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Some guidelines for fuzzy sets application in legal reasoning

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Abstract. As an introduction to our work, we emphasize the parallel interpretation of abstract tools and the concepts of undetermined and vague information. Imprecision, uncertainty and their relationships are inspected. Suitable interpretations of the fuzzy sets theory are applied to legal phenomena in an attempt to clearly circumscribe the possible applications of the theory. The fundamental notion of reference sets is examined in detail, hence highlighting their importance. A systematic and combinatorial classification of the relevant subsets of the legal field is supplied for practical application. Although the use of the fuzzy sets theory is sometimes suggested as a palliative measure (no competition exists), it can also be complementary (serve as a building block to improve modelisation). An Appendix gives a brief recall of the key-concepts of the axiomatic theory of fuzziness and its developments: fuzzy sets, fuzzy logic, fuzzy control and theory of possibility.

Key words: fuzzy logic, gradual knowledge, jurimetrics, rules interpolation

1. Introduction

Although most A.I. and law papers consider vagueness as a crucial element in the modelisation of legal reasoning, surprisingly enough, very few authors focus on the fuzzy sets theory. However it is worth noting here the early intuitions of Leo Reisinger and Ejan Mackaay.[1] Their very existence sets a problem to the scene, 20 years later our paper cannot be ahead of its time but rather marks the 20th anniversary of a marriage not consummated. Paragraph 3. will try to explain why.

An issue of *Informatica e diritto* published the proceedings from "The Computer and Vagueness: Fuzzy Logic and Neural Nets" seminar held in Munich in November 1992. Two papers appear to be devoted to the application of fuzzy sets and set another problem to the scene. One of these (Mazzarese 1993) is firmly based on a sound legal-theory background, however the result is somewhat intriguing, any formalization remains to be found. The second paper (Philipps 1993) refers to one precise application of fuzzy sets, however the author does not advocate the indisputable use of fuzzy sets either in the context of the broader legal area or in the context of developments of the fuzzy sets theory.

[1] Reisinger (1975), in German; Mackaay (1975), in French; Reisinger (1981), in English.

The purpose here is to present an overview in order to highlight the key-concepts. If we are to obtain an accurate view, it's high time we place the subject in perspective. We aim at investigating the pros and the cons of applying fuzzy sets. We *do not* aim at fooling the lawyers into thinking that fuzzy sets are *the* solution to all problems with vagueness.

Paragraph 4. studies which areas might be suited for applications. An attempt to clearly circumscribe the possible applications of the theory must avoid unprofitable debate about the "vagueness of fuzziness". One may avoid the emotive fallacy stemming from the use of "fuzziness" outside of the Fuzzy Sets theory. Fallacy is the very term because the use of fuzzy calculus provides an increased formalization capacity. One enriches the modelisation within preliminary and agreed limits. Nevertheless, one must avoid to engage the lawyers in the daunting task of determining which elements of these specialized formalisms are relevant.

The next paragraph gives a brief recall of the key-concepts of the theory of fuzziness and continues with a parallel interpretation of abstract tools and legal information.

2. Semantic Interpretations of Fuzzy Sets and Legal Reasoning

2.1. WHAT ARE THE MATHEMATICAL TOOLS MADE FOR?

According to the Appendix A, the axiomatic theory of fuzziness is complete with two separate practical developments.

- The purpose of the first one is to use the fuzzy sets to reason on meaning (meaning preserving reasoning). Concepts together with a degree of membership qualify a precise knowledge (numeric, for example). When they are used in linguistic predicates, a fuzzy inference mechanism describes the relation from the fuzzy sets in the premises to the fuzzy sets in the consequence. It is referred to as Fuzzy Logic (F.L.).
- The purpose of the second one is to reason about certainty with a Boolean algebra of precise assertions. A classical inference mechanism (unconcerned about meaning) describes the relation from the truth of the premises to the truth of the consequence. It is referred to as Theory of Possibility (T.P.).

The fuzzy sets interpretation of the T.P. states that a *suggested* proposition corresponding to a situation described with an *asserted* proposition composed of imprecise contents, becomes uncertain when its contents are more precise. A linguistically described vague knowledge acts as flexible constraints on the propositions that may be suggested. According to this interpretation, T.P. deals with both imprecision and uncertainty. The point is the inference mechanism which simulates reasoning.

Whereas F.L. is a multi-valued logic and T.P. is a twin-valued logic; whereas a degree of truth formalizes imprecision and a degree of possibility-necessity formalizes certainty; whereas the certainty of the truth of a fuzzy (partially true) proposition has no meaning; imprecision, uncertainty and their relationships must be inspected.

2.2. UNCERTAINTY AND IMPRECISION

Vagueness will be used as the more generic term in order to qualify a situation open to uncertainty or imprecision. For example, if somebody believes that “the client is 28 years old”, this is an uncertain assertion about a precise fact; “The driver of the car was a child” is an undoubted assertion about an imprecise fact.

J. Wroblewski uses the term “doubt”[2] in his definition of “(legal) interpretation” *stricto sensu*. When uncertainty concerns the meaning, the distinction between uncertain and imprecise vanishes. When commonsense language uses “truly very small” in order to say that the height has a membership degree to “very small” close to the maximum, does it mean that one employs words concerning certainty in order to reduce imprecision? In utterances, imprecise terms are used to increase confidence and certainty is called on to limit interpretation.

We will follow these intuitive remarks with more rigorous reasoning.

A proposition about a given imprecise element under an imprecise state of knowledge could arise. For example, when a noise is known to be loud, the proposition “the noise level is too high” is certain with a truth value of 0.8 or is 0.8-certain (calculi with the membership degrees are taken for granted). The difference between the degrees of truth of the latter and the degrees of certainty of the former is a *syntactical* one. The achieved composition and inference mechanism make the difference.

Before speaking of uncertainty, one has to decide which is the logic at hand. A proposition is certain when there is no opportunity for uncertainty (many-valued logic). It should be emphasized that the theory of possibility does not supply a multi-valued logic but rather a twin-valued logic.

2.3. LEGAL REASONING AND UNCERTAINTY

Assertions and statements found in legal sources are certain. The driven inferences, during law enforcement, judiciary process or document assembly, are affected by conflict and reliability degrees. In coping with imprecision, legal reasoning may produce doubtful assertions (for example, provisional interpretation of norms). Neither the burden of proof nor the grounding of norms is considered here.

The legal domain leaves little room for the usual probabilistic definition of uncertainty.[3] The subjective theory of probability attempts to express the degree

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[2] “…the situations where a doubt exists about the meaning …” (free translation of Dictionnaire encyclopédique de théorie et de sociologie du droit, L.G.D.J., 1988, p. 199).

[3] “The objective concept of probability refers to the calculation of chances of a particular outcome in a series of like events” (Loevinger 1992, p. 341).

of confidence with which an assertion is made. However, in order to translate the speculation into mathematical probability, the degree of credibility of non-A must decrease when the degree of credibility of A increases (see §3.1).

The degrees of the theory of possibility which we suggest as a means of considering the subjective "valuation" of a "level of acceptance" are suited to the legal domain. Except for the burden of proof, legal reasoning leaves little room for the notion of "truth". Partial belief is biased belief. A "high level of acceptance" may be related to:

- "correct way" (reasoning);
- "persuasion" (argumentation);
- "ex lege" (hierarchically superior court).

2.4. LEGAL REASONING AND IMPRECISION

The imprecision of the contents of an assertion is not linked to the qualitative nature of the concepts involved. "Extensional frame referencing" (see §4.1) concerns gradual predicates and open-textured categories. The former imprecision stems from the difficulty to give a quantitative description of a qualitative datum, the latter stems from the difficulty of membership assignment in an intentionally defined (or ill-defined) concept.

The resulting paradox is:

- in search of acceptability, the legal process introduces the use of imprecision by choosing a more generic or a more general term;
- when the fuzzy sets theory allows a manipulation of meaning through the manipulation of imprecision, it is relative to a reference set.

Hence, whilst doubt about the links to a reference set necessarily induces vagueness, the absence of an extensional frame referencing forbids the formalized existence of imprecision.

Syntactically speaking, the notion of "reference set", introduced in Appendix A, enables us to decide: what is precise; what is imprecise. *Intuitively speaking*, the imprecision of a proposition depends on the available knowledge. For example, "the meeting is quiet" has an ambiguous meaning. According to the theory of possibility, the more specific "quiet" is, the more chance that "quiet" will be true (maximal necessity). According to fuzzy logic, the more "quiet" is imprecise, the more chance that the noise measurement will suit. *Pragmatically speaking*, imprecision rests entirely upon the definition of the reference set.

2.5. LEGAL REASONING AND GRADUAL KNOWLEDGE

Several levels of gradual knowledge have to be considered:

- usual predicates that put scales of size into words are referred to as topical fields;
- if-then rules that link topical fields and are referred to as flexible or approximate rules;
- gradual rules that link (monotonic relationships) topical fields and are called topoï (Raccah 1993).

Flexible rules are implications of the form: If x is A Then y is B (A and B are imprecise concepts).

Fuzzy logic interpretation of the degrees resulting from inference mechanisms is: graded conclusion (several rules may be connected together). For example: If retired Then old; If old then mobility allowance.

Topoï are argumentative warrants of the form: the more/less, the less/more. Several interpretations of gradual rules may be suggested (P, Q are propositions):

- graded conclusion; the value varies in direct ratio or in inverse ratio to the premises (the more P, the more Q);
- graded confidence in the conclusion (the more P, the more Q is certain/possible);
- graded importance of the conclusion, (the more P, the more Q is important).

For example, we consider bona fide: the more you speak in good faith, the lower you will be fined; the more you speak in good faith, the more surely you will be acquitted; the more you speak in good faith, the more prevailing your evidence is. A decisional weight requires a possibility-necessity formalization while a graded reply requires a fuzzy formalization.

When using developments of the fuzzy sets theory, the semantic interpretation of the computed degrees depends on what the conclusion aims at.

When reasoning on norms interpretation, implicit doctrinal rules are often used before a clear-cut explicit decision is produced. In this type of reasoning, interpolation is extensively used to cope with:

- the magnitude of the premises and their compensatory features;
- rules available for parallel or sequential conclusions (reinforcement, substitution, modification).

To reach a conclusion in such a process, the respective degrees to which each rule can be applied to the situation are assessed in order to:

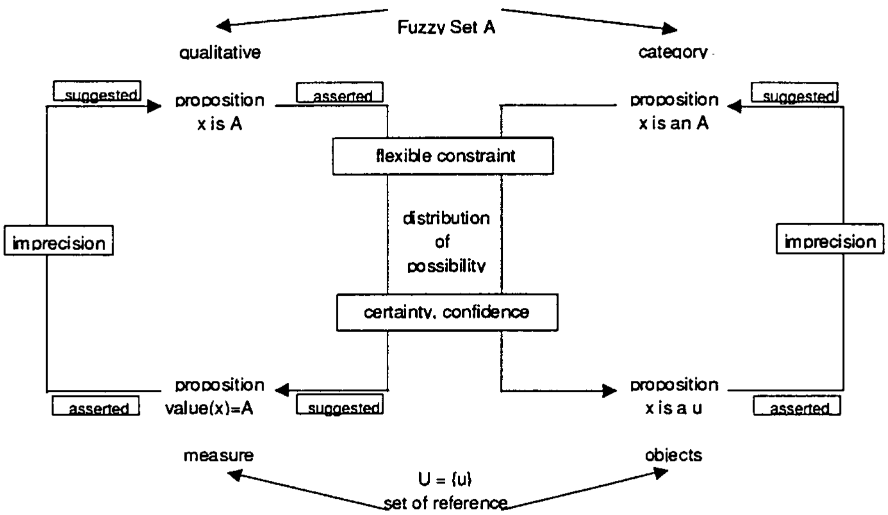


Figure 1. Propositions, uncertainty and imprecision.

– obtain nuanced replies;
– know to what extent the conclusion is recommended or certain.

The first point refers to production of a norm in a reference set. In several areas of judiciary, given numerical facts (or modalities of a qualitative predicate) are processed with linguistic gradual rules in order to produce quantified results. Sentencing or awarding compensatory damages for injury (or loss) are good examples. Furthermore, the "cashectomy" on the defendant the judge would inflict on behalf of the plaintiff has concern with numerous legal topics (usually viewed through a game-theoretic or economic lens). Fuzzy control schema are helpful in such applications.

2.6. THEORY OF POSSIBILITY AND LAW

In any textbook case, uncertainty (twin-valued logic and measure of possibility) is considered when the *asserted* information is a *constraint* on the *suggested* proposition; imprecision (multi-valued logic) is considered when the *asserted* information is a *clue* to the *suggested* proposition. Figure 1 lays out the notions.

In legal reasoning the difficulty lies in the actual concurrence of two sources of knowledge. Let's consider Hart's famous example: "vehicles are forbidden in the park" is a certain proposition. Taking the existence of a "vehicle" fuzzy set for granted, a distribution of possibility may be built since "vehicles are forbidden" is a constraint on the elements of the reference set that are found in the park. We could add "trespassers will be prosecuted" as a certain rule.

The intuitive possibility for Peter (riding a bicycle in the park) to be fined may be turned into a formal measure of possibility for "bicycles are forbidden". The rule is triggered and the degree is propagated to the conclusion.

3. From an Unconsummated Marriage to a Marriage of Convenience

Fuzzy logic is a multi-valued logic, orthogonal to the non classical logics usually adopted in legal reasoning modelisation. We emphasize the relatively poor state-of-the-art existing at present as introduction to our publication. Naturally, alternative answers to problems with vagueness and uncertainty are available.

The first part (§3.1) examines several concurrent modelisations of uncertainty and demonstrates to what extent they suit to the legal field.

In the legal field whether the intended meaning of norms looks clear or not, efficiency is based upon the existence of open-textured concepts. Their meaning is varying along time, points of view, actual situations, ... When the meaning of a legal text has to be assessed in a real or hypothetical situation (St-Vincent 1994) claims that "not only is dialectical reasoning *a* means to explore vagueness but that it is, in fact, the *only* means to do so". The second part (§3.2) is devoted to such grounds on which the current researches are based.

3.1. MODELING UNDER UNCERTAINTY

We shall outline three main approaches to the problem.

- Subjective confidence weighting is a melting pot of empirical solutions derived essentially from the development of Expert Systems. The most famous examples are the belief factors (and their combination formulas) used in the well known MYCIN. Such solutions are not axiomatic theories.
- The success of the probabilistic approach is rooted in the concept of conditional probabilities (Bayesian modeling). The belief in "If E_i Then H_j" is computed from the probability of the hypotheses H and the probability of the evidence E when H .... The prerequisites to bear in mind are:
  - the hypotheses H are an exhaustive description, they are mutually exclusive and must all be given;
  - if the evidences E are not conditionally independent, a combinatorial explosion makes the representation untractable;
  - ignorance has no representation distinct from equiprobability;
  - probabilities are additive measures; the probability of the opposite is computed ($P(\text{non-}A) = 1 - P(A)$).

Jurimetrics made intensive use of this model (see *Jurimetrics Jrl* cumulative indexes) and the above listed points represent the very limits encountered in its widespread use over the whole legal decision-making and reasoning area.[4]

- Evidential reasoning (A. Dempster and G. Shafer, see Shafer (1976), allows two "functions" (plausibility, credibility) to be computed from the belief factors assigned to subsets of hypotheses. The success of the formalization depends upon two major issues:
  - a representation of ignorance is allowed;
  - a formula (Dempster rule) is available to combine independent sources of belief.

The Dempster rule considers as a requisite the independence of separate sources of knowledge, stating that the premises used during an inference process are always different. Opportunities for such can be seen in criminal investigations, with witnesses, or to identify a non-empty set of precedents most similar to the case under consideration (de Korvin 1994).

In conclusion, when a reference set is available together with a set of subsets corresponding to a multi-level scale of valuations (the color of the car is dark, red, bright and gray), the D.-S. theory enables us to cope with separate sources of assertions. Aided by two measures (possibility, necessity), the theory of possibility (see §A.4) provides for the modelisation of ignorance and does not require any of the previously mentioned prerequisites.

A survey of the various logics allowing nonmonotonic reasoning or a list of nonadditive measures are beyond the scope of this paper.

3.2. ARGUMENTATION AND DIALECTICAL REASONING

Since law is not concerned with natural occurring but with imperative norms, rules of reasoning remain the only subject for approximation modeling.

However, reasoning on technology has the advantage of dealing generally with efficiency while legal reasoning is a rational reflection. Legal reasoning aims at reaching the result (the decision) in the correct way. The modelisation of approximate reasoning seems to be inopportune. The legal field tends to take an all or nothing attitude about everything.

However, vagueness (every conditions are not available) and gaps in the knowledge (every conditions are not verified) are the everyday burden of legal reasoning. To reason is not to decide; one may give weight to a binary decision; weighing may originate from a chain of reasoning. The explicit statement of the multiplicity of micro-choices may concern an untractable set of rules.

[4] The necessity of palliative theories is taken for granted, at our own risk and notwithstanding the disapproval of supporters of the contrary (see Cheeseman (1985), "... probability is all that is needed".).

Nevertheless, one pertinent question about fuzzy sets that could be asked is: "where does the membership degree come from?" The deficiencies[5] of numerical fuzzy-valued logic may explain the lack of interest shown by "dialectical rationale" supporters. Some authors suggest avoiding numbers[6]) and replacing them with endorsements (Cohen 1985) or argument-ordering (An 1993). When specific linguistic terms are used in endorsements (combined by using application specific rules), the non-numeric representation of indeterminacy falls into the realm of fuzzy logic.

A legal conclusion is categorical and expressed entirely in verbal terms. It creates a norm, usually beyond of the scope of gradation (excepted when numbers are preceded by a dollar sign), unquestionably without a degree of truth. The use of fuzzy logic as an analogical means of inference (Generalized Modus Ponens) is not suited to legal norms. If A entails B, the lawyer may worry about A' (close to A), not about B' (close ? to B; without a linguistic label ?).

However, preserving continuity is preserving neighborhood (facing slight differences in premises without some discontinuity in results), not that it cleans the gray area of discretion up.

Current research shows a growing interest in studies on argumentation and dialectical processes (Hage (ICAIL 1993, p. 30), Prakken (ICAIL 1993), p. 1), Poulin et al. (ICAIL 1993, p. 90), St-Vincent et al. (ICAIL 1995, p. 137), Gordon (ICAIL 1993, p. 10), Nitta, (ICAIL 1993, p. 20). Systems reasoning with conflicting information define the notion of competing arguments, the ordering of such, and different types of strategy (alternative paths, goal-driven, pleading, decision-making). They differ in the way they formalize these notions using monotonic or nonmonotonic formalisms. The explicit statement of dialectical information is characterized by simplicity.

Dialectical reasoning is not approximate reasoning. Its purpose is not to model under uncertainty and its implementation does not process any interpolation; it deals with inconsistency or disagreement. Neither fuzzy logic nor possibilistic logic focus on inference strategy. They do not provide any inferential meta-rules to identify alternative paths. The handling of exceptions seems to be out of fuzzy modelisation range.

Nevertheless, the next paragraph suggests several guidelines for practical modelisations. We should stress certain cooperative rather than competitive issues in fuzzy sets application.

[5] Actually, in addition to the difficulty to assess such numbers:

– there is no intuitive correspondence of what they represent to the real world;
– there is no proper representation of how changes in the numbers affect the entailments.

[6] Everybody agrees that fuzzy logic is an attempt to preserve continuity; the outcome is rather ordinal; the precision of numbers in [0,1] is a fallacious one.

4. A Methodology for Modeling

A reference set is a scale to represent a vague knowledge (imprecise symbolic term or distribution of possibility). We will now study how fuzzy sets may help the modelisation of imprecision during a "homogeneous contextual extensional frame referencing".

4.1. LEGAL REASONING, IMPRECISION AND THE REFERENCE SETS

An application of fuzzy sets should stress the importance of the following questions:

– what are the reference set U and its elements u?
– where do the labels of the fuzzy sets A come from?
– how do you build the membership functions μ_A?

• **Continuous numerical reference set.**

Gradual fields are the subject here. Fuzzy subsets extend the notion of intervals. A discrete extensional definition is not required, a functional definition may be used.

U is a continuum. The labels of the fuzzy sets come from a qualitative scale (high level, medium level, low level) or from a rich linguistic scale (hot, warm, cold, icy). The membership functions achievement is concerned with arbitrary trapezoid shapes or with statistical laws. When several premises of flexible rules use several levels of qualitative generality, "fuzzy-nested" fuzzy sets may formalize the hierarchical scale of specificity and accuracy.

• **Built up numerical reference set.**

Fuzzy logic allows sets of propositions that *should* be inconsistent over twin-valued logic. Any compositional or comparative action remains valid until it *should* become a dialectical argument. However, arguments are the visible part of the decision-making process; our concern has been to stress the hidden part of the chain of reasoning. We do not believe that arguments are merely sets of rules conditions, or to be less pretentious, we believe that they may stem from a previous evaluation of compromise, which is implicit in the legal discourse. Although one may argue that open-texture has nothing to do with smooth curves or gentle slopes, we suggest that it may be worthwhile to compute for the time being as if it did; discretionary decisions will remain.

Efficiency of statute systems is based upon the existence of open texture concepts, therefore standard rule-based systems that attempt to mimic them may be simplistic. Indeterminacy cannot be totally removed at programming time by pre-interpreting the domain knowledge without some additional rules (the necessary micro-interpretations in order to enrich the isomorphic approach (Bench-Capon

1992)). This principle may be applied when constructing expert systems interface, for example.

One should separate between two cases:

– the situation in which a crisp answer is vectorized over the qualitative concepts of its universe of discourse (e.g. §A.3 example of fuzzification);
– the situation in which one cannot (does not wish to) supply a crisp answer from among a set of exclusive choices. The user must supply estimations.

In the latter case, one has no reference set or membership functions and one has to obtain them and add the modelisation into the interface .

Let $\{A_i\}$ be a set of qualitative predicates with no quantification scale (for example, the estimation of behavior) or with (currently) untractable quantifying (for example, the amount of traffic). A grade (an integer in $U = [0,10]$ for example) may translate a subjective estimation. The membership functions are trapezoid-shaped; the slopes represent the degree of fuzziness, the overlap of the areas represents the linguistic overlapping. The point of this construction is that the grade is vectorized; a weighed vector of estimations (therefore, a more subtle one) can be computed on the fuzzy sets $\{A_i\}$. In a provocative and counterintuitive manner, we suggest:

> A subjective assessment is made more accurate: 1) by avoiding a multiple imprecise-choice question; 2) by using linguistic imprecision.
> A subjective assessment is turned into a vector of assessments.

Fuzzy logic delays action on decision-making and make it more accurate by replacing a subsumption mechanism by a “graded vectorization”.

• **Discrete symbolic reference set.**

This cannot exist outside of a discrete extensional definition for U. Fuzzy sets are fuzzy clusters, the membership functions represent the typicality. Subcategories are often considered as the objects collected in the reference set. Unfortunately, an exhaustive extensional definition is seldom available and the membership degrees are lacking. Although open-textured categories can be graphically dreamed up, the mathematical formalization requires more. A coming point considers the notion of constitutive elements to go forward.

Some textbooks suggest to use such reference sets as the reference sets of qualitative predicates. It’s of no relevance for practical applications but one has to bear in mind that:

– any measurement originated in a universe of discourse (real objects are the yardsticks by which we measure);
– predicating these real objects is the end in view.

The next point inspects the relationships between the reference sets and the predicates.

• **Contextual reference set.**

"Big mice are smaller than small elephants!". A contextual set of reference is the restriction of the universe of discourse U by a property P on the set of the potential subjects x of the assertions. In most examples in textbooks, a property P is always implicitly present. Indeed, x is u (u in U) only has a meaning if a relationship exists between U and the set of x.

When speaking of speed in Appendix examples, we bore in mind that the speed of cars were involved, the fuzzy set "excessive" may also be considered in case of "cars on ice" (locative case for P).

A previous point stresses the burden of explicit fuzziness formalization of vague categories. In case of success, the reference sets are contextual. For example, the fuzzy-valued logic gives a truth degree for "use of a vehicle" from the precise information "use of a bicycle". This degree has a legal meaning only if the reference set is context-sensitive ("in the park").

• **Compositional reference set.**

A concept as "stocky" has no reference set but may be formalized with two reference sets and a composition operator since we consider its definition as "small" *and* "strong".

However, the fuzzy set representation of a bachelor is clearly untractable; what is the reference set ? what is the membership function? Legal reasoning makes it more precise by using flexible definitional rules. An open-textured concept has no single set of necessary and sufficient conditions. When a reference set is not available, the legal concept may be defined with constitutive elements from legal provisions or judiciary processes.

Although "imprudent driver" has no tractable reference set, it may be considered since it has a definition in terms of "car", "driving", "weather", . . . conditions. Necessarily, "in need of repair", "excessive speed", "at dusk", . . . are fuzzy subsets with corresponding membership degrees of the material facts. "imprudent driver" is no more a category. The concept becomes a system, a situation. The properties of the elements of the global object are the fuzzy sets (see Figure 2).

However, the composition of the different degrees of membership requires a skilled advice. The interpolation into the constitutive elements is crucial. The explicit statement of dialectical knowledge must be done by tracing the various types of arguments to their sources in fuzzy logic: the "shape" of imprecision (metaphor of the mountain skyline in (Philipps 1993, p. 39); the compositional nature of the reference set; the contextual nature of the reference set; competition between gradual rules.

For example, the concept of "easily accessible for the citizen" may be a flexible constraint. Opening hours of the form "early in the morning, late in the day" receive a high degree of certainty according to a possibilistic interpretation. Opening hours of the form from "11 a.m. to 3 p.m." *do not* receive a high degree of certainty. The open texture is described within the *form* of the distribution.

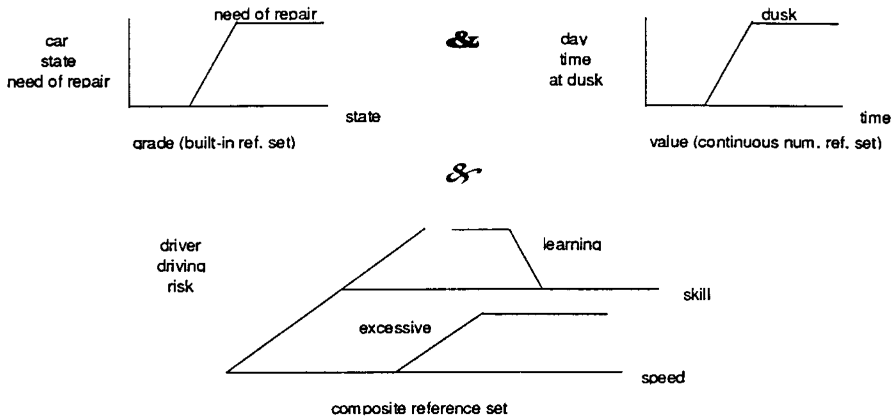


Figure 2. "Imprudent driver" composition with different reference sets.

Some qualitative concepts have to be formalized as systems. For example, "unreasonable", "inchoate suspicion", "hunch", "articulated suspicion" may not receive simple built up numerical reference sets.

To conclude on the notions of context and composition of reference sets, we should stress the importance of level of interpretation. Since interpolations using weights are planned, identifying the constitutive elements has to be understood in terms of identification of authoritative factors and their magnitude. The crucial point is to avoid mixing arguments organized into a hierarchy. Legal reasoning should remain multi-layered. A legal argumentation is essentially multi-layered (Hamfelt 1996).

For example, when reasoning on the dismissal of a pigmy and abnormal size, one has to bear in mind the difference between size estimation, age of the person, birth of the person, nature of work. Each of these elements is a decisional criterion situated at a different level. Fuzzy logic compositional rules contain no unquestionable semantic. (Loui 1995) discusses five kinds of representation of rationales and produces a model of dispute; "no vehicles in the park" is a given example. Interpolation into these rationales would consist of a weighed nonsense composition of "private/emergency vehicle", "tranquillity/disturbance", "on-duty/off-duty", "open/closed park".

The theory of possibility has only one metarule to identify alternative paths: α-cutting (i.e., in case of contradiction, coherent subsets are formed by setting the required degrees of certainty). The measures of possibility must not be used between things which have nothing to do with each other. It increases robustness since we could not claim that all we need to know about a proposition was expressed in the possibility-necessity assessment. What might happen if we had information we do not have? We suggest circumscribing their use to the development of *one* argument (as opposed to a controversial strategy using it).

For example, there is no crisp method by which we may know whether a "waiting time" is "reasonable". On the other hand, how is it possible to assert that formal rules representing the plurality of meaning are always available in a tractable crisp form? Although legal hierarchy provides a suitable means of ordering premises, an assumption about norms is indispensable: one rule prevails over the others. The prior methodologies known to us sweep these elements under the carpet.

It may be necessary to reduce the complexity of an untractable set of rules in order to incorporate an interpretation. For example (Philipps 1993), the "requested waiting period" may be computed from the "time of the accident", the "amount of damage" and the "traffic density".

The process is a compactification of interpretation modeling by reduction of complexity.

Arguing that "the waiting period of 45 minutes is an amount of time adequate in the circumstances, on account of the time of accident and the amount of damage" (from Philipps 1993) may require some case citations about night and slight damage; it does not require a lesson on fuzzification/defuzzification formulas even if a [41,50] minutes period was computed before the choice of the argument. Numerous choices for counterargument are possible: considering the effect of another gradual variable, prevailing exceptional circumstances.

Imprecision is perfectly described when an iterative regression allows compositional and inferential block-building of a concept over basic context-sensitive reference sets. Such a regression may contribute to the formalization of magnitude of imprecision since "perfectly" and "basic" receive a definition (self-issuing a challenge about vagueness!). In short, "perfectly" must be empirically tested, imprecision is described since the end-user answers precise questions; "basic" must be tested likewise, the evidence supplies the answers without ambiguity.

- **A reference set without memberships functions.**

We examine a case where the open texture makes it impossible to achieve a single formalization with the help of fixed definitions. No complex reference set is available but a discrete symbolic reference set may be conceived.

When there is no hope of assessing membership functions, legal reasoning makes extensive use of middle terms during the interpretation process. Considering that A entails B is a norm, when the meaning of A is imprecise because of an "extensional frame referencing", the lawyer does not always try to demonstrate that A' is close enough to A (possibility of compatibility or membership degree). He (she) may prefer to consider a Generalization Concept (Yoshino, (ICAIL 1993, p. 110)) such that A entails GC and GC entails B, then he (she) argues that A' entails GC to conclude.

For example, instead of stating that bicycles are vehicles to conclude that they are forbidden in the park, it is possible to state that vehicles are the cause of accidents, that the cause of accident have to be reduced in the park. Since one argues that bicycles are the cause of accidents, bicycles are . . . Q.E.D.

Things existing in the town are used for an extensional frame referencing; the membership function of the fuzzy set "cause of accident" may have a statistical source. A high membership degree for bicycles may have several uses:

– a preference for the choice of "accident" argument;
– a fuzzy-valued truth for combination (noise, track size, ... are co-occurring fuzzy formalizations) to reach a compromise;
– a measure of possibility to use "bicycles are the cause of accidents" during an inference process.

4.2. A CLASSIFICATION FOR SPECIFIC APPLICATIONS

• **What do we aim at?**

The place of vagueness has to be made clear as well as the nature of the topic:

– a concept (qualitative, category, system/situation);
– a proposition (degree of truth, degree of confidence);
– a rule (classical inference, fuzzy inference, chaining, monotony).

Another point is the direction of the modelisation:

– defining fuzziness (choice for U, A, μ_A of a concept);
– vectorizing crisp information (objective or subjective);
– defuzzification after fuzzy calculi (result of interpolation);
– ordering of propositions under flexible constraints coming from vague knowledge (assessment of meaning in actual situations).

Table I lays out the selected theoretical developments and links them to charts.

Implementation difficulty arises from input factors which are heterogeneous and interact with each other as highlighted by the judiciary processes:

– paragraph 4.1 shed light on the reference set achievement;
– textbooks supply defuzzification formula;
– textbooks supply composition operators for interpolation.

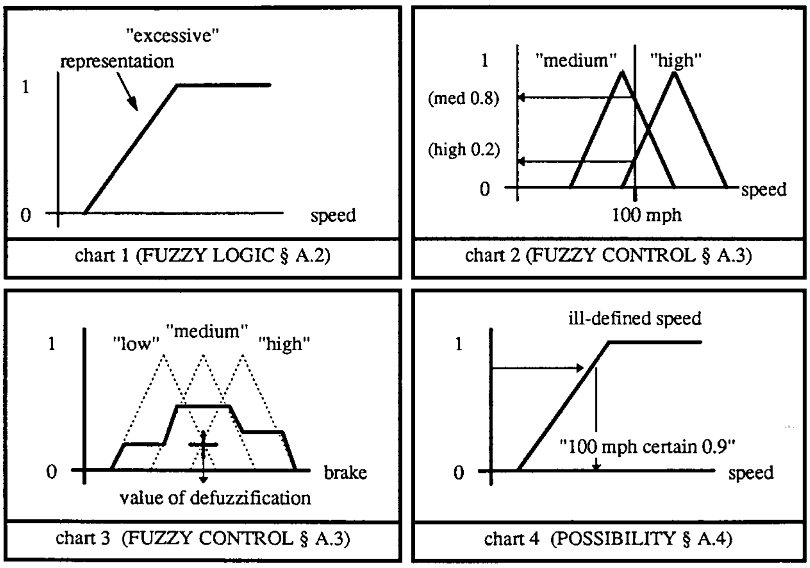
We emphasize that coping with vagueness is not solving hard cases with computer but increasing the flexibility of simplistic present solutions.

• **What's in legal reasoning?**

The conclusions of legal reasoning are expressed in the deductive form, classical syllogism is the paradigm of legal deductive reasoning. The key-problems are:

Table I. Different uses of fuzzy sets theory.

towards imprecision		back from imprecision	
from symbolic to numeric from category to extension	from numeric to symbolic from extension to category	from gradual to crisp	from imprecision to uncertainty
discovery and formalization of fuzziness	use of linguistic labeling	giving up fuzziness no more labeling	ordering among precise propositions
chart 1	chart 2	chart 3	chart 4



– major premise finding;
– minor premise constructing.

The minor premise has to be formulated in the *language* of the major premise. Here is the point. Legal and controversial argumentation debate about language (What is driver skill ? What are law and order issues? What is causae majores?). Naturally, formalization expresses the debate in the deductive form, the different concerned rules are summarized in Figure 3 and Table II lays out the facets of isomorphic rules.

• **Syntactical classification of rules**

Different formalizations exist for the logic connectors:

– *and* may connect two conditions of identical weights;
– *and* may connect two conditions of unequal weights;
– *and* may connect two interacting conditions (reinforcement or compromise aggregation occur).

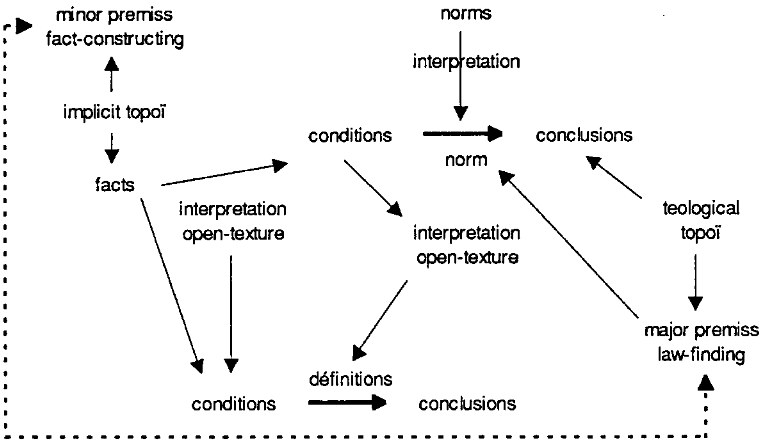


Figure 3. Rule-based model of legal reasoning.

Table II. Isomorphic rule decomposition.

condition					
	definitional rules				
		conditions			
gradual			|	interpolation	
	labelled		|		computing of numbers
	quantified		|		combination of labels
crisp			|	binary	
	vectorization		|		
	grade		|		
		conclusions			
				propagation (graded-fuzzy)	
				alpha-cutting (graded confidence)	
	linguistic variable				
conclusion					
	crisp				
		winner takes all			
		graded confidence			
	gradual				
		labeled			
		quantified			

For example, when buying a new car, it has to be inexpensive and high-performance, inexpensive above all or very high-performance if more expensive.

According to the extended inference mechanisms and the semantic interpretations of *and/or* complexity has set in, Table III offers a combinatorial synthesis, any case of plurality requires a strategy for combination. “Several” means “more than one” and “two of them being different”; by the way, several rules with several conditions and several conclusions on several reference sets are not inevitably distinct.

Table III. A guide for rules and conditions interpolation.

- Single rule
  - single conclusion
    - single condition
    - several conditions
      - single reference set
      - several reference sets
  - several conclusions (see several rules)
- Several rules
  - single conclusion
    - single condition
    - several conditions
      - single reference set
      - several reference sets
  - several conclusions
    - single reference set
      - single condition
      - several conditions
        - single reference set
        - several reference sets
    - several reference sets
      - single condition
      - several conditions
        - single reference set
        - several reference sets

- **Direct reuse of existing results**

Reasoning *under* vagueness is different from reasoning *about* vagueness. The former case engages the lawyers in the task of modeling a specific field, the latter does not. When linguistic edges or endorsements are involved in an explicit legal reasoning, the problem at hand is to choose between axiomatic or nonaxiomatic models in order to combine the assessments. For example, are "many" and "most of" additive, or comparable, in some way? When crisp rules must be triggered with gradual actual information, a new model including vagueness must be worked out. For example, "imprudent driver" may be modeled and graded vectors of features are used to compute a binary condition.

The point is that numerous applications have worked out reasoning about vagueness because natural language is involved. On the opposite, the achievement of legal reasoning under vagueness is still in its infancy.

5. Conclusion

The whole problem seems to be clearly set and circumscribed as follows:

- either the truth value of the implications at each step of the process is decided in an overriding manner;
- or the assignment of a "definitive" truth value is delayed with the help of a formalization of "temporary" (provisional) true assertions; the temporary assertions are then qualified or quantified with:

– dialectical reasoning and argumentative process through the management of preference and contradiction;
– computing the underlying imprecision from which the reliability of the implications follows; a fuzzy calculus may be considered.

Fuzzy sets are a questionable alternative to the classical open-ended process of definition in law. However, they shed light on the concept of “porosity” ((St-Vincent 1994), quoting Waismann) outside of a dialectical approach.

Fuzzy logic is a multi-valued logic, orthogonal to the non classical logics usually adopted in legal reasoning modelisation. It offers an approach to modeling arguments and steps of interpolation in legal reasoning. It is the only model able to deal with gradual rules. It is the only model able to interpolate into the micro-interpretations involved in the expert systems reasoning.

Theory of possibility is a twin-valued logic which permits nonmonotonic reasoning by means of a priority-based preference inference rule. Although more sophisticated approaches have been suggested, the benefit of the theory of possibility stems from the “fuzzy sets” semantic interpretation which converts compatibility of imprecise notions into degrees of acceptance.

Fuzzy control is not transferable as such to legal phenomena. Nevertheless, investigation highlights some fundamentals for A.I. and law, the most important being in the use of linguistic rules to deal with quantified facts. Counterintuitive fuzzification strategy supplies at least one path to jurimetrics and A.I. gathering.

Lawyers that set eventual interpolation against rationale should consider the different potential applications: delaying decision-making, gradation of results, building of arguments. When the interpolation remains local it *does not* concern the explicit rationale.

Although beyond the scope of this paper, it should be emphasized that investigation can also function as a starting point for exciting future fundamental research.

Appendix A: The mathematical model and its developments

A.1. FUZZY SETS THEORY

According to the fuzzy sets theory introduced by Lofti Zadeh (Zadeh 1965): U is a discrete or continuous reference set (or universe of discourse), its elements u being known and well defined.

The subsets A of U are usually defined by a membership function μ_A, the range of which is twin-valued; the most current range is {0,1}; u either is or else is not an element of A.

For example, a book is an element of set of media, a subset of media is the set of written media, a book is an element of written media.

A fuzzy set A (actually a fuzzy subset) of U is defined by a membership function μ_A the range of which is multi-valued; the most current range is the infinite set of reals [0,1] or suitable lattices. The domain of μ_A is U.

For example, a book is a kind of written media (membership value equal to 1); the membership of a photo is less obvious (0.8 may be chosen); so, any of the elements of the set of media are attributed a membership value (a non zero degree of membership may be assigned to CD-ROMs!).

$\mu_A(u)$ is the degree of membership of the element u of the reference set to A, a fuzzy subset of U.

A complete axiomatic theory approach is then achieved by translating the usual set operations and by expanding them when necessary. Different authors suggest different collections of operations (even for the simplest of theoretical cases) and therefore the resulting properties are different; a factor which represents a major obstacle in a direct application.

Fuzzy sets may be used for imprecision modeling. Let x is u be an asserted proposition, x is A be a suggested proposition, $\mu_A(u)$ measures the degree of imprecision of the contents of the latter.

Fuzzy sets provide a basis for the representation of linguistic imprecision. To give an example, if slow, fast, excessive were to be characterized by fuzzy sets in a reference set (speed), a membership function could be interpreted as "fast is a speed above about 70 m.p.h."; $\mu_{\text{fast}}(60) = 0.8$.

A.2. FUZZY LOGIC

The fuzzy sets theory supplies a multi-valued logic. An assertion (x is A) is neither true nor false, as in binary logic, but takes a valuation in the range of the membership function.

The crucial point here is the nature of the subject x predicated in the assertion. The object x is well defined on the reference set and the value $\mu_A(x)$ is known.

Research has provided many ways in which, concurrently and similarly to fuzzy set operators, a fuzzy implication may be expressed.

Fuzzy logic goes from a reference set to a fuzzy set. According to the linguistic interpretation, a concept, together with a degree of membership, qualifies a precise piece of knowledge (numeric, for example).

For example, fuzzy logic enables us to assert that "John's car speed is excessive, value = 0.8" only because John's car speed is known to be 100 m.p.h. (and a membership function has been defined for the term "excessive").

When linguistic rules are used in an expert system to capture the approximate nature of the real world, fuzzy rule-based systems employ inference mechanisms to perform approximate reasoning (cf. Zadeh 1983; Zimmermann 1987). The two patterns of inference are: compositional inference giving a Generalized Modus Ponens and compatibility-modification inference (Cross 1994). The choice of the rule to be triggered is converted to the graded triggering of a set of rules.

A.3. FUZZY CONTROL

Fuzzy control is an effective tool to represent the driver of a physical system and provides a means to regulate the system (when no representation is available).

The method can be summed up as follows:

(1) Fuzzy quantification sets designing:

Fuzzy sets with label on a reference set of quantified measures enabling a fuzzy linguistic partition are provided (for example, vehicle speed may be excessive or rather low).

For example: “If speed is excessive Then slowdown is medium”.

(2) Fuzzification of the input:

Membership functions state a relationship between the labels and the input measures (for example, 100 mph could be slightly excessive with 0.4 and fully excessive with 0.2).

The shape of these functions represents the level of correspondence between a measure and a label.

The slope of the function represents the degree of fuzziness.

The overlap of the areas represents the semantic overlapping of the labels.

(3) Inference with gradual rules:

Linguistic expert rules map input measures onto output actions (where the input fuzzification method applies in the same way).

An inference engine combines the membership degrees of the measures (when multiple conditions exist) and the results (when multiple rules exist for the same action). The final output is a “shape” representing a qualitative conclusion.

(4) Defuzzification for output:

The defuzzification of the result through use of a suitable formula is necessary in order to assign a crisp value to the “action shapes” (obtained as a result of the previous step).

A.4. THEORY OF POSSIBILITY

Two main semantic approaches to possibilistic reasoning with classical propositions have been proposed in the literature. In this appendix, Dubois–Prade’s preference-based approach is taken into account; Ruspini’s (Ruspini 1991) similarity-based approach is ignored.

They serve different inference purpose and it would be fruitful to put them together (Dubois 1995). However, Ruspini’s similarity suggests propositions “not so far of being true”, a metric of closeness defining a kind of extrapolated truth. According to the purpose of the paper, we think that no fundamentals should be lacking.

According to the theory of possibility introduced by Lofti Zadeh (Zadeh 1978; Dubois 1988): A is an ill-defined valuation on a reference set U; π_A is the distribution of possibility for A on U; for any u of U, $\pi_A(u)$ in [0,1] is the measure of possibility of u.

For any subset B of U, the measure of possibility of B, $\Pi(B)$, is the maximum of $\pi_A(u)$, u in B; it states how B and π_A intersect.

For any subset B of U, the necessity of B, $N(B)$, is $1 - \Pi(\text{non-}B)$ (minimum of $1 - \pi_A(u)$, u in non-B); it states how B is included in π_A.

When B is possible, the more impossible non-B is, the more necessary B is.

Ignorance corresponds to $N(B) = 0$ and $\Pi(B) = 1$, B and its contrary are fully possible.

No compositional function is available to compute $\Pi(B \;\&\; B')$. For example, two possible events ($\Pi(B) = 1$, $\Pi(B') = 1$) may have an impossible co-occurrence ($\Pi(B \;\&\; \text{non-}B) = 0$).

For example, U is velocity in m.p.h., A is "excessive but less than 100".

A smooth curve originating in 70 m.p.h. and stopping abruptly at 100 m.p.h. may picture everything one knows about a car speed. It acts as an elastic constraint on the values that may be assigned to the car speed.

For example, $\pi_A(110) = 0$, $\pi_A(80) = 0.5$; B = [80,100], $\Pi(B) = 1$; for example, B = [80,100], $N(B) = 0.5$;

Let x is A be an asserted proposition, x is u be a suggested proposition, $\pi_A(u)$ measures a degree of uncertainty about the truth of the latter.

According to this interpretation, possibility offers a coding of preference for any couple (u, v), the ordering of $(\pi_A(u), \pi_A(v))$ states how u is preferred to v (or the converse). Nonmonotonic reasoning is allowed since an inconsistent knowledge base may be made consistent again by ruling out the weaker assertions.

According to an interpretation where the possibility distribution is chosen as the membership function of a fuzzy set (set corresponding to A, the ill-defined valuation), the theory of possibility goes from linguistic imprecise knowledge to uncertain knowledge.

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