



# Logic Diagrams as Argument Maps in Eristic Dialectics

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Accepted: 22 September 2022  
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## Abstract

This paper analyses a hitherto unknown technique of using logic diagrams to create argument maps in eristic dialectics. The method was invented in the 1810s and -20s by Arthur Schopenhauer, who is considered the originator of modern eristic. This technique of Schopenhauer could be interesting for several branches of research in the field of argumentation: Firstly, for the field of argument mapping, since here a hitherto unknown diagrammatic technique is shown in order to visualise possible situations of arguments in a dialogical controversy. Secondly, the art of controversy or eristic, since the diagrams do not analyse the truth of judgements and the validity of inferences, but the persuasiveness of arguments in a dialogue.

**Keywords** Eristic · Argument maps · Logic diagrams · Dialectic · Argumentation theory · Critical thinking

## 1 Introduction

The philosopher Arthur Schopenhauer was long regarded as an irrationalist and to some extent even as an enemy of logic and rational argumentation. Apart from a few short passages on logic, the best-known text in which Schopenhauer deals with argumentation is an unfinished book fragment on eristic dialectics. This text was written around the year 1830 and translated into English in 1896. At first glance, the popular book fragment underlines the prejudice that Schopenhauer was an opponent of logic and rational argumentation. For it presents 38 stratagems that are logically incorrect but have the potential to be argumentatively convincing. Those who use these stratagems, therefore, do not want to win an argument by rational means, but they deliberately use false arguments only to get their way. For this reason, the eristic dialectics was misunderstood for many years as a sarcastic-prescriptive ‘art of

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(always) being right' and instrumentalised by some politicians, managers and entrepreneurs. The very well-known book fragment and the only short and insignificant passages and chapters on logic have shaped the image of Schopenhauer as a cynical and misanthropic irrationalist (Rocha et al. 2021; Gutenberg et al. 2020; Stoneman 2019).

In recent years, however, this image of Schopenhauer has been greatly revised. Researchers from various fields have, on the one hand, modernised Schopenhauer's book fragment on eristic dialectics in a positive way and successfully used it in fields such as jurisprudence (Stelmach and Brozekh 2006), artificial intelligence (Fouqueré and Quatrini 2012), pedagogy (Hordecki 2021), argumentation theory (Nickerson 2020) and many others. On the other hand, researchers have pointed to hitherto unknown lecture manuscripts by Schopenhauer that shed new light on his oeuvre: In the 1820, Schopenhauer wrote long treatises on logic, sophistry and eristic in his so-called *Berlin Lectures* (Schopenhauer 1911–1942, vol. IX). These lecture manuscripts were written for an academic audience and were underpinned by innovative diagrammatic techniques (Dobrzański and Lemanski 2020). In the scope of an entire book, Schopenhauer first developed in the *Berlin Lectures* a logic in the vein of the diagrammatic approach of Leonhard Euler (Moktefi 2020). This treatise on logic is followed by manuscripts on sophistry and finally by an eristic dialectics, which also employ these logic diagrams.

Whereas diagrams are applied in logic to represent the relationship of concepts, to evaluate the truth of judgements and to prove the validity of inferences, diagrams in eristic dialectics show how arguments are used to represent a dialogue without claiming formal truth or validity. Schopenhauer is not concerned with recommending how to be right in a dialogue at all costs, but with presenting such dialogue situations in order to easily recognise and protect oneself from unjustified arguments (Gutenberg et al. 2020; Chichi 2002; Hordecki 2021; Pedroso 2020). With the help of the diagrams introduced in logic, Schopenhauer wants to develop a unified technique in order to visualise and evaluate justified as well as unjustified arguments. Thus, in Schopenhauer's *Berlin Lectures*, one finds not only a very early form of argument maps but also a unified diagrammatic theory of logic, sophistry and eristic dialectics.

In this paper, I would like to present the lecture manuscripts on eristic dialectics, which are still unknown. I believe that the approaches presented here are relevant for at least two research groups: First, for those who do research on eristic and second, for research on argument mapping. Since many modern approaches in eristic go back to Schopenhauer, as mentioned above, this research could benefit from gaining a more complex picture of Schopenhauer's ideas by considering fragments that are yet unknown. The field of argument mapping is perhaps of interest to the present paper in that Schopenhauer designs a unique kind of argument maps by applying logic diagrams. This results in a technique that has similarities, but also differences from the well-known argument maps by Whately, Wigmore, Toulmin or Dung (Reed et al. 2007).

In Sect. 2, I will first briefly present Schopenhauer's hitherto known texts on logic and eristic. In Sect. 3, I will discuss Schopenhauer's best-known fragment on eristic dialectics and then, in Sect. 4, I will present the lecture manuscript and the diagrams

it contains. Section 5 makes up the main part of the paper. Here I will present the hitherto unknown approaches to eristic dialectics from these lecture manuscripts. It will become apparent that Schopenhauer has undertaken a diagrammatic interpretation of eristic dialectics and argumentation that has hardly been considered so far, resulting in a new type of argument map by using logic diagrams. To what extent the term ‘argument map’ is justified and to what extent Schopenhauer can be historically and systematically attributed to this current will be discussed in Sect. 6.

## 2 Schopenhauer’s Publications on Logic and Eristic Dialectics

During his lifetime, Schopenhauer published only a few short texts on logic and eristic. These shorter published texts are

Book	Chapter	First published	Abbreviation
<i>Fourfold root of the principle of sufficient reason</i>	5	1813	<i>FFR</i>
<i>The World as will and representation, vol. I</i>	9	1818	<i>WWR I</i>
<i>The World as will and representation, vol. II</i>	9, 10	1844	<i>WWR II</i>
<i>Parerga et paralipomena, vol. II</i>	2	1851	<i>PP II</i>

From these texts, only *WWR I* is really significant for eristic. Among Schopenhauer’s many manuscripts, however, there are two other texts that are important here. These two texts are:

Manuscript	Chapter	Written	Abbreviation	References
<i>Berlin lectures</i>	3	1820s	<i>BL</i>	(Schopenhauer 1911–1942, IX, 234–366)
<i>Eristic dialectics</i>	All	1831	<i>ED</i>	(Schopenhauer 1911–1942, VI, 393–428)

In *BL*, the chapter on eristic and logic alone is an extensive manuscript of about 150 pages. *BL* only received any attention a few years ago and was ignored for a long time even by Schopenhauer scholars, although it was first published in 1913. An English translation of *BL* does not yet exist. *ED* was first published in German as early as 1864, translated into English in 1896 and has become famous above all under the misleading title *The Art of (Always) Being Right*.

In later years, Schopenhauer did publish excerpts from these manuscripts in the smaller writings mentioned above. However, a complete understanding of Schopenhauer’s eristic and logic can only be obtained by reading the manuscripts.

Schopenhauer never published them in their entirety, as his published works were primarily intended to appeal to a broad audience. In particular, the texts on logic, which form the basis for eristic, were intended for lecture only to an academic audience (Dobrzański and Lemanski 2020).

Because a reception of the important *BL* did not begin until the mid-2010s, Schopenhauer was often (mis)understood as a simplifier, an irrationalist or even an anti-logician (Young 1988). If one looks at all the texts on logic and eristic, however, it is noticeable that Schopenhauer was intensively involved with these topics until the beginning of the 1830s. The end of this period can be explained by the fact that Schopenhauer gave up his university career in these years and that he therefore no longer saw any need to teach logic. However, since in later years he presented results from the manuscripts of the 1820s and early -30s in a sketchy manner in *WWR II* and *PP II*, the reader who is especially unfamiliar with *BL* gets the impression that Schopenhauer often contradicts himself or has little idea of logic and eristic. I will present some examples in a moment, and now compile the important information from the above mentioned texts. For an even more detailed presentation of Schopenhauerian logic, see (Lemanski 2021).

The first better-known text on logic comes from the dissertation *FFR* from 1813 and comprises two pages dealing with logical truth (§32). In later editions (1847), Schopenhauer marginally expanded this chapter and some further ideas (esp. chap. 5, §§29-34) that had previously been recorded in other writings. More important, however, is §9 of the main work *WWR I*, published in 1819, which was also accepted as a habilitation by the philosophical faculty of the University of Berlin in 1820. In this chapter, Schopenhauer first deals with language and divides it into two particular disciplines: logic and dialectics (or the ‘art of persuasion’). These two parts are based on Kant’s division of the *Critique of Pure Reason*, as logic represents analytic and persuasion represents dialectic (Demey and Lemanski 2021). Analytics or logic deals with the correct formal use of concepts, judgements and inferences, whereas dialectics deals with the illegitimate use of arguments.

In both parts of *WWR I*, §9, Schopenhauer already uses an idiosyncratic diagrammatic method which is to be the cornerstone of the whole theory of rationality. But without further context, this reductionist approach is highly cryptic and hardly comprehensible to the reader. This is already evidenced by the scope and variety of topics: in the English translation, language, semantics, the logic of concepts, judgments, inferences, logic diagrams and the entire dialectic are dealt with on 13 pages (Norman et al. 2010, I, 62–75). However, as Schopenhauer emphasises several times, these topics are only relevant to academic philosophy anyway. Moreover, many topics have already been dealt with in other textbooks, which the reader can consult if necessary. Schopenhauer does not add much information to *WWR I* §9 in the later editions (1844, 1859). Instead, he adds a second volume as a supplement, i.e. *WWR II*. In this volume, Chapters 9 and 10 deal with the logic of judgement and the logic of inference in more detail and also use further diagrams. But Schopenhauer explains here that he no longer wishes to apply this diagrammatic method to all areas of logic. Thus the second volume, conceived as a supplement, creates a contradiction with §9 of *WWR I*. Eristic dialectics is hardly mentioned in the *WWR II*.

In Chapter 2 of the second volume of *PP II*, published in 1851, there are still a few pages on formal logic and eristic dialectics. As in *WWR II* just mentioned, the 1851 passages are mostly of older origin. Schopenhauer has therefore often used old unpublished manuscripts, esp. *BL*, and presented individual passages as new additions. Nevertheless, new information can also be found: For example in *PP II* §26, Schopenhauer reports that he had been working for a longer time on a book fragment on eristic dialectics, i.e. *ED*, but that the subject would always point out to him the wickedness of human beings and that he no longer wanted to treat the subject as it frustrates him too much (Schopenhauer 1911–1942, V, 32). Already here, interpreters should have been made aware that Schopenhauer did not have in mind to recommend to his readers an application of false arguments. After all, Schopenhauer seems to have disliked precisely the irrational or malicious use of an eristic dialectics.

### 3 The Famous Fragment on Eristic Dialectics

Schopenhauer's famous fragment, i.e. *ED*, has often been seen by editors and interpreters as a guidebook on how to win a debate at any cost. These interpreters and editors have therefore given it titles with prescriptive connotations such as *Art of (Always) Being Right*. The text in question is an unfinished book fragment, described by Schopenhauer as a 'first draft' (Schopenhauer 1911–1942, VI, 405).<sup>1</sup>

The imperfection of the fragment also stands out in the text itself. In contrast to Schopenhauer's other manuscripts, which have been often revised by the author, this text changes perspective several times: sometimes Schopenhauer assigns a position to a fictional person, shortly afterwards he puts his literary self in the same position. In addition, there are several repetitions in the document, some of which Schopenhauer would probably have deleted in thorough final editing. Overall, the style suggests that some parts of the text originally belonged to the *BL* that will be discussed below in Sect. 4 (cf. also Schopenhauer 1911–1942, VI, 770f.). And the fact that Schopenhauer later began to write the text anew indicate that he was not satisfied with his first draft.

The prominent title 'Art of (Always) Being Right' is not completely wrong, as it has a reference to the content. As in §9 of *WWR I*, Schopenhauer speaks several times of an 'art of controversy', 'art of persuasion' or 'art of being right' (e.g. Schopenhauer 1911–1942, I, 58; VI 403; IX 363). However, these expressions are not of a recommendatory nature that one might expect. Rather, Schopenhauer uses these expressions neutrally and very likely has in mind the translation of the Greek term 'eristiké téchne' (Dietz 1994). Many editors have taken up the expression 'art of persuasion' or 'art of being right' as a lurid title in order to better market the book. Many interpreters were later seduced by the title and believed that they would

<sup>1</sup> Throughout the paper I follow the Deussen/Mockrauer edition (Schopenhauer 1911–1942). Unless otherwise noted, all translations are my own. With the help of the work concordance (Schubbe and Kossler 2018, 449–470), the reader can also quickly find the parallel passages to most English editions.

find a new guidebook of right-keeping in Schopenhauer. Better editors such as Julius Frauenstädt, Eduard Grisebach, Franz Mockrauer or Arthur Hübscher, however, were clear that the fragment should better bear the title ‘eristic’ (Eristik) or ‘eristic dialectics’ (Eristische Dialektik). Schopenhauer chose the short title ‘Dialectic’ (Dialektik) as the heading of each manuscript page and labelled the envelope of these manuscripts with the title ‘Eristic’ (Eristik) (Schopenhauer 1911–1942, VI, XXXIIIff., 771).

At the beginning of the introduction Schopenhauer states that logic and dialectic have almost always been understood synonymously in history (except by Kant and to some extent by Aristotle). However, he reserves the term ‘dialectic’ for the discipline that does not denote solitary thinking but a dialogue between people of different opinions. (Rhetoric, on the other hand, is for Schopenhauer the communication between a speaker and a larger audience). Dialectics describes how interlocutors are not guided by the facts of the case but by their intentions, their opinions and their wills, and thus get into a dispute. While logic might thus be a science a priori, dialectic is always a posteriori and determined by an empirical situation or even by the empirical character of the arguer (Schopenhauer 1911–1942, VI, 393f.).

However, since for Schopenhauer the term ‘dialectics’ was too often equated with logic in the history of philosophy, he sympathises with the term ‘eristic’ in order to name the projected discipline. But since this expression is a too ‘hard word’, he finally chooses the compound ‘eristic dialectics’ (Schopenhauer 1911–1942, VI, 395), which he uses synonymously with ‘art of controversy’ etc. in several places. In *ED*, Schopenhauer wants to omit logic altogether and concentrate solely on eristic dialectics. Unlike Aristotle, but closer to Kant, he favours that there are only two disciplines of rationality and language: Logic and eristic dialectics. Eristic dialectics is supposed to encompass the other disciplines that were still independent with Aristotle, i.e. also sophistry and peirastics (Schopenhauer 1911–1942, VI, 398). Methodologically, Schopenhauer explains that he first tried to continue the eristics of Aristotle and Cicero, but that he was dissatisfied with this approach (Schopenhauer 1911–1942, VI, 403). Besides the aforementioned empirical observation, he had only one means at his disposal: to put himself in the position of the person who argues not logically but eristically.

To set up eristic dialectics, it is said, one only has to imagine an ‘art of being right’ (Schopenhauer 1911–1942, VI, 403). But this does not mean that Schopenhauer has intended to write a manual of dishonest argumentation as a normative theory. Schopenhauer’s eristic dialectics is not an instruction (Gutenberg et al. 2020). On the contrary, Schopenhauer emphasises several times that people who use eristic arguments transgress ethical principles of argumentation and argue dishonestly. For this reason, too, there are nowadays initial approaches to linking eristics with discourse ethics and examining which rules of rational argumentation are violated in which eristic techniques, called ‘artifices’, ‘stratagems’ or ‘stratagemata’ (Lemanski 2022).

After all, one of Schopenhauer’s basic insights in many of his works is that reason is neutral, it can be used for good or evil, and thus there is an instrumental rationality that leads people away from logical and towards eristic argumentation (Young 1988). For this reason, eristic dialectics as a discipline is a means of prevention for

all those who want or need to defend themselves against fellow human beings who argue unethically: ‘one must know the dishonest artifices in order to counter them’ (Schopenhauer 1911–1942, VI, 403, similar 405). In §9 of the *WWR I*, Schopenhauer therefore also calls a discipline that offers a means of prevention against eristic arguments a ‘scientific dialectic’ (Schopenhauer 1911–1942, I, 56).

For the above reasons, the text ends with a theoretical framework of how argumentation works and with a compilation and description of 38 stratagems of eristic dialectics. This is the second part of *ED*, which will not be discussed in detail here because of its many different artifices (*Kunstgriffe*) and stratagems. These stratagems are linguistic tricks and strategies used by speakers to get an argument accepted, i.e. to win a discussion. Schopenhauer compiled these stratagems from literature or observed them several times in discussions. In part, he imagined a dishonest arguer as a discussion partner in order to identify possible stratagems. (For further details on the second part see (Chichi 2002).)

#### 4 Schopenhauer’s Unknown *Berlin Lectures*

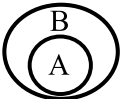



*ED* as described in Section 3 represents an independent and unfinished book fragment written around 1830. At about the same time, however, Schopenhauer also further developed another text on this topic, which he announced in §9 of *WWR I*. This main work, which he submitted as a habilitation to the Berlin University in 1820, was further expanded between the year 1820 and 1832 and concretised in many places for an academic audience and for students. Most importantly, Schopenhauer expanded and improved the theory of rationality and the philosophy of law. In this respect, Schopenhauer’s most detailed approaches to logic, eristic and sophistry can also be found in these lecture manuscripts, nowadays entitled *Berlin Lectures (BL)*. Since the book fragment, *ED*, has elements of the lecture’s content, style and, moreover, consists of similar or even the same material, editors have plausibly argued that the three texts on eristic dialectics are closely related (Schopenhauer 1911–1942, VI, XXXIIIff.): (1) *WWR I* §9 of 1819 announces the project, (2) *BL* of the 1820s continue these ideas, and (3) *ED*, the well-known *ED* extracts and treats a specific part of *BL*.

The manuscripts of *BL* were first published in German in 1913 (reprinted in 1994) and were only rudimentarily known in the Vienna Circle, the Lemberg-Warsaw School, School of Münster in the 1920s and the Mathematical Logic Working Group at the Ruhr University in Bochum in the 1980s (Lemanski and Dobrzański 2020, fn. 2). It is striking, however, that although there were several evaluative statements on the manuscripts, intensive and continuous research on *BL* only slowly began from the mid-2010s onwards. In this context, a new German-language edition of the manuscripts was completed in 2022 (Schopenhauer 2022) and a first English translation is only rudimentary being planned.

In the following, we first look at the content and then at the logic diagrams. First of all, it is striking that Schopenhauer separates logic, eristic and sophistry in the *BL*. Thus, he distinguishes between sophistry and eristic, which is precisely what he tries to combine in *ED*. In fact, even at a cursory glance, one can see that logic is the

largest part of *BL* (Schopenhauer 1911–1942, IX, 234–343) and many of the sophisms and paralogisms Schopenhauer compiles in 13 pages in the *BL* (Schopenhauer 1911–1942, IX, 343–356) are similar to the stratagems in *ED*. However, a precise study of the similarity of the sophisms and stratagems is not yet available. Since *ED* has only two parts, namely a long introduction and a reworking of these sophisms as stratagems, the part that is marked as eristic dialectics in the *BL* (Schopenhauer 1911–1942, IX, 269–284) is missing in the whole book fragment of *ED*. As we will see in the following, eristic dialectics of *BL* also continues the diagrammatic approach of *WWR I*, §9. We can therefore assume that although *ED* successfully integrated sophistry into eristic (as was Schopenhauer’s aim), it did not yet integrate the essential core of eristic at all.

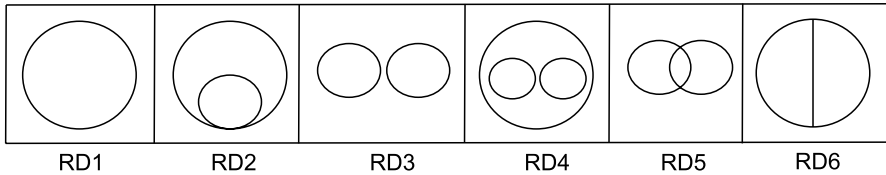
Since this essential core of eristic dialectics in the *BL* uses (logic) diagrams to create an argument map, it makes sense to first introduce the essential principles of logic diagrams here. Schopenhauer first adopts the logic diagrams that were known in his time and which were applied to the four categorical judgements of the syllogistic, as the following table shows.<sup>2</sup>

Abbrev.	Judgment type	Meaning	Logic diagram
A	universal affirmative	All <i>A</i> is <i>B</i> .	
E	universal negative	No <i>A</i> is <i>B</i> .	
I	particulare affirmative	Some <i>A</i> is <i>B</i> .	
O	particulare negative	Some <i>A</i> is not <i>B</i> .	

In these logic diagrams, the concepts are represented by circles or so-called ‘spheres’ and the relations of two or more circles represent judgements. For this reason, the logic diagrams can also be called ‘relational diagrams’ (RD). Schopenhauer also realises that these logic diagrams do indeed represent four judgements of the syllogistic (Aristotelian logic), which can be visualised by three geometric shapes (because the circles at I- and O-judgments have the same position in space). In order to be able to represent further logical relations that do not belong to the syllogistic, he seeks out all geometric shapes that two circles (or semicircles) can have to each other or to a third circle. These can be seen in Fig. 1. With the help of RD1–6, Schopenhauer can also represent the identity of two circles or concepts (RD1), the main

<sup>2</sup> These diagrams are mainly inspired by those of Leonhard Euler and Johann Heinrich Lambert. For details on Schopenhauer’s logic diagrams see (Moktefi 2020).





**Fig. 1** Schopenhauer's Relational Diagrams (RD) taken from (Schopenhauer 1911–1942, IX, 269–284)

connectives of propositional logic as well as partitions (RD4, RD6). For eristic, however, only RD2, RD3 and RD5 are relevant.

One of Schopenhauer's cardinal insights (Schopenhauer 1911–1942, IX, 277) in the use of these diagrams in the logic of judgement and inference is that the diagrams often depict more information than they are actually supposed to express (Lemanski and Schüler 2020). This insight also plays a major role today in the fields of visual reasoning and logic diagrams (cf. e.g. Moktefi 2020). For example, one diagram or geometric shape can display more than one piece of information: RD5 can e.g. be used to show both I- and O-judgements in syllogistic. And, as we will see soon, RD5 can be used to depict judgements in logic as well as in eristic.

Despite this ambiguity of diagrams, some of them are mutually exclusive: The fact that in a diagram one circle cannot completely include and exclude another circle at the same time leads to being able to represent logical oppositions by using circle diagrams: RD2 and RD3 are oppositional to each other because of the topological relations of their circles. In other words, RD2 and RD3 cannot represent the same circles in one diagram at the same time (Schopenhauer 1911–1942, IX, 284–293; Bernhard 2008). Thus, although a diagram can represent several judgements, there are other judgments or assertions that it cannot represent at all. We have thus grasped some of the basic components of Schopenhauer's theory of diagrams, which he also uses in eristic dialectics.

## 5 The Eristic Dialectics of the *Berlin Lectures*

Schopenhauer only ever published one diagram on eristic, in §9 of *WWR I*. As will be shown in Sect. 6, the very brief textual descriptions on logic and eristic in *WWR I*, were still largely comprehensible to his contemporaries. But since other types of diagrams dominated in the 20th century, the diagrams in Schopenhauer's eristic were never treated seriously – perhaps because they remain incomprehensible for non-contemporaries without a more detailed explanation. However, these more detailed explanations and many more diagrams on eristic can be found in *BL* but they have never been presented in research until now.

Having covered logic (including sophistry) to the extent of an entire book in *BL*, Schopenhauer added a chapter there entitled 'On The Art of Persuasion' (Von der Ueberredungskunst) (Schopenhauer 1911–1942, IX, 363–366). This quite obviously continues the eristic dialectics from §9 of *WWR I* and offers several new examples by which he explains his ideas in much more detail.

At the beginning of this chapter, Schopenhauer recalls many theses already presented here in Sect. 3: Logic examines true judgements and valid inferences in monological form, the art of persuasion examines the dialogical form of argumentation guided by intention and will. Right at the beginning, there is a sentence that reminds the diagrammatic method of §9 of *WWR I* on the one hand, and on the other, ties in with the stratagems of *ED*:

As manifold as the arts of persuasion are, they can nevertheless essentially be traced back to the following artifice, and are all ultimately based on the same one. (Schopenhauer 1911–1942, IX, 364)

The extent to which there is a manifoldness of the arts of persuasion is not discussed further in the lectures. However, one can assume that Schopenhauer had sophisms or stratagems in mind, which were treated in the *BL* as well in *ED* (see Sect. 3). Whereas in *ED* it seemed as if the eristic dialectics was a collection of various stratagems (38 in number), here we find a reductionist approach: actually, all artifices, sophisms or stratagems are subject to a single method. This thesis is also found in §9 of *WWR I* (Schopenhauer 1911–1942, I 58) and thus two of the three texts in which Schopenhauer presented the eristic dialectics have a reductionist approach as their core element. One can assume that this reductionist approach was the central thesis of eristic dialectics, which Schopenhauer would have had to integrate in the book fragment of *ED* last if he had completed it. Of course, this also has consequences for the interpretation, because all interpreters who only refer to the popular book fragment of *ED* overlook the core element of the entire eristic dialectic that is still missing there.

So what is the core reductionist thesis? Schopenhauer thinks that stratagems or artifices are based on the fact that they all invert logically correct conceptual relations expressed in judgements. Put simply: one asserts in eristic dialectics something other than what is logically true. To this end, Schopenhauer first recalls the diagrams from logic (RD1–6) and then explains that eristic argumentation always represents oppositional relations to actually valid diagrams. This is explained in the following quotation:

We have seen how the conceptual spheres have multiple communions and intertwine in the most diverse ways because in one concept the content of another concept is thought of completely or only in part. If one does not examine the resulting relations of the concepts thoroughly, but only casually, and sees that two concepts have community but does not examine them, or rather deliberately hides the fact that this community is only partial, then one can represent the sphere of a concept *A*, which lies partly in another *B*, but partly also in *C* that is quite different from it, according to one's subjective intention as lying entirely in the sphere *B*, or in the *C*, as one just thinks is good. (Schopenhauer 1911–1942, IX 364)

To make the quotation clearer, I do not substitute concepts for the variables given, but instead visualise the described relations from the quotation. What Schopenhauer calls 'thorough examination' (*gründliche Betrachtung*) in the above given quote and in other parts of the chapter, I will call *logical perspective* in the following and what he calls 'casual examination' (*oberflächliche*

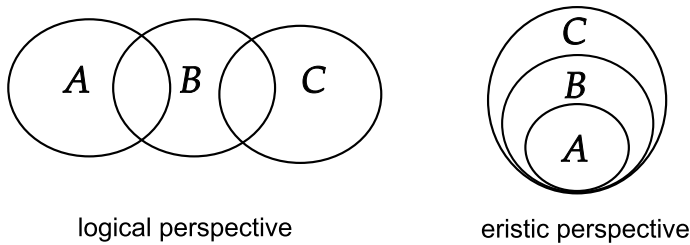


Fig. 2 Schopenhauer's diagram interchange between logic and eristic

Betrachtung) will be named *eristic perspective*. From a logical perspective, the relation between the three terms *A, B, C* is only partial, as in RD5. From an eristic perspective, however, an artifice or stratagem arises in that the relation between the three terms *A, B, C* is universal as in RD2. If one presents the eristic perspective, although the logical one is correct, we can speak of a *diagram interchange*, which is supposed to be the basis of all stratagems. The reductionist approach thus states that all eristic controversies can be understood as mis-handling diagrammable logical relations, e.g. that RD2 is used instead of RD5. This can be visualised as given in Fig. 2 (and this would also work if the same circle would be erased at both perspectives, e.g. *C*).

The whole of eristic, according to Schopenhauer, is based on the fact that one confuses the logical perspective with the eristic perspective, resulting in false RDs. Schopenhauer tries to give several examples of these two perspectives, the most central come from Aristotle and Goethe (Schopenhauer 1911–1942, IX, 365). Here, Schopenhauer interprets Mephistopheles' argument that happiness is a vice. Goethe had thus brought a dialogue response to Aristotle's argument that happiness is a virtue. However, since this fictional dialogue is very complex due to the interpretation of Aristotle and Goethe, I will omit it here and focus on the argument maps, which clarify the logical perspective on numerous eristic ways of argumentation.

Schopenhauer finally expands this theory of eristic into whole *argument maps* in which numerous RDs are connected with each other in the logical perspective. A Schopenhauerian argument map shows a central term in the middle that is up for discussion. One opponent *p* argues that this concept represents something good, another *q* that this concept represents something bad. In the map, Schopenhauer visualises several argumentation paths that *p* or *q* can take in order to claim either that the central concept represents something positive or something negative. The argument map uses RDs to show the logical perspective that a neutral observer can have on many different argumentation paths.

Let me now illustrate this technique with Schopenhauer's examples. The argument map in Fig. 3 shows an example from *BL*, the argument map in Fig. 4 gives an example from §9 of *WWR I*. Figure 3 is written in German, 4 in Latin. Figure 3 shows the central concept 'country life' (Landleben) in the middle and 'good' (Gut) and 'evil' or 'bad' (Übel) in the large semicircles on the right and left. Figure 4 shows another example: Here, too, 'good' (Bonum) and 'evil' (Malum) are in large semicircles at the periphery, but 'travelling' (Reisen) as a concept is in the centre.

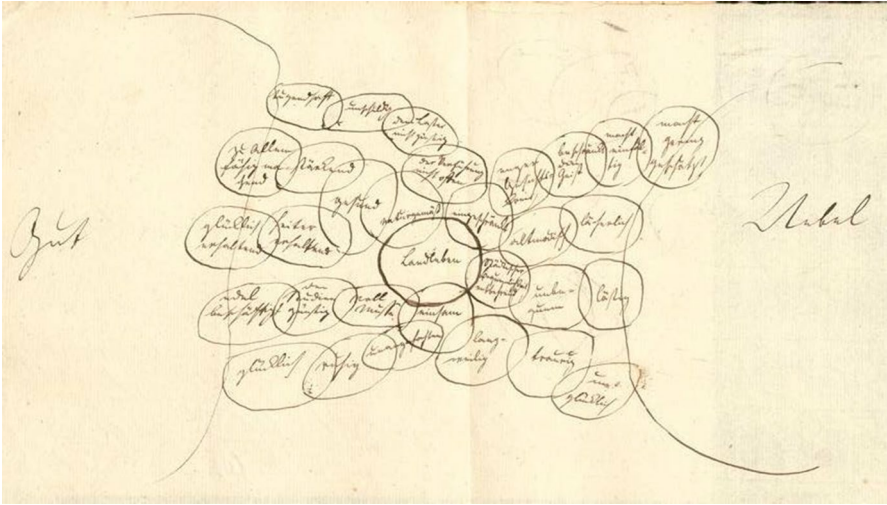


Fig. 3 Schopenhauer’s argument map taken from *Berlin Lectures*, StB PK, Na 50, NL Schopenhauer, 1428, Bl. 170 (urn:nbn:de:hebis:30:2-417557)

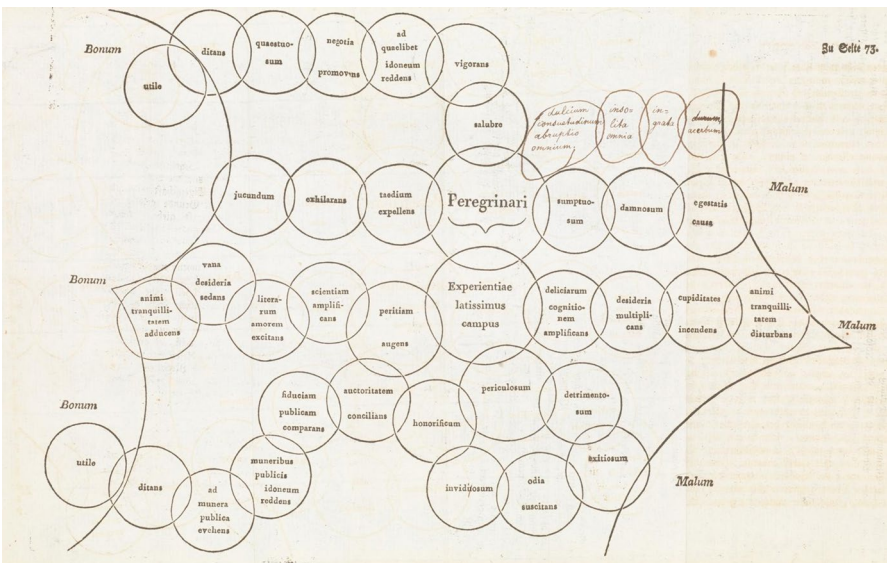
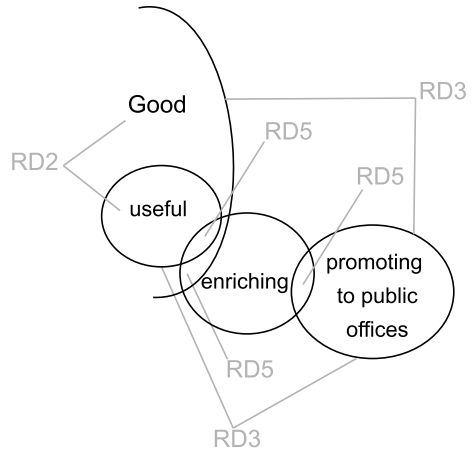


Fig. 4 Schopenhauer’s argument map taken from Schopenhauer’s hand copy of *The World as Will and Representation*, §9, Fondation Martin Bodmer, S. 73 (urn:nbn:de:hebis:30:2-259336)

Schopenhauer dealt with these diagrams mainly before 1820 and in the 1820s. After that, there is no more discussion of the diagrams for a long time. In the last month of his life, however, Schopenhauer made further additions in his hand copy of *WWR I*: For example, in Fig. 4, we see that Schopenhauer has added four circles

**Fig. 5** RD2, RD3, and RD5 in the lower left area of Fig. 4: *bonum = good; utile = useful; ditans = enriching; ad munera publica evehens = promoting to public offices*



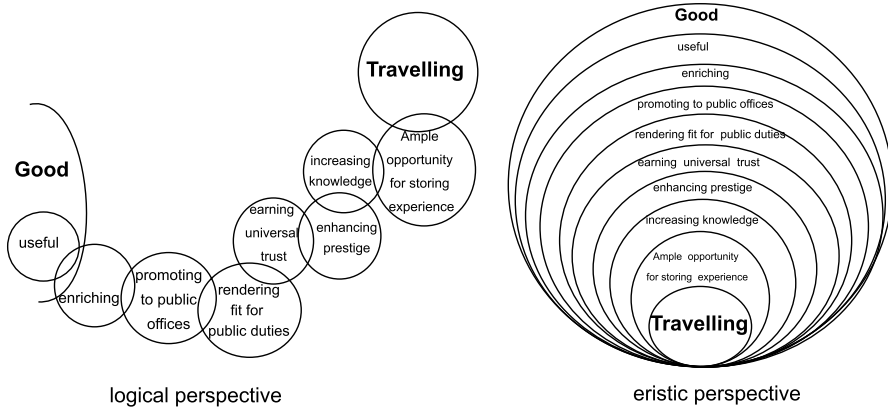
to the upper right edge of the diagram, leading from ‘Peregrinari’ to ‘Malum’. Although Schopenhauer was still interested in argument maps in later years, the chapter from *BL* including Fig. 3 is the most comprehensive on this technique. However, as will be proven later, only Fig. 4 has been received at all.

In both maps, we find three types of RDs: RD5 is obvious; RD3 almost always exists when three terms display two RD5 relations; and RD2 we find exactly twice, namely in Fig. 4 on the top left and bottom left. I will now take the lower left part of Fig. 4 as an example and I redraw this segment as Fig. 5, in which the RDs are given in grey. Here we see four conceptual circles or spheres that show RD5 three times, RD3 twice, and RD2 once.

However, there is a difficulty with both argument maps that Schopenhauer may not have intended: RD3 is almost always interpreted in logic like a diagram for E-judgements, i.e. ‘no *S* is *P*’. Schopenhauer also shares this interpretation in logic (Schopenhauer 1911–1942, IX 274). In this sense, the diagram would say: ‘no promotion to public offices is useful’ and ‘no promotion to public offices is good’. There are conceptual relations to which this RD3 interpretation applies (e.g. ‘nothing good is evil’), but in most cases it does not capture the relation of the depicted concepts. Although Schopenhauer does not explicitly point this out, we, therefore, propose to consider only the relations RD5 and RD2 in the argument maps. Since RD2 is only a special case anyway, appearing only twice in all Schopenhauerian argument maps, the RD5 relation seems to be the normal case (Moktefi 2020, 123).

RD5 means, in terms of intersecting diagrams for *I* and *O* judgments: ‘Some *S* are *P*’ and ‘some *S* are not *P*’. In the case of Fig. 5, it would mean ‘some promotion to public offices is enriching’ and ‘some promotion to public offices is not enriching’. This expresses a specific uncertainty: Something is possibly the case, but not necessarily. If one interprets the relations of the Euler diagrams in terms of alethic modalities, this would express a contingent case (Joerden 2012). It would be different with RD2 judgements, which express a facticity or even necessity: ‘Everything useful is good’ (because if it were not good, it would not be useful).

The argument map only represents the logical perspective, but does not map the eristic dialogue: As Schopenhauer described abstractly in the above quotation and



**Fig. 6** Diagram interchange for one route between ‘travelling’ and ‘good’

concretised in the Aristotle-Goethe dialogue, interlocutors like  $p$  and  $q$  in the argument present contingent paths of argument as factuality or even necessity for different reasons. In other words: In an eristic dialogue, at least one speaker argues with conviction that something is the case or even necessarily the case, although – logically speaking – this something need not be the case.

As Schopenhauer explains, the real argumentation process does not correspond to the logical perspective, but rather to the eristic one. This means that  $p$  or  $q$  argue, for example, that all RD5s (and RD3s) are actually RD2s. Let us therefore contrast once again, using a section of Fig. 4 as an example, the route of argumentation from the central concept of ‘travelling’ to the concept of ‘good’, for which e.g.  $p$  argues. Both perspectives of this route are given in Fig. 6. (In the eristic of the *BL*, Schopenhauer himself drew such diagrams as in Fig. 6, but not with the example of a path of Fig. 4 given here.)

Figure 6 can therefore depict, for example, the following conversation in which  $p$  argues that travelling is a good thing, whereas  $q$  is sceptical and would argue that travelling is a bad thing. The real-life argument could look like this:

- 
- $p$ : Travelling is a good thing!
- $q$ : I don’t think so.
- $p$ : But travelling always gives you the opportunity to store experience and thus increase your knowledge.
- $q$ : But why should travelling therefore be good?
- $p$ : Well, if you increase your knowledge, you also increase your reputation. And that is the only way to gain the trust of the public.
- $q$ : I remain sceptical! That says nothing about travel being a good thing.
- $p$ : Oh yes it does! After all, that’s how you become fit and get promoted to public duties! That in turn is immensely enriching and thus useful for oneself.
- $q$ : That may be so.
- $p$ : But if travelling is useful for me, then it is also something good! Or do you want to contradict that?
-

The dialogue shows that  $p$  tries to set up a route of argumentation that is reminiscent of the eristic perspective and RD2, while  $q$  suspects that these are not necessary arguments. If  $q$  has previously thought about the path of the argument with the help of an argument map such as Fig. 4, for example, she has a preventive means in hand to criticise the contingent proof steps. Thus, the use of the argument map and the awareness of the interchange of the logical with the eristic perspective is a good preventive means against dishonest arguments. This reductionist approach is also summarised in Schopenhauer's words in the *Berlin Lectures*:

All the arts of persuasion and the finer sophisms have the ground of their possibility in this peculiar nature of the spheres of concepts to intertwine and to intersect in manifold ways; thereby they give the will leeway to pass from each concept to this or that other, in which the same possibility occurs again, and so on and on. (Schopenhauer 1911–1942, IX 366)

## 6 Schopenhauer's Diagrams as Argument Maps

In Sect. 5, I introduced the term 'argument map' without defining it more precisely or justifying it, and then claimed that Figs. 3 and 4 represent argument maps. In this section, I will discuss whether it is justified to call Figs. 3 and 4 'argument maps'. Two strategies are conceivable to justify this thesis: (1) one strategy could be to give a universally accepted definition of argument maps, with the help of which one then examines whether Figs. 3 and 4 fall under it; (2) another strategy could be of a historical nature and argue that Schopenhauer's Figs. 3 and 4 influenced later argument maps.

However, both strategies also have certain prerequisites. I focus here on only two: (1) For the first strategy, we must assume that there are sufficient and necessary conditions to classify something as an argument map. (2) For the second strategy, we must assume that all subsequent argument maps are already known in research.

The preconditions for both strategies are not unproblematic and may even be mutually dependent. For both presuppositions I mention only one problem each: (1) Relevant systematic research works prove with examples that so far there is no agreement in research on what exactly an argument map is or that there are pluralistic approaches (Reed et al. 2007; Reed and Rowe 2007). (2) Relevant historical research shows that there are very different approaches to writing a 'history of argument maps' (Dwyer 2017, pp. 211–214; Shum 2003; van Gelder 2013). Both problems seem to be mutually dependent because each decision affects the presupposition: (1) Those researchers who construct a particular definition of argument maps have good reasons to recognise only a very particular history of them. (2)

Whoever constructs a certain history of argument maps has good reasons to accept only the definition that provides sufficient and necessary criteria for all diagrams in this history.

Let us take the relevant research, mentioned above, as an example in order to get a general overview of what argument maps may be and what their history is. Summarising the results of these studies in general, there are three different views on the definitions of argument maps and their history: (a) An argument map is a visualisation of any kind of argument. If this definition is accepted, one can go back, for example, to Socrates, Plato or Aristotle describing that arguments have been visualised in the sand. (b) An argument map is a graphical representation of logical relations. If this definition is accepted, one can use any text that displays a logic diagram. (c) An argument map uses a chain of graphical elements to show the structure of the argument. If one accepts this definition, one can construct a 200-year history of logic diagrams, going back at least to Richard Whately's *Appendix III: Praxis of Logical Analysis* published in the fourth revised edition of *Elements of Logic* in 1831. It is not uncommon to see Whately's approach as a precursor to Toulmin's argument maps, which are widely used today, which is why one can also speak of (Whately-)Toulmin style.<sup>3</sup>

I claim that (a) is the most general or broadest definition of argument maps, whereas (c) is the most concrete or narrowest definition. For each definition there is a very specific history where (a) corresponds to almost the entire history of Western thought, (b) corresponds to the history of logic diagrams going back at least to the Middle Ages, and (c) can be dated to the early 19th century. The most general definition thus has the longest history, the most concrete definition the shortest. Historically and systematically, one can thus establish a rule that can be applied to the question of whether Schopenhauer used argument maps: Whoever accepts that Schopenhauer belongs to (c) should also accept that he belongs to (b) and thus also to (a). But anyone who accepts that Schopenhauer belongs to (a) need not accept that he also belongs to (b) or (c) (and likewise if (b), then not necessarily (c)).

I now claim that Schopenhauer can be assigned to (b) without any problems since there are numerous studies on Schopenhauer's logic diagrams (mentioned in the previous sections). That Schopenhauer used logic diagrams is indisputable, and thus he seems to be assignable in a broader sense, namely (b) also to argument maps. But now the question remains open whether Schopenhauer can also be attributed to (c). After all, in the previous sections I have always spoken of 'logic diagrams' (b), but only in Figs. 3 and 4 of 'argument maps' (c). So our whole discussion in this section so far boils down to one question: Can Schopenhauer's Figs. 3 or 4 be attributed to the so-called Toulmin-style?

<sup>3</sup> In the following, I will speak only of 'Toulmin-style argument maps', as this is probably the best-known method of (c), which goes back to Whately. For a more differentiated way of speaking see (Reed and Rowe 2007, Sect. 2).



Let's start historically: no, Schopenhauer's argument maps in Fig. 3 and 4 should not be assigned to Toulmin-style if only because there is no evidence that Schopenhauer had any influence on any argument map assigned as a precursor to successor of Whately.<sup>4</sup> This does not mean, however, that Schopenhauer's Figs. 3 or 4 were not read or discussed at all. As early as 1824, the first discussion and reprinting of Fig. 4 can be found in Vol. II of Ignaz Denzinger's *Institutiones logicae*. The best-known authors who discuss Fig. 4 at the end of the 19th century are Alois Höfler and Alexis Meinong. In the 20th century, it was received both positively by Alf Nyman and critically by Richard von Mises. So Fig. 4 already had an influence, but not on the authors who are assigned to Toulmin-style argument maps.<sup>5</sup>

Now, however, one can systematically ask whether Figs. 3 or 4 can be assigned to the (c) Toulmin style. This discussion can certainly not be conclusively settled here, since it must remain questionable whether all authors really agree on what (c) is. However, I believe that I can state two criteria that can be found in many relevant texts and are stated there as sufficient *or* necessary conditions for (c): Toulmin-style argument maps show (c1) a graph that maps the (c2) structure of propositions in an argument. We can already state two objections (E) here and make a decision: (E1) Schopenhauer did not know what a graph is; (E2) Figs. 3 or 4 map concepts and not propositions. This seems to settle the case that Schopenhauer did not represent a Toulmin-style argument map.

But one can take a second perspective to argue against (E1) and (E2): (E1) can first be attacked historically, for even Whately, the originator of (a), could not know more about graphs than Schopenhauer. So those who make the case here for Schopenhauer's ignorance of graph theory must also do so against Whately. But now an ahistorical question remains open: Is it possible to interpret Figs. 3 or 4 as graphs by modern means? I believe yes if one represents 'travelling' in Fig. 6 as a source node and 'good' and 'evil' the two sink nodes, each concept between 'travelling' and 'good' or 'evil' then becomes a node in an undirected graph that maps numerous discussion possibilities. The actual mapped discussion, such as the one between  $p$  and  $q$  from Sect. 4, can then be represented as a directed graph from the source node ('travelling') to the sink node ('good').

(E2) Moreover, based on this graph-theoretical interpretation, one can also argue that Figs. 3 and 4 show propositions, but only graphically, not literally. If one draws the graph in Fig. 3 or 4 as just described, each node passes through the intersection of two circles of an  $RD3$ . This intersection represents the decisive relation between two verbally given concepts of the diagram, e.g. 'some  $A$  are  $B$ '. In this respect,

<sup>4</sup> So far, it has been held that Schopenhauer and Whately had little in common (Rigotti and Greco 2019, ch. 5.1). However, only excerpts from *ED* were noted.

<sup>5</sup> Almost all current authors who positively receive Schopenhauer and were mentioned in Sect. 1 focus only on *ED*. By not taking note of the other texts by Schopenhauer on eristic mentioned in Sect. 2 and 3, Schopenhauer's diagrams and argumentation maps are not taken into account. Not infrequently, through the incomplete reception of eristic, Schopenhauer's approach is misjudged, for example as irrational, obscure, ironic. Nevertheless, the stratagemata approach from *ED* is often productively taken further.

Fig. 4 has been described by many researchers as ‘routes’ or ‘paths’ (cf. e.g. Lemanski and Moktefi 2018; Moktefi 2020) and this in turn corresponds to the structure of argument maps described by Thomas and Freeman as ‘serial’ (Thomas 1986, p. 57f.; Freeman 2011, p. 4). Figures 3 or 4 can be interpreted not only as graphs, but also as graphs depicting propositions. Here one could even cautiously argue that Schopenhauer had something like an evaluated graph in mind since the edges express either RD2, RD3 or RD5 and this can be understood as an evaluation of the relation of concepts: RD5 then says, for example, that no or an impossible relation exists between two concepts (‘No *A* is *B*’ or ‘*A* cannot be *B*’), RD3 that a partial or possible relation exists (‘Some *A* are *B* and some *A* are not *B*’ or ‘*A* may be *B*’) and RD2 that a certain relation exists (‘All *A* are *B*’ or ‘*A* is necessarily *B*’).

However, I will stop here at the suggestion of this graph-theoretical interpretation of Figs. 3 and 4, since it should already have become clear that there are ways of interpreting Schopenhauer’s diagrams as a Toulmin-style argument map (at least if one accepts the stated presuppositions). However, I have not claimed that the approach just sketched is the only correct interpretation of Figs. 3 or 4. Since I have now presented some arguments that have placed Schopenhauer’s argument maps in the vicinity of Toulmin-style diagrams, it seems necessary to me, however, to once again explicitly state the differences that have so far only been hinted at. The main difference to Toulmin-style maps can be expressed in one sentence: Schopenhauer does not use graphs for his argument map, but logic diagrams (which can be interpreted as graphs).<sup>6</sup>

Whether the application of logic diagrams to argument maps is advantageous or disadvantageous will be dealt with in detail in Bhattacharjee and Lemanski (2022), together with a precise analysis of the graph structure of Figs. 3 and 4. As a result, it remains to be said that speaking of ‘argument maps’ in Schopenhauer’s eristic makes sense under certain conditions, and that the main difference between Figs. 3 or 4 and many other maps is the use of logic diagrams.

## 7 Summary and Outlook

We have seen in the previous sections that Schopenhauer wrote several texts on logic and eristic, most of which were written between 1813 and 1833. Whereas these texts do not always present a coherent picture, they should at least be taken into consideration and sometimes used as a supplement in order to see Schopenhauer’s statements on individual topics in their proper context. It became clear, for example, that Schopenhauer’s famous fragment on eristic dialectics, i.e. *ED*, was only the beginning of an unfinished book. In it, Schopenhauer integrated the sophisms from *BL* as

<sup>6</sup> Of course there are some other peculiarities of argument maps that are not explicitly found in Schopenhauer maps. I give only a few examples based on Reed and Rowe (2007): Figs. 3 or 4 do not have the datum-warrant-claim (DWC) structure found in Toulmin diagrams. Refutations or attacks, as found in Beardsely maps or Dung graphs, are not recognisable in Figs. 3 or 4. The role of prosecution and defence, as given in Wigmore maps, are also not clearly discernible. Nevertheless, such techniques with already known diagrammatic conventions could be added in the future if Schopenhauer maps were further developed.

so-called ‘stratagems’ or ‘stratagemata’, but the book ended before the core thesis of Schopenhauer’s eristic dialectic began.

This core theme is the attempt to reduce all artifices and sophisms in conversation to a diagrammatic approach. To understand this diagrammatic approach, however, Schopenhauer already presupposes an acquaintance with his logic and the diagrams it contains. The reductionist approach then consists in Schopenhauer’s realisation that all sophisms or stratagemata can be visualised and analysed by choosing the oppositional diagrams instead of those that one would apply from a logical perspective.<sup>7</sup> Instead of contingent or partly relations between concepts (RD5), for example, necessary or complete relations (RD2) are suggested in the eristic argument.

Schopenhauer made this explicit in the text and tried to illustrate it with several techniques. This resulted not only in diagrams, but also in large argument maps that contained many different contingent argumentation paths. This was intended to sketch a neutral and logical perspective on possible arguments, which, however, become decisive when interlocutors favour individual contingent arguments and present them as a true fact. Schopenhauer’s goal of scientific eristic dialectics, to protect against dishonest arguments, was thus to be achieved through the visual means of argument maps that have been produced through the use of logic diagrams.

Despite the interesting and complex technique, however, Schopenhauer’s diagrams and maps raise problems and questions that are just beginning to be addressed in research. One advantage of Schopenhauer’s argument maps seems to be that not only the paths of arguments but also the logical relations are represented with visual techniques. However, the fact that the argument maps contain more logical relations (e.g. RD3) than are actually to be represented already appears problematic. This means that the interpreter needs more precise reading instructions or rules of interpretation than Schopenhauer gave in the text. Establishing such rules and instructions will remain the task of future research.

Furthermore, many questions remain unanswered that could not be clarified here: Can all artifices and stratagems be mapped onto the reductionism of diagram interchange? Is there only the diagram interchange between RD2 and RD5 or can other argument maps be usefully created with other RDs? How did Schopenhauer interpret the argument between Aristotle and Goethe, which is only hinted at here, and is this interpretation meaningful and valid? How can the argument maps be accurately described or formalised? How can Schopenhauer’s argument maps be used today and how can they visualise current concrete paths of conversation? How can Schopenhauer’s argument maps accurately depict the attacks of a discussant? If stratagems are deliberately generated dishonest arguments, how can violations of argumentation and discourse ethics be depicted in them? Is there a tradition of structurally similar argument maps independent of Whately?

These are only some of the most important questions related to Schopenhauer’s argument maps and diagrams. Even though individual fragments on Schopenhauer’s argumentation theory are already popular and used, the paper presented here may have shown that Schopenhauer’s eristic dialectics (and logic) has much more to offer

<sup>7</sup> Concerning oppositional diagrams see the end of Sect. 4

than has been assumed so far. However, a detailed analysis, evaluation and application of Schopenhauer's eristic dialectics are still in their infancy due to the above-mentioned and as yet unresolved issues in research.

**Funding** Open Access funding enabled and organized by Projekt DEAL. The author is supported by the Fritz Thyssen Foundation (project: 'History of Logic Diagrams in Kantianism') and benefited from the ViCom-project 'Gestures and Diagrams in Visual-Spatial Communication' funded by the Deutsche Forschungsgemeinschaft (DFG, German Research Foundation) -- (RE 2929/3-1).

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