Typical Humean worlds have no laws

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The paper uses the concept of typicality to spell out an argument against Humean supervenience and the best system account of laws. It proves that, in a very general and robust sense, almost all possible Humean worlds have no Humean laws. They are worlds of irreducible complexity that do not allow for any systematization. After explaining typicality reasoning in general, the implications of this result for the metaphysics of laws are discussed in detail.

1 What are laws of nature?

Over the past few decades, the best system account has developed into a popular, maybe even the dominant position regarding the metaphysics of laws of nature. In brief, this view holds that laws of nature are merely descriptive, an efficient summary of contingent regularities that we find in the world. Metaphysically, it is based on the thesis of Humean supervenience – named in honor of David Hume's denial of necessary connections – that David Lewis (1986b) famously characterized as "the doctrine that all there is to the world is a vast mosaic of local matters of particular fact, just one little thing and then another." Laws of nature are then supposed to supervene on this Humean mosaic as the deductive system that strikes the optimal balance between simplicity and informativeness in describing the world.

The Humean "regularity view" of laws is opposed to the "governing view," in its various forms, according to which the fundamental laws play an active role in guiding, or

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producing, or constraining the history of the universe. For our discussion (and avoiding a complete overview of the various anti-Humean positions), I will take the main contemporary contenders to be dispositional essentialism (Bird, 2007) – which grounds the laws of nature in dispositional properties instantiated by the fundamental entities – and nomic primitivism (Maudlin, 2007a), which admits "law of nature" as a primitive ontological category, and laws as fundamental entities into the ontology of the world.

Our discussion will only be concerned with fundamental physical laws that govern or summarize the entire physical history of the world, although both Humean and anti-Humean views may be compatible with more deflationary notions of lawhood.[1] On the other hand, I am only going to defend the minimal anti-Humean thesis that laws "govern" or "constrain" that history. If the view that laws "produce" entails more than that (as Schaffer (2016) argues) or is tied to a particular metaphysics of time (see Loewer (2012) versus Maudlin (2007a)), it will require additional justification.

There is one way to phrase the debate between Humeans and anti-Humeans that I find both uninteresting and misleading (for reasons that will become clearer in the course of our discussion): Laws can determine regularities (as their instances), and regularities can determine laws (as their best systematization), and so one may ask: What comes first, what is more fundamental, the regularities or the laws? ("What grounds what?" is how one would put it, more properly, in contemporary metaphysics, see, e.g., Schaffer (2008, 2009).) One might then be skeptical about one of these two grounding relations and choose sides on this basis; e.g., deny that there can ever be an unambiguously best system, or find it utterly mysterious how laws are supposed to "govern" anything. This is not my main concern, however, and my discussion will grant that both the regularity and the governing view of laws are at least conceptually sound. Instead, I consider the debate between Humean and anti-Humean metaphysics to be first and foremost a debate about fundamental ontology – whether there is more to the fabric of the world than the Humean mosaic – and the interesting choice to be one between ontological parsimony and other theoretical virtues.

In this debate, Humeans have had remarkable success in defending a prima facie implausible position against all objections that have been thrown its way (and then claim victory on the grounds of parsimony). In recent years, criticisms of the best system account have focussed, in particular, on the lack of explanatory power of Humean laws (e.g., Maudlin (2007a); Lange (2013)), the alleged subjectivity of the best system (Armstrong, 1983; Carroll, 1994), or the commitment to a separable ontology which is put into question by the entanglement structure of quantum mechanics (Maudlin,

[1]Maudlin's view according to which laws produce the history of the universe is arguably not.

2007a). Humeans have resisted all of these attacks with some, though varying, degree of persuasiveness (see, e.g., Lewis (1994); Loewer (1996, 2012); Cohen and Callender (2009); Hall (2009), for the application of Humeanism to (Bohmian) quantum mechanics see Esfeld et al. (2014); Esfeld (2014); Miller (2014); Callender (2015); Bhogal and Perry (2017)). This is not to say that the objections have no merit, but I believe they have not quite managed to capture the implausibility of Humean metaphysics and turn it into a compelling argument for modal realism.

The present paper aims to do just that. It will thereby elaborate on a fairly common anti-Humean intuition, which is to look at the astonishing order in our cosmos, the uniformity of nature expressed by the simple and successful laws discovered in physics, and ask: *how likely is it that these regularities come about by chance?*

One place where this argument articulated in some detail is in the book "The Divine Lawmaker" by John Foster (2004) (for a more recent discussion along these lines, see Filomeno (2019)):

> What is so surprising about the situation envisaged – the situation in which things have been gravitationally regular for no reason – is that there is a certain select group of types, such that (i) these types collectively make up only a tiny portion of the range of possibilities, so that there is only a very low prior epistemic probability of things conforming to one of these types when outcomes are left to chance ... (Foster, 2004, p. 68)

I agree with the basic intuition but believe that the argument, thus phrased, cannot succeed. Indeed, Humeans have several good points to make in response:

1. We do not have to account for why the law of gravitation – or any other particular law described by physics – holds in our universe. Anti-Humean views don't explain this, either. The debate is about what it is to be a law, not why the laws of our world are what they are.

2. What do you mean by "chance"? The thesis of Humean supervenience holds that the history of the universe, the distribution of "local particular facts", is contingent. But contingency, or the absence of a further metaphysical ground, does not mean randomness. In fact, Humean metaphysics are opposed to all intuitions about the mosaic being "produced" by a chancy process – particles performing random motions, or God playing blindfolded darts and throwing local particulars into spacetime, or anything like that.

3. Where do your "prior probabilities" come from? What determines the right probability measure over possible worlds? All successful applications of probability theory come from *within* science. And according to the most prominent Humean account (Lewis, 1994; Loewer, 2001, 2004; Albert, 2015), the fundamental probability measure that grounds probabilistic predictions and rational priors is itself part of the best system that supervenes on the Humean mosaic. In other words: the actual world determines all relevant probabilities; there are no justified a priori probabilities which could warrant the conclusion that a world like ours is unlikely.

These points are well taken. In particular, I agree that references to probability or chance are dubious in a metaphysical context where subjectivist, frequentist, and regularity interpretations all seem questionable or inappropriate. There exists, however, a different concept that has recently gained increased attention in the philosophical literature and strikes me as a perfect fit for the issue at hand. That is the concept of *typicality* (see, e.g., Goldstein (2001, 2012); Bricmont (2001); Volchan (2007); Lazarovici and Reichert (2015); Maudlin (2007b, 2020); on the differences between typicality and probability, see, in particular, Wilhelm (2019)).

1.1 Typicality

The basic idea is simple: A property P is typical within a reference class W if almost all members of W instantiate P. The property is atypical within W if $\neg P$ is typical.

If P is typical within W, it is also commonly said (with a slight abuse of language) that a typical member of W has the property P.

Why does typicality avoid the objections raised against probability? For one, because typicality statements are extremely robust against variations of the measure used to quantify subsets of W, so much so that, in most cases, the question how to pick the right measure or what it even means to be the "right" measure doesn't even arise (cf. Maudlin (2007b, p. 286)).

A paradigmatic example of a typicality statement is: *almost all real numbers are irrational.* That is, being irrational is typical within the set $\mathbb{R}$ of reals.

In what sense is this true? First and foremost, in terms of cardinalities. The set of real numbers is uncountably infinite, while the subset of rational numbers is only countably infinite. Therefore, $\frac{|\mathbb{Q}|}{|\mathbb{R}|} = 0$. This is a very precise and generally uncontroversial sense in which almost all real numbers are irrational. In principle, nothing more needs to be said here. However, since we will use it later on – and since its more familiar from applications in physics – we can spell out typicality in terms of a measure in the sense of mathematical measure theory. It then seems natural to consider the uniform Lebesgue measure on $\mathbb{R}$, which makes it true that *all real numbers except for a subset of measure zero* are irrational.[2] Note that the Lebesgue measure on $\mathbb{R}$ is not normalizable, so it cannot be confused with a probability measure. But maybe the uniform measure is suspicious as it reeks too much of a "principle of indifference". Fair enough, we can pick virtually any other measure we like. Any non-discrete measure, i.e., any measure that is zero on singletons, will agree that $\mathbb{Q} \subset \mathbb{R}$ has measure zero. (By σ-additivity, a measure can only be non-zero on countable sets if it is non-zero on some one-element sets.) Simply put, we assume nothing more than that a one-element subset is vanishingly small compared to an uncountably infinite set. There is thus a very innocent and intuitive sense in which all reasonable measures agree on the meaning of "typical".

Notably, typicality statements in physics usually admit exception sets of very small (but positive) measure. This introduces a certain vagueness and requires tighter restrictions on typicality measures, leading to some debate about their justification. Here and in our following discussion, we can use a very strict standard of typicality, which provides much stronger results than can be realistically obtained in physics and for which these issues do not seriously arise.

Another crucial difference between typicality and probability is that typicality is not tied to ignorance, randomness, or indeterminism. It is an objective, determinate fact that typical real numbers are irrational. It has nothing to do with anyone's credences, nor with some number being picked at random, or picked out at all.

When applied to a reference class of possible worlds, typicality figures in a way of reasoning about contingency. (And contingency, if anything, is central to Humean metaphysics.) If a fact about the world is contingent, it means that it could have been different. But not all contingent facts are equally surprising or counterfactually robust or deserving of an explanation. Some facts stand out in that they make our world very special. Some facts could have been different but only if God – metaphorically speaking – had meticulously arranged things in the world to make it so. Recently, several papers have explored how typicality facts can ground explanations, predictions, and rational believes, both in everyday life and in the context of fundamental physics and statistical mechanics. We will expand on some of this in the course of our discussion and argue, in particular, that typicality extends to a powerful way of reasoning in metaphysics.

The typicality fact that the best system account has to deal with is then the following: It is typical for Humean worlds to have no Humean laws. Almost all Humean worlds *do not have any regularities in the first place* but are too complex to allow for a complete

[2]This is weaker than the statement in terms of cardinalities; all countable sets have Lebesgue measure zero, but not Lebesgue null-sets are countable.

systematization. (This will be rigorously proven for deterministic laws and in a more hand-waving fashion for probabilistic ones.) The challenge to Humean metaphysics is thus not to account for why we find *these particular* laws in our universe, but why we find *any laws at all*. Conversely, if we do live in a world that is regular enough to be described by physical laws, the best explanation is the existence of something in the fundamental ontology that makes it so.

In the next section, we will clarify the relevant reference class of the above typicality statement. Section 3 will then discuss typicality as a way of reasoning in science and metaphysics, i.e., the normative implications of typicality facts. Section 4 will provide a detailed discussion and proof of the typicality result that almost all Humean worlds have no laws. Section 5 will address the question, in what sense anti-Humean metaphysics do and do not fare better in explaining the lawfulness of our world.

2 Ontological possibility

> *The orthographical symbols are twenty-five in number. This finding made it possible, three hundred years ago, to formulate a general theory of the Library and solve satisfactorily the problem which no conjecture had deciphered: the formless and chaotic nature of almost all the books.*
>
> — Jorge Luis Borges, The Library of Babel

While typicality statements, at least when made with respect to an infinite reference set, require some mathematical tool like measures to give precise meaning to the locution "almost all", their truth-maker – and what is ultimately doing the explanatory work – is not the measure but the reference class with respect to which properties come out as typical or atypical (or neither). In particular, an important – if not the most important – way in which physical laws, however conceived, *explain* or *predict*, is by delimiting a set of nomologically possible worlds that makes certain physical phenomena typical.

Indeed, we learned with the breakthrough of atomism and the development of statistical mechanics that, due to the huge number of microscopic degrees of freedom, the fundamental dynamical laws allow for vast possibilities far beyond what had been thought permissible by natural laws (cf. Albert (2015, Ch.1)). That apples fall to the ground but don't jump up, that planets do not suddenly fly off their orbit (while emitting an ultra-fast particle in the opposite direction), and heaps of dust do not spontaneously transform into dinosaurs, is explained not by the fact that such events are nomologically impossible but by the fact that they are atypical, i.e., that they would require extremely special micro-conditions (of the universe, in the last resort).[3]

The modality involved here can be understood in a largely semantic manner, independent of metaphysical commitments. The content of a physical law (I am referring here primarily to dynamical ones) corresponds, first and foremost, to the modal space carved out by the law. And then there are certain features common to almost all possible solutions (at least given certain macroscopic boundary conditions), while most phenomena that we would consider as falsifying instances are not quite logically incompatible with the law but realized only for atypical models of the theory.

To apply an analogous typicality reasoning in a metaphysical context – evaluating how Humean, respectively anti-Humean metaphysics fare in accounting for the relevant features of our world – we need a reference class of possible worlds that is determined by the respective ontologies and does not a priori coincide with nomic possibilities. The relevant reference class that I propose is generated as follows:

Fix the fundamental ontology of the world as postulated by a metaphysical theory, that is, the fundamental entities with their essential properties, and consider all their possible configurations, i.e., possible distributions of contingent properties (such as spatiotemporal relations) over these "individuals".

Possible worlds thus generated are sometimes called *Wittgenstein worlds*[4]. Allowing for "augmentation" and "contraction" – adding individuals (but not universals) beyond those that exist, or removing some that do – the set Wittgenstein worlds is extended to "Armstrong worlds" (Kim, 1986) and the theory of modality known as Combinatorialism (Armstrong (1986, 1989); see Sider (2005) for a recent discussion). In our discussion, we will not need augmentations and contractions, and if we consider the option that laws of nature may themselves be among the fundamental "entities" in our world, adding or removing them would defeat the purpose. Hence, we shall keep the basic furniture of our world fixed, both in type and in number. Notably, I am not interested in defending this or any other version of Combinatorialism as a full-blown theory of metaphysical possibility. Instead, let us call the relevant notion of modality *ontological possibility*, the crucial point being that a world is ontologically possible (according to a metaphysical candidate-theory) if it has the same fundamental ontology as postulated for ours.

[3]The easiest – but by no means only – way to construct such bizarre examples is to consider the time reversal of entropy-increasing processes, exploiting the time-reversal invariance of the microdynamics.

[4]In reference to the following passage of the *Tractatus*:

2.0271 The object is the fixed, the existent; the configuration is the changing, the variable.

2.0272 The configuration of the objects forms the atomic fact. [...]

2.04 The totality of existent atomic facts is the world.

Here are some examples for the use of ontological possibility: If the fundamental ontology of the world consists in point particles moving in space, it is ontologically necessary for all material objects to be spatially localized. If the fundamental ontology of the world consists of N permanent point particles, it is ontologically impossible for any object to be composed of more than N parts. According to a Super-Humean theory of space or spacetime (Huggett, 2006), it is ontologically possible for spacetime to have more than four dimensions. According to a functionalist theory of the mind – but not according to theories that postulate "minds" as ontological primitives – consciousness is ontologically contingent.

Why should we care about ontological possibility given that it is, as far as I can tell, a notion that we have just stipulated rather than an established philosophical concept?

Most basically, because it seems like a fairly standard semantic interpretation of what a hypothesis about the fundamental ontology of the world means. It is thus important to emphasize that, in this vein, the metaphysical theories themselves determine the class of "ontological possibilities" as they postulate a fundamental ontology.

Intuitively, because the fundamental entities that we believe to exist should have a distinguished epistemic and explanatory role over those that are merely possible or conceivable.

Most importantly, because ontological possibility, thus defined, is the form of modality that captures the disagreement between Humean and many anti-Humean metaphysics. Humeans and anti-Humeans will agree on the set of nomological possible worlds (if they agree on what the best theories of physics are) and they may agree or disagree on metaphysical possibility for all kinds of philosophical reasons that can go beyond their stance on laws of nature. Humeans, however, are committed to a *principle of unrestricted recombination* (Lewis, 1986a): it is possible to change the configuration of fundamental entities or properties in any part of the Humean mosaic while holding fix the rest of the mosaic. This is the positive content of Humean metaphysics, the flip side of the negative theology regarding necessary connections and other "non-Humean whatnots."

The main anti-Humean positions, on the other hand, hold that there exists something in the actual world – be it essential dispositional properties or primitive laws – that restricts combinations; that makes it impossible, let's say, for a world to have the same fundamental ontology as ours but a distribution of masses incompatible with the law of gravitational attraction. Notably, I consider the relevant ontological commitment to be a naturalistic one, i.e., a commitment to the fundamental laws that physics discovers in our universe (and of which, as of today, we have only partial or approximate knowledge). I don't believe that anti-Humeanism is interesting as an a priori thesis, postulating the existence of *some* non-Humean laws regardless of their content or connection to empirical science. Both Humeans and anti-Humeans take a leap of faith when assuming that we live in a lawful universe, that there are true universal laws that physics can, at least in principle, discover (or devise). The interesting debate, to my mind, is whether this justifies, or even compels, ontological commitments over and above the Humean mosaic.

The anti-Humean theories considered here also include the view that the manifestations of the primitive laws or dispositions are essential to them, i.e., that a particular non-Humean law is the same in every world in which it exists.[5] The different meaning of "nomological possibility" under a Humean and anti-Humean understanding is thus manifested in the fact that according to the latter but not the former, ontologically possible worlds form a subset of the nomologically possible ones.Of course, many anti-Humean theories go as far as claiming that nomic possibility coincides with metaphysical possibility, but this is an unnecessarily strong assumption for our purposes.

3 Typicality reasoning and the case against Humeanism

With such a reference class of ontologically possible worlds, typicality can play a similar role in metaphysics as it does in the physical sciences. Any law-hypothesis in physics designates a set of nomologically possible worlds. This set must contain the actual world for the proposed law to have any chance of being true. However, this is not sufficient for us to judge the law-hypothesis as compelling or explanatory or even empirically adequate. For instance, there are very plausibly Newtonian universes which are such that whenever particles are shot through a double slit and recorded on a screen, they form an interference pattern. These and other quantum phenomena are not made impossible by Newtonian laws – they just come out as atypical.

On the other hand, whenever we succeed in explaining (macroscopic) phenomena based on the fundamental (microscopic) laws, we show that they are typical, i.e., obtain for nearly all[6] possible initial micro-conditions, that is, in nearly all nomologically possible worlds (Lazarovici and Reichert, 2015). Among the typical regularities of our world are also statistical regularities, which is where objective probabilities come into play (Maudlin, 2007b; Goldstein, 2012; Oldofredi et al., 2016). If Bohmian mechanics is true, we even understand why quantum statistics are typical in this sense (Dürr et al., 1992).

[5]This may not extend to certain parts of the laws, like the constants of nature figuring in their formulation. Typicality considerations then give rise to the issue of fine-tuning of the constants, which is indeed a big topic in fundamental physics but beyond the scope of our discussion.

[6]At the suggestion of a referee, I am using "nearly all" for "all except for a subset of very small measure," in contrast to "almost all," which means "all except for a subset of measure zero".

The case of the *thermodynamic arrow* is a particularly interesting and much-discussed example. It is argued, based on the insights of Boltzmann's statistical mechanics, that nearly all possible micro-histories, relative to a low-entropy initial macro-state, correspond to an evolution of increasing entropy (see, e.g., Bricmont (1995); Goldstein (2001); Lazarovici and Reichert (2015)). However, it is atypical for the universe to be in a low-entropy state to begin with. This is why we have to invoke an additional theoretical postulate known as the "Past Hypothesis" (Albert, 2000), and there is a big debate about whether this Past Hypothesis is of the right kind to be a basic postulate in a physical theory – even an additional law of nature – or whether it cries out for further explanation (see, e.g., Penrose (1989, Ch.7); Callender (2004); Carroll (2010); Loewer (2012); Lazarovici and Reichert (2020)). Notably, without a Past Hypothesis, a typical universe would look very unordered and chaotic (or utterly boring and unstructured), even it obeyed simple laws on the microscopic scale.

In general, the way in which we evaluate physical theories is thus roughly the following: we consider the set of nomologically possible worlds determined by the laws it postulates and require that the saliant and relevant features of our world – the phenomena which are the target of explanation – come out as typical (or, at least, not as atypical). If our world corresponds, in the relevant respects, to an extremely special and fine-tuned, i.e. atypical, model of the theory, we amend or reject the theory. If we did not follow this standard, we would lose all means to test a theory against empirical evidence since special initial conditions could account for virtually anything.

I submit that a similar standard should apply in metaphysics when we judge proposals for a fundamental ontology of the world. If we want to know what explanatory work a "metaphysical theory" is doing, and how it matches the world that we live in, we should consider the set of ontologically possible worlds determined by the fundamental ontology it postulates and require, at the very least, that the features of our world that fall under the purview of the proposed metaphysics do not come out as atypical.

While will never get around the problem of underdetermination, this does not mean that there are no rational standards by which theories about the fundamental ontology can be judged against the manifest appearance of the world. Typicality provides such a standard. And if we reject it, we could postulate virtually any ontology we like – as long as it gives us enough "degrees of freedom" to play around with – and claim that they are arranged in precisely such as a way as to ground or realize whatever structures we identify in nature (cf. Lazarovici (2018)). In other words: safe for being logically inconsistent, both physical and metaphysical theories cannot do any worse in their respective domains than make the relevant features of our world atypical.

(What makes a feature of our world "relevant" – a valid target of theoretical explanation rather than an acceptably brute fact – is hard to answer in full generality. Every world is atypical with respect to *some* properties, but whether a certain theory is challenged by them depends also on its scope. In the physical context, the relevant features usually correspond to robust phenomena. When we consider proposals for the metaphysics of laws, the existence of laws is clearly a relevant feature of the world that the theory has to account for.)

Typicality is thus associated with the following rationality principle:

> Suppose we accept a theory T and we come to believe that our world has a salient and relevant property P. If it turns out that P is typical according to our theory, there is nothing left to explain. If it turns out that P is atypical according to T, we have to look for additional explanation – or else, in the least resort, reject our theory.

Atypicality, in other words, creates an epistemically unstable situation, and refusal to move means, in effect, to give up on a rational understanding of the world. The idea that our world just happens to be, in some relevant respect, an atypical model of our theory is unacceptable in science. It seems to me that this rationality principle is so deeply rooted in scientific thought that it is rarely made explicit, let alone questioned. As a matter of fact, more authors (see e.g., Putnam (1969)) have questioned the laws of logic than entertained "explanations" based on atypicality.

Very much related to this is another precedent from science, namely that for typicality as a necessary condition for a successful reduction. For instance, we accept the reduction of the thermodynamic theory of gases to the kinetic theory of particles – including the ontological reduction of gases to particle configurations – because the atomistic theory makes the relevant gas properties typical.[7] Conversely, since special micro-configurations could realize almost anything, the typicality standard prevents trivial and spurious accounts. Explanations of the form: Assume the initial conditions are such that P, then P. Or reductions of the form: Assume that X has the right configuration to realize/-ground/serve as the supervenience base of Y, then we can reduce Y to X. Humean supervenience has essentially this character: Assuming that the Humean mosaic is exactly as if governed by laws, we can reduce the laws to the mosaic.

[7] A crucial intermediate result is the Maxwellian velocity distribution, whose derivation, in the words of Ludwig Boltzmann (1896, p. 252), is based on "the fact that by far the largest number of all possible states have the characteristic properties of the Maxwellian distribution, and that compared to this number the amount of possible velocity-distributions that deviate significantly from Maxwell's is vanishingly small."

In a physical context, we would never accept a reduction based simply on deleting some theoretical entity and fine-tuning the heck out of what's left. Why should metaphysicians be held to that much lower standards?

It is admittedly difficult to find genuine examples for typicality reasoning in metaphysics that do not rely on natural laws and hence nomic possibilities. However, it is not too much of a stretch to revisit Leibniz' monadology, which denied the possibility of causal interactions between different monads (fundamental substances that can have either mental or material attributes), and ask: Why did Leibniz have to invoke his infamous doctrine of *pre-established harmony* to account for the coordination between physical and mental states (a form of which David Hume endorsed, as well)? Why could he not have left every monad to itself and claim that it is a contingent fact of our world that they happen to evolve in conformity? Well, because this claim is absurd, because without God's synchronization and in the absence of any causal or metaphysical connection, the conformity of mental and physical events would not be merely unexplained but atypical. Clearly, there would be countless more ways in which the mental and physical history of the world (and each person individually) could be in discord than in harmony, and clearly, discord is thus what Leibnizian metaphysics without pre-established harmony would imply. (Notably, we can make this judgment with high confidence even though we have nothing like a "probability measure" over possible mental states.) In the upshot: because his ontology of monads makes the conformity of mental and physical events atypical (though not impossible), and because giving up on this conformity would lead to absurdity or de facto solipsism, Leibniz had to postulate an additional metaphysical principle, viz. pre-established harmony.

A Humean ontology, as we shall now prove, makes the lawfulness of the world atypical, the harmony (so to speak) between physical events at different times at places that would allow for a systematization of the mosaic. As a consequence, we can accept a Humean ontology and be anti-realists about laws.[8] Or we can believe in true universal laws and look for additional metaphysical principles that account for their existence.

What most advocates of the best system account maintain, however, is that Humean metaphysics are true, and, at the same time, that our world is an atypical instantiation of a Humean ontology – not just with respect to some minor detail, but with respect to its lawfulness, the very feature at the center of their account. And this thesis, as a matter of reason and scientific practice, cannot be accepted.

[8] Which is not *completely* absurd. Maybe Nancy Cartwright (1983) – though no Humean – is right, and we never had good reasons to believe that laws of nature should be exactly and universally true.

4 Typical Humean worlds have no laws

In this section, we will prove the main theorem of this paper. In brief: typical Humean worlds have no laws. We begin with a simple toy model that we call the *Chaitin model*, after Gregory Chaitin (2007) who, based on ideas that strike me as very Humean, proposed a connection between scientific practice and algorithmic information theory.

4.1 The Chaitin model

In our model, a world – with the totality of physical facts – is represented by an infinite sequence of 0's and 1's. Assuming a principle of unrestricted recombinations, the set of ontologically possible Humean worlds thus corresponds to $W = \{0,1\}^{\mathbb{N}}$, the set of all possible sequences.

The *Kolmogorov complexity* of a sequence $w \in W$ is defined as the length of the shortest algorithm that generates it. If w has finite Kolmogorov complexity, i.e., can be produced by a finite algorithm, it is called *algorithmically compressible.*

For instance, the sequence w_0 = 0101010101... can be generated by an algorithm like

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while True:
    print("01")
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(which is just an infinite loop) so that it is algorithmically compressible with Kolmogorov complexity of 22 or less.

We can think of an algorithm as a candidate for a best system law, its role being to provide an efficient summary of the world (i.e., the sequence). In the spirit of the best system account, the length of the algorithm can be thought of as the measure of its simplicity. However, our argument will not even require laws to be particularly simple, they only have to be finite.

One problem, also familiar from the best system account, is that the length of an algorithm depends on the language in which it is written.[9] We will call two languages L_1 and L_2 *intertranslatable* if there exists a finite set of rules translating any algorithm in L_1 into an algorithm in L_2, and vice versa. It is easy to check that intertranslatability is an equivalence relation, and that the Kolmogorov complexity of a sequence with respect to any two intertranslatable languages differs at most by a finite constant. Algorithmic compressibility is thus well-defined on these equivalence classes.

It is well known that the best system account would be trivial without some restriction on the admissible languages in which the systematizations can be formulated. For otherwise, the best system would simply consist in a primitive predicate F such that $F(w)$ is

[9]The short example above is written in *Python*.

true if and only if w is the actual world @, see Lewis (1983, p. 367). "Being intertranslatable with any language known to humanity" seems like a very generous restriction, more so than if we assumed a privileged language of perfectly natural predicates.[10]

Hence, let $\mathcal{L}$ be the set of finite algorithms ("possible laws") in any language intertranslatable with some language known to humanity, and $W^* \subset W$ the corresponding set of compressible sequences. We call any $w \in W^*$ a *lawful world*.

Now, the following are simple mathematical facts:

- The set W of possible sequences ("possible Humean worlds") is uncountably infinite, its cardinality being that of the continuum: $|W| = 2^{\aleph_0} > \aleph_0$.
- The set $\mathcal{L}$ is countably infinite: $|\mathcal{L}| = \aleph_0$. [There are at most countably many admissible languages and countably many finite algorithms that can be formulated in each language. A countable union of countable sets is countable.]
- The set of compressible sequences ("lawful worlds") cannot be greater than the set of possible algorithms ("laws"): $|W^*| \leq |\mathcal{L}| = \aleph_0$. [Since each algorithm generates at most one sequence.]
- We conclude: $\frac{|W^*|}{|W|} = 0$. Hence, almost all sequences are algorithmically incompressible. Or: Almost all Humean world have no laws.

As in our first example of irrational numbers, we could also express typicality in terms of a measure instead of cardinalities. It then holds true that $\mu(W^* \subset W) = 0$ with respect to *all* measures on W that are zero on singletons. In the upshot, "lawfulness" is atypical among Humean worlds under any reasonable interpretation of the concept.

4.2 From the toy model to the real world

While I hope the model to be instructive, the real world is evidently not a sequence of numbers, and fundamental laws of nature are not just algorithms for data compression but, first and foremost, dynamical laws for the microscopic constituents of the world. In order to extend our previous result to realistic physical laws – focusing, for now, on deterministic ones – we proceed as follows:

We fix a slice V of the mosaic, which is sufficiently extended in space and time to fix not just initial conditions for any deterministic dynamics, but also the values of all free parameters, like constants of nature, that may appear in their formulation. (V could

[10] Correspondingly, in our toy-model, we could have defined compressibility with respect to a universal Turing machine.

be the actual history of our universe up to some time t, but a great many other choices will do, as well.) Then there exist at most countably many deterministic laws (if any) compatible with the facts in V – each determining a unique history for the rest of the universe – but uncountably many Humean possibilities to complete the mosaic.

Hence, we conclude that whatever the facts in V, it is atypical for the rest of the Humean mosaic to be consistent with any deterministic law (formulated in any language, formal or natural, that we could ever hope to understand).

As a corollary, we obtain: Given any deterministic law that can correctly describe the world up to time t, Humeanism implies that it will typically fail to be true at later times. This supports and strengthens the argument that Humeanism cannot sustain inductive inferences (Dretske, 1977; Armstrong, 1983; Segal, 2020). Induction may be difficult to justify in general, but Humean metaphysics *predicts its failure* in this sense.

While this (a)typicality result seems serious enough, it is, strictly speaking, a conditional claim "given one part of the mosaic." In general, there are already uncountably many possibilities for the "initial" data, i.e., uncountably many worlds consistent with every single deterministic law. At this point, we need some measure theory, after all, to obtain an unconditional typicality result.

As always, we assume that one-element subsets (and hence, by σ-additivity, countable subsets) of an uncountable set have measure zero. In addition, we require only that this remains true if we conditionalize on the configuration of the Humean mosaic in V and count the possible configurations in some distant region U. This is certainly legitimate considering the Humean principle of free combinations, which holds that one puts no restrictions on the other. (In fact, we assume much less than completely independent configurations in all disjoint spacetime regions, requiring only *some* region U whose possible configurations are not strongly restricted by that in V.)

The proof, given in the appendix, involves one technical subtlety (because we are potentially conditioning on a null-set), but in a nutshell, the argument concludes as follows: Denote by w_U the configuration of the mosaic in a spacetime region U. Then, there are uncountably many possible configurations w_U but, by the previous argument, at most countably many consistent with a deterministic law and the "boundary condition" w_V. Hence, $\mu(w_U \textit{ consistent with a law } | w_V) \equiv 0$ (for some suitable choice of U and V) and thus, with W^* the set of lawful Humean worlds,

$$\mu(W^*) \leq \int \mu(w_U \textit{ consistent with a law } | w_V) \, \mathrm{d}\mu(w_V) = 0$$

according to *any* reasonable measure. We conclude:

Theorem 4.1. *It is atypical for Humean worlds to be consistent with a deterministic systematization.*

Remark. This remains true if one restricts the set of possible Humean worlds to those consistent with intelligent life (in an attempt to appeal to anthropic arguments), or any other boundary conditions that one could impose on empirical grounds. A typical Humean world would still not be more regular than necessary to account for these conditions – which would, at most, require a small island of regularity in otherwise lawless universe.[11]

4.2.1 On typicality measures

Admittedly, the notion of a "reasonable measure" is doing a lot of work here. Mathematically, it is certainly possible to define other measures, but these are so clearly biased or ad hoc that they cannot play the role of a typicality measure. Mathematically, it is also possible to put a delta-measure on the reals and say that "almost all real numbers are zero." But this statement would only be true in the technical sense in which the locution "almost all" is introduced in measure theory. In any other sense, it is simply an abuse of language.

The point is that typicality statements have rational (normative) implications if and only if they are made with respect to a reasonable notion of "almost all." And I claim that the assumptions of our theorem are so weak and well-motivated that they exhaust all measures that could pass for reasonable. It is thereby important to keep in mind that, in contrast to other publications aiming at a similar conclusion, we are not arguing for some assignment of probabilities or a priori believes, but only for a sensible notion of "very large" versus "very small" sets of possible worlds (on which a huge class of measures will, in fact, agree). And to reject our notion is either to deny that a one-element subset is vanishingly small compared to an uncountably infinite set – which seems absurd – or to *presuppose* extremely strong "correlations" between different parts of the mosaic – which means, in effect, to deny Humeanism.

That said, the most common objection I have received remains that Humeans could reject *all* measures as unjustified – pointing, again, to the lack of an empirical basis for quantifying possible Humean worlds. But is it really an *empirical* fact that one out of an (uncountable) infinitude is *almost none*? And is it not a tenet of Humeanism itself that, ontologically, the configuration in one part of the mosaic does not restrict the rest?

[11] The situation here is quite analogous to the so-called "Boltzmann brain problem" in statistical mechanics (absent a Past Hypothesis).

It thus remains unclear to me what premise they could deny, and I suspect that this objection fails to appreciate the different between typicality and probability.

Maybe what some Humeans actually want to deny is not the typicality fact per se but its epistemic/doxastic implications, i.e., the rationality principles associated with typicality facts. Here, this would simply mean to insist that it is not irrational to believe in a certain metaphysical theory of laws, and, at the same time, that our world is unlike almost every model of that theory by having laws in the first place. In this case, we may not be able to settle the issue, although I will say a little more about it in the final section. But we would have at least succeeded in tracing back a key disagreement between many Humeans and anti-Humeans to the rejection/acceptance of a particular way of reasoning.

4.3 Finite systematizations

Our proof that the existence of a deterministic systematization is atypical for Humean worlds relied on the assumption that there are uncountably many possible configurations of the mosaic, or an infinite number of physical facts that the laws have to account for. On what basis could this assumption be denied? One could insist that the world *is* finitary, i.e., that space and time are finite and discrete, and that there are no continuous degrees of freedom in the physical ontology. While this cannot be ruled out in principle, it constitutes a rather strong a priori commitment and a revisionary stance with respect to contemporary physics. Alternatively, one could maintain that laws of nature do not have to provide a complete (microscopic) description of the world but only an approximate or "coarse-grained" systematization of a limited subset of events – e.g., measurement results or empirical observations – that is plausibly finite. This second option is essentially instrumentalism; the view that laws are efficient bookkeepers of empirical data rather than universal truths about the world.

In any case, if laws had to account only for a finite number of physical facts, it would still be true that typical Humean worlds are more or less irreducibly complex – meaning that they cannot be systematized by laws that are significantly simpler than a complete list of the relevant events – but only with respect to a more restricted set of languages in which the systems can be formulated. (Think about the Chaitin model and the question whether the Kolmogorov complexity of a finite sequence is significantly lower than the length of that sequence.) One could thus retreat to the idea that the order in our universe is not objective, but that (instrumentalist) laws – and the regular patterns they summarize – exist because we have adapted our cognitive and mathematical tools to the world that we inhabit (see e.g., Wenmackers (2016)).

Although I find it very uncompelling, I am not going to argue against this possible escape. If one concedes that Humeanism is de facto instrumentalism – or requires revisionary physics –the whole debate would be a very different one.

4.4 Indeterministic Laws

The issue becomes more complicated if we consider the possibility of indeterministic laws. Logically, at least, an indeterministic law (e.g., a stochastic evolution) could be compatible with any mosaic whatsoever – that is, unless there are real propensities in the world that the law is supposed to summarize. In fact, there is even a good case to be made that typical Humean worlds are well described by something like Brownian motion, which can be technically considered a "law" but describes pure noise rather than any kind of regular order.

For a probabilistic law to be *informative*, and allow for something akin to causal inferences, it must predict reasonably high conditional probabilities for a relevant class of events (Lewis (1980) talked, in particular, about "history to chance conditionals"), that is, expressions of the form $\mathbb{P}(A \mid B) \approx 1$ where the conditional probability for A depends non-trivially on B. In our world, the history of the universe up to the present time t should make it reasonably likely that the earth will still be in its solar orbit 10 seconds from now. Kicking a ball from the left/right should make it reasonably likely that the ball flies off to the right/left. More generally speaking, a concentration of masses in a small spacetime region B might make it very likely that masses agglomerate in another region A, or something like that. All laws, or law-candidates, that we take seriously in physics allow for such inferences (at least in relevant "semi-classical" situations).[12]

Now, could such correlations be typical with respect to Humean ontological possibilities? I claim that they can not. For if we take Humean metaphysics seriously, the possible configurations in one part of the mosaic should be independent of the facts in any other part of the mosaic. In effect, any evidence for a robust correlation is evidence that we do not live in a typical Humean world. And the existence of infinitely many correlated events would certainly be atypical with respect to Humean possibilities (while, if there is only a limited number of events that a law has to account for, we are essentially back in the "instrumentalist" scenario discussed in the previous subsection).

This is, admittedly, a less rigorous argument than the one for deterministic laws. And the result is weaker, as well, relying on a distinction between "informative" and "non-informative" laws that would warrant further elaboration, and on typicality measures

[12]The same applies, in effect, if one has a more deflationary conception of laws, according to which even the fundamental laws of physics can allow for exceptions or ceteris paribus clauses.

with strong independence properties. In the Chaitin model, it is also true that typical sequences do not allow for a more informative probabilistic systematization than "The next number in the sequence is 0 or 1 with a 50/50 chance" – but only with respect to a more restricted (though very natural) class of typicality measures (namely those that are absolutely continuous relative to the uniform measure when the set of possible sequences W is embedded into the real interval $[0, 1]$).

That said, at the end of the day, I don't expect the contentious point of our discussion to be whether Humean metaphysics fares much better with respect to probabilistic laws than deterministic ones. As with instrumentalism (or maybe even more so), committing to indeterminism from the get-go does not seem like an attractive option that most Humeans would want to take.

5 On the uniformity of nature

> *It would seem unreasonable ... if the whole universe and each and every part of it were in order..., while there were nothing of the kind in the principles.*
>
> — Theophrastus, Metaphysics 7a10

With the caveats just discussed, I consider it a fact that almost all Humean worlds have no laws. The philosophically more subtle discussion happened in Section 3, where we argued for the *normative implications* of typicality facts.

I take it that any form of rationality is normative. At the same time, I see one of the weaknesses of probabilistic arguments in that they try to shortcut the issue and argue directly in epistemic or doxastic terms. Typicality facts are neither epistemic nor doxastic facts. They hold independently of what we know or believe. Yet, they have implications for what we *should* rationally believe, or accept, or seek to explain. In the present case, that we cannot accept Humean metaphysics and believe in a lawful universe without seeking explanation for its lawfulness.

On the other hand, the account provided by anti-Humean theories is not a bona fide typicality explanation in that non-Humean laws make their instantiation not just typical but necessary. (This is strictly stronger, necessity implies typicality but not the other way around). The primary role of typicality in our argument is thus not to sustain an explanation but to establish that one is required, that the price for declaring the history of the universe to be entirely contingent is unreasonably high. However, what applies here as well as to bona fide typicality explanations is that they do not have to involve an interesting "mechanism" by which the explanandum comes about. There is no interesting story left to tell about *how* laws govern or *how* dispositions bring about their manifestation; the point is that they are a natural part of an ontology that doesn't make the existence of regularities in the world miraculous.

Typicality reasoning thus reveals how the explanatory virtue of an anti-Humean ontology comes from its modal force, from the way in which it restricts ontological possibilities. In contrast, the idea that non-Humean laws fare better in explaining their particular instances has made them vulnerable to the *virtus dormitiva* objection that any explanation they provide over and above the regularity theory is trivial or circular: Why do masses attract each other? Because they have the disposition to attract each other. Or: because it is a law that masses attract each other. In the contemporary literature (see, e.g., Emery (2019)), such statements are often spelled out in terms of grounding relations or as "in virtue of" explanations, which makes them manifestly non-circular, but still ring hollow to people not already sold on the merits of these metaphysical concepts. Thinking in terms of typicality, one appreciates that the real explanatory advantage of non-Humean laws is not that they provide an additional metaphysical ground for individual instances, but that they account for the world being lawful in the first place.

At the end of the day, one can only go so far in compelling someone to accept a certain way of reasoning and the norms that come with it. Some readers may deny that typicality facts have any philosophical implications, that there is even a sense in which Humean metaphysics make the lawfulness of our world surprising or remarkable. But there is no shame in sharing, at least, in a sense of wonder about the order of our cosmos. (After all, according to Aristotle, the sense of wonder is the beginning of philosophy.) The following passage from one of Albert Einstein's letters to his friend Maurice Solovine comes to mind:

> You find it strange that I consider the comprehensibility of the world (to the extent that we are authorized to speak of such a comprehensibility) as a miracle or as an eternal mystery. Well, a priori one should expect a chaotic world which cannot be grasped by the mind in any way. One could (yes *one should*) expect the world to be subjected to law only to the extent that we order it through our intelligence. Ordering of this kind would be like the alphabetical ordering of the words of a language. By contrast, the kind of order created by Newton's theory of gravitation, for instance, is wholly different. Even if the axioms of the theory are proposed by man, the success of such a project presupposes a high degree of ordering of the objective world, and this could not be expected a priori. That is the "miracle" which is being constantly reinforced as our knowledge expands. There lies the weakness of positivists and professional atheists who are elated because they feel that

> they have not only successfully rid the world of gods but "bared the miracles."
> (Cited from Einstein (1987, pp. 132-133).)

What, to their credit, distinguishes most Humeans from the "positivists and professional atheists" that Einstein is talking about, is some acknowledgement that the best system account of laws has to rely on nature being "kind to us" (Lewis, 1994, p. 479), on "a high degree of ordering of the objective world" that cannot be expected a priori. However, this kindness of nature is *so* stupendous and doing *so* much work in the best system account that it is highly unsatisfying, if not intellectually dishonest, to leave it as a footnote or some sort of auxiliary assumption without any basis in the metaphysical theory. If Humeans tried to give it more flesh, and spell it out as a metaphysical principle that makes the uniformity of the world typical (or necessary), their account would be much more sound but also start to look a lot more like anti-Humeanism.

One the other hand, some authors have argued that anti-Humean metaphysics have no advantage when it comes to explaining the uniformity of nature (Hildebrand, 2013). In this vein, advocates of the regularity theory could admit that Humeanism fails to account for a lawful universe but deny that anti-Humean positions fare any better in this respect. In the language of typicality, the relevant argument goes roughly as follows:

"Even if our world contained primitive laws or dispositions that necessitate reasonably simple regularities, this very fact would be atypical, as well. In almost all worlds in which non-Humean laws exist, the laws would be too strange or complex to allow for any meaningful systematization. Hence, the typicality argument can be turned just as well against anti-Humean metaphysics."

I am not sure whether this typicality statement is actually true. At least, most anti-Humean theories do not entail the possibility of arbitrarily complex laws in the same sense in which Humean metaphysics entails the possibility of arbitrarily complex mosaics. Note that if we change the configuration of a lawful Humean mosaic only slightly (that is, in a small spacetime region, not with respect to a Humean similarity relation between worlds that seeks to hold the laws fix by fiat) it will, in general, no longer be a lawful mosaic. If we change a simple law only slightly, it will still be a simple law. The point is that the degrees of freedom of a law are clearly different from those of the world, and the question what metaphysical possibilities we must admit with respect to the type "law of nature" strikes me as a very difficult one. Hildebrand (2013) takes nomic primitivism to mean that there exists a primitive lawhood operator "It is a law that..." that can attach to any proposition P, no matter how gruesome or unnatural. But this is not how physical theories actually look like, or what the anti-Humean positions that we regarded as promising actually commit to.

But even if we grant that typical non-Humean worlds have no simple laws, it is crucial to note that this is a typicality statement with respect to a different reference class than we employed in our discussion; namely metaphysically possible worlds (under a liberal interpretation of metaphysical possibility) rather than what we called ontologically possible worlds (across which the fundamental entities and their essential properties are constant). It is thereby shifting the debate from ontology to meta-ontology, from the question: "What is the fundamental ontology of our world (and does it contain the laws of nature)?" to: "Why is the fundamental ontology (here, specifically, the laws) what it is?". It is much less clear that this is a good and tractable question, and it is, in any case, not the question we set out to debate.[13]

The following analogy may help to illustrate my point: If all matter propagates along three spatial dimensions (not just appears to, but actually does), it is more than reasonable to infer that space – or the fundamental spatial relations, if one pefers, – *are* three-dimensional. (It would possible, yet atypical, that space has more dimensions, while all physical motion happens to occur along a three-dimensional subspace.) But why has space three dimensions when it could, at least mathematically, have had arbitrarily many? I don't know, and this was not the issue. If its three-dimensionality is part of my fundamental ontological commitments, then precisely because I can't reduce it to anything more fundamental.

As emphasized before, the aim of our discussion was not to defend anti-Humeanism as an a priori thesis. No one, I think, holds the view that our world must contain some primitive laws or dispositions, even if they govern only the growth of beetroots, or account for no meaningful regularities at all. My belief in non-Humean laws is very much contingent on the success of the scientific enterprise. And if I wake up tomorrow and find that the law of gravitation no longer holds, I would float through the air and admit that Humeanism was probably right all along.

Certainly, anti-Humean metaphysics do not relieve us of wonder and amazement about the simple and elegant laws that we discover in our universe. The existence of something over and above the Humean mosaic is, instead, an ontological conclusion that we draw from this discovery – with good reason as this paper has argued in detail. That may be as far as we can go. However, if there were a promising chance to take the explanation one step further, to understand *why* the laws are what they are, we should, by all means, follow the evidence where it leads us. It could, in any case, lead us only further away from Humeanism.

[13]It might be worth exploring the idea of meta-laws that constrain the possible non-Humean laws (Lange, 2009), but this goes beyond the scope of this paper and one must worry that it would, at best, be passing the buck (for what explains or necessitates the meta-laws?).

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Appendix: Proof of the main theorem

Theorem. *It is atypical for Humean worlds to be consistent with any deterministic systematization.*

Proof. Let's assume, with David Lewis, that the fundamental ontology is one of "perfectly natural properties" instantiated at spacetime points. (The argument for other ontologies, e.g., continuous particle trajectories, will go more or less analogously.) We can then model the set of possible Humean worlds by $W := \{w : \mathcal{M} \to S \subset \mathbb{R}^n\}$, where $\mathcal{M}$ is the spacetime manifold and the "field values" $w(x)$ describe the magnitudes of the relevant properties at spacetime point x.

We denote by w_V the restriction of w to $V \subset \mathcal{M}$ for a suitable V as explained in Section 4.2 (w_V is "the configuration of the mosaic in V") and by $L_U[f]$ the possible configurations of the mosaic in $U \subset \mathcal{M}$ that are consistent with some deterministic law and the boundary condition $w_V = f$. By the argument given in Section 4.2, $L_U[f]$ is at most countable for any $f : V \to S$, and for any $U \subseteq \mathcal{M} \setminus V$, the set W^* of Humean worlds consistent with a deterministic law must be contained in $\{w \in L_U[w_V]\} \subset W$.

Now, we choose as U a collection of points in $\mathcal{M} \setminus V$; countably infinitely many points if S is discrete, and finitely many if S is continuous. In any case, there are uncountably many possible configurations on U (but at most countably many consistent with a deterministic law and given boundary conditions on V). Let μ be a normalized measure on W (more precisely, on a suitable σ-algebra). Then there exists a regular version of $\mu\,(w(U) \in \cdot\,|w_V)$, i.e., a well-defined measure on the possible configurations in U, even if we conditionalized on a null-set. This holds because the value space of w_U, viz. $S^{|U|}$, is isomorphic to some subspace of $\mathbb{R}^k$, $k \in \mathbb{N} \cup \{\infty\}$ (Ash and Doleans-Dade, 2000, Thm. 5.6.5). By assumption, this conditional measure has no discrete part (for at least some suitable choices of U and V), i.e., it is zero on singletons, and thus by σ-additivity also on countable sets. Hence, $\mu(w_U \in L_U[w_V] \mid w_V) \equiv 0$. Therefore,

$$\mu(W^*) \leq \int \mu(w_U \in L_U[w_V] \mid w_V)\, \mathrm{d}\mu(w_V) = 0. \tag{1}$$

The proof extends at least to σ-finite measures with the conditional "probability" replaced by a Radon-Nikodym density (Ash and Doleans-Dade, 2000, Thm. 2.2.1).

□

References

Albert, D. Z. (2000). *Time and Chance*. Harvard University Press, Cambridge, Massachusetts.

Albert, D. Z. (2015). *After Physics*. Harvard University Press, Cambridge, Massachusetts.

Armstrong, D. M. (1983). *What Is a Law of Nature?* Cambridge University Press, Cambridge.

Armstrong, D. M. (1986). The Nature of Possibility. *Canadian Journal of Philosophy*, 16(4):575–594.

Armstrong, D. M. (1989). *A Combinatorial Theory of Possibility*. Cambridge University Press, Cambridge.

Ash, R. B. and Doleans-Dade, C. A. (2000). *Probability and Measure Theory*. Academic Press.

Bhogal, H. and Perry, Z. R. (2017). What the Humean should say about entanglement. *Noûs*, 51(1):DOI 10.1111/nous.12095.

Bird, A. (2007). *Nature's Metaphysics: Laws and Properties*. New York: Oxford University Press.

Boltzmann, L. (1896). Entgegnung auf die wärmetheoretischen Betrachtungen des Hrn. E. Zermelo. *Wiedemanns Annalen*, 57:773–784.

Bricmont, J. (1995). Science of Chaos or Chaos in Science? *Annals of the New York Academy of Sciences*, 775(1):131–175.

Bricmont, J. (2001). Bayes, Boltzmann and Bohm: Probabilities in Physics. In *Chance in Physics*, Lecture Notes in Physics, pages 3–21. Springer, Berlin, Heidelberg.

Callender, C. (2004). There is No Puzzle About the Low-Entropy Past. In Hitchcock, C., editor, *Contemporary Debates in Philosophy of Science*, pages 240–255. Blackwell.

Callender, C. (2015). One world, one beable. *Synthese*, 192(10):3153–3177.

Carroll, J. W. (1994). *Laws of Nature*. Cambridge University Press, Cambridge.

Carroll, S. (2010). *From Eternity to Here*. Dutton, New York.

Cartwright, N. (1983). *How the Laws of Physics Lie*. Oxford University Press, Oxford, New York.

Chaitin, G. J. (2007). *Thinking about Godel and Turing: Essays on Complexity, 1970-2007*. World Scientific.

Cohen, J. and Callender, C. (2009). A better best system account of lawhood. *Philosophical Studies*, 145(1):1–34.

Dretske, F. I. (1977). Laws of Nature. *Philosophy of Science*, 44(2):248–268.

Dürr, D., Goldstein, S., and Zanghì, N. (1992). Quantum equilibrium and the origin of absolute uncertainty. *Journal of Statistical Physics*, 67(5-6):843–907.

Einstein, A. (1987). *Letters to Solovine*. Philosophical Library.

Emery, N. (2019). Laws and their instances. *Philosophical Studies*, 176(6):1535–1561.

Esfeld, M. (2014). Quantum Humeanism, or: Physicalism without properties. *The Philosophical Quarterly*, 64(256):453–470.

Esfeld, M., Lazarovici, D., Hubert, M., and Dürr, D. (2014). The ontology of Bohmian mechanics. *British Journal for the Philosophy of Science*, 65(4):773–796.

Filomeno, A. (2019). Are non-accidental regularities a cosmic coincidence? Revisiting a central threat to Humean laws. *Synthese*.

Foster, J. (2004). *The Divine Lawmaker*. Oxford University Press.

Goldstein, S. (2001). Boltzmann's Approach to Statistical Mechanics. In Bricmont, J., Dürr, D., Galavotti, M. C., Ghirardi, G., Petruccione, F., and Zanghì, N., editors, *Chance in Physics: Foundations and Perspectives*, pages 39–54. Springer, Berlin.

Goldstein, S. (2012). Typicality and Notions of Probability in Physics. In *Probability in Physics*, The Frontiers Collection, pages 59–71. Springer, Berlin, Heidelberg.

Hall, N. (2009). Humean reductionism about laws of nature.

Hildebrand, T. (2013). Can Primitive Laws Explain? *Philosopher's Imprint*, 13(15).

Huggett, N. (2006). The regularity account of relational spacetime. *Mind*, 115(457):41–73.

Kim, J. (1986). Possible Worlds and Armstrong's Combinatorialism. *Canadian Journal of Philosophy*, 16(4):595–612.

Lange, M. (2009). *Laws and Lawmakers: Science, Metaphysics, and the Laws of Nature*. Oxford University Press, Oxford, New York.

Lange, M. (2013). Grounding, scientific explanation, and Humean laws. *Philosophical Studies*, 164:255–261.

Lazarovici, D. (2018). Super-Humeanism: A starving ontology. *Studies in History and Philosophy of Science Part B: Studies in History and Philosophy of Modern Physics*.

Lazarovici, D. and Reichert, P. (2015). Typicality, Irreversibility and the Status of Macroscopic Laws. *Erkenntnis*, 80(4):689–716.

Lazarovici, D. and Reichert, P. (2020). Arrow(s) of Time without a Past Hypothesis. In Allori, V., editor, *Statistical Mechanics and Scientific Explanation: Determinism, Indeterminism and Laws of Nature*. World Scientific.

Lewis, D. (1980). A Subjectivist's Guide to Objective Chance. In Harper, W. L., Stalnaker, R., and Pearce, G., editors, *IFS: Conditionals, Belief, Decision, Chance and Time*, The University of Western Ontario Series in Philosophy of Science, pages 267–297. Springer Netherlands, Dordrecht.

Lewis, D. (1983). New work for a theory of universals. *Australasian Journal of Philosophy*, 61(4):343–377.

Lewis, D. (1986a). *On the Plurality of Worlds*. Blackwell, Oxford.

Lewis, D. (1986b). *Philosophical Papers*, volume 2. Oxford University Press, Oxford.

Lewis, D. (1994). Humean supervenience debugged. *Mind*, 103(412):473–490.

Loewer, B. (1996). Humean Supervenience. *Philosophical Topics*, 24:101–127.

Loewer, B. (2001). Determinism and Chance. *Studies in History and Philosophy of Science Part B: Studies in History and Philosophy of Modern Physics*, 32(4):609–620.

Loewer, B. (2004). David Lewis's Humean Theory of Objective Chance. *Philosophy of Science*, 71(5):1115–1125.

Loewer, B. (2012). Two accounts of laws and time. *Philosophical Studies*, 160(1):115–137.

Maudlin, T. (2007a). *The Metaphysics Within Physics*. Oxford University Press, Oxford.

Maudlin, T. (2007b). What could be objective about probabilities? *Studies in History and Philosophy of Science Part B: Studies in History and Philosophy of Modern Physics*, 38(2):275–291.

Maudlin, T. (2020). The Grammar of Typicality. In Allori, V., editor, *Statistical Mechanics and Scientific Explanation: Determinism, Indeterminism and Laws of Nature*. World Scientific.

Miller, E. (2014). Quantum entanglement, Bohmian mechanics, and Humean supervenience. *Australasian Journal of Philosophy*, 92:567–583.

Oldofredi, A., Lazarovici, D., Deckert, D.-A., and Esfeld, M. (2016). From the Universe to Subsystems: Why Quantum Mechanics Appears More Stochastic than Classical Mechanics. *Fluctuation and Noise Letters*, 15(03):1640002.

Penrose, R. (1989). *The Emperor's New Mind: Concerning Computers, Minds, and the Laws of Physics*. Oxford University Press, Oxford.

Putnam, H. (1969). Is Logic Empirical? In Cohen, R. S. and Wartofsky, M. W., editors, *Boston Studies in the Philosophy of Science: Proceedings of the Boston Colloquium for the Philosophy of Science 1966/1968*, Boston Studies in the Philosophy of Science, pages 216–241. Springer Netherlands, Dordrecht.

Schaffer, J. (2008). Causation and Laws of Nature : Reductionism. In Sider, T., Hawthorne, J., and Zimmerman, D. W., editors, *Contemporary Debates in Metaphysics*, pages 82–107. Blackwell.

Schaffer, J. (2009). Spacetime the one substance. *Philosophical Studies*, 145(1):131–148.

Schaffer, J. (2016). It is the Business of Laws to Govern. *Dialectica*, 70(4):577–588.

Segal, A. (2020). Humeanisms: Metaphysical and epistemological. *Synthese*.

Sider, T. (2005). Another Look at Armstrong's Combinatorialism. *Noûs*, 39(4):679–695.

Volchan, S. B. (2007). Probability as typicality. *Studies in History and Philosophy of Science Part B: Studies in History and Philosophy of Modern Physics*, 38(4):801–814.

Wenmackers, S. (2016). Children of the Cosmos. In Aguirre, A., Foster, B., and Merali, Z., editors, *Trick or Truth? The Mysterious Connection Between Physics and Mathematics*, The Frontiers Collection, pages 5–20. Springer International Publishing, Cham.

Wilhelm, I. (2019). Typical: A Theory of Typicality and Typicality Explanation. *The British Journal for the Philosophy of Science*.

Wittgenstein, L. (1922). *Tractatus Logico-Philosophicus*. Kegan Paul, London.