (and other authors, including, for instance, Shackel (2005)) consider that such a force is physically impossible under the conditions of the situation and from there deduce that it is impossible for the gods to be able to perform their plan. In our case, similarly, Yablo's strategy would mean denying the possibility of following rule R* on the basis of considering (in some sense) the violation of numerable additivity impossible (at least in the form in which R* proposes to do so). However, in the absence of an argument that provides some support to this basis there would seem to be no alternative than to reject R* on the grounds of the rejection of its supposed violation of numerable additivity, which leads us to a criticism similar to the one already dealt with in my reply (2).

(4) Another criticism could arise from considering the actual process of tossing the coin. It would seem that, as, in particular, the infinite sequences of coin tossings coming up tails infinite times are excluded, there should be some type of strange influence that assures such an exclusion. The response is that no such influence is required: following R* is, as we saw above, sufficient to guarantee that there will be no more than one tails. There is no way of guaranteeing this with a sequence of coin tossings of ordinal type ω (and the present criticism is based on this intuition) but it may be done with a sequence of ordinal type ω^*: by operating in accordance with R*, sequences with more than one tails are impossible. In my view, infinite numerable sequences of events with ordinal type ω^* are a new and interesting area of application of probability theory, the area of the supertask argument against countable additivity.

(5) When specifically considering the physical characteristics of the process of tossing the coin, someone might object that the coin must move increasingly quickly (without limit) as the tossings come closer to $t = 0$, with the consequence of inevitable discontinuities in both its position and its velocity. This observation is true, but we may avoid it, if we wish, by considering a numerable infinity of coins M_1, M_2, M_3, … (of a size sufficiently decreasing with n) such that the coin tossed at $t_n = 1/n$ (should it be required, in accordance with R*) is M_n.

(6) One last criticism could take the following form. Let us assume that the coin I use to follow R* is such that (without me knowing this beforehand) both sides are tails. From (A1) and (A2) it follows that complying with R* is logically impossible in such a case and that therefore I will fail in my attempt to comply with it. Likewise I would fail (the critic would say) when I attempt to observe R* with a normal coin. But the fact that compliance with R* is impossible with a coin with tails on both sides does not imply that it must be impossible with a normal coin (with heads and tails). By way of example, let us consider that I have available a box containing a normal coin. The idea is that every time I take the coin out of the box I shall place it immediately, in principle voluntarily, either with the heads side up or with the tails side up (before returning it to the box). We

also assume that if I take the coin at $t_{n+1} = 1/(n + 1)$ (with n a positive integer) I put it back in the box before $t_n = 1/n$. Let it be that I decide to observe the following rule:

> R^: the coin will only be taken out of the box at the instants $t_n = 1/n$ (with n a positive integer) and, furthermore, for any positive integer n, the coin is taken out of the box at $t_n = 1/n$ if and only if no positive integer $m > n$ exists such that the coin has been taken out of the box at $t_m = 1/m$ and has been immediately placed tails up.

Analogously to what we have seen with regard to R* in the previous section, it is evident from R^ that:

(A1^) If after a given extraction of the coin from the box it has immediately been placed tails up, the coin will not be taken out of the box again. In particular, the coin could not have been taken out of the box twice, where immediately after each extraction it has been placed tails up. If at $t_{n+1} = 1/(n + 1)$ the coin has been taken out of the box and has immediately been placed heads up then it will be taken out again at $t_n = 1/n$.

(A2^) There cannot only be a finite number (zero included) of extractions of the coin.

(A3^) For any positive integer n, if the coin is not taken out at $t_{n+1} = 1/(n + 1)$ then it is not taken out at $t_n = 1/n$ either.

Let us now suppose that the coin I use to follow R^ is such that (without me knowing beforehand) both sides are tails. From (A1^) and (A2^) it follows that complying with R^ is logically impossible in such a case and that therefore I will fail in my attempt to comply with it. But the fact that complying with R^ is impossible with a coin that has two sides showing tails does not imply that it must also be impossible with a normal coin (with heads and tails). Indeed it is evident that it is not: I could, for instance, take the coin out of the box at all instants $t_n = 1/n$ (with n a positive integer) and, in each case, place the coin heads up immediately.

Acknowledgments I would like to thank an anonymous referee for some very helpful suggestions in the writing of this paper. Research for this work is part of the research project FFI2012-33550 (funded by the Spanish Ministry of Science and Innovation).

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