THE PARADOX OF RULE-FOLLOWING

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If S_1 = A course of action can be determined by a rule, it is in accordance with the rule,

S_2 = A course of action cannot be determined by a rule, because other conflicting courses of action are in accordance with the rule,

then, (S_1 & S_2) is paradoxical.

The first parts of S_1 and S_2 are two contrasting assertions. The second parts are reasons for or bases on which the respective assertions are made. Insofar as both of the reasons appear sound, (S_1 & S_2) is paradoxical. Because, the conclusions based on those reasons contrast each other. The virtue of such a paradox is that it reveals the following. Either one of the arguments for the two conclusions is logically invalid or one of the assumptions behind the arguments is false or unacceptable. I argue that Wittgenstein' s[1] *PI*:201 suggests the falsity of the assumption behind S_2.

Assume that $R = \{r_1, r_2, r_3, ... r_n\}$ is a set of rules and $C = \{c_1, c_2, c_3, ... c_n\}$ is a set of courses of action such that none but one member of C, namely, c_k, is in accordance with the rule r_k that belongs to R. Then, S_1 can be presented in the following *modus ponens.*

If c_k is in accordance with r_k, then r_k determines c_k

c_k is in accordance with r_k

Therefore, r_k determines c_k

Assume that $R = \{r_1, r_2, r_3, ... r_n\}$ is a set of rules and $C = \{c_1, c_2, c_3, ...c_n\}$ is a set of courses of action such that every member of C is in accordance with the rule r_k that belongs to R. Then, S_2 can be presented in the following *modus tollens.*

If r_k determines c_k, then, c_k and only c_k is in accordance with r

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It is false that c_k and only c_k is in accordance with r_k
Therefore, r_k does not determine c_k

The above two arguments, being in *modus ponens* and *modus tollens*, are logically valid. Hence, the paradox conceivable in (S_1 & S_2) reveals that one of the assumptions behind the two arguments is false. The common assumption behind the two arguments is that r_k determines c_k if and only if none but c_k is in accordance with r_k. Or at least, no member of C conflicting with c_k is in accordance with r_k if r_k determines c_k. The two conflicting assumptions are as follows. In one case, only c_k is in accordance with r_k whereas, in the other, every member of C is in accordanc with r_k. I attempt to show the falsity of the latter and, thereby, the untenability of the conclusion based on that assumption. In other words, the paradox of (S_1 & S_2) is resolved by making the untenability of the assumption behind S_2 explicit.

Consider the following example in accordance with the assumption of S_2.

Let r_k be the rule of addition.

Given 2 and 3,

c_k be the course of action	$2 + 3 = 5$
c_1 be the course of action	$2 + {*}3 = 5$
c_2 be the course of action	$2 + {**}3 = 5$

Given 6 and 7,

c_k be the course of action	$6 + 7 = 13$
c_1 be the course of action	$6 + {*}7 = 10$
c_2 be the course of action	$6 +{**}7 = 10$

Given 11 and 12,

c_k be the course of action	$11 +12 = 23$
c_1 be the course of action	$11 +{*}12 = 10$
c_k be the course of action	$11 +{**}12 = 20$

When +* and +** are defined as

$x+{*}y = x+{**}y = x+y$,	when $0<x,y<5$;
$x+{*}y = x+{**}y = 10$,	when $5<x,y<10$;
$x+{*}y = 10$, $x+{**}y = 20$,	when $x,y<10$.

What Kripke's[2] Wittgenstein asks is more or less like this: What is the guarantee that one has followed +, Not +* or +**, in answering 5 to What is the sum of 2 and 3?

Unlike the above, different courses of action may render just one answer. For example, 625 is the answer to 25X25=?, even if one takes different courses like $15^2 + 10^2 + 2(15X10) = 625$, adding 25 for 25 times is equal to 625, multiplying 25 with 25 is equal to 625.[3] Given any integers, these three different courses render the same result whereas, for integers greater than 10, results obtained through c_1, c_2, and c_k vary. Since the results vary, to check the correct answer, each of the courses can be put into doubt. It is not so in case of the example of 25 X 25 = 625, no course is put into doubt. In words, the conflicting courses are problematic since they yield conflicting results.

Consider one more example. 2+3 = 5, 2+2+1 =5, 2+2+1/2+1/2 = 5, 2+2+1/2+1/4+1/4 =5 and so on, an infinite alternative courses are there, at least in principle, to answer What is the sum of 2 and 3? But these courses do not conflict, do not render different answers. Contrary to this, C contains conflicting courses of action in the above example of +, +* and +**. How does r_k fail to determine the course of action c_k? Clearly, it is not in the sense that the result 2+3 =5 is not determined in case of the non-conflicting alternative courses. In the non-conflicting cases also it is not determined which exact course is adopted in answering 5. For, even if it goes against certain psychological principles on learning, infinite alternatives are conceptually available to asnwer 5 to the question What is the sum of 2 and 3? Imagine a person who has learnt and mastered the addition of fractions in such a way that, for him, 2+2+1/2+1/4+1/4 = 5 is equally easy or rather, easier than 2+3 =5. What is the interesting difference between a conflicting course like +* or +** and a non-conflicting course like 2+2+1/2+1/4+1/4 = 5 in relation to 2 + 3 =5?

The first difference is that the conflicting courses do not produce the same result to an indefinite extension whereas the non-conflicting cases do. Second, different conflicting rules are employed in case of conflicting courses whereas the same rule or rules consistent with the original are employed in case of non-conflicting courses. In case of non-conflicting cases like 2+2+1/2+1/4+1/4 = 5 , the same rule of addition is employed through the items added are different. In the non-conflicting cases of the example 25 X 25 = 625, through different rules are employed, they are consistent with the rule of multiplying a number with itself. But in case of +* and +** different conflicting rules are employed.

The second difference is more interesting, it shows how in contriving the conflicting cases one makes an illegitimate move, namely, separating rules from their applications. The above difinitions of +* and +** presuppose that rules are separable from their applications. Since this separation is untenable, the very ground of contriving such conflicting courses of action is unacceptable. Rules cannot be separated from their applications because there is an internal relation between rules and their applications.[4] Due to the internal relation between the rule of addition and its application, the rule of addition entails the answer 5 to the question 2+3 = ? and, at the same time, this answer entails the rule of addition. In other words, neither 2+3 = 5 would have been the correct answer if the rule of + had not been the rule of addition, nor the rule of + would have been the rule of addition if 2+3 = 5 had not been the correct answer. Insofar as internal relation holds good between the rule of + and the calculation in accordance with that rule, the rule of + determines that 2+3 + 5. Also, 2+3 =5 guarantees that the rule of + has been followed. A course of action entails the rule as much as a rule entails the course of action. When a course of action entails the rule, what it entails cannot be a different expression of the rule. Since no other expression which might otherwise be a substitution of the rule is allowed, interpreation of that rule has been prevented from playing a role in actual cases of rule-following.

The conflicting courses like +* and +** presuppose that rules are separable from their applications. Otherwise, the purpose of contriving cases like +* and +** is not fulfilled. The paradox is solved once we show that the conflicting courses are illegitimate, that is they contradict the idea that rules are internally related to their applications. A straight-forward way of showing this illegitimacy is derivable from the above discussion of internal relation and non-interference of interpretations. If +* and +** are two different interpretations of the rule addition, they cannot play a role in determining the sum of 2 and 3 insofar as the rule of addition, is internally related to the course of action 2 + 3 =5. However, if + is one of the interpretations of the rule of addition as much as +* and +**, then + too does not determine the sum of 2 and 3.

What does enable one to offer a set of conflicting courses of action, the set C as understood in S_2? Can +* and +** conflict with + if, like +, they are also in accordance with r_k, the rule of addition ? If they are in accordance with r_k as much as + is, they do not condlict with +. They are defined to be so that they go together for limited stepes. But, in defining the courses of action c_1 and c_2 to be such that they conflict with c_k, do not we define some rules, say r_1 and r_2 that conflict with r_k? We. do. Otherwise, the courses of action are not internally related to the rules they accord with. And, to offer different definitions of a rule tantamount to offer different interpretations of the rule. That is, r_1 and r_2 cannot be the same rules as r_k but different interpretations of the rule r_k.

Were a rule be existent outside the courses of action it determines, conflicting courses like +* and +** could have been followed in accordance with the rule of addition. As, for example, if counting is differentiated from the way one counts, counting in reverse is not reverse counting. That is , one may count from 100 to 1, intstead of 1 to 100, but count all the numbers in-between. The two series- 1,2,3,....,100 and 100,99,98...,1-do not conflict when considered without their conflicting rules. Otherwise, they conflict. Similarly, if r_k is separated from the way one is r_k-ing (i. e. , following r_k), then, c_1,c_2 and c_k are not conflicting. That is, if the rule of addition is outside the course of action in accordance with the rule, then, 11+12 = 23, 11+*12 = 10 and 11+**12 = 20 do not conflict as much as 2+3 = 5, 2+*3 = 5 and 2+**3 = 5 do not. In other words, if a course of action does not contain the rule, its conflicting course need not contain a conflicting rule. If c_k does not contain r_k, then c_1 and c_2 need not contain the rules, r_1 and r_2, conflicting with r_k. If 2+3 =5 does not contain the rule of addition, 2+*3 and 2+**3 need not contain the rules conflicting with the rule of addition. It cannot be upheld that a rule is outside its corresponding course of action but its conflicting rules are inside the corresponding conflicting courses of action. Consequently, since a course of action contains the rule it follows, the course of action cannot be redefined without an appropriate redefinition of the rule. The redefined courses of + that conflict with + cannot be legitimately put forth unless their corresponding rules conflicting courses of action are redefined. Insofar as no newly framed rules are mentioned but courses of action are defined to conflict with one for which a rule is mentioned, the defined conflicting courses are illegitimate. Becasue these are the courses of action which have no rules. If they have their rules they are illegitimately defined; but no more are they problematic. For, as courses in accordance with rules, different from and inconsistent with the rule of addition, they are bound to have conflicting answers at some point or other. It is problematic only when either the conflicting courses have the one and same rule or the conflicting rules are followed for ever without producing conflicting answers.

NOTES

1 Wittgenstein, L. W. (1953) *Philosophical Investigations*, (Trans.) Acsombe,G. E.M., Basil Blackewll, Oxford,(2nd edn., 1958). *Philosiphical Investigations* has been abbreviated as *PI*.

2 Kripke, S. (1982) *Wittgenstein on Rules and Private Language*, Basil Blackwell, Oxford.

3 There can be many other course. more simple or complex. As the easiest course for one may be the most difficult for another, no course can be determined on the basis of equating the course actually undertaken with the easiest course. Furthermore, it is not necessary for any one to adopt what he/she considers the easiest course. One can *blindly* undertake a course of action without exercising the power of making a choice. (Cf. , *PI*:219)

4 There are two important interpreations on 'internal relation'. One, the rule and nothing else determines the correctness of rule-following. If anything else determines, then that 'abrogates' the internal relation. Two, social agreement determines the cor rectness of rule-following but it does so without interfering into the internal relation; it remains 'quietly on the background'. Hacker, P. M. S. & Baker, G(1990) "Malcolm on language and rule", *Philosophy*, 65, pp. 167-179 and Malcolm, N. (1989) "Wittgenstein on language and rules", *Philosophy*, 64, pp. 5-28, present arguments for(against) the first(second) and second(first) interpretations respectively.