An epistemological study of recursion and mathematical induction in mathematics and computer science

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[1]IMAG, Univ Montpellier, CNRS, Montpellier, France;
nicolas.leon@umontpellier.fr

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The recent introduction of computer science contents in the mathematics programs for French high schools raises several questions about the means and objectives of such a curricular shift: which concepts are relevant for mathematics and computer science? Are these concepts viewed in the same way by both disciplines? What are the didactical consequences of the potential similarities and differences? These questions are studied within the scope of the ANR DEMaIn research project (Didactics and Epistemology of interactions between Mathematics and Computer Science).

In our thesis, taking place as a part of this project, we concentrate on the notions of mathematical induction (MI) and recursion. We choose them as they are (i) pervasive both in mathematics and computer science, (ii) usually present in high school or first-year undergraduate curricula, and (iii) problematic from a didactical point of view, as abundant literature shows (Michaelson, 2008; Rinderknecht, 2014).

We conjecture that recursion and MI are closely related to each other and that their joint study might help solve some of these didactical problems, an idea that is supported by the existing literature (see, for example, Drysdale, 2011). Methodologically, we adopt the approach of didactic engineering (Artigue, 2014), which prompts us to initially perform an epistemological analysis of both notions. In particular, we would like to pinpoint the different conceptions of recursion and MI that mathematicians and computer scientists have. Therefore, we pose the following research questions:

1. What meanings can be found for recursion and MI in the fields of mathematics and computer science?
2. What relations exist between these two notions?

Methodology

To address these questions, we proceed in three steps:

1. We consult various academic books, either exemplifying or discussing the points of view of mathematics, computer science or logic on recursion and MI. We intend to identify the contexts in which the words “recursion” and “mathematical induction” are used and the links between them that are implicitly or explicitly assumed.

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2. We complement these findings with the information obtained from interviews with mathematicians and computer scientists, in order to gain insight on practices related to recursion and MI that are less visible in the books.
3. We develop a characterization of recursion and MI that is in accordance with the academic literature and expert usage, and appropriate for subsequent didactical analysis.

The books for step 1 have been selected considering that they should be exemplary of a specific look at recursion and MI, that the study of these concepts should take place on an important portion of the book, and that it is desirable for them to have been cited repeatedly in the academic literature. It was not a pre-requisite for them to address undergraduate students, as the epistemological study aimed at characterizing advanced usage.

The choice of experts for step 2 has been made according to availability criteria, and also trying to cover a broad spectrum of research areas. The format of the interview is semi-directed, based on a flexible questionnaire, so as to be able to deepen in specific aspects of the use of the concepts that could appear spontaneously during the conversations.

Results

We discuss preliminary results from the analysis of the first five books and four interviews. New books and interviews will be incorporated into the study before tackling step 3, whose results we also expect to present at CERME.

While the concept of MI seems relatively stable, recursion appears as a weave of fuzzy meanings. Interviewed researchers find it difficult to specify a definition of recursion. Books addressing computability theory will usually put forward the precise mathematical definition of recursive functions, relations, and sets, but even here one can find some ambiguity: "recursive" might also mean, more generally, that something is being defined in terms of (a simpler version of) itself. Computer science books present yet another aspect of the concept, by highlighting its connection with the notions of nested loops or structures: recursion is here viewed as a convenient way to organize procedures or data when certain regularities are observed.

Additionally, the close link between recursion and MI is also noticeable both in the discourse of researchers and book authors, as words like "inductive", "recurrent" and "recursive" are sometimes used interchangeably, depending on the context.

References

Artigue, M. (2014). Didactic engineering in mathematics education. In S. Lerman (Ed.), *Encyclopedia of Mathematics Education* (pp. 159–162). Dordrecht, Netherlands: Springer.

Drysdale, R. L. S. (2011). Mathematical induction is a recursive technique. In T. J. Cortina, E. L. Walker, L. S. King, D. R. Musicant, & L. I. McCann (Eds.), *Proceedings of the 42nd ACM technical symposium on Computer science education* (pp. 269–274). Dallas, USA: ACM.

Michaelson, M. T. (2008). A literature review of pedagogical research on mathematical induction. *Australian Senior Mathematics Journal, 22*(2), 57–63.

Rinderknecht, C. (2014). A Survey on Teaching and Learning Recursive Programming. *Informatics in Education, 13*(1), 87–119.