Why Einstein did not believe that general relativity geometrizes gravity

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Abstract

I argue that, contrary to folklore, Einstein never really cared for geometrizing the gravitational or (subsequently) the electromagnetic field; indeed, he thought that the very statement that General Relativity geometrizes gravity "is not saying anything at all". Instead, I shall show that Einstein saw the "unification" of inertia and gravity as one of the major achievements of General Relativity. Interestingly, Einstein did not locate this unification in the field equations but in his interpretation of the geodesic equation, the law of motion of test particles.

Contents

1 Introduction

What could be more beautiful than the idea that all there is to the world is geometry? What could cause a bigger sense of wonder than finding out that something we do not normally conceive as geometrical is exactly that at its core: a feature of the geometry of space or spacetime. Finally, what could be clearer than that this is exactly what happens in General Relativity (GR for short), and that it is what distinguishes GR most clearly from previous theories of gravity: gravity is being 'geometrized'. In this spirit, Vizgin writes:[1]

> The basic feature of general relativity that distinguished it sharply from all other physical theories, including the first quantum theories, was the inherent idea of the geometrization of a physical interaction (the gravitational interaction). The interpretation of the gravitational field as the manifestation of space-time curvature ... was a departure from the traditional theories of physics.

The sceptic of the geometrization programme, on the other hand, is most prominently represented by Weinberg:[2]

> In learning general relativity, and then in teaching it to classes at Berkeley and M.I.T., I became dissatisfied with what seemed to be the usual approach to the subject. I found that in most textbooks geometric ideas were given a starring role, so that a student who asked why the gravitational field is represented by a metric tensor, or why freely falling particles move on geodesics, or why the field equations are generally covariant would come away with an impression that this had something to do with the fact that space-time is a Riemannian manifold.
>
> Of course, this *was* Einstein's point of view, and his preeminent genius necessarily shapes our understanding of the theory he created. [...] Einstein did hope, that matter would eventually be understood in geometrical terms [...]. [I believe that] too great an emphasis on geometry can only obscure the deep connections between gravitation and the rest of physics.

Even though scholars may disagree on how far the idea of geometrizing physics can be pursued, both admirers and sceptics agree that Einstein was

[1] Vizgin [1994], p. xii-xiii.
[2] Weinberg [1972], p. vii.

the champion of the programme, that he was the man who 'geometrized' gravity and spent the rest of his life trying to do the same with the only other interaction known at the time, electromagnetism. Indeed, Vizgin and Weinberg in particular agree on Einstein's alleged twin goals: geometrization, which would eventually lead to a unification of all known interactions.

However, note that it is not necessary for the two goals to go hand-in-hand; special relativistic electrodynamics gives us a unification of electric and magnetic fields without, it seems, any kind of 'geometrization'. It clearly seems possible to unify two physical fields without relating them in any way to (spacetime) geometry: 'geometrization' and 'unification' are compatible but conceptually distinct research goals.

In this paper, I shall show that Einstein saw himself much more as a traditionalist that as someone who gives a completely new king of gravitational theory via geometrizing gravity. Indeed I will argue that Einstein saw himself as a traditionalist in two important respects: i.) he thought that General Relativity was no more and no less geometrical than Maxwell's theory of electromagnetism; and ii.) that the important achievement of GR was the advancement of the unification programme in direct continuation of special relativistic electrodynamics. Einstein thought that the special theory unified electricity and magnetism, the general theory inertia and gravity. Yet, we shall see that, unbeknown to most scholars, Einstein was emphatic in his belief that this should *not* be interpreted as a 'geometrization' of gravity, especially if 'geometrization' was seen as a reduction of gravity/inertia to spacetime geometry.

The argument will proceed as follows. Section 2 sets the stage by giving a series of almost unknown writings of Einstein that show his strong opposition to interpreting GR as a "geometrization of gravity". The respective quotations range from 1925 to the end of the 1940s, i.e. until near the end of Einstein's life. Despite of stretching almost the quarter of a century, the reader may wonder whether Einstein only acquired this opinion at the beginning of the 1920s, which was a time of conceptual reorientation for Einstein with regard to the interpretation of GR. In section 3, I shall show that even though Levi-Civita's and Weyl's work of giving the modern geometrical conception of the affine connection only took place in 1917, Einstein had all the necessary mathematical and conceptual tools for thinking of GR as a reduction of gravity to geometry, at the latest by 1916. For it will become clear that already then Einstein thought of the geodesic equation as a 'generalized law of inertia' and of test particles subject to arbitrary gravitational fields as moving on geodesics. In section 4, I shall argue that nevertheless Einstein did not adopt this position. Instead, he saw the geodesic equation as manifesting the unification of inertia and gravity in GR, a unification he saw as very similar to the unification of electric and magnetic fields in special relativity. Furthermore, he thought of the geodesic equation as allowing for an arbitrary split into gravitational terms on the one hand and inertial terms on the other. However, he also insisted that such labeling, the very distinction between gravity and inertia in GR, was in principle unnecessary, even though useful when comparing GR to its predecessor theories. This conception of unification brought with it a view of 'gravitational field' that allows the attribution of its presence only relative to a given coordinate system.

That's the story to be told. Let's start with the claim that Einstein geometrized gravity, and see how adamant he was that he did not see his work in that way.

2 What Einstein *did not* believe: the geometrization of gravity

What does it mean to say that GR 'geometrized' gravity? Does it just mean that gravity is *described* by using particular mathematical tools? Or does it mean that gravity has been ontologically *reduced* to (spacetime) geometry in some sense? In this section, we shall see that Einstein believed that at best 'geometrization' means the former — and is thus trivial — and at worst it means the latter and is wrong. We shall see that Einstein saw even Maxwell's and Hertz's use of three-vectors as equally geometrical as the use of metric tensors in GR; and we shall see that Einstein's opinion on this stayed unchanged between the formal completion of GR in 1915 and his autobiographical notes in 1949.

2.1 The Einstein-Meyerson debate: against the reduction of gravity to geometry

One might think that Einstein was unlikely to think of gravity as reduced to geometry in GR as long as he thought that the metric field $g_{\mu\nu}$ itself was reducible to the relationship between material bodies, i.e., as long as he believed in various forms of what he called Mach's principle. However, in the early 1920s, largely fueled by his debate with Willem De Sitter, Mach's principle was facing severe pressure, to the extent that Einstein was forced to recognise the metric field as a fundamental field in its own right according to GR.[3] Interestingly, this did not (yet) make Einstein give up on Mach's principle, but led him to change the role he attributed to the principle: he changed its status from a principle that was supposed to hold for GR as a whole, for every solution to the field equations, to a selection rule by which physically acceptable field equations should abide. This development culminated in the Princeton lectures in 1921, and in a series of notes in which he attacked supposedly 'Anti-Machian' papers afterwards.[4]

Having recognised the metric field as in principle ontologically on a par with the electromagnetic field (considering both of them as fundamental fields as far as GR was concerned) made Einstein more and more entranced with the mission of finding a theory in which both fields would come out as two aspects of one and the same unified field, just as the electric and magnetic field had been shown to be aspects of the electromagnetic field. Many have interpreted this mission as Einstein trying to bestow the geometrization that the gravitational field had allegedly received in GR on the electromagnetic field as well. In this section, I shall show that Einstein did not see GR as a geometrization of the gravitational field, that indeed he insisted that what the claim even meant was utterly unclear. Consequently, Einstein did not see the quest for a unified field theory as an attempt to geometrize both the gravitational and electromagnetic field.

As early as 1925 Einstein insisted, explicitly, that his work should *not* be understood as reducing physics to geometry, either his work on GR or his (and Weyl's and Eddington's) work on a unified field theory of gravitation and electromagnetism. The passage can be found in a review of Émile Meyerson's book '*La déduction relativiste*'.[5] The review was written in German by Einstein, and then translated into French by A. Metz, as solicited by Meyerson, and eventually published as Einstein and Metz [1928]. An English translation was published only as late as 1985, as an appendix to a translation of Meyerson's book. The point regarding 'geometrization' is the main critical point in an otherwise rave review by Einstein. Also, it gives what may be the clearest explication of Einstein's opposition to the idea of 'reducing physics to geometry', although we shall see similar statements from the 1930s and 1940s below. So the passage merits being quoted in its

[3] See e.g. Einstein [1920].

[4] See Einstein [1922*c*], Einstein [1922*b*] and Einstein [1922*a*] and the annotation of these articles in Vol. 13 of the Collected Papers of Albert Einstein (CPAE for short); and compare Hoefer [1994], Hoefer [1995], and Renn [2007].

[5] Meyerson [1925]. See Zahar [1980, 1987] for details on Meyerson's philosophy of science.

entirety:[6]

> Meyerson sees another essential correspondence between Descartes' theory of physical events and the theory of relativity, namely the reduction of all concepts of the theory to spatial, or rather geometrical, concepts; in relativity theory, however, this is supposed to hold completely only after the subsumption of the electric field in the manner of Weyl's or Eddington's theory.
>
> I would like to deal more closely with this last point because I have an entirely different opinion on the matter. I cannot, namely, admit that the assertion that the theory of relativity traces physics back to geometry has a clear meaning. One can with better justification say that, with the theory of relativity, (metrical) geometry has lost its special status vis-á-vis regularities which have always been denoted as physical ones. Even

[6]If not otherwise indicated, translations are either mine or, in the case where Einstein articles have already been published in the translation volumes of the CPAE, those of the translation volumes, sometimes with minor modifications. In the case where the German originals are already available in the CPAE, I do not reproduce them in the footnotes. The passage in the main text is my translation of part of Doc. AEA 91-248 of the Albert Einstein Archives (AEA for short): 'In Descartes' Theorie des physikalischen Geschehens sieht Meyerson noch eine wesentliche Gemeinschaft mit der Relativitätstheorie, nämlich die Zurückführung aller Begriffe der Theorie auf räumliche bew. Geometrische Begriffe; bei der Relativitätstheorie soll dies allerdings erst nach Einordnung des elektrischen Feldes nach Art der Weylschen bezw. Eddington Theorie vollständig zutreffen. Auf diesen letzteren Punkt möchte ich näher eingehen, weil ich hier entschieden anderer Meinung bin. Ich kann nämlich nicht zugeben, dass die Behauptung, die Relativitätstheorie führe die Physik auf Geometrie zurück einen klaren Sinn habe. Ueberhaupt Man kann mit mehr Recht sagen, dass die Relativitätstheorie es mit sich gebracht habe, dass die (metrische) Geometrie ihre gegenüber den Gesetzmässigkeiten, welche man stets als physikalische bezeichnet habe, in der Relativitätstheorie ihre Sonderstellung eingebüsst habe. Auch vor Aufstellung der Relativitätstheorie war es ungerechtfertigt, die Geometrie gegenüber der Physik als eine Lehre "a priori" zu betrachten. Dies kam nur daher, dass man meist vergessen hatte, dass die Geometrie die Lehre von den Lagerungsmöglichkeiten starrer Körper sei. Gemäss der allgemeinen Relativitätstheorie bestimmt der metrische Tensor das Verhalten der Masskörper und Uhren sowie die Bewegung frei beweglicher Körper bei Abwesenheit elektrischer Wirkungen. Dass man diesen metrischen Tensor als "geometrisch" bezeichnet hängt einfach damit zusammen, dass das betreffende formale Gebilde zuerst in der als "Geometrie" bezeichneten Wissenschaft aufgetreten ist. Dies rechtfertigt es aber keineswegs, dass man jede Wissenschaft, in welcher jenes formale Gebilde eine Rolle spielt, als "Geometrie" bezeichnet, auch dann nicht, wenn man sich bei der Veranschaulichung vergleichsweise jener Vorstellungen bedient, welche man aus der Geometrie gewohnt ist. Mit ähnlicher Argumentation hätten Maxwell und Hertz die elektromagnetischen Gleichungen des Vakuums als "geometrische" bezeichnen können, weil der geometrische Begriff des Vektors dabei in diesen Gleichungen auftritt.'

> before the proposal of the theory of relativity it was unjustified to consider geometry vis-á-vis physics as an "a priori" doctrine. This occured only because it was usually forgotten that geometry is the study of the possible positions and displacements of rigid bodies. According to the general theory of relativity the metric tensor determines the behavior of the measuring rods and clocks as well as the motion of free bodies in the absence of electrical effects. The fact that the metric tensor is denoted as "geometrical" is simply connected to the fact that this formal structure first appeared in the area of study denoted as "geometry". However, this is by no means a justification for denoting as "geometry" every area of study in which this formal structure plays a role, not even if for the sake of illustration one makes use of notions which one knows from geometry. Using a similar reasoning Maxwell and Hertz could have denoted the electromagnetic equations of the vacuum as "geometrical" because the geometrical concept of a vector occurs in these equations.

After pointing out that he does *not* think that GR and/or unified field theories are about "geometrizing" things, Einstein comes to what he thinks unified field theories *are* about. After a gap of only one sentence he adds:[7]

> Thus, what is essential about Weyl's and Eddington's theories on the representation of the electromagnetic field is not that they have incorporated the theory of this field into geometry, but that they have shown a possible way to represent gravitation and electromagnetism from a unified point of view, whereas these fields entered the theory as logically independent structures beforehand.[8]

Of course, this is not at all how Weyl himself saw his theory;[9] he saw

[7]'Das wesentliche der Theorien von Weyl und Eddington zur Darstellung des elektromagnetischen Feldes liegt also nicht darin, dass sie die Theorie dieses Feldes der Geometrie einverleibt haben, sondern dass sie einen möglichen Weg gezeigt haben, ~~Geometrie~~ Gravitation und Elektromagnetismus unter einem einheitlichen Gesichtspunkt darzustellen, während vorher jene Felder als logisch voneinander unabhängige Gebilde in die Theorie eingingen.'

[8]For more on Einstein's programme of a unified field theory of gravitation and electromagnetism see Vizgin [1994], van Dongen [2004], Sauer [2009], Goenner [forthcoming] and van Dongen [2010].

[9]Eddington is a more complicated case; see Goenner [2004], section 4.3 and Ryckman [2012], section 5, for details.

his theory as achieving *both* geometrization *and* unification and saw the two aims as intimately related. In 1918, he wrote about his theory:[10]

> [A] geometry comes about, which, if applied to the world, surprisingly *explains not only the gravitational phenomena but also those of the electromagnetic field*. According to the theory thus coming into existence, both emanate from the same source; indeed, *in general one cannot divide gravitation and electricity without arbitrariness.*

This quotation is from one of two papers Weyl wrote on the matter in 1918; this paper was directed at physicists. In the companion paper, aimed primarily at mathematicians, Weyl makes particularly clear what he means by 'the source' from which both gravity and electromagnetism emanate:[11]

> In this theory *everything real, everything that exists in the world, is a manifestation of the world-metric*; physical concepts are none other than geometrical ones.[12]

Weyl clearly thought of gravitational and electromagnetic phenomena as reduced to the geometry of spacetime in his theory. Einstein did not agree, neither with regard to GR nor with regard to any unified field theory. And he was very insistent on the point, as is shown by his correspondence with Meyerson directly after sending him the draft of his review.

[10]Weyl [1918*a*], p.30: '[K]ommt eine Geometrie zustande, die überraschenderweise, auf die Welt angewendet, *nicht nur die Gravitationserscheinungen, sondern auch die des elektromagnetischen Feldes erklärt.* Beide entspringen nach der so entstehenden Theorie aus derselben Quelle, ja *im allgemeinen kann man Gravitation und Elektrizität gar nicht in willkürloser Weise voneinander trennen*'.

[11]Weyl [1918*b*], p. 2: 'Nach dieser Theorie ist *alles Wirkliche, das in der Welt vorhanden ist, Manifestation der Weltmetrik*; die physikalischen Begriffe sind keine anderen als die geometrischen.'

[12]It is important to note that when Weyl speaks of 'the world-metric' or 'the metric of spacetime', he is not referring to a metric tensor field $g_{\mu\nu}$ as we know it from (pseudo-)Riemannian geometry. He refers to what he regards as a generalisation of the concept of a metric obtained in his theory. Given that endowing spacetime with a metric tensor would allow for distant comparison of the lengths of vectors, Weyl took as the fundamental building blocks of his geometry a *conformal structure* and a conception of *length transfer*, thereby forbidding any distant comparison. Together, these two structures define an equivalence class of pairs $[g_{\mu\nu}, Q_\mu]$, where every $g_{\mu\nu}$ is a metric tensor (in the classical sense) and every Q_μ a length connection, which together make it possible to define a unique affine connection. Weyl thus regarded the equivalence class $[g_{\mu\nu}, Q_\mu]$ (rather than any of its members) as 'the metric of spacetime'. For more on Weyl's theory see e.g. Vizgin [1994], Scholz [2001] and Goenner [2004].

Einstein had invited Meyerson to accompany the review with notes of his own, in which he could reply to the points made by Einstein. After Meyerson had sent Einstein the French translation of the German manuscript and his answers to the few points of criticism to be found in the review, Einstein seemed convinced that Meyerson was in the right across the board — in all respects but one. He wrote:[13]

> Your remarks would in principle make it necessary for me to rewrite my review, especially because it seems I did not characterise correctly your standpoint regarding the relationship between the theory of relativity and earlier physics. However, with regard to the second point, the one about "geometrization", I have not changed my mind. I still think that here the word "geometrical" is not saying anything at all.

Einstein emphasized that Meyerson's comments on his review did not change his mind about the fact GR should not be seen as a continuation of Descartes' programme of reducing physics to geometry, and that indeed the theory should not be seen as more geometrical than, e.g., Newtonian gravitation theory or Maxwellian electrodynamics. He would not change his mind on the issue in the years to come either.

In his autobiographical notes, Einstein wrote:[14]

> One is struck [by the fact] that the theory (except for the four-dimensional space) introduces two kinds of physical things, i.e.,

[13]Einstein to Meyerson, 21 August 1927, Doc. AEA 67-694: 'Ihre Bemerkungen würden es nun eigentlich nötig machen, meine Besprechung neu abzufassen, insbesondere deshalb, weil ich Ihre Auffassung der Stellung der Relativitätstheorie gegenüber der früheren Physik wohl nicht ganz richtig charakterisiert habe. Ueber den 2. Punkt der "'Geometrisierung"' habe ich allerdings meine Meinung nicht geändert. Ich denke immer noch, dass hier das Wort "geometrisch" völlig nichtssagend ist.'

[14]I changed the translation of the Schlipp volume by replacing 'inconsistent' by the weaker 'incoherent'; which is closer to Einstein's original word 'inkonsequent'. I also replaced 'intrinsically different' by 'essentially different', which seems closer to Einstein's 'wesensverschieden'. The original (Einstein [1949], p.555/56) reads: 'Es fällt auf, dass die Theorie (ausser dem vierdimensionalen Raum) zweierlei physikalische Dinge einführt, nämlich (1) Massstäbe und Uhren, (2) alle sonstigen Dinge, z.B. das elektromagnetische Feld, den materiellen Punkt etc. Dies ist in gewissem Sinne inkonsequent; Massstäbe und Uhren müssten eigentlich als Lösungen der Grundgleichungen ... dargestellt werden, nicht als theoretisch selbstständige Wesen. [...] [Es bestand die] Verpflichtung, [diese Inkonsequenz] in einem späteren Stadium der Theorie zu eleminieren. Man darf aber die erwähnte Sünde nicht so weit legitimieren, dass man sich etwa vorstellt, dass Abstände physikalische Gegenstände seien, wesensverschieden von sonstigen physikalischen Grössen ("Physik auf Geometrie zurückführen" etc.)"

> (1) measuring rods and clocks, (2) all other things, e.g., the electro-magnetic field, the material point, etc. This, in a certain sense, is incoherent; strictly speaking measuring rods and clocks would have to be represented as solutions of the basic equations..., not, as it were, as theoretically self-sufficient entities.[15] [There was the] obligation, however, of eliminating [this incoherence] at a later stage of the theory. But one must not legalize the mentioned sin so far as to imagine that intervals are physical entities of a special type, essentially different from other physical variables ("reducing physics to geometry", etc.).

An even stronger late statement, which mirrors almost verbatim Einstein's statement in the Meyerson review 23 years earlier, can be found in a letter from Einstein to Lincoln Barnett from June 19, 1948:[16]

> I do not agree with the idea that the general theory of relativity is geometrizing Physics or the gravitational field. The concepts of Physics have always been geometrical concepts and I cannot see why the g_{ik} field should be called more geometrical than f.[or] i.[nstance] the electromagnetic field or the distance of bodies in Newtonian Mechanics. The notion comes probably from the fact that the mathematical origin of the g_{ik} field is the Gauss-Riemann theory of the metrical continuum which we are wont to look at as a part of geometry. I am convinced, however, that the distinction between geometrical and other kinds of fields is not logically founded.

In the draft, Einstein writes first '...that the distinction between geometrical and other kinds of fields *cannot be upheld*' (*kann nicht aufrecht erhalten*

[15]Note that this was Weyl's answer to Einstein's criticism of his (Weyl's) unified field theory, see Einstein [1918*a*].

[16]Doc. AEA 6-58. The English version quoted below is the one actually sent; it differs from the German draft in some points: 'Ich kann nicht mit der weitverbreiteten Auffassung übereinstimmen, dass die allgemeine Relativitätstheorie die Physik 'geometriziere'. Die Begriffe der Physik sind nämlich von jeher 'geometrisch' gewesen, und ich kann nicht sehen, warum das g_{ik}-Feld 'geomtrischer' sein soll als das elektromagnetische Feld oder die Distanz von Körpern in Newtons Mechanik. Wahrscheinlich stammt die Ausdrucksweise aus dem Umstand, dass das g_{ik}-Feld seinen mathematischen Ursprung (Gauss, Riemann) Begriffen entstammt, die man als geometrisch zu betrachten gewohnt ist. Genauere Überlegung zeigt aber, dass die Unterscheidung zwischen geometrischen und anderen Feldbegriffen sich nicht ~~aufrecht erhalten lässt~~ objektiv begründen lässt.'

werden) and then strikes it out to write the above.[17]

In sum, we find that Einstein did not change his mind on the issue. But what exactly did he oppose, and why?

2.2 Analysis of the debate

The above quotations show that Einstein did not believe in there being an interesting distinction between a purely mathematical sense of 'geometrical' and a supposedly more substantive sense many would like to draw between the status of, say, the geometry of a phase space and that of spacetime. For Einstein, there is no interesting distinction between the two cases — the use of a vector in classical electromagnetic theory is as 'geometrical' as the use of a metric tensor in GR. Thus, according to Einstein, the manifold claims that GR teaches us the important lesson that 'gravity *is* geometry',[18] that 'gravity is a manifestation of spacetime curvature',[19] or that in GR gravity is 'geometrized'[20] are just *"nichtssagend"* — they are uninformative, they do not teach us anything interesting about the theory or about the world. Consequently, Einstein would probably not have been surprised at Cartan's managing to formulate Newtonian gravitation theory as a 'geometrized theory'[21] by expressing Newtonian gravity in terms of metric and curvature tensors. For Einstein, it never was the message of GR that it did something new in relating gravity to geometry; GR did not do anything new at all there, it just used mathematical methods to represent gravity that were equally geometrical or ungeometrical as the representation of the gravitational field by scalars or vectors in pre-GR theories. Indeed, this seems quite in line with Trautmann's definition of 'geometric object'; he sets out to define the term in such a way that it "includes nearly all the entities needed in geometry and physics".[22] However, *describing* something in geometrical terms should not be misunderstood as *reducing* something (ontologically) to geometry.

That for Einstein there is a clear distinction between the two possibilities is connected to what Einstein takes the term 'geometry' to refer to. In the

[17] The idea that the metric field in GR is just a field 'like any other' rather than being special because of its alleged 'geometrical significance' has more recent proponents as well; see in particular Brown [2007], chapter 9, and Brown [2009]. Compare also Anderson [1999].

[18] Hartle [2003], p.13

[19] Misner et al. [1973], p. 304

[20] Reichenbach [1957], p. 256

[21] Malament [2012*b*]

[22] Trautman [1965], p.84-5; compare also Anderson [1967] pp.14-16.

1925 quotation from the Meyerson review, Einstein states that geometry "is the study of the possible positions and displacements of rigid bodies", and that thus geometry should never have been regarded as an a priori discipline.[23]

This definition of geometry already occurs four years earlier, in a text from December 1919 / January 1920, which Einstein had intended as an article for *Nature*, telling the story of how he recalled the historical development of both special and general relativity.[24]

> The systematic decoupling of basic geometrical concepts (straight line, distance etc) from the bodies of experience, of which they are abstract representations, must not let us forget that in the end geometry is supposed to tell us about the behavior of the bodies of experience. If there were no practically rigid bodies that can be brought into congruence with one another, we would not speak of the congruence of distances, triangles, etc. It is clear that for the physicist geometry becomes meaningful only as he associates bodies of experience with those basic concepts, for example by associating the concept of distance with a practically rigid body with two markings. Vice Versa, this association makes Euclidean geometry a science of experience in the truest sense, just like mechanics. The sentences of geometry can then be confirmed or falsified, just like the sentences of mechanics.

Geometry defined as the science of possible displacements of rigid bodies is not easily carried over from the Euclidean geometry of space to the geometry of (possibly curved) space*time*. Indeed, in the 1925 quotation from the Meyerson review, Einstein claims that the only reason we call metric tensors geometrical is that they were first used in the context of that area of study which we originally called 'geometry'. And Einstein calls for caution regarding calling these tensors 'geometrical' once we leave that original area of study. Given that this development had already taken place, Einstein effectively diagnoses that the use of 'geometry' has become so broad that

[23] This is clearly directed against Kantian and Neo-Kantian voices which had come up in defense against first special and then general relativity, and which were particularly strong in the 1920s. For details, see Hentschel [1990], section 4.1, Ryckman [2012], section 3 and Ryckman [2005].

[24] See Vol. 7, Doc. 31 CPAE. As Janssen [2012], p.160, states, the article was "but was withdrawn in the end and replaced by a much shorter and less informative piece", which appeared in *Nature* instead. Compare also similar statements in Einstein [1921].

stating that something has been 'geometrized' is just not saying anything — it is *nichtssagend*.

Once this diagnosis has been made, Einstein could have just shrugged his shoulders and said that he does not really care about whether GR is seen as geometrizing gravity or not. The reason why he *did* care is that people who see GR as geometrizing gravity (like Meyerson, Weyl and Vizgin) see this as a profound statement — not as a statement about the mathematical language GR uses to *describe* gravity but as a reductionist claim: gravity has allegedly been *reduced* to geometry. We see the opposition to such reductionist claims (and to the claim that this reduction is the novel feature of GR) in each and every one of the above Einstein quotations. The reason that Einstein opposed this view is that he thought that it diverted attention from what really *was* the important message of GR. We shall return to the latter point in section 4; for now let us look to see if Einstein *could have* endorsed the reductionist claim in all its sophistication.

3 What Einstein *could* have believed: gravity reduced to inertia

Let us pause for a moment to see what 'reducing gravity to an aspect of spacetime structure' could mean in GR. Arguably the most promising way to explicate this statement comes via relating gravity to inertial structure and in turn to spacetime structure. The Newtonian conception of inertial motion is that bodies move inertially if they are subject to nothing but space(time); in particular, they are not subject to forces. In GR, particles subject to gravity move on geodesics, the direct generalisation of straight lines. From here, it is only a small step to regard the motion of particles subject to gravity as inertial motion, and thus gravity as reduced to inertial structure. Given that inertial/geodesic structure is a particular aspect of the structure of spacetime (its affine structure), 'reduction of gravity to inertia' can be seen as a special case of 'reduction of gravity to spacetime structure'.[25] Could Einstein have interpreted GR in this way? If so, *by when* could he have done it, and why did he *not* do it?

It has sometimes been claimed that before Levi-Civita's and Weyl's reconceptualisation of the affine connection in terms of parallel transport in 1917 and 1918,[26] Einstein *could not have* seen GR as 'properly' geometrizing

[25] Wald [1984], p. 67, for example, seems to see GR as doing exactly that.

[26] See Levi-Civita [1917] and Weyl [1918*b*].

gravity.[27] However, it seems that this opinion at least partly rests on not clearly distinguishing between *using* geometrical language in physics (which for Einstein included vectors as much as metrics) on the one hand and ontologically *reducing* physics to spacetime geometry on the other; i.e., showing that gravity is 'just an aspect of spacetime structure'.

In this section, I shall show that i.) already starting in 1913 Einstein saw the extremisation of the line element as describing "straight and uniform motion" and, following a result by Planck from 1906, saw it as the relativistic counterpart of Newton's law of inertia; ii.) Einstein saw these equations as describing the motion of test particles *both* in the absence *and* in the presence of gravity; iii.) by 1916 at the latest Einstein was aware of the idea (presented to him by Friedrich Kottler) that gravity should be seen as reduced to inertia because particles subject to gravity can be described as moving inertially; iv) Einstein rejected this interpretation of GR. Thus, by the end of this section we shall see that by 1916 at the latest Einstein had all the necessary mathematical and conceptual tools to see gravity as reduced to inertia in GR, and indeed to see reduction to inertia as identical to or a special case of reduction to spacetime geometry. Still, he did not adopt this position. In the following section, we shall then see what Einstein believed instead, what he thought the main message of GR was, given that it was not one of reduction of gravity to inertia/geometry.

3.1 The geodesic equation as the equation of motion of test particles subject to gravity

Already in the first papers in which Einstein starts making use of the metric tensor to give an account of gravitation, he is at pains to establish the status of the geodesic equation as describing the motion of particles as "straight and uniform" (*geradlinig und gleichförmig*) even when subject to gravity. This would lead him to call the geodesic equation a "generalised law of inertia"; redefining inertial paths such that the category includes motion under the influence of gravity.

The story begins in the *Entwurf* paper, Einstein and Grossmann [1913]. In section 1 of the physical part, written by Einstein alone, he states that already according to special relativity, the equation of motion of a point particle not subject to forces follows from extremising the line element:

$$\delta \int ds = \delta\{\int \sqrt{-dx^2 - dy^2 - dz^2 + c^2dt^2}\} = 0 \quad . \tag{1}$$

[27] See e.g. Reich [1992, 1994], and Stachel [2007].

In a footnote, Einstein pointed out that this had already been shown by Planck [1906]. In this paper, Planck's sole aim had been to find the relativistic counterpart of Newton's first law of motion (i.e., the law of inertia), and he arrived at $\delta \int ds = 0$. Einstein now stated that this equation "says nothing else but that the material point moves in a straight and uniform line".[28]:

Thus, by 1913 at the latest Einstein clearly follows Planck in seeing equation (1) as the relativistic law of inertia. In the *Entwurf* paper, he then turns to the equivalence principle, and states that as a consequence of the latter he found that in his scalar theory of gravitation (in which the scalar field c represents both the gravitational potential and the local speed of light) the equation of motion for force-free point particles *also* applies to point particles moving in a static gravitational field, as described by that theory; the difference being that in this case c varies with the spatial coordinates in a given coordinate system.[29]

Already in a note added in proof to Einstein [1912], Einstein had stated that equation (1) gives the equation of motion of point particles "not subject to external forces".[30] Thus, it was clear that already in 1912, before even embarking on a metric theory of gravitation, Einstein thought of (static) gravitational fields *not* as invariant force fields diverting particles from inertial motion.[31] Already, in 1912, he thought of equation (1) as describing inertial motion on the one hand, and as describing motion in the presence of (static) gravitational fields on the other.

The natural follow-up question is: what happens if we consider the motion of point particles in the presence of general, non-static gravitational fields? In section 2 of the *Entwurf* paper, Einstein takes the variational principle (1) as a starting point and argues that for non-static gravitational fields, too, we should expect equation (1) to give the equation of motion

[28]Einstein and Grossmann [1913], p. 4.

[29]For more on this theory, see the editorial note "Einstein on gravitation and relativity: the static field", p. 122-128 of Vol. 4 CPAE, and Norton [1995], section 5.1; for a reconstruction of the theory using modern differential geometry see Norton [1989b], section 3.

[30]See Einstein [1912], p. 458.

[31]In section 4, we shall see that this did not stop Einstein from making sense of the attribution of gravitational forces as coordinate-dependent assertions; for him, stating that particles in gravitational fields move on geodesics was completely compatible with saying that in a given coordinate system the presence of gravitational forces can be asserted. Indeed, as Norton [1989b] pointed out, the possibility of asserting the presence or absence of gravitational forces depending on the coordinate system chosen was the conceptual core of the equivalence principle in Einstein's mind.

for point particles. The only difference is that now the line element on the left-hand side of the equation has to be that defined by a general metric tensor $g_{\mu\nu}$; the first time Einstein introduces the latter in a published article.[32] About three months after submitting the *Entwurf* paper, Einstein submitted a paper to the 85th conference of the German Society for Scientists and Physicians, in which he became even more explicit:[33]

> A free mass point moves in a straight and uniform line according to equation [(1)], where
>
> $$ds^2 = \sum_{\mu\nu} g_{\mu\nu} dx_\mu dx_\nu$$
>
> . [...] In general, every gravitational field is going to be defined by ten components $g_{\mu\nu}$, which are functions of x_1, x_2, x_3, x_4. The motion of the material point will always be governed by equations of this form.

Thus, it is clear that already in 1913 Einstein saw the variational principle (1) as describing inertial motion, asserting that point particles would move in this fashion *both* when not subject *and* when subject to gravitational fields. In Einstein [1914], Einstein then used the term "geodesic" for the first time to describe the lines arising from equation (1). He explicitly states, again, that they describe the motion of particles in *arbitrary* gravitational fields:[34]

> In §2 it has already been shown that the motion of a material point in a gravitational field takes place according to the equation $\delta \int ds = 0$. Thus, from a mathematical point of view the motion

[32] Note that in section 4 (p.10) of the *Entwurf* paper Einstein points out, also for the first time, that for the special case of a dust energy-momentum tensor the equations of motion for a single point particle (i.e. one element of the dust) follow from energy-momentum conservation. This remark presents the first instance of what would later be called the geodesic theorem: the possibility of deriving the equations of motion from energy-momentum conservation, given certain conditions on the energy-momentum tensor (and, in theories like the final version of GR where energy-momentum conservation is implied by the field equations, eventually from the latter). The most general version of the theorem to date has been provided by Ehlers and Geroch [2004]. For historical discussion see CPAE V7, p.453, endnote 6; Havas [1989] and Kennefick [2005]; for systematic discussions see Malament [2012*a*] and Weatherall [2011].

[33] Einstein [1913], p. 1256. See Norton [1995], section 5.2 for an analysis of the transition to the Entwurf theory).

[34] Einstein [1914], p.87.

> of a point corresponds to a geodesic line in our four-dimensional manifold. [...] In the original theory of relativity, those geodesic lines for which $ds^2 > 0$ correspond to the motion of material points; those for which $ds^2 = 0$ correspond to light rays. This will also be the case in the generalized theory of relativity.

Most of Einstein's work in 1915 focused on finding the field equations governing the gravitational field; this culminated in his finding what became known as the Einstein field equations in November 1915.[35] After this feat was accomplished, Einstein could take a deep breath, and work on what would become his first major review article about the finalised theory of general relativity. In this treatise, he returns to the question of the equations of motion in an arbitrary gravitational field:[36]

> According to the special theory of relativity, a freely moving body in the absence of external forces moves in a straight and uniform line. This also holds for the general theory of relativity for part of the four-dimensional space, in which the coordinate system K_0 can be chosen such that the $g_{\mu\nu}$ have the special constant values $[\delta_{\mu\nu}]$. If we consider this motion in an arbitrarily chosen coordinate system K_1, then judged from within K_1 the body moves ... in a gravitational field. ... With regard to K_0, the law of motion is a fourdimensional straight line, i.e., a geodesic line. Since the geodesic line is defined independently of the system of reference, the equation describing this line will also be the equation of motion of the material point with regard to K_1,
>
> $$\frac{d^2 x_\tau}{ds^2} = \Gamma^\tau{}_{\mu\nu} \frac{dx_\mu}{ds} \frac{dx_\nu}{ds} \tag{2}$$
>
> We now make the very natural assumption that this generally covariant system of equations determines the motion of the point in gravitational fields also in the case that no reference system K_0 exists with respect to which special relativity holds with respect to a finite space.[37]

[35]See Stachel [1989], Norton [1984], Renn and Sauer [2006], Janssen and Renn [2006], Renn [2007] and Janssen [2007] for the details of the tale of this grand journey.

[36]Einstein [1916*a*], p.801-802.

[37]For a rational reconstruction of Einstein's conception of reference systems and relative spaces, see Norton [1989b], section 3.

Einstein says that with respect to a coordinate system in which the metric is Minkowskian locally, i.e. in which no gravitational fields are judged to be present, a particle moves on a straight line, "i.e. a geodesic line". He then points out that even if one goes to a coordinate system in which a gravitational field is judged to be present, in which $g_{\mu\nu}$ has arbitrary components, the particle will still move on a geodesic; and that we should assume the geodesic equation to govern the motion even if non-vanishing components of the Riemann tensor cannot be neglected in an arbitrarily small but finite neighborhood of the point.[38]

To summarize: by 1916 at the latest Einstein saw the geodesic equation as representing i) inertial motion and ii) motion subject to gravity. Did he make the step to seeing gravity as a reduction to inertia?

3.2 Kottler and the generalised law of inertia

Already in 1916 Einstein was presented with the idea that either gravity diverts particles from inertial paths and is thus a dynamical force field, or bodies move on inertial paths (for Einstein: geodesics) even when subject to gravity, *and thus* gravity is reduced to inertial structure.

The distinction, and the challenge to choose, came to Einstein in the form of an article published by the Viennese mathematical physicist Friedrich Kottler. Kottler had written his *Habilitation* in mathematics on the application of tensor calculus to relativity theory — in 1912, before Grossmann and Einstein started to use the formalism in the construction of the *Entwurf* theory.[39] In 1916, Kottler wrote two articles that were bound to draw Einstein's attention: he claimed that in GR Einstein had effectively abandoned his own equivalence principle and reintroduced gravitational forces as "real" forces, whereas in Kottler's view the equivalence principle required them to be regarded as inertial forces. Kottler starts out by carefully distinguishing between a dynamical and a kinematical conception of gravity:[40]

[38]This is what the final sentence of the above quote amounts to, as elaborated by Einstein in a footnote.

[39]See Call No. 14-329 EA for a Curriculum Vitae written by Kottler himself around 1938, when Einstein helped him to leave Vienna and emigrate to the US.

[40]Kottler [1916], p.955-956: 'Einstein hat seither die Äquivalenzhypothese aufgegeben. Die Gründe liegen im wesentlichen in einer besonderen Auffassung ihrer Ergebnisse durch ihn, die darauf hinausläuft, den Kräften des Gravitationsfeldes einen selbstständigen Charakter zu geben, während *hier* die Bewegung im Gravitationsfeld als kräftefrei angesehen werden soll, also das Trägheitsgesetz abgeändert und die Gravitation als reine Trägheitserscheinung gedeutet wird. Diese Auffassung scheint mir die strenge Konsequenz der Äquivalenzhypothese und daher nur *gleichzeitig* mit ihr verwerflich. [...] Der

> Since then, Einstein has abandoned the equivalence hypothesis. The reasons lie primarily in a particular perception of its results, which amounts to giving an independent existence to the forces of the gravitational field. *Here*, motion in a gravitational field will be seen as force-free. Thus, the law of inertia must be changed and gravitation be seen as a purely inertial phenomenon. This perception seems to me a strict consequence of the equivalence hypothesis; and thus can only be abandoned *together* with the latter. [...] The prime difference [of my approach as compared to Einstein's] is one of principle: the kinematical, rather than dynamical, conception of gravity.

Kottler essentially interpreted Einstein's talk of gravitational forces, and his quest for finding an expression for gravitational energy, as Einstein going back to Newton's conception of gravity: inertial motion is motion on straight lines and gravity is a dynamical force (field) diverting particles from inertial paths. Kottler saw himself as generalising the law of inertia such that gravitational forces count as inertial forces:[41]

> The difference of this interpretation as compared to Einstein's has a kinematical rather than dynamical conception of gravity as its consequence; i.e., in place of the force field we introduce a modification of the Galileian law of inertia. The force-free point does not move in a uniform and straight line anymore, it moves on a curved and non-uniform line... . As paradoxical as it may seem: only this seems to be the consistent conception of Einstein's equivalence! Indeed, if the cause of the equality of gravitational accelerations for all masses is a kinematical one, then gravity itself has to be of a kinematical origin, i.e. it must be an inertial phenomenon!

This seems pretty much in line with the modern conception that particles under the influence of gravity move inertially and thus gravitational

wesentliche Unterschied [zu Einstein] ist ein prinzipieller: die erwähnte kinematische, nicht dynamische Auffassung der Gravitation. '

[41] Kottler [1916], p.961: 'Die ... erwähnte Verschiedenheit der hiesigen Interpretation von der Einsteinschen hat dann eine kinematische an Stelle der dynamischen Auffassung der Schwere zur Folge, d.h. an Stelle des Kraftfeldes tritt eine Abänderung des Galileischen Trägheitsgesetzes; nicht mehr gleichförmig und geradlinig bewegt sich der kräftefreie Punkt, sondern krummlinig und ungleichförmig ... . So paradox es scheint: erst dies scheint die konsequente Fassung der Einsteinschen Äquivalenz! In der Tat, wenn die Ursache der Gleichheit der Schwerebeschleunigung für alle Massen kinematisch ist,so muß die Schwere selbst kinematischen Ursprungs, d.i. Trägheitserscheinung, sein! '

phenomena are reduced to inertial structure.[42] Most importantly, Kottler explicitly claims the law of inertia should be changed so that motion in gravitational fields counts as inertial, unforced motion. Thus, it seems that here at the *very* latest, Einstein was presented with the possibility of saying that since particles move on geodesics even under the influence of gravity and since the geodesic equation is the new law of inertia, gravity should be seen as reduced to inertia. This within Einstein's grasp the possibility of endorsing the idea that GR reduces gravitational phenomena to inertial structure and thus to spacetime geometry.

Whether Einstein identified 'reduction to inertial structure' with (a special case of) 'reduction to geometry' we cannot know for sure, although it seems plausible. What we do know is that Einstein opposed the idea that gravity should be seen as 'reduced to inertia' in GR, just as he opposed the idea that it should be seen as 'reduced to geometry'. It is clear that Einstein *could have* endorsed the line of thought that because the geodesic equation (2) describes test bodies moving in the presence of a gravitational field, the latter cannot be seen as a force field but must instead be seen as an aspect of spacetime structure, namely inertial structure. If movement in a gravitational field is described as a generalisation of inertial motion, is it not clear that thus the gravitational field cannot be a force field, defined as that which diverts particles from inertial motion?

This was not Einstein's view. He could have endorsed it, as it was available for him to do so. Yet he did not adopt it, for the simple reason that he believed something else instead. He believed that even though a particle moves on a geodesic *both* in the absence *and* in the presence of a gravitational field, a coordinate system can be chosen such that the connection components ${\Gamma^\nu}_{\mu\sigma}$ vanish or appear, and thus a gravitational field appears or disappears given a certain choice of coordinates.[43] As we shall see in the next section, this view is, for Einstein, intimately related to the principle of equivalence of inertia and gravity, and to the unification of inertia and gravity achieved in GR.

[42]Note that it is not clear whether Kottler thinks of the 'curved and non-uniform' lines he introduces as the new inertial paths as geodesics; after all, a major difference between Kottler's approach in this paper as compared to Einstein's is that he tries to get along without non-Euclidean geometry. In Kottler [1918] he would change his approach.

[43]Only as late as 1915 had Einstein started to see the connection components ${\Gamma^\nu}_{\mu\sigma}$ as the representative of the gravitational field rather than the derivatives of the metric: see Renn and Sauer [2006] and Norton [2007] for details.

4 What Einstein *did* believe: unification of gravity and inertia

In section 2 I argued that Einstein did *not* believe that gravity is reduced to geometry according to GR. In section 3, I showed that Einstein *could* well have defended this position by 1916 at the latest for he had linked inertial motion to geodesic motion on curved surfaces and saw the geodesic equation as describing the motion of test particles subject to gravity. If it was still needed, Kottler even gave him the last piece by claiming that a generalization of the law of inertia requires that gravity be seen as reduced to inertia. The reason that Einstein did not believe in the geometrization of the gravitational field (understood as reducing gravity to spacetime geometry) as a consequence of all this was that he *did* believe in something else instead, something we want to investigate in this section.

In short, we shall see that Einstein believed i) that the geodesic equation shows that inertia and gravity are unified in GR, analogous to the unification of electric and magnetic fields in special relativistic electrodynamics; ii) that a result of this unification is that the existence of gravitational fields (in contrast to gravitational-inertial fields) becomes coordinate-independent.

Given what we saw in the previous section, it may not be surprising that Einstein did not see himself well represented in Kottler's characterisation of him as having given up the equivalence principle and having described gravity as a force diverting particles from inertial motion. However, he also does not follow Kottler in seeing gravity as reduced to inertia; Einstein refused to make this last step. Indeed, he rejected the very distinction with which Kottler confronted him: either gravity is a dynamical force diverting particles from inertial motion, or it is an inertial phenomenon itself. Einstein wrote:[44]

> Kottler complains that with regard to the equations of motion
>
> $$\frac{d^2x_\nu}{ds^2} + \left\{ \begin{matrix} \alpha\beta \\ \nu \end{matrix} \right\} \frac{dx_\alpha}{ds}\frac{dx_\beta}{ds} = 0 \tag{3}$$
>
> I interpreted the second term as the representative of the influence of the gravitational field on the point mass, whereas I interpret the first term as, so to speak, the representative of Galileian inertia. This, he claims, would introduce "real gravitational forces", which is supposed to contradict the spirit of the

[44]Einstein [1916*b*], p. 641.

> equivalence principle. To this I answer that this equation is, as a whole, generally covariant, and thus consistent with the equivalence hypothesis. The labeling of the terms I introduced does not really matter though and was only meant to accommodate our physical habits of thinking. This is also true, in particular, for the concepts
>
> $$\Gamma^{\nu}{}_{\mu\sigma} = -\left\{ \begin{matrix} \alpha\beta \\ \nu \end{matrix} \right\}$$
>
> (components of the gravitational field) and t_{σ}^{ν} (energy components of the gravitational field). The introduction of these labels is in principle unnecessary, but for the time being they do not seem worthless to me, in order to ensure the continuity of thoughts... .

Here, Einstein gives two reasons for rejecting Kottler's claim that by naming the two terms occurring in equation (3) 'inertia' and 'influence of the gravitational field', respectively, he had given up on the equivalence principle. First, that the equivalence principle must be fulfilled in the theory because of the latter's general covariance. Note that earlier in the paper Einstein had claimed that the equivalence principle is a special case of general covariance (which he saw as the mathematical counterpart of the general principle of relativity). After Kretschmann [1918], Einstein [1918*b*] started to distinguish carefully between the equivalence principle, the relativity principle, and Mach's principle; thus, Einstein's reasoning here would not go through after 1918, when the equivalence principle stopped being a (straightforward) consequence of general covariance in Einstein's mind.[45] However, for our purposes Einstein's second reason for rejecting Kottler's claim is much more important than the first, and, in contrast, it could be defended even today. Einstein points out that the labeling of the two terms of the geodesic equation (3) as 'inertial' and 'gravitational', respectively, was unnecessary, only meant to accommodate our 'habits of thinking' (formed, presumably, by Newtonian theory). Einstein effectively states that the very distinction between 'gravity' and 'inertia' is useful only for relating the theory to its predecessor theories; it is not a distinction *from within* the theory itself. Put differently, if one just looks at the theory without relating it to predecessor theories, there is no need whatsoever to distinguish 'inertial terms' and 'gravitational terms' in the geodesic equation.

Before the above quotation Einstein rejected Kottler's idea that the gravitational field is only 'kinematically determined', precisely by pointing out

[45] See Norton [1999] and Hoefer [1995, 1994] for details.

the limits of the equivalence principle: he states that only *homogeneous* gravitational fields can be transformed away and substituted by *uniform* accelerations, but not *arbitrary* gravitational fields: “Thus, a ‘kinematical, not dynamical interpretation of gravitation’ is not possible”.[46] However, neither does Einstein subscribe to what Kottler had called the dynamical interpretation: the option that gravity is a force field diverting particles from inertial paths. Einstein wants to hold the middle ground: particles move on geodesics in the presence of arbitrary gravitational fields. And indeed, as Kottler had stated, the law of inertia has to be generalised to include motion in gravitational fields. But for Einstein that does not mean that gravity is reduced to inertial structure; instead, the very distinction between gravity and inertia breaks down. As we will see below, Einstein would soon speak of inertia and gravity having been *unified*, just as electricity and magnetism had been unified before.

This line of thought becomes most clear in the Princeton lectures from 1921, which would later be published as the ‘*The Meaning of Relativity*’ in English and as ‘*Vier Vorlesungen über Relativitätstheorie*’ in German. After having shown that the components of the connection become the Newtonian gravitational field in the Newtonian limit, Einstein goes on with a description of how the geodesic equation links inertia and gravity:[47]:

> Formally, the unity between inertia and gravity is expressed by the fact that the entire left side of
>
> $$\frac{d^2 x_\nu}{ds^2} + \left\{ \begin{matrix} \alpha\beta \\ \nu \end{matrix} \right\} \frac{dx_\alpha}{ds}\frac{dx_\beta}{ds} = 0$$
>
> is tensorial (with respect to arbitrary coordinate transformations), whereas the two terms separately are not. In analogy to the Newtonian equations one would have to view the first as an expression for inertia, the second as an expression for the gravitational force.

Note that, just as in his answer to Kottler, Einstein only introduces the labels of an ‘inertial term’ and a ‘gravitational term’ as an “analogy” to Newtonian theory, and only after having pointed towards the “unity” of inertia and gravity, expressed by the tensorial nature of the two terms in the geodesic equation taken together, but not separately.

[46] Einstein [1916*b*], p. 640.
[47] Einstein [1922*c*], p.51.

What is the nature of this unification as Einstein saw it? Is it just like the unification of electric and magnetic fields in special relativity? Or is it a different kind of unification? In Einstein's mind, the unification was very similar indeed, as the December 1919 / January 1920 text on the development of relativity shows. There, he recalls the magnet-conductor thought experiment described in the first paragraph of his 1905 paper on special relativity, from which he concludes:[48]

> The existence of the electric field is a relative one, depending on the state of motion of the coordinate system used; only the electric and magnetic field together can be attributed a kind of objective reality, independent of the state of motion of the observer, i.e. of the coordinate system.

Einstein then describes how he worked on a review article of special relativity in 1907,[49] and links the above realisation regarding the electric and magnetic field to another thought experiment regarding inertia and gravity:[50]:

> Then I had the most fortunate thought of my life in the following form: The gravitational field only has a relative existence in a manner similar to the electric field generated by electro-magnetic induction. *Because for an observer in free-fall from the roof of a house, there is during the fall* — at least in his immediate vicinity — *no gravitational field*. Namely, if the observer lets go of any bodies, they remain, relative to him, in a state of rest or uniform motion, independent of their special chemical or physical nature.

Taken together with the quotation from the Princeton lectures one year later, where Einstein spoke of the "unity between inertia and gravity" as expressed by the fact that the inertial and the gravitational term of the geodesic equation transform as tensors only together, just as the $\vec{E}$ and $\vec{B}$ fields do not transform as (4-dimensional) tensors while the electromagnetic field $F_{\mu\nu}$ does, Einstein seems to think of inertia and gravity as having been unified in GR in quite the same way that electricity and magnetism had been unified in SR.

Of course, suspicion may seem in order regarding whether or not Einstein was right about this last point; arguably, the analogy is not a complete one.

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[48]Vol. 7, Doc. 31 CPAE, p.265.
[49]Einstein [1907].
[50]Vol. 7, Doc. 31. CPAE, p.265.

After all, saying that a *tensor* $F_{\mu\nu}$ represents the unified electro-magnetic field for which we then find field equations is not the same as saying that the *tensor* making up the left hand side of the geodesic equation (in its coordinate-independent form $v^\mu \nabla_\mu v^\nu$) represents the unity of inertia and gravity — for which we then *don't* go on to search for field equations. The analogy would have been more complete if Einstein had claimed that the connection ${\Gamma^\nu}_{\mu\sigma}$ represents the gravitational-inertial field,[51] for this is the (non-tensorial) field which the Einstein equations govern. Instead, Einstein claims that the connection ${\Gamma^\nu}_{\mu\sigma}$ represents the coordinate-dependent gravitational field (more on this below), and the tensor $v^\mu \nabla_\mu v^\nu$ the full gravitational-inertial field of which ${\Gamma^\nu}_{\mu\sigma}$ is the gravitational part. Thus, Einstein locates the unity of inertia and gravity entirely in a mathematical object occuring in the equation of motion of test particles, rather than in a mathematical object occuring in the field equations, as in the case of $F_{\mu\nu}$ and the Maxwell equations. However, just as the split between electric and magnetic fields, the split between gravity and inertia in equation (3) is coordinate-dependent given Einstein's labeling of gravitational and inertial terms in (3).

Either way, our main line of investigation here is historical: find out what Einstein *did* think on the relationship between inertia and gravity as compared to electricity and magnetism; leave aside considerations of what he maybe *should have* thought.

So let us now bring the two strands together: Einstein's interpretation of the geodesic equation as giving the equation of motion of test particles in the presence of gravitational fields on the one hand, and his claim that the unity of inertia and gravity is expressed in the tensorial nature of combined inertia and gravity terms in the geodesic equation.

At the beginning of the fourth Princeton lecture, Einstein starts the discussion of the motion of point masses, and we see him using language similar to Kottler [1916] when stating that in GR the law of inertia has to be generalised by generalising the concept of a straight line:[52]

> According to the principle of inertia, the motion of a material point in the absence of forces is straight and uniform. In the fourdimensional continuum of special relativity, this is a real straight line. The natural, i.e. the simplest, generalisation of the straight line making sense in the conceptual scheme of the general (Riemannian) theory of invariants is the straightest (geodesic) line.

[51] This position is advocated e.g. by Ehlers [1973] and Giulini [2002].

[52] Einstein [1922*c*], p. 51.

Following this, he links the generalised law of inertia to the equivalence principle and thereby relates inertia and gravity:

> Following the equivalence principle, we will have to assume that the motion of a material point subject only to inertia and gravity is described by the equation
>
> $$\frac{d^2x_\mu}{ds^2} + \Gamma^\mu{}_{\alpha\beta}\frac{dx_\alpha}{ds}\frac{dx_\beta}{ds} = 0 \tag{4}$$
>
> Indeed, this equation becomes that of a straight line if the components $\Gamma^\nu{}_{\mu\sigma}$ of the gravitational field vanish.

Einstein then shows that in the Newtonian limit the geodesic equation becomes

$$\frac{dx_\mu}{dl^2} = \frac{\partial}{\partial x_\mu}(\frac{\gamma_{44}}{2}) \tag{5}$$

where γ_{44} is defined by

$$g_{\mu\nu} = \delta_{\mu\nu} + \gamma_{\mu\nu} \quad .$$

Following this, he points out a link between the geodesic equation and the Newtonian equation of motion for particles subject to gravitational fields:

> This equation [5] is identical with Newton's equation of motion of a point in a gravitational field if one identifies $-\frac{\gamma_{44}}{2}$ with the gravitational potential. ... One look at equations [4] and [5] shows that the quantities $\Gamma^\nu{}_{\mu\sigma}$ play the role of the field strength of the gravitational field. These quantities are not tensorial.

Here we see Einstein making precise what he had alluded to in his answer to Kottler: the labeling of the two sets of terms in the geodesic equation as 'inertial' and 'gravitational', respectively, comes about only by comparing the theory to Newtonian theory; more precisely, by comparing the Newtonian limit of GR with the Newtonion equation of motion of a point in a gravitational field. In a way, there is no reason even to distinguish between gravity and inertia in GR unless one is concerned with comparing it to Newtonian theory (or, indeed, special relativity).

We found that i.) Einstein thought of the geodesic equation in GR as a *generalisation of the law of inertia*; ii.) in which inertia and gravity were *unified* so that iii.) the very *labeling* of terms as 'inertial' 'gravitational' becomes in principle "*unnecessary*", even if useful when comparing GR to Newtonian theory. Finally, we found that Einstein saw the status of the gravitational field ${\Gamma^{\nu}}_{\mu\sigma}$ as quite analogous to that of the electric field $\vec{E}$ in special relativistic electrodynamics: their attribution only makes sense relative to a chosen coordinate system. Michel Janssen has recently investigated this *relativity of the gravitational field* in detail, and sees the arbitrariness of the inertia/gravity split as *the* main difference between GR and other theories:[53]

> [W]hile the slide into general covariance turns the relativity of non-uniform motion of space-time coordinate systems into a feature general relativity shares with older theories, it does not so trivialize the relativity of the gravitational field. Even in generally covariant reformulations of these older theories, there will be an inertial field and a gravitational field existing side by side. The unification of these two fields into one inertio-gravitational field that splits differently into inertial and gravitational components in different coordinate systems is one of Einstein's central achievements with general relativity.

To sum up, Einstein did not accept that gravity must *either* be a force field, *or* an aspect of spacetime structure. He did not believe that *either one* of the two options needs to be true, or that denying the truth of one implies the truth of the other. Indeed, we saw that for Einstein gravity is a force field in a different sense from that envisaged by modern authors: it is a frame-dependent force field. Thus, asserting that gravity is a force field for Einstein does not imply that it diverts particles from inertial motion: all motion in gravitational fields is motion on geodesics, and if a body moves on a geodesic in one frame of reference, it does so in all frames. Even so, every frame of reference chosen involves an arbitrary split, making part of the trajectory 'inertial' and part of it 'gravitational'. However, Einstein saw this labeling as ultimately unnecessary — unnecessary but useful for certain purposes, especially the comparison between GR and its predecessor theories. Finally, Einstein saw the arbitrariness of this split as the expression of having unified inertia and gravity in the general theory of relativity.

[53] Janssen [2012], p.162

5 Conclusion

In the introduction, I took a quotation from Vizgin which pointed to (some kind of) geometrization as being what "distinguished [GR] sharply from all other physical theories". We have seen that for Einstein the important achievement of GR was *not geometrization* of gravity *but unification* of gravity and inertia. Furthermore, Einstein did not see this as something that "distinguished [GR] sharply from all other physical theories"; he saw his theory in direct continuation of previous unificationary successes, especially of the unification of electric and magnetic fields in special relativistic electrodynamics. As in many other respects, while almost everybody else saw Einstein as a revolutionary, he saw himself as a traditionalist.

We also saw that Einstein could well have argued, by 1916 at the latest, that GR shows that gravity is reduced to inertial structure and thus ultimately to spacetime geometry. Could he have thought that GR *both* unifies gravity and inertia *and* reduces them to spacetime structure? Of course he could have; indeed, that was Weyl's interpretation of GR: unification *via* reduction to spacetime structure. But for Einstein, it might have been a bit like not wanting to eat pudding because he had already had a big entrée for dinner. It's not that it's impossible to eat both — it's not even impossible to eat both of them at once. It's just that eating one may make you find the other one less attractive: you just don't have any appetite for it, and maybe you find pudding dubious from the start. Einstein certainly thought of 'geometrization' as dubious, of 'unification' as the ultimate goal. Other people may want to eat both dishes at once, even identify what you see as two kinds of dish as, in fact, one and the same kind ('the best main meal is a big pudding') or see one as a necessary consequence of the other ('no dinner without pudding'). Again, for Weyl and those of like mind unification and geometrization went hand in hand: the former was a consequence of the latter. But not for Einstein: unification was all he wanted.

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