On the measurement process in Bohmian mechanics (with a reply to Gao)

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This note clarifies several details about the description of the measurement process in Bohmian mechanics and responds to a recent preprint by Shan Gao, wrongly claiming a contradiction in the theory.

1 Measurements in Bohmian mechanics

A prototypical measurement in Bohmian mechanics is an interaction between a system S and measurement device D resulting in one of several, microscopically discernible, configurations of D ("pointer positions") which are correlated with certain possible quantum states of S. Schematically, the interaction between the measured system and measurement device is such that, under the Schrödinger evolution,

$$\varphi_i \Phi_0 \xrightarrow{\text{Schrödinger evolution}} \varphi_i \Phi_i \ , \tag{1}$$

where the wave function Φ_0 is concentrated on pointer configurations corresponding to the "ready state" of the measurement device, and Φ_i are concentrated on configurations indicating a particular measurement result, e.g., by a pointer pointing to a particular value on a scale, a point-like region of a detector screen being darkened, a detector clicking or not clicking, etc. The Schrödinger time evolution is linear, so that the superposition

$$\varphi = c_1\varphi_1 + c_2\varphi_2, \qquad c_1, c_2 \in \mathbb{C}, \qquad |c_1|^2 + |c_2|^2 = 1,$$

leads to

$$\varphi\Phi_0 = (c_1\varphi_1 + c_2\varphi_2)\Phi_0 \xrightarrow{\text{Schrödinger evolution}} c_1\varphi_1\Phi_1 + c_2\varphi_2\Phi_2. \tag{2}$$

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At this point, standard quantum mechanics is hit by the measurement problem. In Bohmian mechanics, however, the system is described not only by the wave function but also by the actual spatial configuration $(X, Y) \in \mathbb{R}^m \times \mathbb{R}^n$ of measured system and measurement device, given by the positions of their constituent particles.

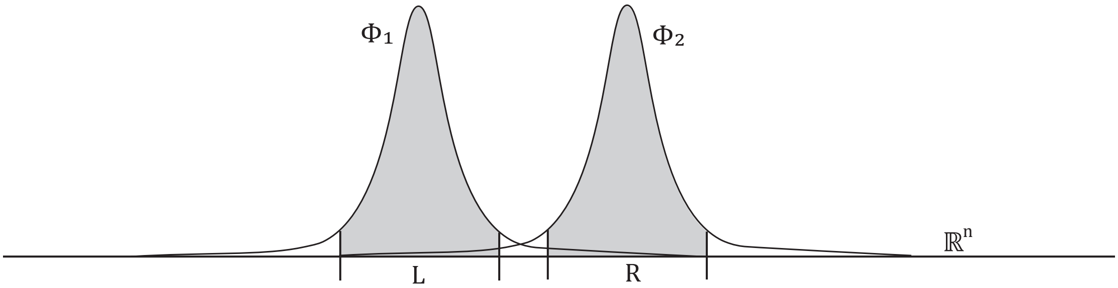

Figure 1: Sketch of the pointer wave functions on configuration space.

For illustrative purposes, let's say that Φ_1 is concentrated on a region $\mathrm{L} \subset \mathbb{R}^n$ of the configuration space of D corresponding to the pointer of the measurement device pointing to the left, while Φ_2 is concentrated on a region $\mathrm{R} \subset \mathbb{R}^n$ of the configuration space of D corresponding to the pointer pointing to the right. Obviously, the two regions are disjoint, i.e. $\mathrm{L} \cap \mathrm{R} = \emptyset$. By assumption, Φ_1 and Φ_2 must be well localized in the respective regions (otherwise, the measurement device is no good), i.e. almost zero outside. This implies, in particular,

$$\int_{\mathrm{L}} |\Phi_1|^2 \, \mathrm{d}^n y \approx 1, \quad \int_{\mathrm{L}} |\Phi_2|^2 \, \mathrm{d}^n y \approx 0 \tag{3a}$$

$$\int_{\mathrm{R}} |\Phi_1|^2 \, \mathrm{d}^n y \approx 0, \quad \int_{\mathrm{R}} |\Phi_2|^2 \, \mathrm{d}^n y \approx 1. \tag{3b}$$

As Gao (2019) rightly points out, it's not realistic to assume that Φ_1 and Φ_2 have compact support in L, respectively R, i.e., that they are precisely zero outside.[1] Hence the "$\approx$" in the above equations. The better these pointer states are localized, the better the approximation. In practice, this will depend on the details of the experiment, such as the makeup of the measurement apparatus, and the strength and duration of its interaction with the microscopic system.

Now, according to Bohmian mechanics, the probability of the pointer *actually* pointing

[1]It is also not realistic to assume, as he does, that the pointer wave functions evolve freely. Usually, there will be some potential keeping the pointers in place, and decoherence, through interactions with the environment, leading to further localization. Nonetheless, the pointer wave functions will be spread out, in general, and may even have infinite tails.

to the left is:

$$\begin{aligned}\mathbb{P}(Y \in \mathrm{L}) &= \int_{\mathbb{R}^m \times \mathrm{L}} |c_1\varphi_1\Phi_1 + c_2\varphi_2\Phi_2|^2 \,\mathrm{d}^m x \mathrm{d}^n y \\ &= |c_1|^2 \int_{\mathbb{R}^m \times \mathrm{L}} |\varphi_1\Phi_1|^2 \mathrm{d}^m x \mathrm{d}^n y \\ &+ |c_2|^2 \int_{\mathbb{R}^m \times \mathrm{L}} |\varphi_2\Phi_2|^2 \mathrm{d}^m x \,\mathrm{d}^n y \\ &+ 2\,\mathrm{Re}\Big(c_1 c_2 \int_{\mathbb{R}^m \times \mathrm{L}} (\varphi_1\Phi_1)^* \varphi_2\Phi_2 \mathrm{d}^m x \,\mathrm{d}^n y\Big) \approx |c_1|^2.\end{aligned} \tag{4}$$

The final approximation follows from eqs.(3a) (together with the Chauchy-Schwarz inequality $|\int_L \Phi_1^*\Phi_2| \leq \sqrt{\int_L |\Phi_1|^2}\sqrt{\int_L |\Phi_2|^2}$). Similarly, the probability of the pointer pointing to the right is $\mathbb{P}(Y \in \mathrm{R}) \approx |c_2|^2$. If φ_1 and φ_2 are eigenstates of some quantum observable, $|c_1|^2$ and $|c_2|^2$ are the statistical predictions of standard quantum mechanics for an *ideal* measurement. The better the pointer states Φ_1 and Φ_2 are localized in disjoint regions of configuration space, the closer the measurement is to "ideal".

Since a realistic measurement is not quite ideal, we see that there is also a very small – yet non-zero – probability that the final pointer position is inconclusive, e.g. because it remains roughly in the ready state, or because the measurement device is blown into pieces. Tough luck, measurements can fail[2].

1.1 Some remarks and observations

1. After the measurement (assuming it was not destructive), the system S will be guided by the wave function $\varphi_1(x)\Phi_1(Y) + \varphi_2(x)\Phi_2(Y)$. If the pointer actually points left (let's say), i.e. $Y \in \mathrm{L}$, we have $\Phi_2(Y) \approx 0$ and hence (after normalization) the *effective wave function* φ_1 describing the System S after the measurement. This is the effective collapse in Bohmian mechanics.[3]

2. In many papers – including some of my own – it is said with regard to eq. (7) that we are integrating "over the support of Φ_1". This is indeed a little abuse of mathematical language. For non-idealized situations, one should read the statement like a physicist, not like a mathematician, namely as saying: we integrate over a region of configuration space – here L – that contains almost the entire L^2-weight of Φ_1 (and which corresponds to configurations in which the pointer points to the left). Let's call this the *FAPP[4]-support*.

[2]though other sources of error would seem more likely, in practice, than an atypical pointer position due to the tails of the pointer wave functions

[3]See Dürr et al. (2013, Ch. 2) for more details.

[4]For All Practical Purposes

3. If the pointer after the measurement is actually pointing to the left (let's say), i.e. $Y \in \mathrm{L}$, then the contribution of Φ_2 to the Bohmian guiding field at Y will be negligible small, as well – provided the tails are "well-behaved". This justifies the statement that the configuration of the measurement device is effectively guided by the wave packet Φ_1 only. "Well-behaved" means that not only Φ_2 itself but also its gradient – more precisely $\operatorname{Im} \nabla_y \Phi_2$ – is very small outside the FAPP-support (think, for instance, of Gaussian tails). This assumption is commonly made in physics, and well justified in general. Moreover, the contribution of the other branch will further diminish (even for not so well-behaved tails) as *decoherence* is progressing through interactions with the environment, leading in effect to the situation of the effective collapse for the pointer states (see remark 1 and 4 below).

4. Suppose we go one step further and consider a "measurement of the pointer position" by another system E. You may think of an "observer" looking at the measurement device, resulting, ultimately, in a particular particle configuration in her brain (though I prefer a camera or some other system under no suspicion of *consciousness*). In any case, the spatial resolution of such an observation can easily be finer than the localization of the initial pointer wave functions, thus corresponding to a Schrödinger evolution of the form

$$\Phi_i \longrightarrow \sum_j \Phi_{ij} \Psi_j,$$

where $\sum_j \Phi_{ij} = \Phi_i$ and the Ψ_j are well-localized in disjoint regions of the configuration space of E. This then leads to further decoherence and localization (by effective collapse) of the apparatus wave function into one of the wave packets Φ_{ij}.

Hence, clearly, the accuracy of an observation of the pointer position is not limited by the spread of the pointer states Φ_i prior to observation (contrary to what Gao (2019) seems to suggest). Moreover, we didn't even have to consider another "measurement"; environmental decoherence – if only by scattering of air molecules, photons, etc. – occurs everywhere and all the time (unless one takes very special precautions to prevent it), causing localization of the macroscopic wave function.

If some of these points (like 2) are rarely spelled out in detail in the "Bohmian" literature, then because they involve fairly standard physical arguments. No deep foundational issues are hiding behind the mathematical details here. If there's a lesson to learn, then that serious physics is a bit messier and a bit more subtle than the sterile operator-formalism of quantum mechanics reveals.

2 What Gao's objection gets wrong

So, if not mathematical nitpicking, what is the point of the objection formulated by Shan Gao (2019)? My best attempt at a reconstruction of his argument goes as follows: The possible measurement results are first and foremost given by the pointer states Φ_1 and Φ_2. The role of the Bohmian particle configuration Y is to pick out one of the two results (by ending up in the support of one of the two wave functions) and thus determining a unique outcome. However, since Φ_1 and Φ_2 actually overlap, the Bohmian configuration is not able to do so – at least not always. Gao (2019) writes: "This means that there is no one-to-one correspondence from the particle configurations of a measuring device to the result wave functions of the device or the measurement results." (p. 3)

This objection is based on a superficial understanding of Bohmian mechanics and misses its mark for several reasons. First of all, the pointer configuration *IS* the measurement result (and thus always unique). It is important to take the ontological commitment of Bohmian mechanics to particles seriously.

Second, the pointer configuration Y *does* typically pick out either Φ_1 or Φ_2 as the wave function guiding the system, namely in the sense explained above (remark 3) that the contribution of the other wave packet becomes negligibly small.[5] In particular, it will typically lead to an effective collapse of the measured system S into either φ_1 or φ_2.

Finally and most basically, there is no postulate in Bohmian mechanics stating a "one-to-one correspondence" between the pointer configuration and the "result wave function" of the measurement device. The connection that the theory *predicts*– rather than assumes – is the one explained above.

It is not clear what theory or principle Gao has in mind when he insists that the wave packets Φ_1 or Φ_2, which he calls "result wave functions" (and emphatically these wave packets *as a whole* (p. 2)), correspond to the measurement result. Apparently, he thinks of the wave packets Φ_1 and Φ_2 as an (observationally) "preferred basis", or something like that, but this has nothing to do with Bohmian mechanics.

Actually, any quantum theory I can think seems to agree that the wave functions Φ_1 and Φ_2 overlapping – on configurations roughly in between "pointer left" and "pointer right" – means that it is possible (though maybe very unlikely) for the pointer to end up pointing to the middle rather than left or right. For this reason alone, the assertion that there are exactly two possible outcomes of the measurement, corresponding to the pointer states Φ_1 and Φ_2, seems planly wrong – and not just in Bohmian mechanics.

[5] It should also be noted that the description of the measurement process in *any* precise quantum theory relies on decoherence, in one way or another, and that this decoherence is never perfect for finite times and a finite number of degrees of freedom.

Moreover, any (serious) quantum theory I can think of seems to agree that decoherence would lead to further branching (and, in whatever sense, "collapse") of the apparatus wave function, which can result in wave functions other than Φ_1 or Φ_2 (see remark 4 above). From this point of view also, insisting that there are exactly two possible outcomes, corresponding precisely to Φ_1 and Φ_2, seems wrong or at least arbitrary – and not just in Bohmian mechanics.

A more subtle point is that the question how wave functions, in general, are connected to measurement results – or any physical facts at all – is at the core of the measurement problem and, in fact, most debates in quantum foundations. Different quantum theories answer this question differently (though I still don't know which one answers it in a way consistent with Gao's analysis).

Bohmian mechanics is a theory about particles moving in physical space. The empirical content of the theory lies in the spatio-temporal configuration of matter, constituted by particles. The role of the wave function is first and foremost to determine how the particles move, and also (though this is a theorem rather than an additional postulate) to describe typical statistical distributions in ensembles of subsystems. The wave function of a closed system always evolves unitarily (in and of itself, it is thus insensitive to different "measurement results"). The actual configuration of a system (and its environment), however, determine which part of the wave function is effectively guiding the subsystem, and which decoherent branches, if any, can be ignored FAPP. In this sense, a particular effective wave function can result from a measurement process, but it would be misleading to say that the wave function *is* the measurement result.

In the present case, the wave packet Φ_1 (Φ_2) "corresponds" to a pointer pointing to the left (right) in the sense that it is well-localized in a region of configuration space whose points realize a pointer configuration pointing to the left (right) and thus assign very high probability to the respective pointer position. This (and only this) is also what justifies common notations such as $|\text{left}\rangle$ and $|\text{right}\rangle$ for the pointer states. Nonetheless, Φ_1 may be consistent with a pointer actually pointing to the middle or to the right (at least for a brief period of time), and Φ_2 may be consistent with a pointer actually pointing to the middle or to the left (at least for a brief period of time). So again, it is unclear what Gao has in mind when he insists that "the whole result wave function [not just a truncated part] ... corresponds to the result" (p. 2). If the relevant result is "pointer left" or "pointer right", the statement is clearly wrong.

In Bohmian mechanics, the complete state of the measurement device (if there is no relevant entanglement with other subsystems[6]) is always described by a pair (Y, Φ),

[6] Otherwise, we have to consider the *conditional* wave function, see Dürr et al. (2013, Ch. 2).

where $Y \in \mathbb{R}^n$ is the configuration of the particles and $\Phi \in L^2(\mathbb{R}^n)$ the (effective) wave function. In principle, there is thus a continuum of possible outcomes of the measurement, as there is a continuum of possible configurations Y. In practice, we care only about a coarse-grained description (e.g. $Y \in \mathrm{L}$ or $Y \in \mathrm{R}$) which leads to the statistical analysis sketched above. There are, however, no postulates about "measurement results", in particular no "result assumption" that Gao repeatedly alludes to.

The most charitable reading of Gao's objection is that the final pointer position "left" ($Y \in \mathrm{L}$) or "right" ($Y \in \mathrm{R}$) is not *perfectly* correlated with the quantum states φ_1 or φ_2 of the measured system. Indeed! The pointer states Φ_1 and Φ_2 having a finite overlap (as Gao insists) means precisely that the detector is imperfect in this sense, that the measurement is not quite "ideal". Not only is this a realistic limitation of measurements (that more sophisticated quantum measurement formalisms capture, as well,) it is actually put to good use in physics, namely in so-called weak measurements, in which the possible pointer states overlap a lot, thus providing very little information from a single measurement event but also affecting the state of the measured system as little as possible (cf. e.g. Wiseman (2007)). In the upshot, far from being contradictory, the Bohmian description of the measurement process gets things exactly right, and it took the operator formalism some time to catch up with it.

At this point, I also have to warn against thinking of the quantum states φ_1 or φ_2 – even if they are eigenstates of some relevant observable – as corresponding to pre-existing properties of the system that the measurement is supposed to reveal. This idea, which is conclusively dispelled by Bohmian mechanics, lies behind many of the alleged paradoxes of quantum mechanics or misguided talk about "quantum logic" (Bell (2004, Chs. 17, 23), Lazarovici et al. (2018)). Thus, contrary to what Gao seems to assume (p. 5), there is no "measured quantity" with pre-existing values that the Bohmian particle configuration registers (cf. Norsen (2014) for the particular example of spin). Bohmian particles have a position and nothing else[7], while different wave functions or quantum states have to be understood through their dynamical role for the particle motion. John Bell (2004) summarized this important insight brilliantly:

> "[I]n physics the only observations we must consider are position observations, if only the positions of instrument pointers. It is a great merit of the de Broglie-Bohm picture to force us to consider this fact. If you make axioms, rather than definitions and theorems, about the 'measurement' of anything else, then you commit redundancy and risk inconsistency." (p. 166)

[7] We can leave open the status of dynamical parameters such as mass and charge.

Possibly the most basic mistake committed by many critics is to think of Bohmian mechanics essentially as standard quantum mechanics plus an ad hoc addition of particle positions to solve the measurement problem. In fact, the measurement formalism of quantum mechanics *reduces* to Bohmian mechanics as an effective statistical description of the fundamental microscopic theory. Simply put, Bohmian mechanics is to textbook quantum mechanics what Hamiltonian mechanics is to thermodynamics. There would thus be a lot more to learn by studying the measurement process from a Bohmian point of view: the status of Born's rule (Dürr et al., 2013, Ch.2), the role of observables (Dürr et al., 2013, Ch.3), the meaning of the no-hidden-variables theorems (Lazarovici et al., 2018) – all this and more is explained and demystified by Bohmian mechanics.

What Bohmian mechanics doesn't provide – and what a serious physical theory shouldn't provide, as we learned, most notably, from Bell – are postulates about "observables", "measurements", "measurement results", etc. The theory describes what is going on in the world, and we have to analyze the theory to know what can or will happen in a specific physical situation, what can be measured and how, and what we can infer from a particular outcome.

One of the many intellectual harms done by operational quantum mechanics is that this way of doing serious physics is no longer common ground.

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