Historia Logicae and its modern Interpretation. Ed. by Ingolf Max, Jens Lemanski. London: College Publications, 2023. ISBN: 1848904088

Seneca's and Porphyry's Trees in Modern Interpretation

Jens Lemanski
University of Münster, University of Hagen, Germany
jens.lemanski@fernuni-hagen.de

1 Introduction

Traditional tree diagrams, as developed in Platonic and Aristotelian doctrine (see e.g. Fig. 1) are regarded as precursors of modern techniques of visualisation in philosophy, biology, mathematics, linguistics, computer science, music theory etc. [16], [44], [10, chap. 5]. These visualisations are used to constitute ontologies, taxonomies or, generally speaking, to perform conceptual analysis [17, p. xiii]. If we take only computer science as an example, these tree diagrams have been used in areas such as ontology engineering, semantic web, object-oriented programming, knowledge representation, artificial intelligence, etc. [8]. John Sowa, for example, wrote that the first semantic network in the form of a tree diagram can be found in the work of the Neoplatonist Porphyry (c. 234–305 AD) [41, p. 4]. This assessment was also reproduced in *AIMA*, today's standard textbook on artificial intelligence [3, p. 471]. Nowadays, we also know that perhaps the first design of a logic machine in the Baroque era was inspired by the form of tree diagrams [30, p. 10], [Sect. 5].

The following information often reappear in connection with tree diagrams: Logicians and metaphysicians in earlier times generally (1) used only one (porphyrian) syntax of tree diagram, (2) usually made

Parts of this paper were presented at a lecture on 14 January 2021 at Sun-Yat Sen University and on 8 October 2021 during the conference 'Concepts of Logic, Logic of Concepts' at the University of Leipzig. I would like to thank the discussion partners and especially Ingolf Max for many valuable suggestions. The paper benefited from the project 'History of Logic Diagrams in Kantianism' (Thyssen Stiftung).

no nominal distinction between different types of trees and (3) almost always illustrated one semantics, namely the concepts depending on the Aristotelian category of `substance` as the highest genus in the tree diagram. However, many of these prejudices have been revised in detailed studies: (1) Barnes [29, p. 108ff.] Mansfeld [25, p. 78ff.] and further also Verboon [46, pp. 44ff.] argue, for example, that one should distinguish between at least two syntactic forms of traditional tree diagrams. (2) Barnes and Mansfeld also argue that these two diagram types can be traced back to two classical philosophical texts, i.e. Seneca's *58th letter to Lucilius* and Porphyry's *Isagoge*. (3) Blum [5, p. 301] and Sowa [41] note that tree diagrams can be used to depict more categories than just the one of `substance`.

My original aim was to support all these three points from the aforementioned studies. I had the idea to focus on many unusual diagrams from the scholastic and early modern periods and to show that they depict and represent much more information than one would expect. In doing so, however, I had to realise that a modern interpretation of the classical texts and diagrams, which are connected to so-called Seneca's and Porphyry's trees, is so wide-ranging that one does not get to advance from antiquity to modern times in the scope of an ordinary paper.

The most serious problem I have seen is the way traditional tree diagrams have been treated in the literature: While there is a growing body of historical work on tree diagrams in the medieval and early modern periods, none offers a – from a logical standpoint – satisfactory way of describing them: The approaches of Sommers and Englebretsen are profitable for term logic, but perhaps only serve to a limited extent for the analysis of traditional tree diagrams [12]. There are many good logical approaches from the field of formal ontology, but they often only examine individual sub-questions of tree diagrams [15]. Hacking [16], who is one of the few to combine a historical and logical approach to tree diagrams, sees a continuous development from antiquity to modern graph theory, but he too only outlines some difficulties in applying modern graph theory to traditional tree diagrams. But that is exactly what one needs to look at more complex tree diagrams, which have been around since the early Middle Ages.

For this reason, I would like to argue in this paper for a modern interpretation of traditional tree diagrams, continuing mainly the approach of Hacking. In Section 2, I will first introduce the two classical texts by Seneca and Porphyry. Section 3 draws on the interpretations of Barnes and Mansfeld, who have the most convincing approach to explain how the texts of Seneca and Porphyry are transformed to tree diagrams from the early Middle Ages onwards. In Section 4, I will then discuss the semantics and syntax of traditional Seneca and Porphyry trees. Section 5 will then sketch some problems and examples of the given syntax and semantics. Finally, Section 6 will give a summary and an outlook.

2 Trees in Seneca and Porphyry

If the intention is to examine ancient texts that could be a trigger for the great flood of tree diagrams that have come down to us from the early Middle Ages at the latest, one must actually look into the sources before Seneca and Poryphyry. Plato and Aristotle are relevant authors to whom later philosophers such as Seneca and Porphyry explicitly referred. Certainly, however, traces can already be found in the Pre-Socratics. Nevertheless, the texts of Seneca and Porphyry are considered seminal for the visualisation of conceptual structures, which then became known as tree diagrams. In this respect, it makes sense to examine not the entire genealogy of tree diagrams, but the most important passages of the texts.

In this section, I would first like to discuss the relevant text passages by Seneca (2.1) and Porphyry (2.2). As will be shown in Section 3, both text passages are the foundations for later tree diagrams due to their metaphorical way of speaking, but do not contain any visualisations themselves.

2.1 Seneca's *Letter 58*

Seneca's 58th letter has two central themes, especially from the perspective of logic, i.e. Platonic concepts of οὐσία (essentia) and το ὄν (quod est) [48, p. 622ff.] and the relationship between *genus* and *species*. In Seneca's treatise on genus and species two different methods are involved: (1) perductio and (2) deductio.

(1) In perductio, singuli or single items are picked up backwards (*coeperimus singula retro legere*). By collecting and connecting more and more singuli higher and higher species and genera emerge bottom-up. The fact that we are picking up the singuli backwards indicates that there was already a forward movement that distributed these singuli. The method of perductio, which Seneca unfortunately does not explain in detail, is particularly reminiscent of *inductio by simple enumeration*, but also to some extent of the processes that today are called *backward chaining* [6, Chap. 13.2], [28].

(2) Once Seneca has arrived at the highest genus, i.e. the `being` or `quod est`, he deduces all subsequent subconcepts top-down with the help of the division [2, p. 223], [48, I, p. 98f.]. In doing so, Seneca uses three theoretical terms that are intended to structure a set of concepts that depend on the Platonic term `quod est`. In order to make the theoretical terms clear in the following, I insert the Latin expressions in italics in curly brackets and use them in the following:

> For by using this term [sc. `quod est`] they will be divided into species, so that we can say: that which exists either possesses, or lacks, substance. This, therefore, is what genus is, — the primary, original, and (to play upon the word) 'general' {*genus generale*}. Of course there are the other genera: but they are 'special' genera {*genera specialia*}: 'man' being, for example, a genus. For 'man' comprises species: by nations, — Greek, Roman, Parthian; by colours, — white, black, yellow. The term comprises individuals {*singuli*} also: Cato, Cicero, Lucretius. So 'man' falls into the category genus, in so far as it includes many kinds; but in so far as it is subordinate to another term, it falls into the category species. But the genus 'that which exists' [sc. `quod est`] is general {*genus generale*}, and has no term superior to it. It is the first term in the classification of things, and all things are included under it. [37, p. 393ff.]

The quoted passage, which deals with the division of the genus `being` (`quod est`) into several species and singuli, comes from the part on deductio, in which Seneca proceeds top-down. In this quote, Seneca introduces three theoretical terms: (1) The *genus generale* is the `quod est` and stands highest. (2) Below the genus generale are many *genera specialia*, which are both species for higher genera and genus for lower species. (3) At the bottom are the *singuli*, which are contained by only one particular species. Each genus is usually divided into two subspecies or -concepts (dichotomic), sometimes in three or more (polytomic) using the divisio method.

To make it easier to assign the concepts to the corresponding technical terms, I have created Table 1. The terms in Tab. 1 are only a selection of the concepts mentioned in Seneca's text and are mainly oriented towards Mansfeld's selection and interpretation, which plays an important role in Sect. 3. In the last column, '1' indicates that it is true that a concept is a genus or species for something else; '0' indicates the opposite, i.e. that it is false or not the case that a concept is a genus or species for something else. As we can see, combinatorics of '1' and '0' is not exhaustive.

Although Seneca does not describe or draw a tree in the treatise on genera and species, there are metaphors of subordination that lend meaning to the text only through their arrangement in a vertical scheme (suspensa; sub se habere; superiorem locum; superius; supra se habet; sub illos; etc.): In the method called perductio, the text describes the ascent from the singuli to the genus generale. In the method of divisio, the descent from the genus generale to the singuli is given. The vertical image of the bottom-up or top-down movement can evoke the picture of the trunk of a tree, the diviso from one to many (or the perductio from many to one) the respective branches.

What is astonishing about Seneca's text is that he is very imprecise in his choice of terms, as some are nouns (e.g. `animal`, `horse`), others adjectives (e.g. `corporeal`, `animate`). But he is very precise in his use of logical connectives: He often uses the logical connective *exclusive or* to relate to a dichotomic pair, suggesting an oppositional relationship between a positive concept and its negation. Seneca uses the phrase *either…or*, i.e. *aut…aut*, aptly in a total of five passages and also explicitly refers in this context to the law of excluded middle ('Nihil tertium est.'). However, in the passages where Seneca subsumes three or more concepts under one generic one, he uses the metaphor of subordination

technical term	concepts	is genus/species for sth.
genus generale	quod est	1/0
genera specialia	corporeal/ incorporeal, animate/ inanimate, animal/ plant, man/ horse/ dog, Greeks/ Romans/ Parthians	1/1
singuli	Cato, Cicero, Lucretius	0/0

Table 1: Seneca's Technical Terms

or containment (comprehensa sunt, complectatur, in se, continet, etc.). The above quotation proves these relations.

Thus one finds in Seneca a meaningful connection of four themes that are again being discussed intensively and in context in logic today: classical negation, contradictory, dichotomy/ polytomy, and the laws of thought [4], [36]. Seneca thus seems to have been much better aware of logical relations than most modern historians of logic give him credit for.

2.2 Porphyry's *Isagoge*

Porphyry's relevant text, which later became known under the title *Introduction* (εἰσαγωγή), is a letter to a student named Chrysaorius. Porphyry explains at the beginning of the letter that he would like to introduce five terms in order to present a concise exposition to the Aristotelian *Organon* in the manner of an introduction (ὥσπερ ἐν εἰσαγωγῆς τρότῳ). It is usually thought that Porphyry only wanted to give an introduction to the first book of the *Organon*, namely the treatise on the concepts or *Categories*. However, since he also lists topics concerning the doctrine of judgement (τῶν ὁρισμῶν, τά περὶ διαιρέσεως) and inference (καί ἀποδείξεως), his introduction is not limited to the categories: Porphyry also has in mind the other books that are traditionally also counted as part of the Aristotelian *Organon*. Moreover, Porphyry indicates that the introduction is a summary of the knowledge of the ancients (πρεσβυτέροις), by which Seneca, among others, may be meant.

To this end, he explains five central concepts that became canonical from the Middle Ages onwards as *quinque voces* or *predicabilia*, i.e. genus, species, difference, property, and accident. Porphyry explains that he does not intend a metaphysical treatise but a logical one (λογικώτερον), as he omits topics of metaphysics and focuses mainly on the logical relations of the predicabiliae. The text can be divided into five chapters, each corresponding to one of the praedicabilia. In the first chapter. i.e. on genera and species, he defines four theoretical terms that can be distinguished by their combinatorial relation to genus and species. In order to make the theoretical terms clear in the following, I insert the Latin expressions (which became common by the translation of Boethius) in italics in curly brackets and use them in the following:

> For of predicates, some are said of only one item—namely, individuals {*individua*} (for example, Socrates and 'this' and 'that'), and some of several items—namely, genera and species [...]. In each type of predication there are some most general items {*genus generalissimum*} and again other most special items {*species specialissima*}; and there are other items between {*inter alia*} the most general and the most special. Most general is that above which there will be no other superordinate genus; most special, that after which there will be no other subordinate species; and between the most general and the most special are other items which are at the same time both genera and species (but taken in relation now to one thing and now to another). [29, p. 4–6]

The four theoretical terms can be distinguished by the extent to which they are genus or species for something else. (1) The *genus generalissimum* is genus for all other concepts, but it is not itself a species in relation to a higher concept. (2) The *inter alia* are both genus for some concepts and species for other ones. (3) The *species specialissima* are species for other concepts, but not genus for any other one. (4) The *individua* are neither species nor genus for other concepts. Porphyry introduces the technical term *species specialissima* that did not exist in Seneca. This is already made evident by the fact that Porphyry exhausts the possible combinations of genus and species, which was still incomplete in Seneca. To make the combinatorics of the theoretical terms even clearer, their relationship can be tabulated. I follow the same method as in Sect. 2.1.

Table 2 shows not only the four technical terms and the extent to which they are genus or species, but also which concepts are assigned to the four technical terms. The assignment goes back to another passage in the text, which was decisive for later interpreters and commentators in constructing a Porphyrian tree in the first place. Like Seneca, Porphyry does not visualise a tree, but only evokes a vertical scheme of concepts with his figurative terminology. In Porphyry, too, one finds a strong use of metaphors of subordination (e.g. ὑπὸ τὸ γένος) and containment (e.g. περίεχειν). In the relevant passage, Porphyry explains the four technical terms and their relationship to genus and species on an Aristotelian category:

> What I mean should become clear in the case of a single type of predication. Substance is itself a genus. Under it is body, and under body animate body, under which is animal; under animal is rational animal, under which is man; and under man are Socrates and Plato and particular men. Of these items, substance is the most general and is only a genus, while man is the most special and is only a species. Body is a species of substance and a genus of animate body. Animate body is a species of body and a genus of animal. Again, animal is a species of animate body and a genus of rational animal. Rational animal is a species of animal and a genus of man. Man is a species of rational animal, but not a genus of particular men—only a species. [29, p. 6]

Barnes rightly claims from this passage that the concepts mentioned in Tab. 2 would evoke a subsuming line rather than the idea of a tree [29, p. 109]. It is only in the second chapter on difference (διαφορὰ, lat. differentia) that another method appears that suggests a dichotomous division of concepts [29, p. 177ff.]: `Substance` can e.g. be divided by this dichotomy into `corporeal` and `incorporeal` or `body` into `animate` or `inanimate` etc. This results in a strict division between nouns and adjectives, sometimes called the division between extensional and intentional terms, since extension means the set of objects contained under a

technical terms	**concepts**	**is genus/species for sth.**
genus generalissimum	substance	1/0
inter alia	body, animate body, animal, rational animal	1/1
species specialissima	man	0/1
individua	Socrates, Plato, etc.	0/0

Table 2: Porphyry's Technical Terms

noun and intension means the properties contained under an adjective [15, p. 540],[1, pp. 45ff].

It is striking that these adjectives brought about by division are classical negations of each other: `rational` is the negation of `irrational`, `mortal` of `immortal`, etc. One can imagine the vertically arranged line of nouns as the trunk of a tree, and the dichotomously ordered adjectives as the branches that descend from this trunk. We will analyse this in more detail in Section 5.

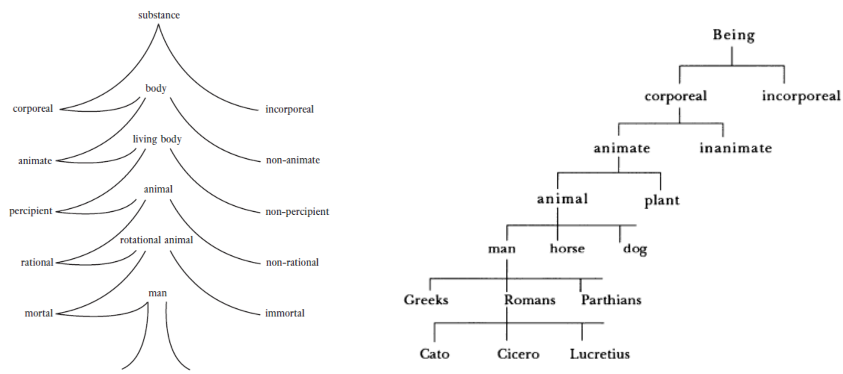
Porphyry describes the relation of these adjectives with the expression of dihairetic or divisive difference (διαιρετικαὶ διαφοραὶ, lat. divisivae differentiae). If two adjectives stem from one noun for which it is true that it is a genus for another, then the positive adjectives describes a property of the next lower noun specie, e.g. `animate` describes `living body`, `rational` describes `rational animal`, and so on. This description is called *constitution* (συστατική, lat. constitutiva) [29, p. 179]. Porphyry thus makes a more precise distinction between nouns or extensional and adjectives or intensional concepts than Seneca and adds further relations to the logical connectives, which we will define more precisely in the following sections using the tree diagrams.

3 Typical Diagrams of S- and P-Trees

Tree diagrams have been handed down to us at least since the early Middle Ages. Although philosophers, theologians and scientists have dealt with these diagrammatic structures for many centuries, with the end of traditional Aristotelian logic in the modern age, more intensive occupations with the historic diagrams have become rare.

The studies by Barnes and Mansfeld are, in my view, the best to be found on our subject, even if I am not convinced about every detail. Barnes and Mansfeld argue that both text passages quoted in Sect. 2 are canonical for tree diagrams, but neither manuscript of these classical texts shows a tree diagram. Barnes and Mansfeld thus provide an ideal type, a kind of average or standard tree of the kind most often drawn between the early Middle Ages and the modern age to visualise the texts of Seneca or Porphyry. This can be easily seen by comparing the two trees with the historical illustrations in e.g. [46], [14], [40].

When any tree diagram was first drawn is a strong point of debate among historians of logic or art, but should not concern us here. For our main topic, it is first more crucial that Barnes and Mansfeld agree that there are two types of tree diagrams, one corresponding to the Senecaic text and one to the quotes of Porphyry. Both use similar logical connectives, differing in their structure. According to Barnes, the Porphyrian tree (P-tree) looks like Fig. 1a [29, p. 110] and according to Mansfeld, the Senecaic tree (S-tree) looks like Fig. 1b [25, p. 96].



(a) **(P-tree)** (b) **(S-tree)**

Figure 1: Tree Diagrams

Fig. 1a is made by Barnes, Fig. 1b by Mansfeld, and both figures correspond as far as possible to the diagrams that have been found again and again in many centuries since the Middle Ages. I therefore take both diagrams as the basis for the following analysis. In both diagrams, one has to take into account that Barnes and Mansfeld have considered more text than was discussed here in Section 2: Therefore, the S-tree of Fig. 1b shows more concepts than discussed in the quote of Sect. 2.1 and the P-tree of Fig. 1a also shows branches that do not correspond to the quotes discussed in Sect. 2.2. Note that the S-tree only shows the concepts that Seneca also discusses in his text, thus omitting many subconcepts that could have been developed on many genera on the right side of the diagram.

Nevertheless, the correspondence between the quotes from Sect. 2 given above and the respective trees in Fig. 1 should be apparent. For example, one can accurately identify in Fig. 1 the technical terms from Sect. 2: (1) In both diagrams, the *genus generalissimum* or *genus generale* is at the top, (2) followed by several levels of subspecies (*inter alia* or *genera specialia*). (3) The P-tree shows the concept `man` as *species speciallissima*. As stated above, however, Seneca does not introduce a technical term like *species specialissma* in his text. If he had introduced it, it would indicate the concepts `Greeks`, `Romans`, `Parthians` in the S-tree. (4) The P-tree does not include *individuals* that would have to be integrated at the trunk or roots of the tree.[1] (In almost all traditional textbooks including P-trees individuals such as `Socrates`, `Plato`, `Petrus` are given at this position.) The S-tree shows *singuli* at the bottom such as included in `Romans`, e.g. `Cato`, etc.

At the first glance, there is a clear diagrammatic difference between S-trees and P-trees: S-trees indicate the division of the genus generale into at least two extensional subspecies. One has to take into account that the first two levels in the S-trees under `being` are, strictly speaking, `corporeal being, incorporeal being` as well as `animate body, inamimate body`. Thus, on the one hand, all concepts mentioned in the S-tree are extensional. On the other hand, one would not have any problems showing the subtypes in the diagram for `incorporeal being`.

This is different in the P-trees. Starting from the top, the extensional concepts of the genus generalissimum and inter alia are divided

[1]Please also note that in Fig. 1 not only the individuals at the bottom of the diagram are missing, but that it should also read ‘rational animal’ and not ‘rotational animal’.

into exactly two intensional subconcepts. In almost all the diagrams I know, there is a positive intensional side and a negative intensional side. In Fig. 1a, the positive intensional side is on the left and includes the concepts `corporeal`, `animate`, `percipient`, `rational`, `mortal`. The negative intensional side is on the right in Fig. 1a and shows the concepts `incorporeal`, `non-animate`, `non-percipient`, `non-rational`, `immortal`. Only the concepts in the middle of the diagram, i.e. between `substance` and `man`, are extensional. The concepts in the P-tree are thus mostly intensional and only partly extensional. As we will see in a moment, this gives rise to various difficulties in extending the P-tree.

4 Modern Interpretation of S- and P-trees

Our aim is not only to consider diagrams as good visual tools, but also to be able to give as exact a definition of them as possible, so that we can examine the differences between canonical diagrams more closely. In though we do not intend to develop a formal logical system with tree diagrams, it is still useful to follow Shin's method and distinguish between the syntax and the semantics of diagrams [38].

The *syntax* of a tree diagram can largely be described by using graph theory. Hacking [16] had already not only made a historical connection between the diagrams of ontology and graph theory, but also made some considerations about graph-theoretical interpretation, which, however, must be much more detailed. The *semantics* of a tree diagram is determined by the concepts whose relation can be visualised by a graph. In simplified terms, by syntax we mean here the form and appearance of the tree diagram, by semantics the meaning of all parts of the diagram. In sum, a tree diagram is a representation of concepts and their relations with the help of a graph.

So far, we have mainly worked with trees whose syntax was determined by the semantics of Seneca's and Porphyry's text. Barnes and Mansfeld provided two typical visualisations of the two texts, which were semantically occupied with the concept used by Seneca or Porphyry. In the following, however, let us try to look at the graph of the diagrams.

4.1 Syntax of Trees

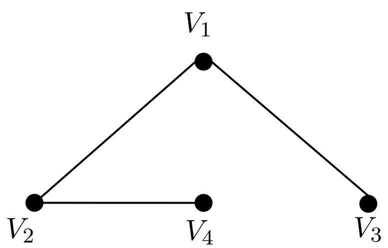
We begin by presenting a set of simplified definitions from graph theory, taking [11] as our guide:

Graph. Let G = (V,E) be a *graph* if V is a finite set of vertices or nodes and E is a set of relations on V represented as edges. Two graphs G_1 and G_2 are called *isomorphic* if they are structurally the same. *End vertices* in a graph G are the two vertices x and y if they are connected by an edge xy or yx. Two edges are *connected* if they have a common vertex, and two vertices are connected if they have a common edge. The *degree* deg(V) denotes the number of vertices connected to a vertex. The *input degree* deg-(V) of a vertex V is the set of edges leading to this vertex. The *output degree* deg+(V) of a vertex V is the set of edges leading away from this vertex. In an *undirected* graph, the edges xy and yx are equal, $\overline{xy}$ for short. In a *directed* graph, the edges xy and yx are unequal, $\overleftarrow{xy}$, $\overrightarrow{xy}$ for short. In an *edge-weighted graph*, each edge is assigned a real number. G' is a *subgraph* of G if G' is a graph and every set of G' is a real subset of G, i.e. V(G') $\subseteq$ V(G), E(G') $\subseteq$ E(G). In this case, G is also called a *supergraph* of G'. A *path* P is a composite of vertices whose *length* k is denoted by the number of connected vertices, i.e. P^k. A path with connected vertices a, b, c, d would then be $P =$ abcd. A path P that contains two times a vertex connected by at least two edges is called a *circle*. If a, b, c are vertices, $P =$ abca is a circle. A *line* L is a path over several vertices and edges, where each vertex is connected to another by only one edge and the direction of each edge is continued by the next one.

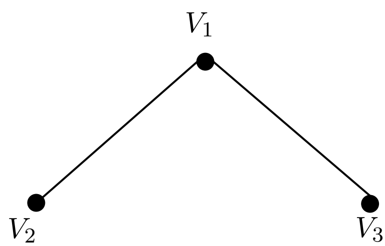
Tree. A graph G is called a *tree* T if it is connected and contains no circles. In T there can exactly be one *root* that is a node of degree 2 and does not form a line with the end vertices of its edges. Any tree with a root is a *rooted graph*. In T, a vertex that has a degree of exactly 1 is a *leaf*. If a vertex in T has a degree of 2 and forms a line with the end vertices of its edges, this vertex is called a *non-branching node*. In T, a vertex that has a degree of ≤ 3 is a *inner vertex*. In a directed graph, which is a tree, the vertex x in the edge $\overrightarrow{xy}$, is called a *child* and the vertex y is called a *parent*. An *out-tree* is a rooted directed graph where the root has an output degree of ≤ 2 and an input degree of 0, the leaves have an input degree of 1 and an output degree of 0 and where there is only one direct directed path to each leaf starting from a root. An *in-tree* is a rooted directed graph where the leaves have an initial degree of 1 and an input degree of 0, the root has an input degree of ≤ 2 and an output degree of 0, and where there is only one direct directed path from each leaf, ending at the root. If in an out-tree each inner vertex has the same output n and an input of 1 or in an in-tree each inner vertex has an input of n and an output of 1, then it is called *regular*, otherwise irregular. A regular tree with n= 2 is called a *binary tree*, with n= 3 a *ternary tree*, with n= 4 a *quaternary tree*, etc. If the length of the path between each leaf and the root in a tree is always the same, the tree is called *balanced.* If the length of P is different, it is called *non-balanced.* A set of disjoint trees is a *forest.*

In contrast to e.g. formal logic, we find in traditional tree diagrams no rules of construction: We can create innumerable graphs, which we may also name as trees with the help of modern graph theory, but a traditional S- or P-tree has a special form, which roughly corresponds to either Fig. 1a or Fig. 1b. If we look at these ideal-typical trees of Fig. 1, we quickly realise that there is a syntactic iteration in both P- and S-trees. We therefore define these iterations as schemes as follows:

Schemes. P-trees and S-trees are two supergraphs composed of several isomorphic subgraphs, which are called *schemes.* A *P-scheme* consists of four vertices and three edges, an *S-scheme* of three vertices and two edges. In a P- and in a binary S-scheme there is one vertex (V_1) at the top that is named *top root* and two vertices (V_2, V_3), each of which is below V_1, so that V_2 is below V_1 on the left and V_3 is below V_1 on the right. In P- and S-schemes, $\overline{V_1V_2}$ and $\overline{V_1V_3}$ holds. In a P-scheme, there is a vertex V_4 that is vertically below V_1 so that it is approximately on the same plane as V_2 and V_3. For P-schemes holds $\overline{V_2V_4}$.



P−Scheme

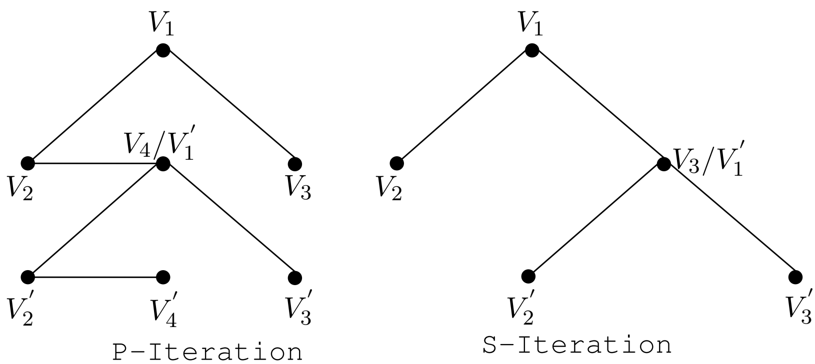


S−Scheme

If S- and P-trees are nothing but repetitions of the same scheme (which only deviates from it at its upper and lower ends, as in Fig. 1), then we can give something like rules to construct iterations of these schemes.

Iteration. An *iteration* is a substitution of a schema at a certain vertex of an already existing schema. The graph of a P- or S-trees, which consists of at least two schemes, is named *P-iteration* or *S-iteration.* In P-schemes, only V_4 of an existing graph may be substituted by V_1 of a graph to be substituted if an outtree is to be constructed. In an intree, V_1 of an existing graph may be substituted by V_4 in a P-scheme. In S-schemes, only V_2 or V_3 of an existing graph may be substituted by V_1 of a graph to be substituted if an outtree is to be constructed. In an intree, V_1 may be substituted by either V_2 or V_3 in an S-scheme. The outtree thus grows upwards, the intree downwards.

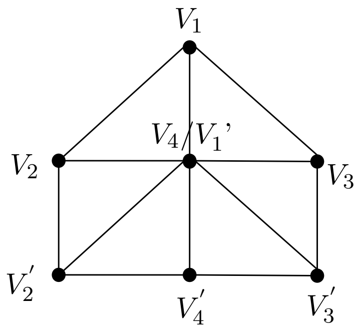
Perhaps we have now already found a sufficient way to describe a large part of the trees of Fig. 1 with the existing syntax. Of course, it must be noted that we have only constructed binary S-schemes so far. In this respect, it must be added, for example, that each n-ary S-scheme increases the number of vertices and edges by 1 for n > 2. In this case, V_1V_n applies in each case.



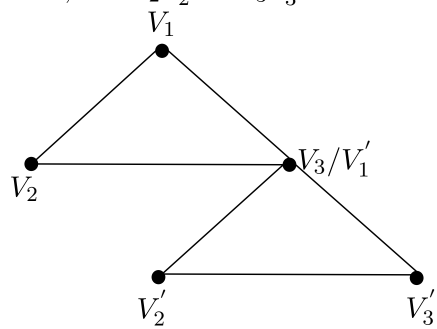
Furthermore, logicians have been pointing out since the Middle Ages that there are semantic relations between vertices that are often not even drawn in the traditional tree diagram (e.g. in [27]). In the case of P-trees, this concerns possible edges between two vertices in two schemes, in the case of S-trees possible edges between two vertices in already one scheme. To make this implicit information explicit, we now extend

S-schemes and P-iteration.

Extended iterations and schemes. Isomorphic subgraphs of a tree are called *extended* if they contain edges between vertices that are not defined by a scheme or an iteration. In this case, we speak of S-extensions or P-extensions. In an S-scheme it is possible to draw the edge $\overline{V_2V_3}$. In a P-scheme it is possible to draw the edge between the vertices $\overline{V_1/V_4}$ and the vertices $\overline{V_3V_4}$. In a P-iteration, edges can be drawn between all V_2 or all V_3 vertices, i.e. $\overline{V_2V_2^{'}}$ or $\overline{V_3V_3^{'}}$.



P-Extension



S-Extension

4.2 Semantics of Trees

The separation of syntax and semantics in tree diagrams is not quite simple. We have so far used graph theory as syntax. By semantics we can now understand the meaning of vertices and edges in a tree diagram: the meaning of a vertex is a concept, the edges between two terms is a relation. As Frické has argued, the relations within the tree are logical in nature, even if modern logic often dispenses with them [15].

We have already sent some important information about the concepts in Section 3, but we can now make them a little more precise. We will use Porphyry's classification of concepts from Table 2.

Concepts. In an S- or P-tree, the V_1 vertex that functions as the top root is the *genus generalissimum.* In a P-tree, any vertex that functions simultaneously as a V_4 in one schema and as a V_1 in another schema is an *inter alia* concept. Any vertex that is V_4 but is not substituted again by another schema and thus does not function again as V_1 in another schema is a *species specialissima.* In P-schemes, the vertex under the species specialissima are the *individua*; but there is no uniform syntactic notation for drawing individuals. In S-schemes, any vertex that functions simultaneously as V_2 or V_3 in one schema and as V_1 in another schema is an *inter alia* term. Any vertex that is V_2 or V_3 but is not substituted again by another schema and thus does not function again as V_1 in another schema is an *individual*. In an S-tree, any inter alia vertex that is directly connected by edges to vertices of the individuum is a *species specialissima*.

We can add that a genus generalissimum in P- and S-trees is usually called `substance` or `being` or a synonym of these concepts. However, as described in the introduction, recent research has shown that there are indeed more semantic forms. In any case, all V_2 terms in a P-tree are adjectives with positive connotations, all V_3 terms are adjectives with negative connotations (*intensional concept*). All V_4 concepts in a P-tree are positive nouns (*extensional concept*). S-trees usually contain only one grammatical form of concepts, even if Seneca mixes adjectives and nouns, as described above.

There is usually a semantic relation between most of the concepts in a tree. We have already taken these relations in Sect. 2 from the texts of Seneca and Porphyry, namely: divsion, subordination, exclusive disjunction, constitution.

Relation. The edge relation between $\overline{V_1V_2}$ and $\overline{V_1V_3}$ in a P- or S-scheme of an outtree is the *division*. If it is an intree, both relations are *constitutions*. The edge relation $\overline{V_3V_4}$ in a P-scheme and $\overline{V_2V_3}$ in a S-scheme is an *exclusive disjunction*. In P-schemes, the edge relation $\overline{V_2V_4}$ is a *constitution*. The edge relation $\overline{V_1V_4}$ in a P-scheme as well as $\overline{V_2/V_2'}$ and $\overline{V_3V_3'}$ in a P-iteration is a *subordination*. *Division* is the separation of either a genus into at least two species or of a species into at least two individuals. *Constitution* is the semantic correspondence between an adjective and a noun. *Exclusive disjunction* denotes the fact that if the concept of one vertex is true, the other is false (taking into account only the relationship to each other and not to other concepts). *Subordination* denotes the relation of a genus to its next lower species, provided that both concepts are either extensional or intensional.

It is already evident from the above-mentioned relations that there is a certain order, which we determine more precisely below under Suppes's definition ([43, p. 210ff.]). This also allows us to create an ordered graph of the traditional trees:

Ordered relations. *Subordination* and *division* are transitive, irreflexive and not symmetrical: If $\overline{V_1V_4}$ are in subordination and so are $\overline{V_4V_4'}$, them $\overline{V_1V_4'}$. If $V1$ is genus for a kind V_x, then $V_x = V_1$ cannot hold. For the *subordinations* in a P-scheme or a P-iteration this means $\overrightarrow{V_1V_4}$, $\overrightarrow{V_2V_2'}$, $\overrightarrow{V_3V_3'}$. If the *division* $\overline{V_1V_2}$ and $\overline{V_1V_3}$ applies, $\overline{V_1V_2'}$ must also apply if also the division $\overline{V_3V_2'}$ and $\overline{V_3V_3'}$ applies. If V_1 divides into V_x and V_y, then it must hold that $V_x \neq V_1$ and $V_y \neq V_1$. If V_2 and V_3 are divided by V_1, then V_1 cannot be divided by either V_2 or V_3, so $\overrightarrow{V_1V_2}$ and $\overrightarrow{V_1V_3}$ hold. Under the same condition, however, the *constitution* relation $\overrightarrow{V_2V_1}$ and $\overrightarrow{V_3V_1}$ holds. Since nothing contradicts the self-constitution of a concept, constitution is transitive and reflexive. The ordered relation of the *exclusive disjunction* is known (symmetrical, irreflexive), but it must be pointed out that there is a certain transitivity in P-schemes: If the exclusive disjunction $\overline{V_3V_4}$ and the *constitution* $\overline{V_2V_4}$ exist, then the exclusive disjunction $\overline{V_2V_3}$ also applies. Something similar also applies in S-schemes if they have at least 3 schemes and are balanced: If the *divisions* $\overline{V_2V_2'}$, $\overline{V_2V_3'}$ as well as $\overline{V_3V_2''}$, $\overline{V_3V_2''}$ apply, then the *exclusive disjunctions* $\overline{V_2'V_3'}$ and $\overline{V_2''V_3''}$ must also apply. If this is the case, $\overline{V_3'V_2''}$ also applies, which can be expressed syntactically by a further edge, but implicitly also the exclusive disjunction of all further vertices among each other on the same level.

5 Problems and Examples

We have now made an offer in Section 4 to be able to analyse tree diagrams better. However, this is not without problems and, moreover, concepts without intuition remain, as Kant says, empty. We will therefore first discuss some problems in this section and then go into some examples to test the definitions and results of the previous sections. This allows us to fill the empty definitions of the previous chapters with intuitions.

Since there are clear differences between traditional trees and the terms of graph theory, in the following we will only explicitly refer to them when traditional trees and not trees in the sense of graph theory are meant.

5.1 Problems with Trees

There are some similarities, but also many differences between today's graph theory and the traditional trees in the vein of Seneca and Porphyry. Only a few comparisons and observations can be made here before we sketch some examples:

(1) An icon resembles an object, without having to be physical itself, to a certain degree [42]. Symbols need not bear any resemblance to what they represent. By using such definitions, we can yet say that traditional trees are iconic, whereas trees of graph theory are usually symbolic, or iconic only by accident. This can be seen, for example, in the fact that in the traditional trees the root is at the bottom of the diagram, the leaves always at the side and the tree top always at the top of the diagram. In graph theory, the root of a tree is usually at the top of the figure, while the leaves are at the bottom. Nevertheless, trees in graph theory can have any conceivable shape, provided they are circle-free.

(2) The method of perductio described in Seneca corresponds to an in-tree in graph theory, the method of dihairesis corresponds to an out-tree. But not every intree or outtree of graph theory corresponds to a traditional tree.

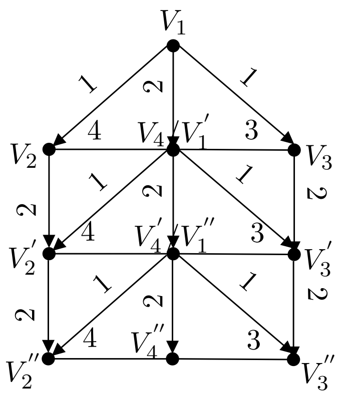
(3) The P-trees and S-trees shown in Fig. 1 are not isomorphic. Both are unbalanced trees because the length of the paths between root and leaves is shorter on the left side than on the right side, but the structure of both trees is different. If one were to arrange the P-tree vertically like an S-tree, it would be more noticeable that the adjectives or intensional expressions form a half-leaf that is missing in the S-tree.

(4) As mentioned above, several relations are missing in the illustrations of Fig. 1, which are described in Seneca and Porphyry and can also be found in some illustrations of P- and S-trees from the Middle Ages onwards. If these relations were added, from a graph-theoretical point of view, the P- and S-trees would no longer be trees at all, because the graphs would have circles. We will see this at the end of this section in the extended graphs.

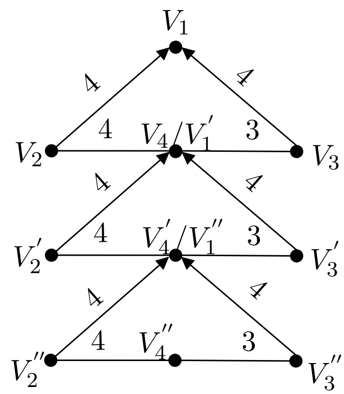
5.2 Examples of P-Trees

In this subsection, we will take two P-trees as examples, analyse them using the syntax and semantics mentioned above, and include some interpretations of classical texts. The two examples are P-T1 and P-T2.

In the following, we will weight the edges in order to abbreviate the relations: division (1), subordination (2), exclusive disjunction (3) and constitution (4). We will not fill the vertices conceptually, as this can be done, for example, with the help of the comparison with Fig. 1.

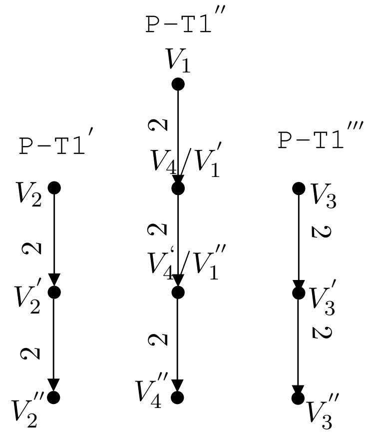


P-T1



P-T2

P-T1 and P-T2 show graphs that can be found in a similar way in several textbooks of the early modern period, e.g. in [13, p. 84] or [35]. We see three subordination paths in P-T1, which are often regarded as the most important information, as they visualise the relation between homogeneous concepts (extensional or intensional, positive or negative concepts). If we take out the relations between heterogeneous concepts, i.e. the edge weights 1, 3, 4, we are left with a forest of three trees, which is displayed in P-F1. We have in F1 the disjoint union of the trees P-T1$'$, P-T1$''$, P-T1$'''$, where P-T1$'$: $\{V_2, V_2', V_2''\}$ indicates the positive intensional, P-T1$''$: $\{V_1, V_1'/, V_1''/V_4', V_4''\}$ the positive extensional and P-T1$'''$: $\{V_3, V_3', V_3''\}$ the intensional negative concepts. P-T1$''$ is traditionally referred to as *linea directa* and P-T1$'$ and P-T1$''$ as *linea indirecta* [7, p. 276, p. 400].



P-F1

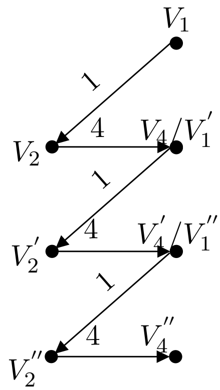
However, if one focuses in P-T1 on the relations, a process is often highlighted in the outtree P-T1, which runs top-down through a directed tree from V_1 to V_4. With this focus, the negative side of P1 is completely ignored and the constitutional process is interpreted as a directed vertex in the P-scheme $\overrightarrow{V_2V_4}$. The result of this process focus of the outtrees is the subgraph P-Sub1, where V_1 is called *terminus a quo* and V_4'' *terminus ad quem*. Traditional concepts of the description of the top-down process, going from the genus generalissimum to the individua or species specialissima, are κάθοδος, παραγωγή, deductio, and many others.

The exact opposite concept is called ἄνοδος, ἐπαγογή, inductio, etc., and is visualised by the directed graph P-Sub2, which is a subgraph of P-T2, where the constitution relation $\overrightarrow{V_4/V_1V_2}$ has been directed.[2] In P-Sub2, V_4 is now the *terminus a quo* and V_1 the *terminus ad quem*. So the process is *bottom-up*, from individua or species specialissma to genus generalissimum. The two subgraphs and the description of the associated processes can be found, for example, in [47, p. 309].

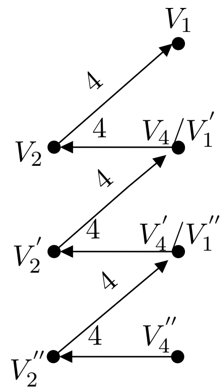
We see in both tradional tree graphs that they have an iconicity that many modern graphs that are trees do not have: As Barnes says, the P-trees remind us above all of pine trees [29, p. 110]. In P-T1, for example, one can clearly see the trunk in the middle, i.e. P-T1'' in P-T2 one can see mainly the fir shape of the branches. These terms, however,

[2] A history of concepts and ideas of these bottom-up and top-down processes from antiquity to modern philosophy of science can be found in [23].

do not correspond to the definitions of modern graphs any more than the term ‘forest’ does to traditional trees.



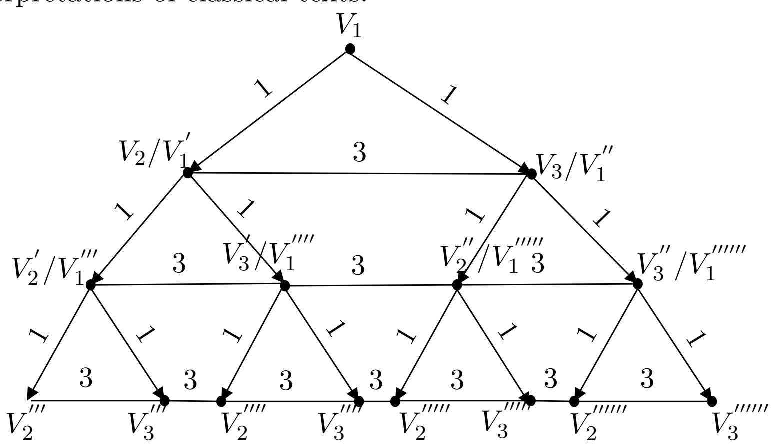
P−Sub1



P−Sub2

5.3 Examples of S-Trees

In this subsection, I will take an S-tree S-T1 as an example, analyse it using the syntax and semantics mentioned above, and include some interpretations of classical texts.



S−T1

Like P-T1 and P-T2, S-T1 is not a tree in the sense of graph theory. However, if one were to remove the edges for the exclusive disjunction (3), S-T1 would be a balanced outtree. If the directionality of the edges were then reversed, the outtree would become an intree in which the edge weight 1 no longer stands for the exclusive disjunction, but for the constitution. This can be seen in some interpretations of the early modern period, in which constitution was represented by the inclusive disjunction: *aut…aut*, then becomes *vel…vel* in edge-weighting 3.[3] In the latter case, the moment of constitution of edge-weighting 1 rests on the fact that V_2 and V_3 can both hold in V_1. Thus, the interpretation of the disjunction relation also has an impact on the constitution relation in bottom-up processes.

In any case, V_1 represents the genus generalissimum in S-T1, the vertices $V_2V_1^{'}$ and $V_3V_1^{''}$ being the inter alia, $V_2^{'}V_1^{'''} \dots V_3^{''}V_1^{'''''}$ the species specialissima and $V_2^{'''} \dots V_3^{'''''}$ the individua. The relations are limited to the dihairesis (1) and the exclusive disjunction (3). In particular, through the dihaireses, the process is seen as top-down, with the genus generalissimum being the terminus a quo and the individua the terminus ad quem.

6 Conclusion and Outlook

In this paper, it was shown what the traditional trees in philosophy looked like, which were used especially in the roughly mentioned period between the 9th century and the 19th century. First, we analysed the relevant texts of Seneca and Porphyry and, using the ideal-typical interpretation of Barnes and Mansfeld. Then, we showed how these ancient texts were diagrammatically reworked. In the process, several difficulties were identified that have so far stood in the way of a modern interpretation of these trees.

Nevertheless, a method was proposed in Section 4 to investigate the syntax and semantics of traditional tree diagrams. It has been shown that graph theory offers a possibility for interpreting the syntax, whereas for the semantics one has to explain above all what the meaning of the vertices and edges is, i.e. concepts and (partially) logical relations. It must be emphasised that traditional trees are organized by the same structures that are repeated again and again: We have therefore spoken

[3] Compare, for example, the relation of 3 in P. Ramus, who sees it as an exclusive disjunction [32, pp. 95ff.], with the relation of 3 in the S-tree of B. Keckermann's, who interprets it as an inclusive disjunction [20, l. I, pp. 5f.].

of schemata and of iterations. In addition, however, graph theory also offers the possibility of representing the otherwise only implicit information of relations in the form of edges.

The greatest difficulties in the modern interpretation arise from the fact that the concept of tree is used differently in traditional ontology than in modern graph theory. Although modern graph theory also grew out of metaphysics, the modern definition of the concept of tree no longer emphasises iconicity: Modern trees of graph theory no longer need to have the form of a tree, although there are also problems with traditional iconic diagrams, for example, that in outtrees growth is from the tip to the root.

The modern interpretation proposed here, however, has attempted to bring the traditional tree diagrams closer to graph theory again, that is, a use of graph-theoretical definitions in the language of analysis. To what extent each detail proposal must be considered a success remains to be seen. However, the examples shown in Section 5 should be sufficient evidence to show that the definitions developed in Section 4 provide an effective means of constructing and analysing traditional trees.

From a graph-theoretical point of view, the success of this paper is likely to be very manageable. From a philosophical point of view, the methods proposed here could offer a way to present certain historical and systematic topics in a new light. As examples, consider some topics related to traditional tree diagrams. For example, philosophers such as Murmellius and Sfondrati made syntactic and semantic extensions to diagrams such as P-T1 in order to carry out holistic language analysis [5]; Johann Christian Lange designed the first logic machine on the basis of tree diagrams [24, chap. 2.2.3]; in Kant's or Hegel's systems tree diagrams play implicitly an essential role [31], [45, §6]; Tree diagrams had an influence on modern logics, as can be seen in Peirce and Gentzen for example [1]; and today's ontologies in the field of knowledge representation tie in with the methods of traditional tree diagrams [41].

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