

Number and Natural Language

One of the most important abilities we have as humans is the ability to think about number. Without it, modern economic life would be impossible, science would never have developed, and the complex technology that surrounds us would not exist. Though the full range of human numerical abilities is vast, the positive integers are arguably foundational to the rest of numerical cognition, and they will be our focus here. Many theorists have noted that although animals can represent quantity in some respects, they are unable to represent precise integer values. There has been much speculation about why this is so, but a common answer is that it is because animals lack another characteristic feature of human minds—natural language.

In this chapter, we examine the question of whether there is an essential connection between language and number, while looking more broadly at some of the potential innate precursors to the acquisition of the positive integers. A full treatment of this topic would require an extensive review of the empirical literature, something we do not have space for. Instead, we intend to concentrate on the theoretical question of how language may figure in an account of the ontogeny of the positive integers. Despite the trend in developmental psychology to suppose that it does, there are actually few detailed accounts on offer. We'll examine two exceptions, two theories that give natural language a prominent role to play and that represent the state of the art in the study of mathematical cognition. The first is owing to C. R. Gallistel, Rochel Gelman, and their colleagues; the second to Elizabeth Spelke and her colleagues. Both accounts are rich and innovative, and their proponents have made fundamental contributions to the psychological study of number. Nonetheless, we will argue that both accounts face a range of serious objections and that, in particular, their appeal to

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language isn't helpful. Of course, this isn't enough to show that the acquisition of number doesn't depend on natural language. But it does raise the very real possibility that, although language and number are both distinctively human achievements, there is no intrinsic link between the two.

1 Gallistel and Gelman

We will begin with Gallistel and Gelman's treatment of the positive integers. As they see it, the power of language stems from the way it interacts with an innate and evolutionarily ancient system known as the *Accumulator*. Before explaining their theory, it will help to have a basic understanding of what this system is and how it is motivated.

1.1 *The Accumulator*

Much of the motivation for the Accumulator derives from the study of nonhuman animals (for a review, see Gallistel, 1990). It turns out that many animal species are able to selectively respond to numerosity (that is, numerical quantity) as such, though not, it seems, to precise numerosity. For example, in one experimental design, a rat is required to press a lever a certain number of times before entering a feeding area to receive food. The rat can press more than the correct number of times, but if it enters the feeding area early it receives a penalty. On experiments of this sort, rats were shown to respond appropriately to numbers as high as 24 (Platt & Johnson, 1971; see also Mechner, 1958). While they don't reliably execute the precise number of required presses, they do get the approximate number correct, and their behavior exhibits a predictable pattern. First, they tend to overshoot the target, pressing a few more times than necessary rather than incurring the penalty. Second, and more important, their range of variation widens as the target number of presses increases (see fig. 13.1).

What makes this data interesting is that it looks as though the rats really are responding to numerosity rather than some closely related variable, such as duration. In a related experiment, Mechner and Gueverkian (1962) were able to control for duration by varying the hunger levels of their subjects. They found that hungrier rats would press the lever faster but with no effect on the number of presses. So the rats weren't simply pressing for a particular amount of time. Moreover, rats are equally good with different modalities (e.g., responding to numbers of lights or tones) and can even combine stimuli in two different modalities (Meck & Church, 1983). In short, the evidence strongly points in the direction that rats are able to respond to number; they just don't have precise numerical abilities.

Related studies with pigeons suggest that animals can respond to even larger numbers and that their discriminative capacity, though not as precise as the positive integers, is surprisingly fine grained. In these experiments, pigeons face a panel with three buttons and have the task of pecking the center button while it is illuminated. The experimenter controls things so that the illumination ceases after either 50 pecks or some other specified number, n . If the pigeon ends up pecking 50 times, it is supposed to peck the right button next, but if it pecks n times, then it is supposed to peck the left button next. Under these conditions, whether the

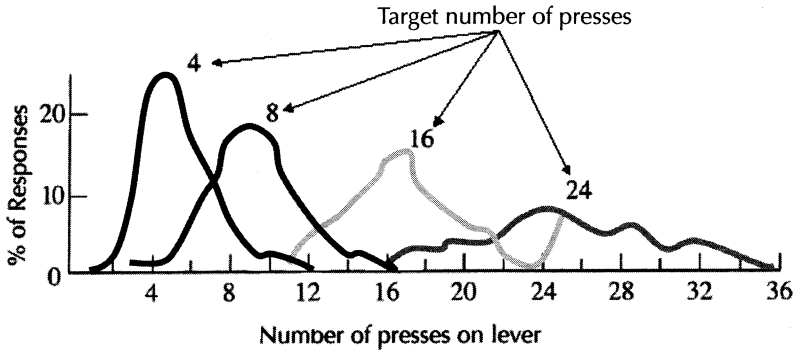


FIGURE 13.1 Data from Platt & Johnson's Experiments. In Platt & Johnson's experiments, rats were required to press a lever a certain number of times before moving to a feeding area. As the target number of presses increases, the range of variation in the number of presses widens (adapted from Platt & Johnson, 1971).

pigeons are able to reliably peck on the left or the right in appropriate circumstances indicates whether they are able to discriminate n from 50. Rilling and McDiarmid (1965) found that pigeons are able to correctly discriminate 40 from 50 90 percent of the time and 47 from 50 60 percent of the time.

The data from these sorts of experiments conform to two principles—the *magnitude effect* and the *distance effect* (see Dehaene, 1997).

The Magnitude Effect

Performance for discriminating numerosities separated by an equal amount declines as the quantities increase. For instance, it's harder to tell 10 from 12 than to tell 2 from 4, even though the difference between the two pairs is the same.

The Distance Effect

Performance for discriminating two numerosities declines as the distance between the two decreases. For instance, it's harder to tell 3 from 4 than to tell 3 from 8.

Together these principles illuminatingly describe the approximate character of animals' numerical abilities.

Gallistel and Gelman, following others, posit the existence of the Accumulator to explain the animals' pattern of results (Gallistel, 1990; Gallistel & Gelman, 2000). As we'll see, the interpretation of this system is a matter of some disagreement, and Gallistel and Gelman have their own peculiar way of understanding it. What is widely agreed upon, however, is that the Accumulator represents numerosity via a system of mental magnitudes. In other words, instead of using discrete symbols, the Accumulator employs representations couched in terms of a continuous variable.

Gallistel and Gelman employ an analogy to convey how the Accumulator works (Gallistel & Gelman, 2000; Gallistel et al., forthcoming). Imagine water being

poured into a beaker one cupful at a time and one cupful per item to be enumerated.¹ The resulting water level (a continuous variable) would provide a representation of the numerosity of the set: the higher the water level, the more numerous the set. Moreover, with an additional beaker, the system would have a natural mechanism for comparing the numerosities of different sets. The set whose beaker has the higher water level is the larger set. Similarly, the Accumulator could be augmented to support simple arithmetic operations. Addition could be implemented by having two beakers transfer their contents to a common store. The level in the common store would then represent their sum.

The Accumulator's variability has several possible sources. One is an inaccuracy in the measuring cup. Perhaps slightly more or less than a cupful gets into the beaker on any given pouring. Another possibility is that the beakers are unstable. Perhaps water sloshes around once inside them. In any event, the suggestion is that the variability is cumulative, so that the higher the water level, the greater the variability. This would explain why a system along these lines is only approximate and why pairs of numbers separated by equal distances are harder to distinguish as the numbers get larger.

Gallistel and Gelman make a good case for the importance of the Accumulator in accounting for the numerical abilities of nonhuman animals. But, as they note, rats and pigeons aren't the only ones who employ approximate representations of numerosity (Gallistel & Gelman, 2000). Humans do as well, and this suggests that humans have the Accumulator as part of their cognitive equipment too. In an important recent study, Fei Xu and Elizabeth Spelke set out to test the view held by many psychologists that preverbal infants aren't capable of discriminating numerosities beyond the range of one to three (Xu & Spelke, 2000). They presented six-month-old infants with displays of dots. One group of infants saw various displays of 8 dots while the other group saw displays of 16. After reaching habituation (i.e., a substantial decrease in looking time), both groups of infants were shown novel displays of both 8 and 16 dots and their looking times were measured (see fig. 13.2). In both the habituation phase and the test phase, Xu and Spelke were extremely careful to control for features of the stimuli that correlate with numerosity—display size, element size, stimulus density, contour length, and average brightness. What Xu and Spelke found was that the infants who were habituated to one numerosity recovered significantly more to the novel numerosity, indicating that they *are* able to distinguish 8 from 16 after all. However, infants under the same experimental conditions showed no sign of being able to discriminate 8 from 12. Xu and Spelke's conclusion was that infants at this age can discriminate between large sets of differing numerosity "provided the ratio of difference between the sets is large" (p. 87). Within the framework of the

1. Put without the analogy, the model maintains that a fixed amount of energy is stored for each item enumerated and that the process is iterative in that only one unit is stored at a time. However, a major point of disagreement among defenders of the Accumulator is whether the process is in fact iterative. For a noniterative model, see Church and Broadbent (1990). Another point of disagreement worth mentioning is whether one and the same mechanism—the Accumulator—underlies both numerical and temporal discriminations. Gallistel and Gelman maintain that the Accumulator, functioning in different modes, underlies both types of discriminative ability.

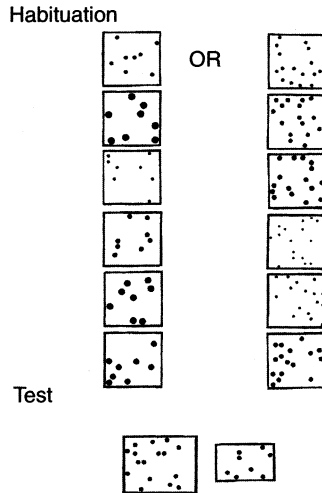


FIGURE 13.2 Sample stimuli from Xu & Spelke's experiments. In Xu & Spelke's experiments 6-month-old infants were habituated to displays of either 8 dots or 16 dots. In the testing phase they were shown new displays with both 8 and 16 dots. The infants dishabituated more to displays with the novel numerosity, indicating that they were able to discriminate 8 from 16 (Xu & Spelke, 2000). Reprinted from *Cognition*, vol. 74, no. 1, F. Xu et al., "Large Number Discrimination in 6-Month-Old Infants," p. B5, copyright 2000, with permission from Elsevier.

Accumulator model, this all makes sense. Like the rats and pigeons, infants are able to discriminate some numerosities from others. It's just that their Accumulator isn't fully developed and so isn't as sensitive as the one found in (mature) rats and pigeons.

Evidence for the accumulator can also be found in adult humans. For example, Whalen, Gallistel, and Gelman (1999) gave adults tasks comparable to the ones previously given to rats. In one of their experiments, adults had to respond to a displayed numeral by tapping a key the corresponding number of times as rapidly as possible. The speed of the tapping ensured that the subjects couldn't use subvocal counting, and Whalen and colleagues were able to rule out a reliance on duration as well. The results were that Whalen et al.'s subjects performed in much the same way as Platt and Johnson's rats. Their responses were approximately correct, with the range of key presses increasing as the target numbers increased. The conclusion Whalen et al. drew was that adults employ "a representation that is qualitatively and quantitatively similar to that found in animals" (p. 134).²

So there is substantial evidence for the existence of an innate number-specific system of representation that provides humans and animals with an ability to respond to approximate numerosity by means of a system of mental magnitudes. This system explains the distance and magnitude effects and a wealth of experimental results (of which we have only been able to present a small sample here). Though the Accumulator's

2. For further evidence concerning the Accumulator's role in adult human cognition, see Dehaene (1997) and Barth, Kanwisher, and Spelke (2003).

representational resources may seem rather crude compared to the concepts for the positive integers, Gallistel and Gelman's position is that they form the basis for how we acquire the positive integers. We are now in a position to turn to their theory.

1.2 *The Theory: Getting the Integers from the Reals*

Psychologists typically assume that the positive integers form our most basic system of precise numerical representation. Systems incorporating zero, negative integers, fractions, real numbers, and so on are thought to be cultural inventions. Indeed, the cultural origin of many of these systems is taken to be part of the historical record.

Gallistel and Gelman's theory boldly challenges this conventional wisdom. As they see it, the Accumulator plays a foundational role in the acquisition of the positive integers. But they offer a distinctive interpretation of the Accumulator and what its states represent that provides the point of departure for a truly radical account of the relationship between the integers and the reals. For Gallistel and Gelman, it's the *reals*, not the integers, that are the more basic:³

We suggest that it is the system of real numbers that is the psychologically primitive system, both in the phylogenetic and the ontogenetic sense. (Gallistel et al., forthcoming, p. 1)

Our thesis is that this cultural creation of the real numbers was a Platonic rediscovery of the underlying nonverbal system of arithmetic reasoning. The cultural history of the number concept is the history of learning to talk coherently about a system of reasoning with real numbers that predates our ability to talk, both phylogenetically and ontogenetically. (Gallistel et al., forthcoming, p. 3)

For Gallistel and Gelman, the integers are a psychological achievement but one that occurs only against the background of representational resources that most others take to be a *far greater* psychological achievement.

On the standard interpretation of the Accumulator, its representations are of approximate numerosity (see, e.g., Carey, 2001; Dehaene, 1997). They represent, in Elizabeth Spelke and Sanna Tsivkin's useful phrase, "a blur on the number line" (2001, p. 85). Instead of picking out 17 (and just 17), an Accumulator-based representation indeterminately represents a range of numbers in the general vicinity of 17. A good deal of the evidence in favor of this interpretation—and likewise, a good deal of evidence in favor of the Accumulator—comes from the variability in animal and human performance under a variety of task conditions. But Gallistel and Gelman have a different take on this variability. Their interpretation is that it traces back to problems with memory. "[T]he reading of a mental magnitude is a noisy process, and the noise is proportional to magnitude being read" (forthcoming, p. 5). That is, the accumulator represents precise numerosities that are systematically distorted when stored and retrieved. Mental magnitudes, as they see it, aren't approximate. It's the processes that are defined over them that make them seem as

3. See also Gallistel and Gelman (2000) and Gelman and Cordes (2001).

if they are. How precise are the representations that feed into memory? Gallistel and Gelman's answer is that they are extremely precise, that mental magnitudes by their very nature are so fine grained as to represent the real numbers.⁴

Given this understanding of the Accumulator, arriving at representations of the positive integers is not a matter of trying to make precise the approximate representations used by the Accumulator. The representations in the Accumulator are already perfectly precise; in fact, precise representations of all the positive integers are already present in the Accumulator, since the positive integers are a subset of the reals. What's needed is some way to pick out the positive integers from among the reals. This is where Gallistel and Gelman appeal to natural language.

One of Gallistel and Gelman's major contributions to the study of numerical cognition is the characterization of a set of principles whose mastery is constitutive of learning to count. There are four principles in all (see Gelman & Gallistel, 1978):

Gelman and Gallistel's Counting Principles

1. *The one-one principle*: one and only one tag is to be used for each item in a count.
2. *The stable-order principle*: the tags used in counting must be applied in a fixed order.
3. *The cardinal principle*: the final tag in a count gives the cardinality of the set of items being counted.
4. *The abstraction principle*: principles 1–3 apply to any collection of entities; in other words, there is no restriction on the sorts of things one can count.

For Gallistel and Gelman, counting plays a critical role in the acquisition of concepts of positive integers. They argue that the preverbal system—the Accumulator—effectively embodies the counting principles⁵ and that children may come to perceive the correspondence between nonverbal and verbal counting processes. This leads children to conclude that counting terms represent the same thing as the preverbal mental

4. Gallistel and Gelman's claim that mental magnitudes represent the reals isn't a metaphor. It's to be taken quite literally. Oddly, though, they are not entirely explicit about why they think this is so. We suspect that their reasoning may be something like the following. Since a single system, the Accumulator, functions to represent both number and duration (see note 1), the representations involved must have the same basic features when representing number and time. And since time can be measured in terms of arbitrarily finer and finer units, the representations must be capable of being divided in ever finer ways, ultimately to the point of representing any real numbered unit of time. Anything less would be to impose a discrete structure on what is by all accounts a continuous, non-discrete vehicle of representation. The upshot is that it is supposed to be intrinsic to the format of representation that it picks out quantities in terms of real numbers. So when the Accumulator is working with numerosities, that can hardly change. It's built into the nature of the representations themselves.

5. Returning to the beaker analogy, each water level resulting from adding a cupful of water corresponds uniquely to the next item enumerated (one-one principle). Likewise, the beaker states occur in a fixed order (stable-order principle), with the final beaker state giving the cardinal value of the set (cardinal principle). Finally, the Accumulator is not tied to any particular modality; it can be used to evaluate the numerosity of visual stimuli, auditory stimuli, tactile stimuli, and so on (abstraction principle).

magnitudes, namely, numerosities. What's more, language, according to Gallistel and Gelman, acts as a kind of filter.⁶ Its discrete character invariably selects the integers from the rest of the reals:

[T]he integers are picked out by language because they are the magnitudes that represent countable quantity. Countable quantity is the only kind of quantity that can readily be represented by a system founded on discrete symbols, as language is. (Gallistel et al., forthcoming, p. 19)

For Gallistel and Gelman, the nonverbal system gives children a head start in learning the verbal system, in that it directs them to the verbal system and shapes their understanding of its significance. But in learning the verbal system, children are able to go beyond the limitations of the preverbal system and beyond the capacities of animals and infants. Language brings the positive integers into focus and eliminates the variability that is so characteristic of the preverbal system.

1.3 *Objections*

Unfortunately, Gallistel and Gelman's theory faces a number of serious objections, and ultimately, we believe, it cannot be made to work.

Let's start with their understanding of the Accumulator and its representational states. Granting that the representations in the Accumulator are given by mental magnitudes, should we take the system to be capable of representing the full range of real numbers? The answer quite simply is *no*. For example, there is no reason to suppose that Platt and Johnson's rats are capable of representing 3.5, much less 7.4121326769 or $\sqrt{2}$. Certainly the rats' *behavior* doesn't show sensitivity to these numerosities. To be sure, they can't reliably determine whether they should press 7, 8, 9, or 10 times, when the required value is precisely 8. But this would only seem to indicate a failure to discriminate among various *whole number* values.

Of course, it may be that experiments that are sensitive enough to detect the presences of more fine-grained representational capacities have not yet been conducted. Perhaps future experiments will show that the rats' representations of numerosity do encompass the full range of the real numbers and that they can distinguish between, say, 7.4121326768 and 7.4121326769. Similarly, we suppose one could try to insist that the rats have the far more powerful representational system embodying the reals but are unable to manifest it in their behavior. At present, however, we have no reason to take either of these possibilities seriously.⁷

6. Alternatively, Gelman and Cordes describe the process as making explicit what was previously implicit through a process of "rerepresentation" (2001, p. 294).

7. Though we don't have space to discuss it here, there is reason to doubt whether the mental magnitudes employed in measuring duration are as fine grained as the reals either. It's hardly obvious that we ever represent to ourselves durations of π or $\sqrt{2}$ seconds. Certainly, there is no behavioral evidence for this. Nor is there evidence that for any two durations there is always a representable duration between them. Much the same is true of other mental magnitudes. There is no reason to believe that the visual system can always represent a length between any two lengths no matter how fine-grained, or that the auditory system can always represent a volume between any two volumes.

Moreover, the situation isn't just that there is a lack of evidence to support Gallistel and Gelman's position. There is also an inherent tension in their account. Assuming that the Accumulator's states do represent the reals, it's hard to see how the Accumulator could embody the counting principles. The idea that there is a "next tag" makes no sense with respect to the reals. The problem is that the reals are *dense* in that between any two real numbers there is always another real number. So "2" is no more "the next tag" after "1" than "1.5" is (or, for that matter, than any other number greater than 1 is). Putting this problem aside, even if there was some sense in which "the next tag" could be defined for a system representing the reals, the Accumulator would still have to operate with impossibly perfect precision to ensure that the same accumulator levels are applied in the same order for each count. In all likelihood the level corresponding to "1" would rarely be followed by the level corresponding to "2"; rather, it would sometimes be followed by "2.0000000000103," sometimes by "2.000010021," and so on. But that's just to say that the stable-order principle wouldn't hold. And if two items were being counted and the final tag were anything other than precisely "2," the cardinality principle wouldn't hold either, since the cardinal value of a two-membered set is *precisely* 2, not, 2.0000000000103 or 2.000010021.⁸

What has gone wrong? Our diagnosis is that Gallistel and Gelman have taken features of the representational format to necessitate features of the content of the representation. In particular, they have assumed that if the vehicle of representation is a continuous magnitude, then what it represents must also be a continuous magnitude. However, this assumption is mistaken. There is nothing at all incoherent about mental magnitudes representing discrete values.

What about the second half of Gallistel and Gelman's model, namely, the role that they assign to language? Recall that on their view natural language acts as a sort of filter, selecting the positive integers from among the reals. Natural language is able to do this because it is discrete, and discrete representations are supposed to readily represent only countable quantities. Unfortunately, this feature of their theory is indefensible, quite apart from the troubles with their interpretation of the Accumulator.

The main problem is their assumption about what language can and cannot readily represent. The fact that language is discrete does not in any way limit it to representing discrete contents. Language has no difficulty representing imprecise, nondiscrete properties such as *being bald*, *being red*, or *being tall*. Far from it; vagueness is a pervasive feature of language (Keefe, 2000). Likewise, language isn't limited to terms like "pencil," which pick out countable entities. It can happily

8. These problems also undermine Gallistel and Gelman's claim that the correspondence between verbal and nonverbal counting will help in picking out the integers from the reals. Since there won't be any Accumulator states consistently correlated with verbal counting symbols, there won't be any correspondence to notice. Moreover, this problem remains on the alternative interpretation of Accumulator states, where such states represent a "blur on the number line." In that case, the "correspondence" would be between "*n*" and a blur somewhere in the general vicinity of *n*. But this isn't really a correspondence at all. Indeed, the problem remains even if we suppose that the Accumulator states represent *precise integer values*—albeit ones that can only be accessed via the noisy and distorting process of memory. Since the precise values cannot be accessed as such to be compared with the verbal counts, again it seems there would be no correspondence that the child could notice.

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accommodate mass terms, such as “salt,” which pick out substances or stuffs. Mass terms can also be incorporated into expressions of quantity (“more salt,” “less salt,” “a little salt,” “a lot of salt,” “loads of salt”). And it should also go without saying that language has numerous devices for expressing inexact quantities of differing sizes (“some,” “plenty,” “a few,” “a handful,” “a bunch of,” “an army of”).

Language can also readily represent specific real number quantities via names and descriptions (“pi” and “the square root of two”). And by incorporating a system of decimal notation, language can of course represent arbitrarily fine-grained real values, allowing us to discuss such things as whether the current interest rate of 5.867 percent is likely to rise.

We take it that these considerations undercut any hope that the discrete character of language accounts for how the integers emerge from the reals. Once again, the difficulties for Gallistel and Gelman’s theory appear to stem from a conflation of representational formats, or vehicles, and representational contents. In this case, the problematic assumption is that discrete vehicles—linguistic symbols—can only readily express discrete contents. But it should now be abundantly clear that this assumption is false. Discrete systems like language are not limited to representing countable quantities. The relation between vehicles and contents just isn’t as tight as Gallistel and Gelman would have us assume.

We have argued that Gallistel and Gelman’s account of the ontogeny of the integers faces a number of serious objections. Their interpretation of the Accumulator as representing the reals is unwarranted, their commitment to this interpretation is in direct conflict with their claim that the Accumulator operates in accordance with the counting principles, and their view about language’s role as a filter is based on mistaken assumptions about what language can and cannot readily represent. These objections go to the heart of Gallistel and Gelman’s account. Without their interpretation of the Accumulator and without their view of language acting as a filter, their account simply cannot be made to work. All the same, Gallistel and Gelman are right to emphasize the importance of the Accumulator. It is a number-specific system that is plausibly innate and likely to play a role in the ontogeny of the integers. In the next section we will examine another theory that also makes use of the Accumulator, but in very different way.

2 Spelke

We turn now to Elizabeth Spelke’s theory of the positive integers. Like Gallistel and Gelman, Spelke makes use of the Accumulator, but she also emphasizes a second cognitive system. And, importantly, she identifies a new and interesting role for natural language to play.

2.1 *Language as the Basis for Conceptual Change*

Spelke’s treatment of the positive integers is based on a general account of conceptual change that aims to explain, among other things, why the human conceptual system is far more expressive and flexible than that of other animals. At the center of Spelke’s account is natural language. According to Spelke, human beings are endowed with a variety of innate domain-specific, task-specific modules. These modules function

independently of one another, and their internal workings are inaccessible to other parts of the mind. As Spelke sees it, the richness of adult human thought isn't a matter of the contents of any particular module; most of these modules are supposed to be present in other species. Rather, the key difference is owing to the human ability to bring together the contents of two or more modules. Crucially, the way this is done is through natural language. "Natural languages provide humans with a unique system for combining flexibly the representations they share with other animals. The resulting combinations are unique to humans and account for unique aspects of human intelligence" (Spelke, 2003, p. 291). Language's power stems from two of its central features—its domain generality and its compositionality:

First, a natural language allows the expression of thoughts in any area of knowledge. Natural languages therefore provide a domain-general medium in which separate, domain-specific representations can be brought together. Second, a natural language is a combinatorial system, allowing distinct concepts to be juxtaposed and conjoined. Once children have mapped representations in different domains to expressions of their language, therefore, they can combine those representations. Through these combinations, language allows the expression of new concepts: concepts whose elements were present in the prelinguistic child's knowledge systems but whose conjunction was not expressible, because of the isolation of these systems. (Spelke & Tsivkin, 2001, p. 71)

Spelke's primary and most developed illustration of this account focuses on spatial reorientation (Shusterman & Spelke, chapter 6 here; Spelke, 2003; Spelke & Tsivkin, 2001). In reorienting, one could rely on geometrical information about the layout of the environment, landmark cues, or both. Surprisingly, many nonhuman animals seem unable to combine these two types of information; for example, they don't take advantage of concepts like *LEFT OF THE BLUE WALL*.⁹ Moreover, while adult humans *do* employ combinations of this sort, children who have yet to master the spatial vocabulary don't, and neither do adults who are engaged in tasks that interfere specifically with language processing. These results seem to provide strong support for Spelke's general account of conceptual change. Natural language, as she puts it, has the "magical property" of compositionality. "Thanks to their compositional semantics, natural languages can expand the child's conceptual repertoire to include not just the preexisting core knowledge concepts but also any new well-formed combination of those concepts" (Spelke, 2003, p. 306).

2.2 *The Theory of Positive Integers: Old Concepts, New Combinations*

Spelke's account of how the positive integers are acquired is supposed to follow the same pattern as the spatial reorientation case, once again drawing upon the domain generality and combinatorial structure of language.

9. Here and below we employ the standard small capitals notation for concepts and mental representations.

The foregoing analysis of spatial orientation prompts a different [i.e., novel] account of number development. Children may attain the mature system of knowledge of the natural numbers by conjoining together representations delivered by their two preverbal systems. Language may serve as a medium of this conjunction, moreover, because it is a domain-general, combinatorial system to which the representations delivered by the child's two nonverbal systems can be mapped. (Spelke & Tsivkin, 2001, p. 84)

What, then, are the two preverbal systems on the basis of which the positive integers are formed? Unfortunately, Spelke doesn't have a lot to say about them. The first she and Sanna Tsivkin characterize as a *small-number system*, saying that it "serves to represent small numerosities exactly" (p. 83). The second, in contrast, is supposed to be a *large-number system*, one that "serves to represent large sets" but whose "accuracy decreases with increasing set size in accord with Weber's Law" (p. 83). We take it that the large-number system is the Accumulator. Though Spelke doesn't come right out and say this, the evidence that she and Tsivkin cite on behalf of the large-number system is exactly the sort that is generally associated with the Accumulator. Things are a little trickier with their so-called small number system. But the sort of evidence they cite in connection with this system suggests that what they have in mind is what is elsewhere known as the *object indexing system* (or the *object file system*).

The object indexing system is a psychological mechanism that supports the visual tracking of a small number of objects. Several similar models have been proposed, but the basic idea in each case is to have reassignable indexes that function as abstract representations of individual objects (see, e.g., Leslie et al., 1998). In adult humans, the number of indexes is about four—a number that derives from work on object-based attention studies in vision (Trick & Pylyshyn, 1993). The indexes are abstract in that they don't inherently represent the color, shape, texture, or any of the features of an object. They are sometimes likened to fingers, which can point to a thing without thereby conveying any of its features. Object indexes are able to do this because they track objects, in the first instance, by responding to their spatial-temporal properties.¹⁰ As a result, once an index is assigned to an object, it "sticks" to it simply on the basis of such things as the object's maintaining a continuous path (with allowances for brief occlusions).

The object indexing system has a great deal of explanatory power. Here we have space for only one example—its ability to account for an influential finding of Karen Wynn's. Wynn (1992) showed five-month-old infants scenes that instantiated simple additions and subtractions, followed by outcomes that were either arithmetically correct or incorrect. In one experiment, after a doll was placed on an empty stage, a screen came up to hide the doll from view. While the screen was still up, a second doll was visibly added. The screen was then withdrawn, revealing either two objects

10. This isn't to say, however, that an object's features aren't represented by the object indexing system. Leslie et al. (1998) emphasize that features may be recorded and may even be used in the assignment of object indexes. It's just that the use of spatial-temporal properties is more basic and can govern the assignment of indexes independently of information about features.

(the correct outcome) or one object (an incorrect outcome). The infants' looking time (relative to their base preference levels) was significantly greater for the incorrect outcome, suggesting to Wynn that five-month-olds know that $1 + 1 = 2$ (see fig. 13.3). Wynn's conclusion is controversial, but for present purposes the interesting fact is that her results hold only for small numbers. This is part of the reason Spelke and Tsivkin claim that there is a system that represents only small numerosities.

The object indexing system explains this cap in terms of its limited stock of indexes; it can track no more than four objects simultaneously. The looking-time patterns in Wynn's experiments can also be explained under the assumption that attention is allocated when an active index loses its object or when a new object necessitates the activation of a new index. In the $1 + 1$ scenario, infants look longer at the incorrect outcome ($1 + 1 = 1$) because they end up with an active index that has lost its object.

Having introduced Spelke's two preverbal number modules, we turn now to her account of how they come together to yield the integers. Representations from the small-number system (the object indexing system) are supposed to be conjoined with representations from the large-number system (the Accumulator), through the power of natural language. According to Spelke and Tsivkin, exposure to number words leads children to notice that representations from the two systems apply to the same sets of entities for small numbered sets:

[B]ecause the words for small numbers map to representations in both the small-number system and the large-number system, learning these words may indicate to the child that these two sets of representations pick out a common set of entities, whose properties are the union of those picked out by each system alone. This union of properties may be sufficient to define the set of natural numbers. (2001, p. 85)

A variety of cues then suggest that all number words should be treated alike, even though the small-number system is limited to very small sets:

Because all the number words appear in the same syntactic contexts (see Bloom & Wynn, 1997) and occur together in the counting routine, experience with the ambient language may lead children to seek a common representational system for these terms. (p. 85)

And finally it all comes together, the result being representations of the positive integers:

[B]ecause the terms *one*, *two* and *three* form a sequence in the counting routine, children may discover that each of these number words picks out a set with one more individual than the previous word in the sequence, and they may generalize this learning to all the words in the counting sequence. (pp. 85–6)

In support of this account, Spelke cites two further sources of evidence linking language to number. One source of evidence involves cases of brain-damaged patients who have impaired language and are also unable to perform exact calculations (yet retain the ability to approximate). The other source of evidence involves experimental work on bilinguals who were trained to do certain sorts of exact calculations and approximations in one of their languages and then tested on these tasks in both of their languages. Interestingly, the bilinguals were able

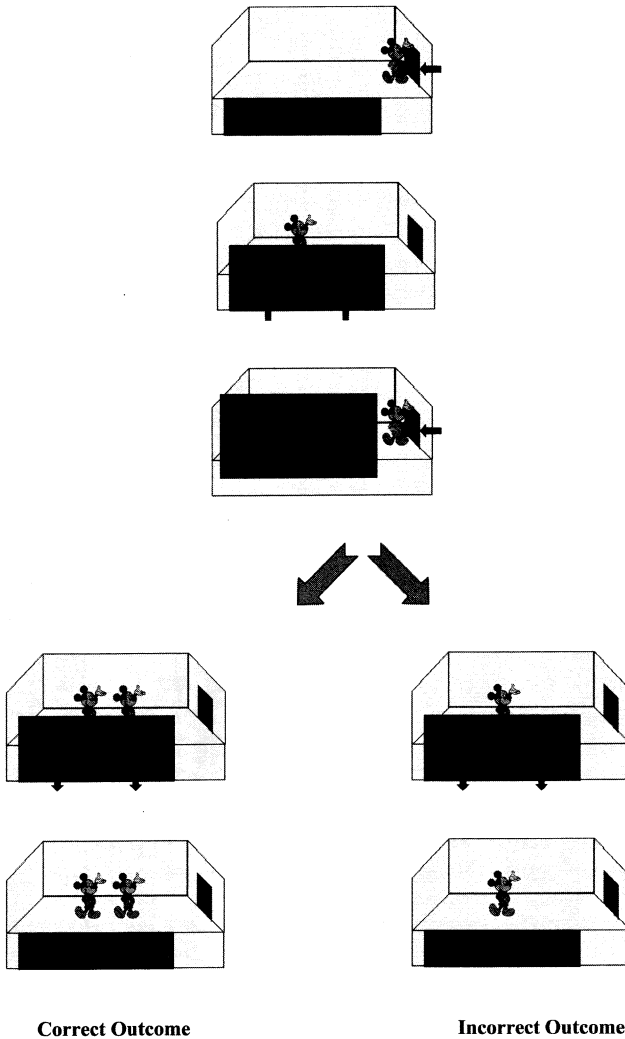


FIGURE 13.3 Schematic Depiction of one of Wynn's Addition/Subtraction Experiments. After a doll was placed on an empty stage, a screen came up to hide the doll from view. While the screen was still up, a second doll was visibly added. The screen was then withdrawn revealing either two objects (the correct outcome) or one object (the incorrect outcome) (adapted from Wynn, 1922).

to transfer the new approximation skills across languages but were unable to transfer their new skills with exact calculations. Spelke and her collaborators take this to suggest that language is essentially involved in the representation of large exact numerosities—a view that is a natural corollary of her theory of development.

2.3 *Objections*

Spelke's account faces a number of serious objections, and, ultimately, we believe it is no more promising than Gallistel and Gelman's. Much of the trouble with Spelke's account comes right at the beginning. In particular, it isn't clear which representations are to be drawn from the two modules. Spelke gives several answers that are significantly different from one another if not simply inconsistent.

As we showed in section 2.2, Spelke and Tsivkin (2001) claim that the small-number system "serves to represent small numerosities exactly." This remark is embedded in a larger discussion where they introduce the small number system by noting that "the capacity for representing the exact numerosity of small sets is common to humans and other animals and emerges early in human development" (pp. 82–3). Likewise, writing with Marc Hauser, Spelke refers to "a system for representing the exact number of object arrays or events with very small numbers of entities" (Hauser & Spelke, 2004, p. 9). Yet in a related discussion, Spelke says that the system "does not permit infants to discriminate between different sets of individuals with respect to their cardinal values" (2003, p. 299). These claims, if not simply inconsistent, are in strong tension with one another. How could a system represent the exact numerosity of different small sets without at least permitting infants to discriminate among them with respect to their cardinality?

Other times the concern isn't inconsistency but rather that what are supposed to be the same components of the theory are presented in ways that aren't at all equivalent. For example, at one point Spelke and Tsivkin say that the small-number system represents a two-member set as "an object x and an object y , such that $y \neq x$," whereas the large-number system represents it as "a blur on the number indicating a very small set" (Spelke & Tsivkin, 2001, p. 85). Elsewhere, however, they suggest that what the two contribute is something very different:

From the small number system may come the realization that each number word corresponds to an exact number of objects, that adding or subtracting exactly one object changes number, and that changing the shape or spatial distribution of objects does not change number. From the large-number system may come the realization that sets of exact numerosity can increase without limit, and that a given symbol represents the set as a unit, not just as an array of distinct objects." (p. 86)

Given all of these different pronouncements, it's hard to say which should be taken as Spelke's considered view of the representations that the two modules are supposed to deliver.

If that weren't bad enough, it's doubtful that any of her answers are especially promising. For instance, take the representations (i) and (ii):

(i) AN OBJECT x AND AN OBJECT y , SUCH THAT $y \neq x$

(ii) _____ ["_____ " indicates a specific blur on the number line corresponding to approximately 2]

Spelke and Tsivkin talk repeatedly about “conjoining” representations from the small and large number systems. But conjoining these two representations results in the bizarre representation (iii):

(iii) AN OBJECT X AND AN OBJECT Y, SUCH THAT $Y \neq X$ AND ———

The problem is that it is anything but clear what this representation means.

Since the target is a concept like SEVEN (exactly seven, not approximately seven), perhaps a more promising suggestion is to combine the generic concept of EXACT NUMEROSITY with a given approximate numerical range. The generic concept may be what Spelke has in mind when she emphasizes that the small number system “represents small numerosities exactly.” Suppose, then, that the combination is a representation of exact numerosity with a blur corresponding to approximately 7—SEVENISH, for lack of a better expression. The question is what the result would be. We see no reason to think that there is a determinate answer to this question or one that Spelke would find particularly favorable. To see why, consider a close analogy. RED indeterminately applies to a range of colors with no precise boundary separating red and its neighboring colors, such as orange. What happens when the concept RED is combined with the concept EXACT COLOR? What would the content of this concept be? The answer isn’t at all clear. Notice that adding COLOR to RED doesn’t add anything at all, so in combining EXACT COLOR and RED, EXACT does all the work. But what does EXACT add to RED? Something can be such-and-such percentage red, or such-and-such shade of red, but not exactly red. Perhaps the best that can be said here is that EXACT + RED just means red. In that case, EXACT NUMEROSITY + SEVENISH would just mean sevenish. This hardly brings us closer to SEVEN.

What’s more, the situation doesn’t improve even if one insists that EXACT NUMEROSITY + SEVENISH must refer to some more specific numerosity, since there are many specific contents that would be candidates. These include (but aren’t limited to) the range 7–8, the range 6–7, the range 6–8, the number 7.5, the number 8, and so on. All of these are different ways of making SEVENISH more precise. Modifying SEVENISH by EXACT NUMEROSITY does nothing to single out seven.

Things get even worse in that Spelke can’t assume that a concept of numerosity is in the small-number system in the first place. If this system is the object indexing system, as we suggested earlier, then its representational powers are far more modest. What it does is attend to a small number of objects by employing a small number of indexes, one per object. Its representations are the indexes, each of which only represents the object it temporarily tracks. Of course, whenever the system responds to two objects, it will activate exactly two indexes. But that doesn’t mean that the system is employing the concept EXACTLY TWO or representing the two-ness of the set. Rather, it’s just a reflection of the parallel activation of two indexes, each of which continues to represent no more than its object. The same considerations extend to other numerical or quasi-numerical concepts that Spelke may wish to appeal to—EXACT NUMEROSITY, EXACT, NUMEROSITY, ONE, TWO, EXACTLY ONE, EXACTLY TWO, and so on. None of

these are present in the object indexing system, and none can be taken for granted.¹¹

Up until now we have been taking at face value Spelke's claim that her treatment of the positive integers follows the same model as her treatment of spatial reorientation. It may be, however, the two aren't so closely related and that what Spelke ought to say is that the common ground between them is just the importance given to language. In that case, it may be that the compositional structure of language is what's important for spatial reorientation but that language functions rather differently when it comes to number. If this is right, then Spelke's view of number isn't grounded in her general theory of conceptual change (or else that theory is described very misleadingly). On the other hand, the departure from her general theory of conceptual change would make sense of the fact that Spelke suggests a variety of different contributions from the preverbal number modules. It would also make sense of Spelke and Tsivkin's remarks about different "realizations" coming from the two number systems.

Suppose, then, that the theory isn't that the representations of the small and large number systems are combined compositionally. The remarks about realizations suggest a more intellectual process where information made available by the two modules is subjected to reflection and a certain amount of theorizing takes place, leading somehow to a new stock of concepts. One problem that this raises for Spelke is *where* the reflection takes place. Spelke's inventory of innate mechanisms includes the modules we share with animals plus language. Clearly reflection of the required sort isn't something that could occur in a domain-specific, task-specific module; and language, while it may provide a domain-general medium, isn't a mechanism that can be counted on to embody any inference you like. So it may turn out that the seat of conceptual change has yet to be identified.

More generally, though, we need to ask what exactly the initial information to be combined looks like, how exactly the process works, and how any new concepts emerge from it. Since the alternative model of conceptual change that we are considering is not explicitly discussed in Spelke's work, it cannot be evaluated in any detail. But to get a feel for the difficulties it is likely to face, consider just the question of what initial information is to be combined. In several places, Spelke indicates that the small-number system may contribute something like the concept of an individual, while the large-number system contributes something like the concept of a set. For instance:

One system represents small numbers of persisting, numerically distinct individuals exactly and takes account of the operation of adding or removing one indi-

11. One might try to argue that, though these are not explicitly represented in the object indexing system, one or more are *implicitly* represented. We should note that we don't think that this is a promising suggestion. Part of the problem is that Spelke would then need a mechanism that could make them explicit. Moreover, such a mechanism would threaten to make her language-based theory of conceptual change superfluous. Any cognitive mechanisms that could render a concept explicit in the envisioned sense would be capable of formulating an entirely novel concept. Language would no longer be the driving force for conceptual change.

vidual from the scene. It fails to represent the individuals as a set, however, and therefore does not permit infants to discriminate between different sets with respect to their cardinal values. A second system represents large numbers of objects or events as sets with cardinal values, and it allows for numerical comparison across sets. This system, however, fails to represent sets exactly, it fails to represent the members of these sets as persisting, numerically distinct individuals, and therefore it fails to capture the numerical operations of adding or subtracting one. (2003, p. 299)

Learning the meaning of small number words is supposed to bring these two representations together, thereby laying the groundwork for concepts of the positive integers:

To learn the full meaning of two, however, children must combine their representations of individuals and sets: they must learn that two applies just in case the array contains a set composed of an individual, of another, numerically distinct individual, and of no further individuals. (p. 301)

One point to note here is that it is puzzling how the combination of such varied information is supposed to be achieved. The suggestion is that the likes of (1) and (2) are brought together to yield (3):

- (1) The information that there is a set consisting of a small indeterminate number of individuals that aren't represented as persisting or as being numerically distinct from one another
- (2) The information that there is a persisting individual and a different persisting individual
- (3) The belief that there is a set consisting of a persisting individual and a different persisting individual and no other individuals

A major problem with this proposal, to the extent that we understand it, is the very different assumptions about "individuals" in the two systems. In one case the individuals are persisting and numerically distinct. In the other, they are neither of these. There would seem to be little point of contact between the two, making it difficult to see how they could come to support a common belief, short of equivocation. Similarly, the notion of set that is supposed to be derived from the large-number system is a peculiar one. Our ordinary notion of a set is one that is defined in terms of its members (where these are numerically distinct, persisting individuals). But Spelke can't avail herself of this notion. Another concern is that, while Spelke may be right that the small-number system doesn't represent the set of objects as such—that it only represents the individuals in the set—whatever justification there is for this claim could be applied to the Accumulator as well. The only thing the Accumulator patently represents is a property of sets, namely, their approximate numerosity. This no more requires that the sets themselves be represented than representing the redness of an individual requires representing the individual as such. As a result, Spelke isn't in a position to assume that the Accumulator has any explicit representation of a set to begin with.

Together these considerations cast doubt on Spelke's theory insofar as it breaks away from the spatial reorientation example. Because Spelke says so little about how the imagined combination proceeds, it's hard to say more. Still, we do want to

mention one final potential difficulty. The current model requires that both the small-number system and the large-number system are responsive to smaller numbers, each in its own way. For example, both are supposed to be able to respond to sets of two items, particularly in the course of learning the word “two.” The result is supposed to be that learning the first few number-words precipitates, and in some sense causes, a conceptual shift, giving rise to the positive integers. It goes without saying that for any of this to work, the large-number system—the Accumulator—has to function for small numbers. Our last concern is that there is a very real possibility that it doesn’t. In the Xu and Spelke study cited in section 1.1, it was found that infants who could distinguish 8 from 16 couldn’t distinguish between 8 and 12 (Xu & Spelke, 2000). And in a subsequent work, Xu has found that infants who can distinguish between 4 and 8 nonetheless can’t distinguish between 2 and 4 (Xu, 2003). Xu concludes that infants at this age have an Accumulator that requires a 1:2 ratio but, in addition, doesn’t respond to small numbers (thus the failure with 2 v. 4). Why not? There are several possibilities. One is that, as Xu puts it, the Accumulator’s “computations are unstable or undefined for small values” (p. B23). This would be a likely outcome, particularly if its operations aren’t iterative—as is assumed by Gallistel and Gelman—but instead compute approximate number in some other way.¹² Another possibility is that “the output of the object tracking system inhibits the output of the number estimation system [the Accumulator]” (p. B24). Either way, Spelke’s treatment of the positive integers would be problematic, since she couldn’t assume that children have representations from both preverbal systems at the level at which they are supposed to be compared. The result is that they would have no basis for formulating concepts for the integers 1, 2, and 3, and the account wouldn’t even get off the ground.

Finally, before closing this section, we should say a word or two about the evidence linking language to number. This includes evidence from brain-damaged patients and from bilinguals, both pointing to a link between language and the representation of exact number, including exact calculation. The question is whether the link is so strong that it argues that language is intrinsic to the representation of the positive integers, making language a condition for their emergence. We would suggest that the evidence is, at best, inconclusive. This is for the simple reason that, among language users, language may come to play an important role in the representation of the integers without being the original source of these concepts. Though extremely interesting, the data aren’t developmental data; consequently, they don’t tell one way or the other about the fact of ontogeny.

Of course, even if these data did establish that language is essential to number, this wouldn’t argue for Spelke’s theory in particular. The data are equally compatible with Gallistel and Gelman’s theory or any of a large number of different possible theories that take language to play a crucial role in the ontogeny of number. Moreover, there are also data suggesting that number isn’t essentially dependent on language. Though we lack the space to go into much detail here, it’s worth mentioning in this context that there are cases of patients with severe

12. Spelke herself has argued for a noniterative model in Barth, Kanwisher, and Spelke (2003).

linguistic deficits who can perform exact calculation. For instance, Hermelin and O'Connor (1990) describe a speechless autistic man who can identify five-figure prime numbers and who can factorize numbers of the same magnitude, all based on exposure to a few examples. The examples involve the use of symbols—standard Arabic notation. However, the important point is that Arabic notation isn't anything like a natural language and can hardly vindicate Spelke's model of development. At the very least, it lacks the domain generality that is supposed to allow language to bring together representations from distinct modules.

In this section we have argued that Spelke's account faces a number of serious objections. Many of these concern the representations that are supposed to be contributed by the preverbal number modules. In particular:

- It isn't clear what these representations are.
- Spelke's suggestions aren't always consistent.
- The reasonable candidates involve concepts that aren't explicitly represented (EXACTLY ONE, NUMEROSITY, SET, etc.).
- The reasonable candidates aren't able to get us closer to the positive integers when combined via the compositional semantics of natural language.

Further, if compositionality isn't the mechanism of conceptual change, then it just isn't clear what the alternative is supposed to be. And finally, all of the suggestions and hints that Spelke makes assume that both preverbal systems contribute representations in connection with the first few integers. But there is evidence to suggest that the Accumulator doesn't function for these numbers, in which case Spelke's account can't even get off the ground.

In light of these problems, Spelke's account of the positive integers is not promising. At the same time, Spelke does identify an innate cognitive mechanism (the object indexing system) that, like the Accumulator, may well play an important role in the ontogeny of the integers. But the question remains of how exactly the two could be combined to yield the integers and what other ingredients might be needed.¹³

3 Conclusion

Are language and number essentially linked? In this essay we have examined two of the most important current accounts of the origins of number concepts. Though they have their own distinctive commitments, both identify language as one of the core innate capacities that subserve the development of number. We have argued that neither account is defensible. Still, work by Gallistel, Gelman, Spelke, and others has done much to advance our understanding of the origins of number. So the answer to our question is, so far as anyone knows, *no*. Though it is still too early to say whether the ontogeny of number depends on language, the situation at present is that we have little reason to suppose that it does.

13. For our views on these questions, and a more detailed discussion of the ontogeny of number, see Laurence and Margolis (in prep.).