The Equivalence Principle(s)

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Abstract

I discuss the relationship between different versions of the equivalence principle in general relativity, among them Einstein's equivalence principle, the weak equivalence principle, and the strong equivalence principle. I show that Einstein's version of the equivalence principle is intimately linked to his idea that in GR gravity and inertia are unified to a single field, quite like the electric and magnetic field had been unified in special relativistic electrodynamics. At the same time, what is now often called the strong equivalence principle, related to the local validity of special relativity, can also be found in Einstein's writings, albeit by a different name and clearly separated from what he calls the equivalence principle. I discuss both the development of Einstein's thoughts on the different versions of the equivalence principle, their relationship to the relativity principle, as well as later reflections and variants proposed.

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[1]This chapter was written for the Routledge Companion in 2017; the current version is an updated version from 2019. Participating in the companion included adhering to a strict page limit, as well as aiming to write a text that would be accessible both to advanced undergraduates and to graduate students, while at the same time aiming to be useful to researchers in physics on the one hand and history and philosophy of physics on the other. As a result, I don't delve as deeply into the technical details of Einstein's publications as I might otherwise have, and I discuss the important developments between 1912 and 1913 only very briefly, espeically the development of the different equivalence principles, as well as their relationship to one another, in the context of Einstein's scalar theories of gravity. For a comprehensive treatment of this I recommend Norton [1992*a*, forthcoming] and Giulini [2008]. With regard to the more recent philosophical work on the strong equivalence principle, I wish I could have included a throrough discussion of Fletcher [forthcoming].

1 Introduction: A manifold of principles

The equivalence principle of general relativity, the Einstein equivalence principle, or the strong equivalence principle, is a curious beast. Or rather, it is not really a beast at all, but a whole bunch of beasts, as there sometimes seem to be more versions of the equivalence principle than physicists and philosophers who have thought about it. For some it is one of if not *the* main pillar upon which general relativity (GR) rests, for others it is but a chimera, or a ladder that was helpful in finding GR but to be thrown away now that we have the theory in hand.

Despite the plethora of different versions of the equivalence principle, they can be divided into two classes; both classes originate with Einstein, though he himself only discusses the first class under this name. This first class postulates an equivalence between gravitational and inertial effects, or between the very essences (as Einstein put it in 1918) of gravity and inertia. This class typically allows for homogeneous gravitational fields, or certain gravitational effects, to be 'transformed away' by changing the coordinate system. In contrast, the second class states that GR is 'locally special relativistic', in the sense that locally there is no evidence of gravity; the different members of this class differ primarily in how they spell out what it means to be 'locally special relativistic', and under which conditions the respective principle holds. I will call the first class of principles Einstein equivalence principles (EEPs), and the second strong equivalence principles

(SEPs).[2]

Einstein himself introduced what others have come to call (a form of) the strong equivalence principle as a premise of the theory in his celebrated 1916 review paper in the following way:[3]

> [L]et us now introduce the following premise: For infinitely small four-dimensional regions the theory of relativity in the restricted sense [i.e., special relativity] holds, if the coordinates are suitably chosen.

Einstein goes on to elaborate that the suitably chosen coordinates are to be such that no gravitational fields appear; from elsewhere in the same paper (p. 802) we know that this means that the components of the affine connection are to vanish in those coordinates.

The Einstein equivalence principle appears much earlier than the strong equivalence principle in the 1916 review paper; in the section entitled 'On the reasons to extend the relativity principle'.[4] Rather than formulating it as a principle (as in previous and later papers), Einstein here introduces the EEP as a "well-known physical fact", and takes it as a reason to expect that the relativity principle applies not *only* to uniform motion: an observer has no way of distinguishing between being uniformly accelerated but not subject to gravity as compared to being at rest but subject to a (homogeneous) gravitational field. The similarity to Galileo motivating the relativity principle by appeal to the inability of an observer to distinguish between being at rest and moving with constant velocity is immediate. In section 3, I shall return to how this principle was related to the other principles Einstein saw at the heart of GR.

[2]Many modern sources call the second class (or a member thereof) 'the Einstein equivalence principle', and many use the two labels as synonyms. But Norton [1989b] showed that Einstein not only carefully distinguished the two principles but also only called (a member of) the first class 'equivalence principle' in the context of GR. Norton [forthcoming] shows how Einstein transitioned from what we here call the EEP to the SEP for a while in the context of his 1912 scalar theory of gravity, only to return to the EEP in GR, now taking a version of the SEP as a starting point if not a precondition to motivate the EEP. This makes sense in that in the context of his 1912 scalar theory he came to think of the SEP as a an emergency solution, only to be accepted in place of the EEP if there was no other choice, and clearly logically weaker than the EEP in the same sense that the WEP was logically weaker than the SEP. Both could thus serve as motivators and even preconditions for the EEP; but the latter was the only true equivalence principle (in the sense of a true love rather than a clearly true belief) in Einstein's heart.

[3]Einstein [1916*a*] p. 777.

[4]For a comprehensive discussion of the 1916 review paper see Sauer [2004]; for a beautiful facsimile reproduction and commentary see Gutfreund and Renn [2015].

Einstein was adamant in defending his version of the principle against much of the rest of the community, and against the claim that it was of *only* heuristic importance in the search for GR. As we shall see, for Einstein it was also intimately related to what he saw as the main result of GR: the unification of inertia and gravity in a sense to be specified.

In contrast, others argued that the Einstein equivalence principle is *false* according to GR, but that one version or the other of the strong equivalence principle (related to the local validity of special relativity) *does* hold in the theory.

In the following, I shall distinguish the two classes of the equivalence principle and the different versions within each class that have been proposed. I shall put particular emphasis on the role the respective principles play within GR according to their proponents. In both cases, the relationship to GR's Newtonian and special relativistic predecessors plays a major role. For many proponents of the EEP the link to Newtonian theory shows *how* GR unifies inertia and gravity and how it overcomes the distinction between real and fictitious forces; for many proponents of the SEP the local validity of special relativity imbues the $g_{\mu\nu}$ field with its chronometric significance (i.e. relates it to the measurements of rods and clocks).

But before I can elaborate on all that, I will speak about the beginning of it all, the precondition for this mess, if you will: the *weak* equivalence principle.

2 The Weak Equivalence Principle: How it all began

The rest of this paper will be about the Einstein equivalence principle and different versions of the strong equivalence principle. These two categories have often been seen as rivals: many authors seem to believe that you have to subscribe to one or the other. However, there is widespread consensus that the weak equivalence principle is logically weaker than both the EEP and the SEPs, and even a precondition to both of them.

The weak equivalence principle (WEP), too, comes in two major forms. One appeals to the Newtonian concepts of inertial and gravitational mass and asserts an equivalence between the two of them. The other form tries to get by without reference to these concepts, and instead asserts the 'universality of free fall': all bodies fall at the same rate (of acceleration). The latter version is historically prior, partly because, when it was first formulated by Galileo, the concepts of inertial and gravitational mass (indeed, arguably the notion of 'mass') did not yet exist. In this form the WEP is more a generalised observation, akin to Galileo's principle of relativity, in that it generalises and abstracts away from observations with a minimum of theoretical concepts.

One of the most careful ways of stating this version of the WEP is due to Clifford Will. He defines the weak equivalence principle thus:[5]

> [WEP1:] [I]f an uncharged test body is placed at an initial event in spacetime and given an initial velocity there, then its subsequent trajectory will be independent of its internal structure and composition.

Will goes on: "By 'uncharged test body' we mean an electrically neutral body that has negligible self-gravitational energy (as estimated by Newtonian theory) and that is small enough in size so that its coupling to inhomogeneities in external fields can be ignored".[6]

The second version of the WEP draws more strongly on Newtonian concepts. It says:[7]

> [WEP2:] For any body the gravitational mass of the body is equal to its inertial mass.

Note that both Ohanian and Will drop the restriction to test bodies and claim that WEP2 applies to *any* body.

What is the relationship between WEP1 and WEP2? Given careful definitions of the terms "inertial mass" and "gravitational mass", WEP2 implies WEP1.[8] However, especially given the restriction to test bodies

[5]Will [1993], p.22

[6]Ohanian [1977], p. 904, calls something very close to WEP1 above "Galileo's principle". He follows up with the suggestion to define 'test particle' by the limiting case of a particle of small size R ($R \to 0$) and small mass m ($m \to 0$), resulting in a vanishing gravitational self-field ($\frac{Gm}{Rc^2} \to 0$), alongside the vanishing of spin and higher multipole moments.

[7]See again Ohanian [1977], p. 904 and Will [1993], p.22. Ohanian gives definitions of both "inertial mass" and "gravitational mass", quite in line with Mach [1872] in defining them both as ratios of relative accelerations.

[8]Will [1993], p. 12 could be read as stating that WEP2 and WEP1 are equivalent, but I believe he would agree that given the more precise definition he gives on p.22 entails that the arrow of implication goes only from WEP2 to WEP1. After all, WEP1 could be true even if we did not even have the concepts of 'inertial mass' and 'gravitational mass'.

present in WEP1 but not in WEP2, the reverse is not true.[9]

Loránd Eötvös would later make much more precise measurements within the second based on the second way of thinking about the WEP, measuring the difference between the inertial and gravitational mass of a body using the torsion balance he himself had invented. The precision of these results had a direct influence on Einstein; he himself stated that Eötvös' results both inspired and were a precondition for the Einstein equivalence principle to hold.[10]

3 Gravity and Acceleration: Einstein Equivalence Principles

3.1 Two construction sites instead of one

After Einstein completed the founding paper of what we today call the special theory of relativity in 1905, he faced a bit of a conundrum. On the one hand, it was immediately clear that the theory was in conflict with Newton's theory of gravity, which presupposed action at a distance and the absoluteness of simultaneity. Many of the early readers of Einstein [1905a] were aware of this and immediately set out to develop a (special) relativistic theory of gravity.[11] But for Einstein, there was a second problem, a perspective that made him see the theory as but a stepping theory to a more general theory. Einstein was convinced that the correct theory would relativize not only motions at constant speed but *all* motions: it was supposed to be a matter of which frame of reference was chosen whether a body moved not at all, at constant speed, or in an accelerated manner. It was obvious that this second project demanded a major trick; after all we *feel* the effects of acceleration, we feel 'inertial forces' when we are accelerated. However, it had long been known in classical mechanics that one could 'create' inertial forces by transforming to particular coordinate systems, and that one

[9] It has been shown that some alternatives to General Relativity, like Brans-Dicke theory, predict the inequality of inertial and gravitational mass for sufficiently large, extended objects. Thus, such a theory would violate WEP2 but not necessarily WEP1. Indeed, Brans-Dicke theory is an example of a theory in which WEP2 is violated while WEP1 holds. For details see Lehmkuhl [2017].

[10] For particularly clear (and early) statement to this effect see the introduction of Einstein and Grossmann [1913] . For a careful reconstruction and analysis of Galileo's and Newton's conceptions of the weak equivalence principle, both linked to Newton's Corollary VI, derived from Newton's three laws of motion in his *Principia*, see Saunders [2013].

[11] See Renn [2007*b*], Volume 3.

could 'transform them away' in the same manner. Indeed, the German word for 'inertial forces' is '*Scheinkräfte*', which literally translates to 'fictitious forces'. Any conceptually minded student studying physics at a German speaking university, as Einstein had, would likely wonder about the exact status of these forces as compared to allegedly 'real' forces like gravity.

Thus, Einstein saw two construction sites, two projects spreading from his work of 1905: reconciling relativity theory with Newton's theory of gravity, and extending the relativity principle to arbitrary motion. This was the state of affairs that Einstein found himself in in 1907 when, while performing his duties in what would be his final year as a patent clerk in Bern, Johannes Stark asked Einstein to write a review of relativity theory. It was likely during the course of writing this review paper (or while he was gearing up to do so) that a thought occurred to Einstein that he would come to call 'the happiest thought of [his] life' in 1922; see Vol. 7, Doc. 31, of the Collected Papers of Albert Einstein (CPAE from now on). Essentially, the thought was that the two construction sites are connected, that it might be possible to kill two birds with one stone. Einstein realised that it might be possible to reconcile Newton's theory of gravity with relativity theory *in such a way* that the relativity principle would be extended; not to arbitrary states of motion but to uniformly accelerated motion. Remember Galileo's thought experiment involving the ship: he argued that there is no way for an observer to tell the difference between being in the bowel of a ship at rest or in the bowel while the ship is moving at a constant velocity. It would be possible to argue that it would be similarly impossible to decide whether the observer is in a uniformly accelerated elevator or in an elevator at rest; as long as he was subject to a homogeneous gravitational field in the latter case.

3.2 'Just' a midwife?

It is clear that the Einstein equivalence principle was absolutely crucial for Einstein's path towards the general theory of relativity. Given this, and given that Einstein himself often emphasised the *heuristic* value of the equivalence principle, one might have some sympathy with Synge [1960] when he writes the following oft-quoted passage in the preface to his book on GR:

> I have never been able to understand this Principle [of Equivalence]. [...] Does it mean that the effects of a gravitational field are indistinguishable from the effects of an observer's accelera-

> tion? If so, it is false. In Einstein's theory, either there is a gravitational field or there is none, according as the Riemann tensor does not or does vanish. This is an absolute property; it has nothing to do with any observer's world-line. Space-time is either flat or curved, and in several places in the book I have been at considerable pains to separate truly gravitational effects due to curvature of space-time from those due to curvature of the observer's world-line ... . The Principle of Equivalence performed the essential office of midwife at the birth of general relativity ... . I suggest that the midwife be now buried with appropriate honours and the facts of absolute space-time faced.

First, let us note that Synge captures well Einstein's equivalence principle, in the second sentence quoted, if one adds the qualification of an indistinguishability between *homogeneous* gravitational fields and *uniform* accelerations. So what would Einstein have answered to this criticism? Fortunately we don't have to wonder, for both Friedrich Kottler (in 1916) and Max von Laue (in 1950) posed very similar questions to Einstein. We shall see in the following that Einstein essentially rejected both the identification of gravity with the curvature tensor and the distinction between real and fictitious gravitational fields that Synge presupposes in the above quote; and that in addition to the *heuristic* role the principle played in finding GR, Einstein saw it also as having an *enduring* role in how it unified gravity and inertia. Thus, he saw the principle as bringing about a breakdown of the old distinction of an allegedly real force like gravity and an allegedly fictitious force like the centrifugal force. But before we discuss the enduring role Einstein saw for the equivalence principle, let us speak about its heuristic role in helping Einstein climb the mountain on which he found the gravitational field equations of late November 1915.

3.3 The heuristic role: midway to a general principle of relativity

Synge has created one of the most memorable metaphors in the conceptual analysis of spacetime theories with his picture of Einstein's equivalence principle as a midwife: absolutely indispensable to bringing GR into the world, but not needed anymore once it has been brought to life. Indeed, Einstein himself stressed again and again the heuristic importance of the EEP in his search for what came to be GR. This role of the principle is intimately connected to Einstein thinking of it as a relativity principle. He clearly saw it as extending the special principle of relativity, that states that all inertial motions, including rest, are empirically indistinguishable and thus equivalent in an important sense. Likewise, Einstein's equivalence principle asserted that uniformly accelerated motion is empirically indistinguishable from rest, if the observer at rest is also subject to a homogeneous gravitational field. Thus, Einstein's equivalence principle as relativity principle was midway between the special principle of relativity that treats all inertial motions as on a par, and the general principle of relativity that does so with *all* motions, including non-uniformly accelerated motions.[12]

This role becomes immediately clear from almost any formulation of the equivalence principle in Einstein's texts; here is a particularly clear example:[13]

> Starting from this limiting case of the special theory of relativity, one can ask oneself whether an observer, uniformly accelerated relative to K in the region considered, must understand his condition as accelerated, or whether there remains a point of view for him, in accord with the (approximately) known laws of nature, by which he can interpret his condition as "rest". Expressed more precisely: do the laws of nature, known to a certain approximation, allow us to consider a reference system K' as at rest, if it is accelerated uniformly with respect to K? Or somewhat more generally: Can the principle of relativity be extended also to reference systems which are (uniformly) accelerated to one another? The answer runs: As far as we really know the laws of nature, nothing stops us from considering the system K' as at rest. If we assume the presence of a gravitational field (homogeneous in the first approximation) relative to K'; for all bodies

[12] Norton has argued that it is possible to make precise Einstein's way of thinking of the equivalence principle as an extension of relativity principle, and of this extension being "halfway" between the Lorentz covariance of special relativity on the one hand and the general covariance of general relativity on the other, even in the face of Kretschmann's objection that every theory can be brought into generally covariant form (Kretschmann [1918]). See Norton [1993] and especially Norton [1992*b*], section 7.3. At the same time, it has been widely argued that GR did not indeed succeed in relativizing all types of motion, that it is indeed not a theory that implements the general principle of the relativity of motion. See Janssen [2014] for a comprehensive and accessible review.

[13] Einstein [1916*b*], p. 639-40; judged by Norton [1989b], whose translation of the passage I adapt, as the clearest expression of the Einstein equivalence principle in Einstein's oeuvre. Note that what I call the ontological version of Einstein's equivalence principle below (and what Janssen [2014] calls the mature version) is much more concise, but also less self-contained.

> fall with the same acceleration independent of their physical nature in a homogeneous gravitational field as well as with respect to our system K'. The assumption that one may treat K' as at rest in all strictness without any laws of nature not being fulfilled with respect to K', I call the "principle of equivalence".

Note how within this short quote Einstein emphasises thrice the approximate nature of our knowledge of the laws of nature, and how he generalises the principle of equivalence twice within the paragraph, while naming a version of the weak equivalence principle as a precondition. Two years later, he would make yet another step of generalisation. In Einstein [1918], Einstein started to distinguish between the equivalence principle and the general principle of relativity more strongly, while at the same time choosing different words to define the principle:

> The equivalence principle: Gravity and inertia are the same in their very essence ('*wesensgleich*').

One might interpret this as Einstein moving from an *epistemological* form of the principle (it's about states that we can't distinguish) to an *ontological* one (it's about identifying gravity and inertia). However, in his 1921 Princeton lectures, the published version of which is the closest thing Einstein ever wrote to a textbook on GR, he related the wording of the 1916 version to that of the 1918 version:[14]

> ... there is nothing to prevent our conceiving this gravitational field as real. That is, we can consider as equally justified the point of view that K' is 'at rest' and a gravitational field is present, and the point of view that only K is a 'justified' system of co-ordinates and no gravitational field is present. The premise of the complete physical justification of this point of view we call the 'principle of equivalence'; it is obviously suggested by the equality of inertial and gravitational mass and signifies an extension of the principle of relativity to co-ordinate systems which are in non-uniform motion relative to each other. Through this point of view one reaches a theory in which inertia and gravity are the same in their very essence (*wesensgleich*).

We see that in 1918 Einstein uses the term '*wesensgleich*' to describe the relationship between gravity and inertia established by the equivalence

[14] My translation.

principle. It is notoriously difficult to translate. I chose above to translate *'wesensgleich'* as 'the same in their very essence'; Norton translates it as "equality of essence", Janssen as being "of the exact same nature".[15]

However one translates the term, it is clear that it is at the core of what Janssen [2014], p.177, calls "the mature version of the [Einstein] equivalence principle." To appreciate it fully we have to recall that Einstein had gone through an important development between 1916 and 1918, a development directly related to the question of what the term 'gravity' or 'gravitational field' that appears in every version of the equivalence principle, means.

3.4 Gravitational fields as fundamental

Throughout his search for GR, Einstein saw himself in line with Mach's criticism of Newton's concept of absolute space. In GR, he wanted to eliminate from the category of fundamental constituents of the world what he saw as the successor of Newton's absolute space: the metric field $g_{\mu\nu}$. For a long time, Einstein thought that what makes a body move inertially according to GR should be entirely determined by the distribution of the other material bodies.[16] Like the equivalence principle, these thoughts were tightly connected to Einstein's aim of generalising the relativity principle; after all, if the distribution of material bodies in the universe did determine which motions are inertial motions, then the distinction between inertial and non-inertial motions would become an entirely contingent one. It should be noted that though this line of thought resonates with the idea of relativizing all motion, it is not as straightforward to think of Mach's principle as a relativity principle as it is for the equivalence principle. In Einstein [1918], Einstein defined 'Mach's principle' as the requirement that the metric $g_{\mu\nu}$ be uniquely determined by the mass-energy-momentum tensor $T_{\mu\nu}$. But if this were the case and $T_{\mu\nu}$ would thus, via the metric determining which curves are geodesics, also determine which motions are inertial motions, then there would still be a fact of the matter about which motions are inertial and which are not. It would be a contingent fact rather than an absolute fact as in Newtonian theory, but it would still be a fact, in contrast to what the general principle of relativity demands.[17] Likewise, Einstein expected that the

[15]See Norton [1992*a*], footnote 42 and Janssen [2014], p. 219.

[16]See Hoefer [1994, 1995], Barbour [2007] and Renn [2007*a*] detailed descriptions of the role Machian thoughts (and different versions of 'Mach's Principle') played for Einstein in the development of GR.

[17]Note, however, that Einstein resisted seeing the metric field as *primarily* the successor of inertial structure as established by absolute space in Newtonian theory; more on this in the next section.

inertial mass of each and every body should be determined by its relations to the other material bodies. He did not sharply distinguish between this demand and the demand that which motions are inertial should depend on the mass distribution. The connection might be that he saw inertial mass, like Newton, as the measure of resistance of a body against being diverted from inertial motion. This might have made it seem natural to expect that which makes inertial motion contingent to go hand in hand with making inertial mass contingent.[18]

Consequently, Einstein expected that there should not be non-trivial solutions to the Einstein field equations if the mass-energy-momentum tensor of all the matter in the universe is equal to zero.[19] Indeed, in Einstein [1917], Einstein suggested a modified form of the field equations (introducing the cosmological constant) in part to ensure that Minkowski spacetime (the only vacuum spacetime he knew about at the time) would not be among the solutions of the field equations. Alas, Willem de Sitter showed within months that even the modified field equations *did* have non-trivial vacuum solutions: GR allowed for non-trivial inertial structure, non-trivial spacetime structure, even in the absence of matter.

This brought about a severe change in Einstein's thinking, though one that is difficult to pin down. Einstein [1918] contains the first explicit definition of "Mach's Principle" as the demand that $T_{\mu\nu}$ was to uniquely determine $g_{\mu\nu}$, even though that principle had already come under pressure by de Sitter. However, we have to note that through much of the debate with de Sitter Einstein looked for a way to dismiss the solution as non-physical, which would have rescued Mach's principle to some extent.[20]

Indeed, this is how Einstein would come to operate during the 1920s: Mach's principle became a selection principle to choose between physical and non-physical solutions both of the original field equations and of the field equations with cosmological constant.[21] At the same time, he slowly came to acknowledge that his field equations as such demanded of him to see $g_{\mu\nu}$

[18]See Barbour [1990], Norton [1995] and Brown and Lehmkuhl [2017] for studies in how these ideas that Einstein always attributed to Mach differed from Mach's actual ideas; and how fruitful this flexibility in reading Mach became for Einstein in the development of GR.

[19]This is strictly speaking an additional demand to expecting a unique determination of $g_{\mu\nu}$ by the energy-momentum tensor; for if there were *one* solution corresponding to $T_{\mu\nu} = 0$, the latter condition would have been fulfilled.

[20]For more details on the debate between Einstein and De Sitter, see CPAE Vol. 8, the editorial note "The Einstein-De Sitter-Weyl-Klein-Debate," on pp. 351357; and Smeenk [2015], section 4.

[21]See section the Introduction to CPAE Vol. 13, p.xlv.

as a fundamental element of reality, as something not reducible to something else, like the distribution of material bodies in the universe. Starting with Einstein [1919], Einstein began to embrace fields, and especially $g_{\mu\nu}$, as the fundamental stuff that constitutes material bodies, rather than as something that is reducible to the distribution of material bodies.

It has to be said that with regard to the electromagnetic field Einstein had already thought along these lines by 1909, as we see in a letter to Lorentz, in which he considers a non-linear generalisation of the Maxwell equations to which he expected to find particle solutions, in particular solutions capable of representing electrons and photons.[22] So why did Einstein not think of $g_{\mu\nu}$ as equally fundamental, as on a par with the electromagnetic field, much earlier?

I believe that part of the answer is that Einstein saw $g_{\mu\nu}$ as the successor to Newton's absolute space, and thus, following Mach, as something that should not be fundamental in the next step of theoretical development. But another part of the answer may be that through relating gravity and acceleration, and thus gravitational forces and inertial forces, via the equivalence principle, Einstein had opened up the possibility of thinking of gravitational forces as just as 'fictitious' as inertial forces.[23] Indeed, this was exactly Kottler's challenge, alluded to above. As we shall see in the following section, Einstein's answer to this challenge (and to Max von Laue's, which mirrors Synge's) will allow us to make the puzzle pieces fit together and to answer: i.) why Einstein did not start out thinking of $g_{\mu\nu}$ as on a par with the electromagnetic field, even though he used the analogy with the electromagnetic field equations all the time in his search for new gravitational field equations; ii.) why Einstein did not think that GR *reduced* gravitational forces to inertial forces; iii.) how he thought that GR *unified* gravity and inertia and how this point of view demanded an enduring role for the equivalence principle in GR.

3.5 The enduring role: unifying gravity and inertia

The *heuristic* value of Einstein's equivalence principle was that it guided Einstein towards what he thought of as a general theory of the relativity of motion, embodied by the fact that the theory took its simplest form in a gen-

[22] Einstein to Lorentz, 23 May 1909; Vol. 5, Doc. 163 CPAE.

[23] Remember my digression on the German word for 'inertial force' corresponding to 'fictious force' or 'apparent force' in section 3.1. It is also worth noting that the same argument allows to say that inertial forces have turned out to be just as real as gravitational forces.

erally covariant representation.[24] Even Einstein might have admitted that once such generally covariant field equations were at hand, the equivalence principle had fulfilled *this* purpose. But the *enduring* value of the principle in Einstein's mind is connected to a *second role* of the principle, and it is connected to what Einstein regarded as the main achievement of the theory. The second role of the principle concerns the unification of inertia and gravity in GR, which Einstein saw as a direct analogue to the unification of electricity and magnetism in (special relativistic) electrodynamics.

But first let us come back to Kottler's challenge, hinted at towards the end of the previous section. Kottler claimed in 1916:[25]

> Since then, Einstein has abandoned the equivalence hypothesis. The reasons lie primarily in a particular perception of its results, which amounts to giving an independent existence to the forces of the gravitational field. *Here*, motion in a gravitational field will be seen as force-free. Thus, the law of inertia must be changed and gravitation be seen as a purely inertial phenomenon. This perception seems to me a strict consequence of the equivalence hypothesis; and thus can only be abandoned *together* with the latter. [...] The prime difference [of my approach as compared to Einstein's] is one of principle: the kinematical, rather than dynamical, conception of gravity.

So for Kottler the equivalence principle entailed that gravitational forces are *nothing but* inertial forces: the concept of inertia is fundamental, and gravity has been reduced to inertia. Unsurprisingly, Einstein resisted; for him the point of the equivalence principle was exactly the indistinguishability, and eventually the essential unity, of inertia and gravity. Michel Janssen has dubbed this Einstein's insight of the relativity of the gravitational field, in contrast to the relativity of motion:[26]

[24]See Einstein's answer to Kretschmann [1918], Einstein [1918].

[25]Kottler [1916], p.955-956.

[26]Janssen [2012], p. 159. See also Janssen [2005] and Janssen [2014]. In these sources, Janssen also argues that Einstein did not indeed succeed in relativizing motion in the sense demanded by the general principle of relativity. Before the passage quoted in the main text he writes: "It is so obvious today that the general theory of relativity does not extend the relativity principle from uniform to arbitrary motion that it has become something of a puzzle how Einstein could ever have claimed it did ... . [...] [T]o a large extent, the solution of this puzzle is simply that what Einstein called the relativity of non-uniform motion is more appropriately called the relativity of the gravitational field."

> Two observers in non-uniform motion with respect to one another can both claim to be at rest as long as they agree to disagree about whether or not there is a gravitational field. This requires a coordinate-dependent definition of the gravitational field, but, unlike modern relativists, Einstein opted for such a definition, representing the gravitational field by the so-called Christoffel symbols.

For Einstein, seeing the presence of gravitational fields as a coordinate-dependent state of affairs was not a price to be paid but a major achievement of the theory. It was tightly linked to how he interpreted the geodesic equation which, like Kottler, he saw as the successor to the Newtonian law of inertia, as a "generalised law of inertia". But for him, in contrast to Kottler, this generalisation lay exactly in the fact that the clear separation of inertia and gravity was overcome:[27]

> Kottler complains that with regard to the equations of motion
>
> $$\frac{d^2 x_\nu}{ds^2} + \left\{ \begin{matrix} \alpha\beta \\ \nu \end{matrix} \right\} \frac{dx_\alpha}{ds} \frac{dx_\beta}{ds} = 0 \tag{1}$$
>
> I interpreted the second term as the representative of the influence of the gravitational field on the point mass, whereas I interpret the first term as, so to speak, the representative of Galilean inertia. This, he claims, would introduce "real gravitational forces", which is supposed to contradict the spirit of the equivalence principle. [...] The labelling of the terms I introduced does not really matter though and was only meant to accommodate our physical habits of thinking. This is also true, in particular, for the concepts
>
> $$\Gamma^\nu{}_{\mu\sigma} = -\left\{ \begin{matrix} \alpha\beta \\ \nu \end{matrix} \right\}$$
>
> (components of the gravitational field) and t_σ^ν (energy components of the gravitational field). The introduction of these labels is in principle unnecessary, but for the time being they do not seem worthless to me, in order to ensure the continuity of thoughts... .

[27]Einstein [1916*b*], p. 641.

So Janssen is right: Einstein does operate with a coordinate-dependent concept of "gravitational field". But at the same time, what *makes* Christoffel symbols the representatives of the gravitational field is only the comparison to the predecessor of GR, Newton's theory of gravity; in GR taken by itself, there is only "the unity between inertia and gravity [as] expressed by the fact that the entire left side of [equation (1)] is tensorial (with respect to arbitrary coordinate transformations), whereas the two terms separately are not".[28]

3.6 Is there a spacetime free of gravity?

Ehlers, Stachel and Giulini have identified the affine connection with this unified inertio-gravitational field, partly following in the footsteps of Weyl, who saw the affine connection as a "guiding field" because of how it establishes the distinction between geodesic and non-geodesic motion, and because of how it 'makes' particles move on geodesics.[29]

Giulini especially contrasts this view with that expressed by Synge, namely, that Minkowski spacetime corresponds to the gravity-free case, while a curved metric (i.e. a non-vanishing curvature tensor) signifies the real, invariant, presence of a gravitational field. Indeed, Synge thus implicitly upholds an objective distinction between inertia and gravity, which Einstein believed GR had overcome.

Luckily, Max von Laue challenged Einstein in a way very similar to Synge during a lengthy correspondence in the early 1950s. Von Laue was working on a revised edition of his classic relativity textbook from 1921, and his reengaging with the subject brought about a detailed correspondence between the two old friends. In a letter from 8 September 1950,[30] von Laue questioned Einstein's treatment of the rotating disc in the 1921 Princeton lectures. In particular, he challenged Einstein's use of co-rotating measuring rods in the treatment, and claimed that one should only use rods that move on geodesics. In Einstein's answer, currently dated as from 12 September 1950, he writes:[31]

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[28]Einstein [1922], p.51. For further analysis see Lehmkuhl [2014], section 4. Janssen appreciates the above; indeed, he calls the following statement the equivalence principle in its mature form: "There is only an inertio-gravitational field that breaks down differently into inertial and gravitational components depending on the state of motion of the person making the call." Janssen [2014], p. 178. (Note that Janssen [2014] was written in 2008, though only published in 2015; thus it was written well before Lehmkuhl [2014]).

[29]Ehlers [1973], Stachel [1986, 1995, 2007] and Giulini [2002], Weyl [1922], p. 219-224.

[30]Einstein Archive Call No. (in the following abbreviated to EA) 16-143.

[31]EA 16-148. Translation from Norton [1989b], p. 39.

> It is true that in that case the R_{iklm} vanish, so that one could say: "There is no gravitational field present." However, what characterises the existence of a gravitational field from the empirical standpoint is the non-vanishing of the Γ^l_{ik}, not the non-vanishing of the R_{iklm}. If one does not think intuitively *(anschaulich)* in such a way, one cannot grasp why something like a curvature should have anything to do with gravitation. In any case, no reasonable person would have hit upon such a thing. The key for the understanding of the equality of inertial and gravitational mass is missing.

Remember that the position considered in the first sentence is exactly that advocated by Synge. In what follows, we see Einstein rejecting it, emphasising the heuristic role of the EEP, which takes the components of the affine connection Γ^l_{ik} as representing a coordinate-dependent gravitational field; but we also see him insisting that only in this way WEP2, the numerical equivalence of inertial and gravitational mass, can be understood. We will come back to the latter point. For now let us note that von Laue did not drop the issue. In a follow-up letter he attacks Einstein's interpretation of the geodesic equation as featuring components of the gravitational field. On 8 January 1951 he writes:[32]

> So in the general theory of relativity, g-brackets are supposed to represent something like the field strength of the gravitational field. Now let us look at the normal pseudo-Euclidean metric of the special theory of relativity, and transform to spatial cylinder coordinates, or indeed any other curved coordinates, without changing the temporal coordinate. Success: g-brackets show up in the geodesic equation. But surely it does not make physical sense to say that one has created a gravitational field by this purely mathematical operation.

Here von Laue makes the ingenious move of looking at the geodesic equation in special relativity, where in pseudo-Euclidean coordinates the geodesic equation looks just like Newton's law of inertia, rewritten in four-dimensional language. Von Laue points out that by transforming e.g. to cylinder coordinates, Christoffel symbols will enter the geodesic equation — which according to Einstein's position would correspond to the 'creation' of a gravitational field in virtue of a coordinate transformation.

[32]EA 16-152, my translation. Note that when von Laue speaks of g-brackets he means Christoffel symbols Γ^l_{ik}, i.e. components of the affine connection.

This sounds like the type of letter that Synge might have written had he received Einstein's previous letter. But note that both Synge and von Laue might not have objected to saying that by virtue of the coordinate transformation *inertial* forces had been 'created'. After all, creating inertial forces by coordinate transformations was an old hat in Newtonian physics; and they were not *real* forces, so an ability to create them by help of coordinate transformations was not seen as problematic. One the other hand, likewise in Newtonian physics, gravitational forces were supposed to be real forces that could not be brought about in this way. In his answer, Einstein compared this Newtonian conception of inertia and gravity with what he saw as that of GR. On 16 January 1951 he writes:[33]

> Now to the gravitational field. Here, one has to properly distinguish between different concepts. In Newtonian theory, everything that is built from the potential counts as the gravitational field. In particular, we would understand the first derivative of the potential as the field strength. In the relativistic theory of gravity, the gravitational field is everything built from the symmetric g_{ik}. Now, it is clear that we cannot do justice to the interpretation of the relativistic gravitational field by appeal to Newtonian theory. Of course, the interpretation of the field of a system that is accelerated and moving parallel to an inertial system (equivalence principle) was of the utmost heuristic importance, because such a field is equivalent to a Newtonian gravitational field with parallel force lines. In this case, the Newtonian field strength is equal to the spatial derivative of g_{44}. One can thus, if one wants to, speak of the first derivatives of the g_{ik}, i.e. of the affine connection Γ, as the gravitational field strength. Of course, these quantities are not tensorial. With this manner of speaking, it is indeed the case that introducing cylinder coordinates in a Galilean space [i.e. Minkowski spacetime] would bring with it the appearance of field strengths. This is only a manner of speaking. But what is essential is that also in the case of a Galilean, i.e. Minkowskian, space there is, according to the general theory of relativity, a gravitational field, even if its field strengths vanish in the sense defined above. For in the theory of relativity the dimensionality of the field is the only thing that remains from the earlier, physically independent, (absolute) space.

[33] EA 16-154. I quote in some detail because the letter is not yet available in the CPAE, and will not be for some decades.

> I think that most have not understood this main achievement of the theory.

Note the similarity to Einstein's answer to Kottler in Einstein [1916*b*], where Einstein spoke of the terms "gravitational field" and "Galilean inertia" as an unnecessary (but useful) attempt at accommodating our "habits of thinking". Similarly, in his letter to von Laue 45 years later Einstein sees these notions as just "a manner of speaking" when used in the context of GR.[34] But what is *essential* for Einstein is not to hold on to these Newtonian distinctions, but to accept that Minkowski spacetime is just as much a gravitational(-inertial) field as solutions to the Einstein equations for which curvature does not vanish. Thus, for Einstein there is no distinction anymore between 'real gravitational forces' and 'fictitious inertial forces'. For Einstein, GR, if interpreted in light of the EEP, unified gravity and inertia, and thereby gave up the very distinction between gravity and inertia.

Now, does this mean that the unified field is real or fictitious? Pauli addressed this question when he wrote, right after pointing out that the geodesic equation in GR replaces Galileo's law of inertia:[35]

> In Einstein's theory, gravitation is just as much a *fictitious force* as the coriolis and centrifugal forces are in Newton's theory. (However, it is equally justified to say that in Einstein's theory neither of these two forces is a fictitious force.)

In GR, as understood by both Einstein and Pauli, the distinction between inertial structure and gravity, the distinction between gravitational and inertial forces, and more generally between real and fictitious forces, is overcome through the equivalence principle. As a result, a spacetime without curvature has exactly the same status as a spacetime with curvature: it is a particular state of the unified gravitational-inertial field.

4 The local validity of special relativity: Strong Equivalence Principles

4.1 Einstein's version of the strong equivalence principle

The strong equivalence principle (SEP) has taken centre stage in modern philosophy of physics primarily through the work of Harvey Brown. Brown's

[34] See Norton [1989b], section 8, for further analysis.

[35] Pauli [1921], p. 709. I have translated '*Scheinkraft*' as fictitious force in this instance; the emphasis is Pauli's.

project started from the so-called dynamical approach to *special* relativity that he had pioneered together with Oliver Pooley.[36] In that context, Brown and Pooley argued that the non-dynamical Minkowski metric of special relativity can be reconceptualized as being merely a way of expressing features of the dynamical properties of matter field equations, especially their universal Lorentz covariance. It is then the universal Lorentz covariance of all these laws that brings about the chronometric significance of the metric, by allowing to interpret rods and clocks as "waywisers" of the Minkowski metric.

Brown [2007], chapter 9, then argues that *in GR* it is precisely through the strong equivalence principle, which guarantees the local validity of special relativity in GR, that the metric obtains its chronometric significance, its relationship to rods and clocks. In other words, since the SEP does not follow from the Einstein field equations, the equations governing the dynamics of $g_{\mu\nu}$, the geometric interpretation of $g_{\mu\nu}$ is made possible by a principle that goes beyond gravitational dynamics.[37]

I have already alluded to the fact that Einstein himself stated and discussed the role of the SEP in GR, though not under this name. One of the most telling discussions of the principle can be found in a little known letter from Einstein to Paul Painlevé from 7 December 1921. There he writes:[38]

> According to the special theory of relativity the coordinates x, y, z, t are directly measurable via clocks at rest with respect to the coordinate system. Thus, the invariant ds, which is defined via the equation $ds^2 = dt^2 - dx^2 - dy^2 - dz^2$, likewise corresponds to a measurement result.
>
> The general theory of relativity rests entirely on the premise that each infinitesimal line element of the spacetime manifold physically behaves like the four-dimensional manifold of the special theory of relativity. Thus, there are infinitesimal coordinate sys-

[36] See Brown and Pooley [2001, 2004]. For details on the dynamical approach see the chapter by Brown and Read in this book, Brown and Read [2018], and references cited therein.

[37] Note that in contrast to the dynamical view in the context of special relativity, Brown does *not* claim that the metric field in GR can be reduced to the dynamics or the symmetries of the matter fields. Like Einstein after 1917 (see section 3.4), Brown accepts that $g_{\mu\nu}$ has irreducible dynamical degrees of freedom. What is at issue is how this newly dynamical field obtains its relation to the measurements of rods and clocks.

[38] Vol.12, Doc. 314 CPAE. Unfortunately, the letter was not selected for translation in the translation supplement to Vol. 12. Thus, the translation given here is my own; and I quote in more detail than I otherwise would.

> tems (inertial systems) with the help of which the ds are to be defined exactly like in the special theory of relativity. The general theory of relativity stands or falls with this interpretation of ds. It depends on the latter just as much as Gauss' infinitesimal geometry of surfaces depends on the premise that an infinitesimal surface element behaves metrically like a flat surface element ... .

It is easy to read this quote as Einstein seeing a (geo)metric interpretation of GR as absolutely essential to the theory. But it is important to note that in 1921 one of the things he was preoccupied with was to differentiate GR from the first attempt at a unified field theory, delivered by Hermann Weyl. Einstein believed that the main problem of Weyl's theory was that it gave up on the direct relationship between the line element and the readings of rods and clocks. However, Einstein came to concede Weyl's riposte that rods and clocks should come about as solutions of the fundamental field equations governing matter and gravity, rather than being treated as primitive entities.[39] Moreover, he came to actively oppose the idea that GR 'geometrizes the gravitational field', or that it shows that the latter is reducible to spacetime geometry.[40]

Of course, Brown, whose position is partly foreshadowed by Einstein's letter to Painlevé, is far from believing that the local validity of SR in GR makes the chronometric interpretation *essential* to GR. Indeed, both Einstein and Brown can be interpreted as putting their finger on the fact that the SEP makes a chronometric interpretation of $g_{\mu\nu}$ *possible*, but that this interpretation is not brought about by the dynamical equations of GR, the Einstein field equations, but by an extra premise, the SEP.

4.2 Different versions of the strong equivalence principle

Though we have seen clearly that, under a different name, the SEP was present in Einstein's thought from early on, the honour of forging a relationship between the local validity of SR on the one hand and the Einstein equivalence principle on the other hand in the context of GR belongs to Wolfgang Pauli. In his seminal encyclopaedia entry on relativity theory, Pauli writes:[41]

[39] See Giovanelli [2013, 2014].
[40] See Lehmkuhl [2014] and Giovanelli [2016].
[41] Pauli [1921], p. 705-706 (Pauli's emphasis, my translation).

> **General Formulation of the Equivalence Principle. Link between Gravitation and Metric.** Originally, the equivalence principle was posited only for *homogeneous* gravitational fields. In the general case we can formulate it as follows: *For every infinitely small region of the world (i.e. a region so small that the spatial and temporal variation of gravity can be neglected therein), there always exists a coordinate system* $K_0(X_1, X_2, X_3, X_4)$ *such that there is no influence of gravity either on the motion of mass points or on any other physical processes.* In short, in an infinitely small region of the world the gravitational field can be transformed away. [...] It is obviously natural to assume that the special theory of relativity holds in K_0. [...] In this sense, one can thus say that the invariance of the laws of nature with respect to Lorentz transformations continues to hold in the infinitely small.

Knox takes the italicised part of this quote as a starting point to distinguish between different ways of restating the principle.[42] Casting the net slightly more widely, one might distinguish between two broad classes: pointy SEPs and neighbourly SEPs. The first class makes assertions about the local validity of SR at a point, the second about its validity in the neighbourhood of a point.

Pauli's own version of the SEP is a member of the class of neighbourly SEPs, distinguished from its pointy rivals by the claim that the coordinate system whose existence is postulated is taken to exist in some neighbourhood of a point p of the spacetime manifold. Different versions of the neighbourly SEP differ with respect to the demands placed on the coordinate system, and with respect to the demands placed on the neighbourhood in question.

Regarding the coordinate system, many authors stipulate that in the coordinate system in question (I shall keep calling it K_0 for convenience) the 'gravitational field' or 'the influence of gravity' can be 'transformed away' or 'neglected'. Other authors have complained about the talk of transforming gravity away, for, they say, if gravitational fields are real then one ought not be able to transform them away. Talk of being allowed to 'neglect' the gravitational field in certain circumstances has caused less ire, presumably

[42]Knox [2013], p. 351. Knox motivates all of these types by the aim of restating the SEP in such a way that no reference to 'gravity' or 'gravitational field' is necessary. She distinguishes between what she calls 'the geometrical SEP', attributed to Trautman [1966], which restricts the SEP to the claim that "all possible experiments determine the same affine connection"; the 'pointy SEP' which I subsume under the first class of SEPs described in the main text, and her own 'effective SEP', which I shall subsume under the second class about to be described.

because it suggests that the principle is supposed to be merely an approximation; we shall come back to this. Other versions of the neighbourly SEP, like Knox' own, try to avoid reference to 'gravity' or 'gravitational fields' altogether and instead demand that in K_0 the metric field should "take on Minkowskian form"[43], or they demand that K_0 be a normal coordinate system, which is defined as a coordinate system in which the components of the affine connection vanish.[44]

Regarding the demands placed on the neighbourhood in question, Pauli's version of a neighbourly SEP states that the neighbourhood for which K_0 can be defined is to be "infinitely small", others speak of it being "arbitrarily small", which I take to be a cautious way of expressing the same idea. The general idea of neighbourly versions of the SEP has been criticised by way of pointing out that tidal gravitational effects, or effects of curvature, cannot in general be transformed away or neglected in arbitrarily small neighbourhoods. The two claims are related by the fact that in Newtonian theory tidal gravitational effects are described by equations involving second derivatives of the Newtonian gravitational potential, while in GR the same effects are described by equations involving second (covariant) derivatives of the metric, which form the curvature tensor. This observation has led to different reactions. As outlined in section 3.2, it led Synge to claim that the SEP is false and should be abandoned altogether, and it led Ohanian [1977] and Ghins and Budden [2001] to versions of the pointy SEP.[45]

[43]Knox [2013], p. 352.

[44]Arguably, most if not all of these statements can be translated into one another. If, following Synge, "gravitational field" is identified with "non-vanishing curvature", then the demand for "no influence of gravity" (Pauli) and that for the metric "taking on Minkowskian form" coincide. Likewise, if we restrict our attention to theories like GR where there is a unique affine connection and a unique metric which are compatible according to the non-metricity condition $\nabla^\mu g_{\mu\nu} = 0$, then the demand for K_0 to be a normal coordinate system implies that the metric compatible with the connection looks like a Minkowskian metric in that coordinate system. The 'looks like' is important here, for higher derivatives of the metric (second derivatives and above) don't necessarily vanish in normal coordinates (see Read et al. [2018] for details on this).

[45]Norton [1989b] has been widely lauded for clarifying what Einstein's own conception of the equivalence principle actually was. But the article did even more than that: it also constructed a major challenge to the pointy strong equivalence principle, a problem that Einstein had alluded to in correspondence with Schlick and that Norton formalised using modern differential geometry and concepts developed by Robert Geroch. Norton [1989b], section 10, concludes: "The results of this section vindicate Einstein's objection to Schlick. If we understand the infinitesimal principle of equivalence [i.e., the SEP] to assert that special relativity holds at a point to second-order quantities only, then it follows that we cannot formulate special relativity's requirement that the world line of a free point-mass is a geodesic."

Another possible reaction would have been to claim that the neighbourly SEP is only *approximately* true in GR: true if and only if tidal gravitational effects / spacetime curvature can be neglected *given* a particular region of spacetime, a particular physical system in that region, and an interest in particular properties or behaviour of said systems. The most important thing to note is that the approximation is of a particular kind: it would be wrong to say that the SEP is approximately true for all regions and physical systems in question. It is taken to be approximately true for some of them, namely exactly those where curvature / tidal effects are negligible with respect to the properties of the physical system under investigation.

Even though both the pointy and the approximate neighbourly SEP can be (and have been) motivated by criticising quotations of Einstein's equivalence principle (as described in section 3) and Pauli's version of the strong equivalence principle, it ought to be noted that both Einstein and Pauli were perfectly aware that there may be physical systems for which second derivatives of the metric, and thus the components of the curvature tensor, will not be negligible given a particular small region of spacetime and a particular question posed, even an arbitrarily small but still extended region. An example of such a system, discussed in detail first by Eddington [1923], was recently discussed by Read et al. [2018]: the wave equation for the electromagnetic field $F_{\mu\nu}$ that can be derived from the standard Maxwell equations contains second derivatives of $F_{\mu\nu}$, which in a general relativistic context brings about terms containing the curvature tensor. Thus the wave equation becomes

$$F^{c}_{ab;c} = 2\left(F^{e}_{[b}R_{a]e} - R_{abcd}F^{cd} + J_{[a;b]}\right). \qquad (2)$$

Going to a sufficiently small region (Knox), an infinitely small region (Pauli), or even a point (Ohanian) won't help with such a system if the components of the curvature tensor don't vanish at that point. For, as noted above, the components of a tensor cannot be transformed away, even at a point.[46] Pauli was most explicitly aware of this when he stated that in

[46]Note, however, that the curvature tensor needs more than a point in order to be defined. Given that the definition of the curvature tensor involves second covariant derivates that only make sense in terms of parallel transport along a curve, one *does* need an extended (though arbitrarily small) region to define the curvature tensor. This is just a special case of the fact that properties of spacetime regions that are represented by vectors or tensors (rather than merely by scalars) cannot be defined at points alone; at least an implicit reference to points in the neighborhood are necessary. See Butterfield [2005] for details.

SR special relativity is locally valid *if* second derivatives of the metric (and hence curvature) can be neglected.[47]

So far so good: we have distinguished between quite a few versions of the SEP. But what is the role of the principle in GR? What property does a theory in which (a version of) the SEP holds gain by the principle holding, how does the principle constrain the theory? Equally present in Pauli (and Einstein) but more strongly emphasised in recent literature (and pioneered by

One example of focusing on this role of the SEP is Knox [2013], who proposes the following version of the (neighbourly) SEP after dismissing other versions:[48]

> To any required degree of approximation, given a sufficiently small region of spacetime, it is possible to find a reference frame with respect to whose coordinates [K_0] the metric field takes Minkowskian form, and the connection and its derivatives do not appear in any of the fundamental field equations of matter.

Equally focused on the matter field equations is a recent version by Read et al. [2018]:[49]

> The dynamical equations for non-gravitational fields reduce to a Poincaré invariant form, with no terms featuring the Riemann tensor or its contractions, in a neighbourhood of any $p \in M$.

It is not entirely clear whether Knox [2013] believes that it is possible for *any* physical system to find a K_0 for which her SEP will be fulfilled (given a particular required degree of approximation and a sufficiently small region of spacetime); if so, then the electromagnetic wave equation (2) would be a counterexample.[50] On the other hand, Read et al. commit to the claim that the principle only holds approximately, where *how well* the approximation is depends on the strength of curvature, the strengths and dynamics of the

[47] Pauli [1921], p. 707.

[48] Knox [2013], p. 352.

[49] Read et al. [2018], p. 12. The authors call this version of the SEP EP1', distinguished from EP2'. The latter does *not* demand the absence of curvature terms in the matter laws as formulated in the neighbourhood in question.

[50] One open avenue to rebut this would be to demand that the principle only holds for the *fundamental* equations governing the matter fields, in this case the Maxwell equations.

matter fields under investigation, and the measurement apparatus used to investigate them.[51]

But is this all there is to say — the SEP constrains how the matter fields couple to the metric? What does the SEP teach us about the metric itself?

4.3 The role of the strong equivalence principle

Let us come back to Einstein's letter to Painlevé from 7 December 1921, discussed in section 4.1 above. The core of it is Einstein's statement that it is the locally Minkowskian form of the GR metric that allows us to discern local inertial coordinate systems, with the help of which the metric can be related to the measurements of clocks exactly like in SR.

Pauli and Brown likewise stated that it is via the local validity of special relativity within general relativity that the dynamical $g_{\mu\nu}$ of GR can be related to readings of rods and clocks, just like in SR.[52]

Equally important is the idea that through the local validity of SR we can 'import' the SR-notion of inertial frames into GR, which, according to Pauli, allows us to think of the motion of a mass point subject to gravitational fields as force free.[53] Brown [2007] and Janssen [2014] point out that in a sense this existence of local inertial frames, brought about by the local validity of SR, means that GR is *not*, truly, a general theory of the relativity of motion.

It is hard to say whether Einstein and Pauli would have been convinced by this argument. After stating that the concept of inertial frames can be 'imported' from the locally valid special theory of relativity, Pauli hastens to add that Galileo's law of inertia (and, one might deduce, the concept of inertial frames inherent in it) was to be replaced by the geodesic equation in GR. Einstein consistently spoke of the geodesic equation in GR as the "generalised law of inertia", even while stating that, by help of the EEP and the link to Newtonian physics described in section 3.5, one can think of a particle moving on a geodesic as subject to gravitational forces. How exactly does all this fit together?

Note that in discussing the role of the SEP in GR we have now been led to return to the role of the EEP, albeit having gone to such lengths to distinguish them from each other. But distinguishing does not mean entirely separating. What exactly is the relationship between the EEP and (versions

[51]Note that they do not include any reference to a particular frame of reference in their formulation of the SEP; though the existence of a frame of reference in which the metric takes Minkowskian form follows if their version of the SEP is fulfilled.

[52]See Pauli [1921], p. 708 and Brown [2007], chapter 9.

[53]Pauli [1921], p. 708.

of) the SEP?

5 The equivalence principles as a bridge between theories

We have seen in section 3.5 that the role of the Einstein equivalence principle, at least in Einstein's mind, was to unify the Newtonian concepts of gravity and inertia, concepts that Einstein himself thought were not necessary to GR in and of itself. Thus, one might look at the EEP as a bridge principle, a principle forming a bridge from GR to Newtonian theory, a bridge that allows us to see the shadows of Newtonian theory in GR.[54] But this bridge is not *just* about accommodating our "physical habits of thinking" in allowing us to keep operating with the terms 'gravity' and 'inertia'; it also implies that a curvature-free spacetime is just as 'gravitational' as a strongly curved spacetime.[55]

While the Einstein equivalence principle can be seen as a bridge from GR to Newtonian theory, the strong equivalence principle can be seen as a bridge from GR to SR. But the nature of this bridge is different: while the EEP relates GR to a predecessor theory that is not really needed anymore once we have GR, the SEP has a double role. On the one hand, it likewise relates GR to another theory, SR, but contrary to Newton's theory of gravity that theory plays a role *within GR itself*. So the SEP is a bridge principle like the EEP, but here the bridge is to (part of) the internal structure of GR itself.

What about the practical role of the SEP and the EEP, respectively? We have seen that the role of the EEP is to unify the Newtonian concepts of gravity and inertia, and in turn to provide the possibility of interpreting the equations of motion, the geodesic equation, of GR in a certain way. The role of the SEP, as we have seen in section 4.3, seems likewise interpretational: it allows us to carry over certain interpretational possibilities from SR. In particular, it allows us to transfer the interpretation of rods and clocks as waywisers of the metric tensor from the special case of the Minkowski metric to the case of a generically curved (but locally Minkowskian) metrics, and it allows us to interpret the frames of reference in which the metric is locally Minkowskian as local inertial frames in the sense of 'inertial frame'

[54] It is possibly needless to say this, but I don't mean 'bridge principle' in the Nagelian sense of the term; the bridge formed here is between theories.

[55] Recall section 3.6, in we saw Einstein making this point in correspondence with Max von Laue, and see also section 12 of Norton [1989b].

we are wont to use from SR.[56] The local validity of SR allows a 'trickling up' of interpretations from SR to GR. I said that this makes the role of the SEP seem interpretational, but we have to be careful not to see its role as 'merely' interpretational. The SEP *explains* why rods and clocks can serve as waywisers of the metric field.

So both the EEP and the SEP can be thought of as bridge principles in the sense described above. But are the bridges themselves connected to one another? Pauli clearly thought of the SEP as a generalisation of the EEP. And indeed, if we restrict ourselves to neighbourly versions of the SEP (as described in section 4.2), then the EEP lives quite close by. For given that neighbourly versions of the SEP make statements about extended regions, the transformation between frames of references described in Einstein's original statement of the EEP is possible in that region. Thus, the demand that the SEP approximately holds for a particular system in a particular neighbourhood is a necessary condition for the EEP to be applicable to that system and that neighbourhood.

The attentive reader may realise that in a way we have come full circle. After all, in Einstein's statement of the EEP (cited in section 3.3), Einstein begins with "Starting from this limiting case of the special theory of relativity, one can ask oneself...", i.e. he motivates the EEP with the SEP. Have we learned anything new? Well, we have seen different versions of the SEP, implications of both EEP and SEP, interpretations and cross-theoretic links made available because of them. True, Einstein may have packed all that into one paragraph, and it took us a whole book chapter to unpack it all and ponder its implications. But we're in good company: Einstein himself likewise wrote dozens if not hundreds of letters unpacking all this for his contemporaries.

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[56]For a pathbreaking paper making specifying the circumstances under which matter fields in a curved spacetime "behave" locally as if they were in a Minkowski spacetime see Fletcher [forthcoming].

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