Metaphysical implications of causal nonseparability

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Implications métaphysiques de la nonséparabilité causale

Metaphysical implications of causal nonseparability

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Declaration of Authorship

I, Laurie LETERTRE, declare that this thesis titled, “Metaphysical implications of causal nonseparability” and the work presented in it are my own. I confirm that:

- This work was done wholly or mainly while in candidature for a research degree at this University.
- Where any part of this thesis has previously been submitted for a degree or any other qualification at this University or any other institution, this has been clearly stated.
- Where I have consulted the published work of others, this is always clearly attributed.
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- I have acknowledged all main sources of help.
- Where the thesis is based on work done by myself jointly with others, I have made clear exactly what was done by others and what I have contributed myself.

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Abstract

Laurie LETERTRE

Metaphysical implications of causal nonseparability

In quantum mechanics, quantum nonseparability is at the core of philosophical debates regarding its meaning. Interestingly, the more general context of the process matrix formalism features another kind of nonseparability, called causal nonseparability. It characterises quantum processes (connecting the inputs and outputs of different local quantum operations) that are incompatible with any definite causal structure among interacting parties. The present work discusses the possible interpretations of causal nonseparability under the assumption that it points towards novel objective features of nature.

It is first defended that a scientific realist approach towards the process matrix formalism, which is an operational theory generalising quantum mechanics, is as much legitimate as any possible antirealist reading, contrary to certain views found in the literature. The reason is that operational formalisms are ontologically and epistemically neutral. From there, the theoretical concepts of interest, namely causal nonseparability and its model-independent counterpart called noncausality, are analysed in more details, in order to highlight in what sense they are distinct from the standard notions of quantum nonseparability and nonlocality in quantum mechanics. The discussion then focuses on noncausality. It is argued that noncausality has an interesting connection with a notion of temporal nonlocality, which is a more constraining principle than that of local causality used in Bell's theorem. In the same way that Bell nonlocality is given different underlying explanations depending on the details of the chosen quantum mechanics' account, noncausality can be given a variety of underlying descriptions depending on the exact way to interpret the process matrix formalism. The last chapter focuses precisely on this particular point, namely on the various ways to understand process matrices and causal nonseparability in a realist context. In order to explore the potential impact of causal nonseparability on spacetime, we shift from the notion of (indefinite) causal structure to (indefinite) spatiotemporal ones. This shift is allowed under a set of reasonable assumptions regarding the properties of a physical spacetime manifold and the connection between an operational and relativistic notion of causal relations. While different readings are suggested for indefiniteness of spatiotemporal relations, we insist in particular on an objective understanding appealing to the concept of metaphysical indeterminacy. It is argued that such an approach could prove useful in a more general theoretical context such as quantum gravity, while being already partly supported in standard quantum mechanics.

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Contents

List of Abbreviations

FTL	**F**aster **T**han **L**ight
GR	**G**eneral **R**elativity
ICO	**I**ndefinite **C**ausal **O**rder
MI	**M**etaphysical **I**ndeterminacy
PMF	**P**rocess **M**atrix **F**ormalism
QFT	**Q**uantum **F**ield **T**heory
QG	**Q**uantum **G**ravity
QM	**Q**uantum **M**echanics
QS	**Q**uantum **S**witch

Chapter 1

Introduction

Quantum mechanics has stirred numerous debates in philosophy, touching both epistemic and metaphysical issues. This theory, developed in the beginning of the twentieth century [1], allowed to successfully predict the behaviour of physical systems belonging to the sub-microscopical realm [2]. Due to its impressive empirical success and to its description of matter at very small scales, quantum mechanics is considered as an important pillar of modern physics (Ismael, 2021). Its predictions led to important innovations, such as superconductors and lasers, which are widely used in today's technologies (Jaeger, 2019). Yet, in spite of this theoretical and practical success, the theory remains puzzling on a conceptual level.

The core reason for this is that the theory, in its standard form, leaves many questions unanswered regarding the way one should understand a quantum measurement and the mechanisms it involves. It remains silent about the exact nature of quantum systems, their non-classical properties and dynamics, and the reason underlying the fact that quantum systems seem to lose their quantum behaviour when observed. This explanatory gap is referred to as the *measurement problem* (Maudlin, 1995). There are many different ways to solve the measurement problem. In all cases, they imply to adopt both a particular epistemic stance towards the theory, and a particular interpretation of its formalism (which, to complicate the matter, exists in different variants [3]).

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[1] See (Cushing, 1998) for an overview of the historical development of the theory.

[2] Although the size of systems displaying quantum behaviours is not necessarily small (as the phenomena such as superconductivity and superfluidity demonstrate (Annett, 2004; Blundell, 2009)), most quantum systems indeed belong to small scales.

[3] See, e.g., (Lewis, 2016; *Philosophy of Physics: Quantum Theory*) for an overview of the different versions of quantum mechanics.

More specifically, the debate between scientific realism and antirealism questions the ability of science to describe accurately the objective world (and, in particular, the fundamental layer of the world in the case of fundamental physical theories) [4]. This debate has taken a pressing significance in the context of quantum physics, as it heavily conditions the strategy to answer the measurement problem. Antirealists will *dissolve* the problem by adopting an instrumentalist approach towards the theory, or by reducing quantum measurements to some manipulation of the observer's knowledge. This has the advantage to locate the non-classical features of quantum mechanics primarily at the level of the relation between observers and objects. On the contrary, a scientific realist will *solve* the measurement problem by providing an account of quantum measurements in physical terms, and locate non-classical behaviours of quantum systems in nature itself. For those embracing a realist attitude, science constrains, to a certain extent, the ontology of the world and its dynamics. As a result, a form of naturalised metaphysics is practised [5]. The question arises of the exact way the articulation between naturalised metaphysics and science takes place, and of whether this new methodology should conflict with other ways to pursue metaphysical enquiries. The realist exploring the nature of reality as constrained by a specific scientific theory will have to develop metaphysical theories suiting these constraints. In the case of quantum mechanics, one will speak of quantum ontologies to refer to such metaphysical pictures of fundamental reality. Those ontologies are rather non-intuitive due to their non-classical features. In particular, quantum features such as entanglement and nonlocality need indeed to be accounted for by innovative metaphysical theories [6].

While these issues are discussed since the early developments of the theory of quantum mechanics, i.e. more than a century ago, physicists have continued to expand the theoretical apparatus of quantum physics by developing more general theories (or approaches thereof) in a relativistic setting (e.g. quantum field theory [7]), and including gravitation in the picture (i.e. a theory of quantum gravity [8]). These developments yield further interpretative challenges as they introduce novel theoretical features and principles to be considered.

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[4] See (Psillos, 2005; Agazzi, 2017; Lyons and Vickers, 2021) for a presentation.

[5] See, e.g., (Ladyman et al., 2007; Morganti, 2013; Ross, Ladyman, and Kincaid, 2013) for a discussion.

[6] See, e.g., (Lewis, 2016) for an overview.

[7] See, e.g., (Peskin and Schroeder, 2019) for an overview.

[8] See (Oriti, 2009) for a recent presentation of the current developments.

In that context, philosophy of quantum mechanics has become a significant sub-field of philosophy of science (and of physics in particular). This area provides useful case studies and novel constraints feeding the debates about (i) scientific realism, about (ii) the connection existing between science and metaphysics, and about (iii) metaphysical theories for fundamental ontologies. One sees that philosophy of quantum physics lies at the intersection of physics, epistemology, and (for scientific realists) metaphysics. The scope of this work will be mainly restricted to metaphysical questions, as the specific theoretical framework used to formulate quantum mechanics (namely a generalisation of quantum mechanics called the *process matrix formalism* (PMF) [9]) on the one hand, and the epistemic stance (namely scientific realism) assumed throughout the work on the other hand, will be adopted as working hypothesis.

There is currently no consensus among the realist accounts of quantum mechanics as to what ontology, or dynamics is to be preferred. Instead, a large variety of pictures have been developed to describe the quantum world. As regards to the fundamental ontology, it can, e.g., display holistic or structuralist characteristics, or even incorporate a form of metaphysical indeterminacy [10]. The fundamental ontology can be constituted by entities localised in a fundamental 3+1-dimensional spacetime (primitive ontology [11]) or non-spatiotemporal (i.e. constituted by entities located in a different space $\mathcal{S}$ than our familiar 3+1-dimensional spacetime, which is instead derivative from that more fundamental space $\mathcal{S}$ [12]). The macroscopic world emerging from the quantum realm can coincide with our experience of the classical world, or can be seen as dynamically structured into causally disconnected branches [13]. As regards the dynamics of the fundamental ontology, it can be deterministic (e.g. Bohmian mechanics [14]) or stochastic (e.g. the GRW theory [15]). It can involve linear equations (e.g. the many worlds theory [16]) or nonlinear ones (e.g. the GRW theory). The choice among these numerous possibilities is based mainly on personal preferences rather than being exclusively motivated by a decisive

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[9] See (Oreshkov, Costa, and Brukner, 2012) for a presentation.

[10] See, e.g., (Lewis, 2016) for a presentation.

[11] See, e.g., (Allori, 2013).

[12] See, e.g., (Albert, 2013).

[13] See, e.g., (Wallace, 2012).

[14] See (Bohm, 1952).

[15] See (Ghirardi, Rimini, and Weber, 1986).

[16] See (Wallace, 2012).

philosophical or physical superiority of one account over the others [17].

The rich theoretical framework of quantum mechanics and the large array of possible philosophical stances that one can articulate to make sense of it allow for a diversity of metaphysical theories to be explored under the constraints of quantum physics. Deepening the metaphysical understanding of a given theory can impact the approach towards future theoretical progress, by making certain metaphysical commitments explicit and conferring a structuring role on them within the overall theoretical apparatus of the future theory. The diversity of metaphysical accounts developed in the context of physics can also prove useful when applied to different realms of nature. As such, pursuing a metaphysical analysis of our current best theories can yield useful tools for scientists (Chakravartty, 2017b).

Yet, these metaphysical readings of quantum physics still deserve refinement and development, as they spark off numerous debates related to their various implications. Moreover, the incomplete status of fundamental physics creates a gap within the metaphysical work of the scientific realist. While the ontologies assigned to quantum mechanics are themselves plentiful, the different programmes developing a theory of quantum gravity are possibly underpined by different philosophical stances about the world. It is legitimate to ask whether metaphysical views in standard quantum mechanics can survive the transition to quantum gravity, and, if so, to what extent [18]. It is also interesting to see whether the suggested metaphysical features in the context of quantum gravity could shed some light on the conceptual issues of quantum mechanics.

The present work aims at exploring a bit closer this conceptual gap between quantum mechanics and theories of quantum gravity. The key difference between these two frameworks is that the latter unifies the quantum description of matter with the relativistic description of spacetime including gravity, yielding a quantum description of gravity. As such, it is expected that an accurate description of spacetime at the fundamental level could be reached only within a theory of quantum gravity. Yet, it is our intuition that the way spacetime is constrained in a non-gravitational, yet quantum, theory is likely to expose existing tensions between potentially incompatible features of classical spacetime with quantum physics. For this reason, our methodology will be to use quantum mechanics as a starting point, and explore to what extent this theory sets possible constraints on the way spacetime is conceived. More precisely, we will focus on the above-mentioned extension of quantum mechanics

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[17] See (Chakravartty, 2017b) for a discussion.

[18] See (McKenzie, 2020).

called the process matrix formalism, in which correlations between multiple parties can be described without specifying *a priori* their spatiotemporal locations. As such, this framework allows to explore the way quantum theoretical features can impact, to a certain extent, spatiotemporal relations among interacting parties, while imposing minimal constraints on the characteristics of spacetime itself (it is literally unspecified at the formal level). It is an open question whether the reflections led in the context of the process matrix formalism would remain relevant once gravity is included in the picture [19]. Yet, this research will allow to have a first look at the possible impact of quantum features on spacetime, independently of the way gravity is conceived at the quantum level. This could provide a basis for more advanced reflections in quantum gravity. Retrospectively, it could also shed some new light regarding the way standard quantum mechanics is interpreted. In other words, the conceptual issues arising from non-gravitational generalisations of quantum mechanics have the potential to act like the missing link connecting metaphysical studies in standard quantum mechanics and the yet to be completed theory of quantum gravity.

More precisely, this work will focus on a central theoretical feature of the process matrix formalism, called *causal nonseparability* [20]. It is defined, to a certain extent, in analogy with quantum nonseparability, which characterises the quantum state of a composite quantum system that cannot be expressed as a probabilistic mixture of tensor products of the subsystems' quantum states. Causal nonseparability, by contrast, characterises quantum processes (connecting the inputs and outputs of different local quantum operations) that are incompatible with any definite causal structure among interacting parties. One talks about *indefinite causal orders* (ICOs). A famous example of causally nonseparable processes is called the *quantum switch* (QS). It is extensively studied in the literature in virtue of its simple architecture and its various implementations in laboratories. The present work will discuss the possible interpretations of QS's causal nonseparability under the following assumptions: (i) a scientific realist approach towards quantum processes, and (ii) the physicality of causal nonseparability for at least certain processes (including the quantum switch), i.e. that causal nonseparability is seen as pointing towards novel objective features of nature. The objectives of this work will be then to provide an overview of possible realist attitudes towards causal nonseparability, and to discuss the connections that this characteristic establishes with spacetime. We will reflect on the extent to which these views might remain relevant across

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[19] See (Zych et al., 2019; Paunković and Vojinović, 2020).

[20] See (Oreshkov, Costa, and Brukner, 2012).

different theoretical contexts, namely standard quantum mechanics and quantum gravity. The results will aim at emphasising an existing tension between quantum theoretical features such as causal nonseparability and the idea of a classical spacetime.

In that context, and following the above-mentioned methodology in order to reach the announced objectives, this dissertation will be structured as follows:

The **second chapter**[21] provides an overview of the discipline called *naturalised metaphysics*, which is about taking science as the (best) guide for metaphysical enquiry, and various related questions about the way this connection between science and metaphysics (should) take place. The chapter will also describe the specific area of naturalised metaphysics applied to quantum mechanics. This will set the background conceptual stage of quantum ontologies against which the implications of novel concepts such as causal nonseparability will be presented. The **third chapter**[22] will then introduce this new theoretical feature and the overall framework in which it is defined, namely the process matrix formalism. In order to establish more firmly this work against current tendencies in the literature promoting an antirealist attitude towards such a theoretical formalism, we will defend the legitimacy of a realist attitude towards the PMF. The **fourth chapter** will then analyse causal nonseparability at a purely formal level, emphasising in what sense it is different from standard quantum nonseparability. This is a first important step to avoid potential interpretative shortcuts that could affect further metaphysical discussions. From there, an operational point of view will be adopted, i.e. that causal nonseparability will be replaced by the notion of noncausality [23], which characterises experimental correlations among interacting parties that are two-way signalling. Similarly to causally nonseparable quantum processes, two-way signalling correlations are incompatible with a definite causal structure among the involved parties. Possible underlying physical situations behind noncausality are discussed. The differences between noncausality and standard quantum nonlocality are emphasised. It is then argued that noncausality is closely related to a form of *temporal* nonlocality, carefully defined.

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[21] Of which section 2.2.2 has been published in (Lam, Letertre, and Mariani, forthcoming). In particular, subsection 2.2.2.2 has been jointly developed.

[22] Of which sections 3.2 and 3.4 have been published in (Letertre, 2021), and sections 3.1 and 3.3 have been published in (Lam, Letertre, and Mariani, forthcoming).

[23] See (Oreshkov, Costa, and Brukner, 2012).

Finally, the core results of this thesis will be presented in the **fifth chapter**[24], in which causal nonseparability is discussed from the metaphysical point of view according to various possible stances. The accent is put on the way one can connect causal nonseparability and spatiotemporal relations. In that context, indefinite causal orders can be seen as pointing towards a form of indefiniteness [25] of spatiotemporal relations. The nature of this indetermination, as well as the way these results can possibly connect with standard quantum mechanics on the one hand, and a future theory of quantum gravity on the other hand, are discussed. Chapter 6 concludes and provides further perspectives.

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[24] Of which section 5.3's ideas have been jointly developed (see (Mariani and Letertre, forthcoming)), and section 5.6 has been jointly developed and published in (Lam, Letertre, and Mariani, forthcoming).

[25] The terms *indeterminacy* and *indefiniteness* will be used interchangeably throughout this dissertation, in spite of the former being more often used by metaphysicians in a specific technical sense and the latter by physicists in a vaguer sense.

Chapter 2

Quantum mechanics and naturalised metaphysics

This chapter's aim will be to present the overall background context in which the present work takes place. Because the main subject of this research is the metaphysical implications for spacetime of recent developments in quantum mechanics, it is relevant to review the literature about the relation between physics and metaphysics, which will be done in section 2.1.1. From there, a closer look at the metaphysics of quantum mechanics will be taken in section 2.2 to provide a general overview of the field.

2.1 Naturalised metaphysics

2.1.1 Historical background

It is notoriously known that philosophy of science has had a complicated relation with metaphysics (see Chakravartty (2010), Callender (2011), and Kistler (2020) for an overview). Until the seventieth century, natural philosophy was the field studying the nature of the world by pursuing methods encompassing those of the disciplines now known as science and philosophy. Upon the birth of modern science, a progressive divide[1] appeared within natural philosophy, with science adopting its own empirical methods. Although no consensual and clear-cut demarcation can be found between science and philosophy (see section 2.1.4), it is established that science and philosophy constitute distinct academic disciplines.

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[1] See Chakravartty (2010).

Given their shared origin and the fact that they (sometimes) explore the same subject (e.g. what the world is like), it is not surprising to find a close connection between science and philosophy. This relation has evolved across time, and in particular, during the twentieth century.

In the late 1920s, a philosophical current, named logical empiricism [2], emerged among philosophers of science. This movement is characterised by a strong distrust towards metaphysics, placing instead the sensory experiences as the only viable source of knowledge. Many philosophers of science contributed to the logical empiricist current, among them are Carnap[3], Reichenbach[4] and Hahn[5], just to name a few [6].

However, important objections have been raised against this logical empiricist movement (see, e.g., (Alston, 1954; Friedman and Michael, 1999)). A major objection to logical positivism is the fact that it requires a language devoid from metaphysical posits and should appeal to observable notions, which proved to be a failure (see (Chakravartty, 2010) for an overview of the debate). Quine (1951), by casting doubts on the validity of the distinction between analytic and synthetic truths[7], provided a strong argument against a clear distinction between the empirical content of a theory and its conceptual formulation, hence, between observational and theoretical statements. Yet, this distinction was crucial for the inner coherence of logical positivism.

These attacks against logical empiricism and its anti-metaphysical views allowed, during the second half of the twentieth century, for a revival of metaphysical work within philosophy of science, which was until then focused on epistemological issues[8]. This led to the emergence of a novel academic field of enquiry, called “scientific metaphysics”, or “naturalised metaphysics”. This discipline can be broadly defined as the pursuit of metaphysical questions

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[2] I take here the term ‘logical empiricism’ to encompass that of ‘logical positivism’ (see Creath (2021)).

[3] See Carnap (1931).

[4] See Reichenbach (1949).

[5] See Hahn *et al.* (1929).

[6] A comprehensive discussion of the logical empiricist movement can be found in (Creath, 2021).

[7] While an analytic truth is true in virtue of both language’s rules and the meaning of the terms involved, a synthetic truth is true in virtue of our experience only.

[8] Although philosophical stances arguing against metaphysical commitments in sciences remain discussed, as it can be seen in the work of Van Fraassen (1980) that yielded to constructive empiricism. See (Chakravartty, 2010) for a critique.

within the framework of contemporary science. In other words, it is, to a certain extent, engaged with, informed by, or continuous with science (French and McKenzie, 2015).

Naturalised metaphysics evolved in opposition to what is called "armchair metaphysics" (Jackson, 1994), which undertakes the study of the nature of the world with little consideration for contemporary science, and its methods heavily rely on *a priori* conceptual analysis, intuitions and common sense (Chakravartty, 2010). The next section will review the arguments opposing naturalised metaphysics to "armchair" metaphysics, and how a naturalised conception of metaphysics can be articulated.

2.1.2 A rationale for naturalised metaphysics

As mentioned previously, there have been attacks against armchair metaphysics from philosophers of science defending instead a naturalised approach to the field (Callender, 2011; Maudlin, 2007a). Those criticisms are developed in particular in Ladyman et al. (2007), where it is argued that armchair metaphysics is frivolous, relies too much on intuition to justify its proposals, or is overly committed to an outdated science and ontological view of the world (namely a classical one).

Another way to pursue metaphysical enquiry was then promoted by Ladyman et al. (2007), offering a radical account of naturalised metaphysics which states the following:

> "[...] a metaphysics that is motivated exclusively by attempts to unify hypotheses and theories that are taken seriously by contemporary science. For reasons to be explained, we take the view that no alternative kind of metaphysics can be regarded as a legitimate part of our collective attempt to model the structure of objective reality. (Ladyman et al., 2007, p. 1)"

While this account is detailed, motivated and justified at length in Ladyman et al. (2007), their proposal can be summarised in two claims called the "negative claim of naturalised metaphysics" (NC-) and the "positive claim of naturalised metaphysics" (NC+), as suggested in McKenzie (2020, p. 2-3):

> "(NC-) : Metaphysics not informed by science is not worth doing."
>
> "(NC+) : Metaphysics that is informed by science *is* worth doing."

This idea resonates in other works [9], such as in (Esfeld, 2007) who argues that metaphysical theories that are not naturally supported by (although not logically incompatible with) the current best scientific theories display an *ad hoc* character, compared to other metaphysical theories fitting more naturally these physical theories (e.g. a block universe view arguably fits relativity in a more natural way than presentism).

Yet, the radical stance of Ladyman et al. (2007) has been objected by various philosophers. For example, Morganti and Tahko (2017) worry that it might be too minimally different from the position of radical empiricists rejecting metaphysics completely. They also provide a range of objections affecting various principles at the basis of Ladyman et al. (2007)'s rationale. Chakravartty (2010) argues that the demarcation between empirical and metaphysical aspects of a theory is not always clear-cut, which might be used against (Ladyman et al., 2007)'s work. He also raised how the very idea of "grounding" metaphysics in science lacks a compelling articulation (Chakravartty, 2013). More recently, Chakravartty (2017a) objected that the negative and positive claims of naturalised metaphysics are too vague to effectively establish constraints to acceptable metaphysics. In a related spirit, Williamson (2013) accused the concept of "science" to be not well delimited enough as to really allow for a meaningful definition of naturalised metaphysics. Guay and Pradeu (2020), for their part, defend that metaphysics is a rich discipline that cannot be simply categorised as "a priori metaphysics" against "naturalised metaphysics". As a result, radical naturalised metaphysics should be articulated more precisely with regards to the whole array of conceptions of metaphysics, instead of rejecting a restrictive view of traditional metaphysics as the sole existing variety. Another type of objection disagrees with the idea that metaphysics not informed by science is useless. Instead, metaphysics that is not constrained by science can still provide benefits by developing conceptual tools that can prove useful if applied to scientific theories (French and McKenzie, 2015; Bryant, 2020).

In light of those various criticisms, it appears necessary to develop alternative accounts of naturalised metaphysics. While those do not necessarily address all the above-mentioned criticisms, the next section will still present such more moderate variants.

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[9] See (Maudlin, 2007a; Esfeld, 2013).

2.1.3 Alternative accounts of naturalised metaphysics

In view of the unsatisfactory aspects of Ladyman et al. (2007)'s account, several authors have proposed a more moderate view of naturalised metaphysics. Among them are Morganti and Tahko (2017), who mapped the relation between science and metaphysics according to two axes, namely whether they are understood as sharing the same methodology or not, and whether they are understood as investigating the same subject matters or not. This resulted in 4 categories of stances: either science and metaphysics are seen as distinct disciplines with neither the same methodology nor the same subject matters (this corresponds to a more "traditional" view of those fields); or science and metaphysics share both the same methodology and subject matters (this can be seen as corresponding to the radical proposal of (Ladyman et al., 2007)); or science and metaphysics share only a common methodology (this has been defended, in the opinion of Morganti and Tahko (2017), by Goldman (2007) and Goldman (2015)); or science and metaphysics share only a common subject matter. The latter option is the one developed by Morganti and Tahko (2017). Indeed, they see science and metaphysics to enquire about the same domain, i.e. the ontology of the physical world and its dynamics. However, they have distinct methodologies since, according to them, there are elements of metaphysics that are prior to science (namely the exploration of the possibility space serving as a ground for interpreting scientific theories) and reciprocally (namely shaping the possibility space along with metaphysics, as well as gathering information from the actual world to be fed to the metaphysical theories). Hence, while (Ladyman et al., 2007)'s account sees metaphysics as subordinated to science, Morganti and Tahko acknowledge a sort of symmetric relation between both fields. More precisely, the methodologies of the two fields are considered to be entangled and one discipline cannot be pursued without the other if one aims at exploring the structure of the world. It results from this a view in which non-instrumentalist approaches to science will see science as the access door to the fundamental nature of reality, yet, containing non-eliminable *a priori* element from metaphysics allowing to shape the theory, identify possible ways for reality to be, and to provide a basis for the interpretation of scientific theories. Both disciplines are then used in a back and forth dynamic as the investigation of reality's nature is pursued.

This view was later summarised in Morganti (2020a):

> "[...] the purely philosophical analysis of fundamentality and the structure of reality can certainly go a long way in identifying *possible* ways things *could* be like, that is, alternative philosophical hypotheses. But when it comes to making claims specifically about

> the *actual* world an at least moderate dose of naturalism seems in order, allowing both for science to "flesh out", as it were, philosophical hypotheses, and for philosophical theories to be used for interpreting our best current theories." (Morganti, 2020a, p. 6)

Once that naturalised metaphysics is granted a larger methodological autonomy from scientific investigation, one can still refine the question as to how exactly distinguishing between naturalised metaphysics and non-naturalised one, and what is the worth of the latter. This question has been discussed by several authors, and in particular in Callender (2011). His view is basically compatible with the moderate conception of naturalised metaphysics advocated by (Morganti and Tahko, 2017). On the one hand, it grants a distinct methodology for metaphysics and science as suggested in the following quote: "Does a scientific metaphysics have room for philosophy, for metaphysics, or does metaphysics become the "handmaiden" of science on my picture? My reply is that there is definitely room for philosophy, indeed, a demand for philosophy and metaphysics. [...] The methods of any particular science at any particular time don't exhaust the ways of properly studying the world." (Callender, 2011, p. 22). On the other hand, it suggests a way to discriminate between naturalised metaphysics and non-naturalised metaphysics:

> "With these two divisions – that between epistemically worthy and unworthy pursuits and that between metaphysics and science – I can make two claims.
>
> First, the metaphysics we ought to strive for should fall on the epistemically worthy side of the first divide. Or using older terminology, it ought to count as "science" rather than pseudo- or non-science.
>
> Second, I then claim that the metaphysics on the right side of this criterion nearly inevitably will be responsive to and deeply connected with the science also falling on the right side of this line." (Callender, 2011, p. 2)

Callender argues that a metaphysical hypothesis is to be taken as seriously as the scientific theories in which it is applied. A given metaphysical hypothesis will be judged according to whether it allows to provide a good systematisation of the world, just like good scientific theories are expected to do. In that case only, metaphysics will be seen as "epistemically worthy", i.e., treated like a science. On the contrary, isolated metaphysical claims in themselves are not epistemically worthy, because they are not (indirectly) sensitive to the criteria that select good scientific theories. To summarise, Callender claims the following:

> "There are possibilities and necessities related to principles found in our putatively best theories of the world and those that are not. Only the former need attract our attention."

French and McKenzie (2015) seem more moderate, by exploring the distinction between legitimate and illegitimate non-naturalised metaphysics, suggesting that metaphysics performed outside the field of philosophy of science remains worth doing. Their view is compatible with the moderate account of naturalised metaphysics provided by Morganti and Tahko (2017), as they embrace a heuristic approach to metaphysics in which metaphysics developed independently of scientific input can still prove useful to science. According to them, the criterion to distinguish good from bad non-naturalised metaphysics is found in what they call the *compatibility principle*:

> "**The compatibility principle**: the constraint that any metaphysical theory invoking entities x and deployed at some time t should be compatible with at least some independent, well-supported, overall 'serious' scientific theory that directly describes or that is otherwise relevant to those entities, should such a theory exist at that time. " (French and McKenzie, 2015, p. 15)

Yet, they also point towards a tension existing between this criterion and their heuristic view of metaphysics, since the compatibility of a given metaphysical hypothesis with science is contingent to present and future states of science, which cannot be foreseen. They conclude that non-naturalised metaphysics remains supported by their heuristic view of metaphysics, yet in a conditionalised way upon two factors that are *external* to metaphysics itself: (i) naturalistic metaphysicians need to use conceptual tools developed by non-naturalistic metaphysicians instead of developing the integrality of their own tools, and (ii) those conceptual tools need to prove useful to science as it evolves.

We see, in light of the above overview, that the precise way science and metaphysics could and/or should interact, and under which kind of justification, is a debated topic, with still unanswered questions (see, e.g., section 2.1.2 which listed some open challenges for naturalistic metaphysics such as defining a sufficiently precise constrain for metaphysics to count as naturalised/acceptable). More generally, the very question of how metaphysical science needs to be for the proper interpretation of scientific knowledge has been deemed unsolvable by Chakravartty (2010) for purely philosophical reasons. Indeed, it is argued that answering this question amounts to choosing among different epistemic stances (e.g. scientific realism or instrumentalism) that are all inherently coherent and ultimately motivated by personal values impacting what is considered as needing an explanation in science, and what a satisfying explanation is.

Yet, while there is still room for debates and improvement regarding the assessment of the epistemic worth of armchair metaphysics and the demarcation with a precisely defined notion of naturalised metaphysics, the benefits and virtues of making metaphysical investigations dialogue with scientific theories is very natural when the aim is to interpret realistically a given physical theory. This will be the posture adopted in this work.

It is now time to turn to further foundational issues regarding naturalistic metaphysics that will resonate more specifically with the present research.

2.1.4 Naturalised metaphysics and the fundamentality of physical theories

The metaphysical consequences of fundamental physics can be dramatically revisionary, as discussed, e.g., in (Norton, 2020) (see also chapter 5). Hence, the stakes carried by naturalised metaphysics are high. Yet, what is considered as *fundamental* physics today[10] is either the current best theories of matter (i.e. quantum theories such as quantum mechanics or quantum field theory) or of spacetime (i.e. general relativity), or a not yet fully developed theory of quantum gravity. In that context, McKenzie (2020) raised the worry that, while the idea of epistemic progress in physics can be successfully defended, a corresponding progress towards the truth in naturalised metaphysics is more complicated to justify, on the grounds that metaphysical claims are not easily conceived as potential "approximations" of reality, due to their often dichotomous and crude formulation. Assessing whether claims in naturalised metaphysics can be approximated would require to specify more carefully the motivations behind naturalised metaphysics, on the basis of which the very possibility of metaphysical progress will be evaluated.

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[10] An interesting discussion of what is meant by "fundamental" physical theory can be found in Morganti (2020b).

Mckenzie's argument identifies a possible threat to naturalised metaphysics: if epistemic progress in the metaphysical interpretations of the theories is not warranted, then why pursue naturalised metaphysics at all? While not responding directly to this challenge, it seems that some authors put forward several virtues of naturalised metaphysics that could possibly remain unaffected by a possible lack of epistemic progress in naturalised metaphysics. First, Ismael and Schaffer (2020) argued that metaphysical readings of physical theories could have the benefit of highlighting existing tensions between incompatible physical principles at the basis of the theory under consideration, which itself could serve as a hint towards a future more fundamental theory. Second, Chakravartty (2017b) argued in favour of a pragmatic attitude towards the pursuit of naturalistic metaphysics, on the grounds that theorising about the metaphysics underlying physics possesses virtues. Indeed, metaphysical theories can serve as a catalyst for future theoretical developments, by providing a better understanding of various possible commitments to be implemented in future theories. Moreover, when different metaphysical theories are developed to account for a given theory, the resulting concepts could be well suited for different physical realms and areas of scientific descriptions. Hence, the exercise itself can lead to useful tools for describing different aspects of the world.

In the context of this work, we are precisely facing Mckenzie's worry by engaging in metaphysical implications of recent developments generalising quantum mechanics, while remaining outside of the scope of a more fundamental theory of quantum gravity. We will therefore see, in chapter 5, how these reflections could prove relevant in a future theory of quantum gravity, and more generally, what could be their general usefulness. More precisely, we will show how the upcoming discussions will allow emphasising existing tensions between potentially incompatible physical principles in standard quantum mechanics.

Before presenting these recent developments generalising quantum mechanics (see chapter 3), it is useful to zoom on the main metaphysical accounts of quantum physics, as they represent current central results in naturalised metaphysics. This broad overview will serve as the state of the art background against which this work will be presented.

2.2 Quantum metaphysics

We will now provide an overview of the metaphysical hypotheses that can be adopted in order to assign a realist meaning to quantum mechanics [11]. As we will see, this can be done in two different ways. Since there exist different theoretical variant of quantum mechanics presenting each different dynamics (namely, a linear version of quantum mechanics, a spontaneous collapse version, and a hidden-variable version called Bohmian mechanics), one can position oneself in either one of these accounts, and interpret the formal machinery of the theory by assigning it an ontology. The alternative would be to focus on specific theoretical features of the theory that remain present in all of its variants, such as quantum entanglement, and assign a metaphysical meaning to it. Both ways will be discussed below, along with the major metaphysical hypotheses having been suggested so far.

2.2.1 Interpretations of quantum mechanics

The Heisenberg's uncertainty principle demonstrates that there are pairs of "incompatible" observables which are such that measuring one of the observables prevents us from knowing simultaneously and precisely the other (Heisenberg, 1927). It is therefore impossible to know simultaneously the values of all the observables of a system. When the quantum state describing the system is associated with a definite value of a given observable O incompatible with another observable O′, we say that the system is *in a quantum superposition* of definite values for O′. Upon measuring that observable O′, the standard formulation of quantum mechanics claims that the quantum state "jumps" from the superposition to a state associated with a definite value of O′. That exact value is random and cannot be predicted with certainty. The theory only predicts the probability of obtaining a given possible result.

In short, the measurement of a superposed quantum state disturbs this state, which "jumps" towards one of the definite quantum states associated with a possible value of the physical observable under consideration. We cannot predict which definite outcome will be observed. More precisely, a measurement induces an evolution of the quantum state that is indeterministic and nonlinear. When no measurement is performed on the system, its quantum

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[11] This section will limit the discussion to standard, non-relativistic quantum mechanics, since non-relativistic quantum theories is the framework in which this work takes place.

state evolves linearly and deterministically. There are therefore two distinct dynamical regimes for the quantum state of a system, and what triggers the random quantum "jump" is the act of measurement. However, the notion of measurement is vague and imprecise, and does not provide us with an objective, physical criterion to decide which dynamics applies when a quantum state evolves (Bell, 1990). As a result, it is considered that in order to have a satisfactory (interpretation of the) theory, more has to be said about the quantum measurement, its meaning and possibly underlying mechanisms. This is called **the measurement problem** in quantum mechanics (Maudlin, 1995).

There exist different ways to "solve" the measurement problem. One possibility is to "dissolve it" by adopting an antirealist attitude towards the theory and locating the quantum jump of the quantum state at the level of the observers, which confers to the phenomenon a purely epistemic status (see e.g. (Cabello, 2017) for an overview). The realist alternative is to provide a solution to the measurement problem by explicating the underlying (objective) mechanics underlying the quantum jump upon measurements. There are three main such realist accounts (often called *interpretations* in spite of the ambiguity of such a term), each based on a different dynamics and mathematical machinery to encode it. The *many worlds* interpretation relies on a linear dynamics of the quantum state (see (Wallace, 2012) for a prominent development), the *GRW theory* (and its variants called GRW-flash and GRW- matter density) relies on a stochastic dynamics describing the quantum state as spontaneously collapsing to a definite state (see (Ghirardi, Rimini, and Weber, 1986) for a prominent account), and *Bohmian mechanics* relies on hidden variables following, along with the quantum state, a deterministic evolution (Bohm, 1952).

Each of those accounts has been developed within a realist spirit, and various ontological systems can be assigned to them. Indeed, providing a realist "interpretation" (in the sense of solving the measurement problem by giving a physical, objective account of quantum measurements) does not fix the ontology of the theory. In other words, a given realist solution to the measurement problem may not be sufficient to specify the precise meaning of the quantum state and the entities described by the theory, as we will see below.

Either the fundamental space in which lies the whole of reality is our familiar 3+1 dimensional spacetime, or it is the high-dimensional configuration space in which lives the wavefunction of the entire universe. In the former case, a *primitive ontology* will populate spacetime and constitute the fundamental "bricks" of what exists in the world (see, e.g., (Allori, 2013)). In the latter case, often called *wavefunction realism* (Albert, 2013), whatever is perceived and defined in 3+1 dimensions does not correspond to the fundamental ontology, which is itself constituted from the wavefunction (i.e. the quantum state) and possibly other entities (as in a wavefunction realist account of Bohmian mechanics, in which the universal wavefunction and a universal particle in configuration space are the only two fundamental entities of reality (Ney and Albert, 2013, Chap. 1)). Primitive ontologies in quantum mechanics can be point particles having a continuous and deterministic dynamics across spacetime (as in Bohmian mechanics). They can also be, as in variants of the GRW theory, matter flashes obeying a stochastic dynamics, or a matter density field continuously filling space, and characterised by various degrees of density varying according to a stochastic dynamics. The nature of the wavefunction in each of these systems is highly debated and several proposals have been developed. For example, the universal wavefunction has been given a physical status (Hubert and Romano, 2018), a nomological status (Ney, 2013, p. 3), the status of physical property (Monton, 2006), or is classified as a new ontological entity of a new kind (Maudlin, 2007b).

It will not be of interest here to develop further these options, as the remainder of this work will not take place within a particular account of quantum mechanics. For that reason, the second approach (presented below) to assigning a meaning to quantum mechanics is more promising, as it is precisely interpretation-independent.

2.2.2 Interpretation-neutral metaphysics

In recent years we have witnessed a novel tendency in the research on the metaphysical implications of quantum theory. Given the difficulties in providing a shared ontological picture of how the world is like if quantum theory is true (in large part due to the many ways in which we could address the measurement problem) researchers have attempted to focus on the features of the theory that can be considered to some extent "interpretation-neutral", as expressed by Wallace (2019). Phenomena such as entanglement and superposition [12], along with the mathematical features underpinning them, seem to be essential for how we define what a quantum theory is (Janotta and Hinrichsen,

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[12] These phenomena play a central role in famous quantum experiments, such as the double-slits experiment (Young, 1804; Carnal and Mlynek, 1991; Eibenberger et al., 2013) or Bell experiments (Georgescu, 2021). In the former case, quantum superposition and entanglement are formal concepts underlying the observed interference pattern on the detection screen intercepting quantum particles. In the latter case, entanglement underlies operationally detected nonlocal correlations. In order to account for these peculiar observed manifestations, a precise understanding of superpositions and entanglement needs to be provided, whatever the preferred interpretation.

2014), and this is arguably true independently of one's preferred approach to the measurement problem. The metaphysical characterisations of those features are not in the business of providing novel solutions to the measurement problem. Rather, the idea behind them is to refine the overall metaphysical understanding of the theory. Indeed, there are now several concrete proposals on how to implement these readings within the context of specific interpretations of the theory (see section 2.2.2.2), and under the overall assumption of scientific realism towards physics. In the eventuality where the core "interpretation-neutral" theoretical features are preserved in more general theories (e.g. quantum gravity), their metaphysical readings might remain relevant, to a certain extent, beyond standard quantum mechanics.

The next section will present the notions of quantum entanglement and a closely related phenomenon called nonlocality, while section 2.2.2.2 will introduce the various metaphysical readings that can be assigned to those features.

2.2.2.1 Quantum entanglement and nonlocality

In many ways, quantum entanglement and non-locality are central features of quantum mechanics—and, to some extent, of any quantum theory (such as quantum field theory). Within the standard quantum formalism, entanglement is encoded in the ubiquitous entangled quantum states for composite systems. Quantum states can be represented by a vector in a Hilbert space, noted $|\psi\rangle$. A more general representation is provided by appeal to density matrices, noted ρ, which are linear operators acting on the Hilbert space assigned to the system under consideration. A density matrix encodes either pure quantum states, i.e. vectors in a Hilbert space, or mixed quantum states, i.e. probabilistic mixture of vectors. In the remainder of this dissertation, we will mostly appeal to density matrices to represent a system's quantum state, not only because this mathematical object connects more naturally with the forthcoming discussions, but also because it provides a more generalised framework.

Let a composite system, labelled 1-2, be composed of two subsystems, labelled 1 and 2. The quantum states of the subsystems 1 and 2 are said to be *nonseparable*, or *entangled*, if the global quantum state of system 1-2 cannot be expressed as follows:

$$\rho_{1-2} = \sum_i q_i \, \rho_1^i \otimes \rho_2^i \tag{2.1}$$

where the index i sums over classical probabilities (q_i) to have the subsystem x in the (pure or mixed) quantum state described by ρ_x^i. This notion of entangled states is purely formal at this stage, and needs to be interpreted to get assigned a meaning. If one adopts a realist approach towards quantum mechanics, the quantum state is considered as pointing towards objective features of nature. Yet, there is a debate regarding the exact nature of these objective features (as demonstrated by the variety of developed quantum ontologies). Most accounts see the wavefunction (i.e. the pure quantum state expressed in a particular basis) as the mathematical object representing the objective content of the quantum state, while density matrices are seen as encoding a mere epistemic information about the quantum state. This is however debated, and previous work emphasises that there is no need for an epistemic interpretation of density matrices (Aharonov, Anandan, and Vaidman, 1993). Several authors have defended a view called *density matrix realism*, in which it is the density matrix that represents the objective content of the quantum state (Chen, 2020). Such a strategy would yield ontological differences compared to accounts based on the reality of the wavefunction. For example, a monist approach to the universal density matrix would mean that a different mathematical object from the universal wavefunction is reified. On the other hand, a nomological reading of the universal density matrix has the ability to provide us with more explanatory power, as it is argued that the initial universal density matrix can be uniquely determined in agreement with the Past Hypothesis postulated to account for the arrow of time in time-symmetric theories (Chen, 2020).

Importantly, quantum entanglement can lead to empirically verified non-classical correlations violating Bell-type inequalities among spacelike separated entangled subsystems (Hensen et al., 2015). A Bell inequality, as famously defined in Bell's theorem (Bell, 1964) (see Fig. 2.1), is an algebraic inequality, the violation of which by any given probability distribution pointing towards the violation of the premise called "local causality". According to this premise, causes precede their effects, and causal influences travel continuously through spacetime at subluminal speeds. Such quantum correlations violating a Bell inequality are said to be *nonlocal*. More precisely, nonlocal correlations are said to display *Bell nonlocality*. An account of Bell nonlocality (i.e. of the violation of a Bell inequality, hence, of the violation of local causality) can involve a form of *nonlocality* which is an underlying mechanism appealing to some superluminal influences. Nonlocal correlations are not determined by and do not supervene on the respective states of the entangled subsystems[13] or by additional local variables not encoded in the entangled states. They are also independent of the distance between the spacelike separated subsystems.

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[13] This failure of supervenience is often referred to as a form of non-separability in the philosophical literature (see recently Ismael and Schaffer 2020); in the physics literature, quantum non-separability often specifically denotes the non-factorizability of entangled quantum states, as it is reflected by Eq. (2.1).

In this context, quantum entanglement is naturally considered as involving some form of non-locality.[14] Since Bell inequalities can be defined in a purely operational way—i.e. by appealing exclusively to notions such as inputs and outputs of quantum operations treated as black boxes[15]—nonlocal correlations are said to be model-independent. For this reason, Bell nonlocality (i.e. the existence of nonlocal correlations) is naturally taken as reflecting some objective fact about the physical world that any quantum theory has to account for.[16]

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[14] Indeed, we here focus on entanglement leading to the violation of some Bell-type inequalities, and hence to some non-local correlations among the entangled subsystems—in particular, note that all pure entangled states lead to the violation of a Bell-type inequality, whereas mixed entangled states may not violate any Bell-type inequalities (Gisin, 1991; Werner, 1989).

[15] In this context, a model-independent notion is one that does not appeal to any specific machinery, tool or apparatus; the experimental setup is reduced to a black box fed with some inputs and returning some output.

[16] By this, of course, I do not mean to argue that nonlocality is unavoidable. As a matter of fact, even within a broadly realist approach, there are accounts that may escape this conclusion. Examples include the acceptance of retrocausality (Price, 2012; Leifer and Pusey, 2017; Friederich and Evans, 2020), versions of superdeterminism (Hooft, 2016), and perhaps some versions of the many-worlds approach to QM (Vaidman, 2021). It is highly debated whether any of these strategies really help us avoiding nonlocality—see Myrvold, Genovese, and Shimony, 2020 for a discussion.

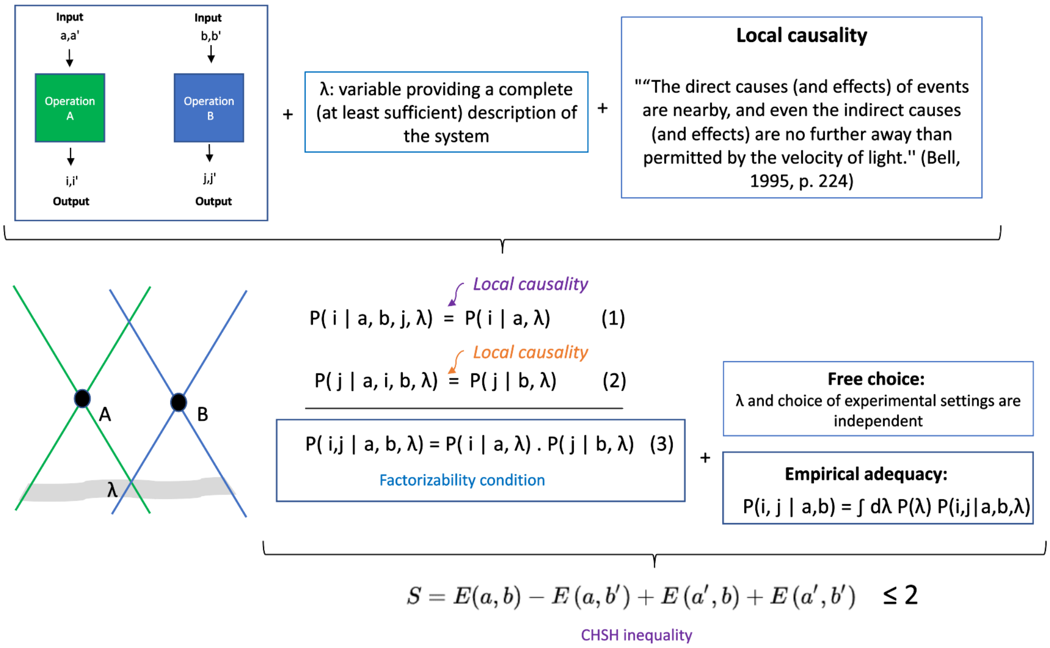

FIGURE 2.1: 2^d version of Bell's theorem (Bell, 1995). Diagrams represent past and future lightcones, the indices a, a′, b, b′ represent the experimental inputs, the indices i, i′, j, j′ represent the experimental outputs, and the functions E(x,x′) represent the expectation values of the product of the measurements' outcomes for the two inputs x and x′.

2.2.2.2 Holism, structuralism and indeterminacy

Several metaphysical tools can account for quantum entanglement and the related non-local correlations.

A first approach that can be used to interpret quantum entanglement is ontic structural realism (OSR), according to which relations (or structures) have a fundamental status in the world's fundamental ontology. The metaphysical details of the relationship between relations (or structure) and relata within OSR can be articulated in different (and sometimes controversial) ways and have been much discussed in the literature (e.g. see the references in Lam 2017, §1). The *moderate* structuralist conception according to which the relations are on a par with their relata—forming together 'structures'—seems especially appropriate for many situations in (fundamental) physics, including the entanglement case.

Indeed, the fact that the modal connections that these quantum correlations exemplify cannot be understood in terms of intrinsic properties of the entangled subsystems (as encoded in their reduced density matrices or with the help of possible additional—'hidden'—variables)[17] provides a strong motivation for a structuralist interpretation in the sense of ontic structural realism.

In the context of quantum entanglement, a natural structuralist understanding takes the novel, experimentally verified non-local correlations among entangled subsystems as the manifestation of a new fundamental physical relation—often simply called 'entanglement relation'—connecting the subsystems (whatever these latter precisely are according to the quantum theory under consideration and the preferred quantum ontology). In this structuralist perspective, the entanglement relation connects the entangled subsystems such that these latter have no independent existence. On this view, the existence of the entangled subsystems (ontologically) depends on the entanglement structures they are part of, that is, on there being entanglement relations, but also on there being other subsystems to which they are entangled to—these latter being conceived as (ontologically) interdependent on one another. This characterisation of entanglement in terms of (symmetric) ontological interdependence or mutual dependence has been recently nicely discussed in Calosi and Morganti (2018).

A second approach to account for quantum entanglement is quantum holism (recently defended in Ismael and Schaffer 2020), which consists in arguing for the ontological priority of the quantum whole (that is, the total composite quantum system) over its entangled parts (the entangled subsystems).

Such an interpretative move gets direct inspiration (and support) from the fact that the quantum state of the total composite system determines those of its entangled subsystems, while the converse fails. Besides this characterisation in terms of ontological priority, various holistic aspects of quantum entanglement have been articulated for some time in the physics and philosophy literature (see (Healey, 2016) for a review), some of which more or less explicitly encode structuralist elements (to some extent, certain types of holism can be considered as precursors of the recent structuralist conceptions in the quantum context). However, it is not the place to discuss the commonalities and the disanalogies between quantum holism and structuralism (see Calosi and Morganti 2018 for a recent critical look).

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[17] Arguably, even within Bohmian mechanics, the non-local modal connections among Bohmian particles cannot be accounted for only in terms of intrinsic (and local) properties of the particles (Lam 2016).

What is especially interesting is the common ground argument recently put forward for quantum holism (Ismael and Schaffer, 2020). In many ways, the structure of this argument is similar to the familiar Reichenbach's common cause principle, which roughly states that two correlated events, where neither is the cause of the other, have a common cause that screens off the correlations between them. In view of the well-known difficulties of this principle in the quantum context (the issue is subtle though, see Hitchcock and Rédei 2021 for a recent review), Ismael and Schaffer, 2020, (section 4) articulate a common ground account of quantum entanglement, which relies on the principle that if "non-identical entities a and b are modally connected, then either (i) a grounds b, or (ii) b grounds a, or (iii) a and b are joint results of some common ground c" (4137)—where grounding is understood, in the way advocated by Schaffer, 2009 as a (metaphysical) asymmetric dependence relation between more fundamental and less fundamental entities.[18]

The application of this principle to entangled quantum subsystems then naturally leads to consider the total composite quantum system as their common ground, which clearly amounts to a form of holism since the whole (the total composite system) is then considered as ontologically prior to (more fundamental than) its parts (the entangled subsystems).

Finally, quantum entanglement can be read as an instance of what is called "quantum indeterminacy". Quantum theory seems to violate a rather standard principle regarding the way in which properties are instantiated by physical systems. Contrary to what happens in classical physics, where every quantity gets assigned a definite value at all times, quantum theories are affected by what has been called *lack of value-definiteness* (LVD). Following (Calosi and Wilson, 2019), we can provide a threefold classification of cases of LVD in quantum theory, which can be understood according to the standard way of assigning values to physical systems given the quantum formalism, namely the so-called Eigenstate-Eigenvalue Link (EEL):

EEL. A physical system s has a definite value v of an observable $\mathcal{O}$ *iff* s is an eigenstate of $\mathcal{O}$.

The three categories are the following:

- **Superposition**: A linear combination $|\psi\rangle = q_1 |\phi_1\rangle + q_2 |\phi_2\rangle$ (with $q_1 \neq$

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[18] Note, however, that *grounding* is more commonly understood as an explanatory relation between facts (Fine, 2012; Correia and Schnieder, 2012). On Schaffer's view, instead, *grounding* applies to every kind of entity (not just facts), which is also why his notion of grounding closely resembles that of *ontological dependence* (Fine, 1994; Tahko and Lowe, 2020).

q_2) of different eigenstates $|\phi_1\rangle$ and $|\phi_2\rangle$ of an observable $\mathcal{O}$ is not always an eigenstate of $\mathcal{O}$. If a system S is in $|\psi\rangle$ it does not have a definite value of $\mathcal{O}$.

- **Incompatible Observables**: Consider two observables $\mathcal{O}_1$ and $\mathcal{O}_2$. The observables commute *iff* $[\mathcal{O}_1, \mathcal{O}_2] = \mathcal{O}_1\mathcal{O}_2 - \mathcal{O}_2\mathcal{O}_1 = 0$. If they do not, they are *incompatible* (see section 2.2.1). If two observables are incompatible, they do not share all the same eigenstates. Thus, if S is in one such non-shared eigenstate of $\mathcal{O}_1$ ($\mathcal{O}_2$), it follows that it does not have a definite value for $\mathcal{O}_2$ ($\mathcal{O}_1$).

- **Entanglement**: Consider an *entangled* system S_{12} composed of S_1 and S_2 with corresponding Hilbert space $\mathcal{H}_{12} = \mathcal{H}_1 \otimes \mathcal{H}_2$. S_{12} might be in an eigenstate $|\psi\rangle$ of $\mathcal{O}_{12} = \mathcal{O}_1 \otimes \mathbb{1}_2 - \mathbb{1}_1 \otimes \mathcal{O}_2$ that is neither an eigenstate of $\mathcal{O}_1$ nor an eigenstate of $\mathcal{O}_2$—with $\mathcal{O}_1$ and $\mathcal{O}_2$ defined on $\mathcal{H}_1$ and on $\mathcal{H}_2$ respectively. Both S_1 and S_2 will therefore lack a definite value for the corresponding observables.

In each of the above cases, applying the EEL entails that one or more observables do not always possess a definite a value. Such lack of definiteness has been taken at face value as to indicate the existence of an ontological kind of indeterminacy (called *metaphysical indeterminacy* (MI)), namely one that we cannot explain away as due to our ignorance or to semantic indecision.

According to Calosi and Wilson, 2019, MI is pervasive in quantum theory, and affects in one way or another every interpretation of quantum theory. We shall notice, however, that this claim is not so straightforward, and requires many details that we cannot enter here. For one, consider that the argument leading from LVD to the existence of quantum indeterminacy is essentially based on the EEL. The EEL, however, is far from being unanimously accepted within the various accounts of quantum mechanics (Wallace, 2019). Therefore, in order to establish the existence of quantum indeterminacy in the context of various interpretations of the theory, it seems that much more needs to be said. Yet, similar arguments have been put forward in recent years in many of the existing interpretations[19], thus showing that, if not forced upon us, MI is at least to some extent suggested by quantum theory, and could then be a useful explanatory tool.

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[19] In particular, notice that in the context of spontaneous collapse interpretations of QM, indeterminacy can arise even by revising the EEL. See, e.g.: Lewis, 2016; Albert, 1996; Mariani, 2020. For a critique, see Glick, 2017.

Several proposals have been developed as consistent ways to make sense of QI. Two quite distinct families of approaches should be mentioned: the *meta-level* views, and the *object-level* views. Very roughly, the distinction between them is the following. According to the *meta-level* view, indeterminacy is understood as wordly unsettledness between fully precise alternatives. Therefore, on this view, there is MI when it is indeterminate which determinate state of affairs obtains. In the context of quantum entanglement, it is indeterminate which determinate quantum state is assigned to entangled subsystems. According to the *object-level* view, indeterminacy is understood as the obtainment of an indeterminate state of affairs. Thus, on this view, there is MI when an indeterminate state of affairs (determinately) obtains. In the context of quantum entanglement, it is determinate that the quantum states assigned to entangled subsystems are indeterminate. Both approaches to MI have been exploited in order to make sense of the lack of value-definiteness in quantum theory, although it is fair to notice that the *object-level* view seems to be preferred (see Calosi and Mariani 2020 for a discussion).

From this discussion, we see that there are different ways to read entanglement, from a metaphysical point of view. Each approach can yield a different world ontology (structuralist, holistic or indeterminate) as the result of the presence of entanglement, although there exist some possible connections between those perspectives. For example, while holism and structuralism are distinct proposals (one emphasises the prevalence of the whole over the parts, and the other emphasises the fundamental status of relations) a radical version of structuralism as proposed by Ladyman (1998) assigns to the entanglement relation a more fundamental status than that of the quantum states of the entangled sub-systems, which coincides with a holistic view in which the quantum state of the whole does not supervene on those of the parts. Structuralism and indeterminacy can also meet in the context of entanglement when it is read as a fundamental relation irreducible to intrinsic properties of the relata, which, for their part, are left undetermined. Such strategies can, in principle, be applied independently of the particular solution to the measurement problem.

2.3 Conclusion

This chapter's first section reviewed the discussions concerning the relation between science and metaphysics, highlighting the arguments in favour of a close dialogue between the two fields. While this work will not aim at addressing specific issues pertaining to the field of naturalised metaphysics, it is important to keep in mind the various open questions raised in that literature that could affect the very motivations and legitimacy of the present research, as section 2.1.4 showed. For that reason, we will come back to, and discuss, these particular questions in due time across the remainder of this dissertation.

The second section provided a broad overview of the metaphysical pictures of reality that were discussed in the context of quantum mechanics, either in the context of a full solution to the measurement problem, or as a coherent reading of a particular (and central) theoretical feature of the theory, namely quantum entanglement. This global picture will constitute a state-of-the-art metaphysical toolbox to guide the development of the present work.

Chapter 3

Recent developments in quantum physics: exploring quantum causality

This chapter will now present the specific formalism containing the concepts of which the philosophical implications will be discussed in this work. Sections 3.1 and 3.2 will present the general theoretical context in which certain formalisms are developed. Section 3.3 will present the particular formalism of interest here, namely the *process matrix formalism*. Finally, section 3.4 will defend at length the methodology followed in this research, consisting in adopting a scientific realist attitude towards the process matrix formalism, as this goes against a somehow antirealist trend in the field of quantum foundations.

3.1 The field of quantum foundations

In the previous chapter, the discussion focused on the theory of quantum mechanics (and its variants) as expressed in the formalism of Hilbert spaces. Yet, the theory can be expressed in the language of other formalisms, of which the frameworks can potentially prove more convenient to investigate a range of technical or conceptual questions.

The field of quantum foundations seeks precisely to reformulate quantum mechanics in order to investigate, as the name suggests, the foundations of the theory. More precisely, reformulations of quantum mechanics can be used to explore the very *structure* of the theory. The main aim is to discover theoretical features that are genuinely *quantum*, i.e. the features that characterise the theory and that are not present neither in classical theories, nor in theories that are more general than quantum mechanics. Such a knowledge is important to distinguish what makes quantum mechanics distinct from other theories, and what is specific to the quantum realm and genuinely non-classical. In doing so, one can search an answer to the question "Why is quantum mechanics as it is?" (instead of answering the question "how is the world according to quantum mechanics?"). The answer lies in the theory's core characteristics, in its foundational architecture.

A direct byproduct of the above-mentioned research lines is the obtaining of insights regarding possible generalisations of quantum mechanics. By studying first the foundations of the theory, one can aim at generalising quantum mechanics by modifying one or more of its foundational features. This allows to explore wider theoretical frameworks that may or may not describe physical phenomena. In the first case, the post-quantum theory would be an actual physical theory describing a broader realm than that of quantum mechanics. In the second scenario, the post-quantum theory would inform us about important limits inherent to the physical world. In both cases, such results are expected to provide us with new knowledge about our physical reality.

The formalism used to reformulate quantum mechanics is that of *operational theories*. It offers an interesting framework as it anchors the theory in a set of physical principles (from which the theory is recovered).

The next sections will develop each of these above-mentioned elements. Section 3.2 will present the notion of operational probabilistic theories (OPT) and how they can be used to study the basic foundations (hence, structure) of quantum mechanics. Section 3.3 will present a specific OPT that aims at generalising quantum mechanics. It will then discuss the conceptual questions arising from such a theoretical discovery. These discussions will be of first importance in chapters 4 and 5, where philosophical questions will build on the technical and theoretical notions exposed in this chapter. Finally, to complete setting the stage for the philosophical discussions of chapters 4 and 5, section 3.4 will argue for the neutrality of the field of quantum foundations regarding the realist/antirealist debate. Indeed, as a realist stance will be a central working hypothesis for our discussions, and since antirealism is often associated with the field of foundations of quantum mechanics, it is important to clear the path from potential "in principle" objections coming from the antirealist camp and explicitly demonstrate the viability of our realist approach towards recent developments in the field of quantum foundations.

3.2 Operational probabilistic theories (OPT)

The framework of operational probabilistic theories is also called the operational framework for physical theories. In that framework[1], a physical experiment is entirely encapsulated in a probability distribution correlating the states of the system with the outcomes of the possible measurement's procedures. The state of a physical system is associated with the preparation procedure leading to that state. More precisely, a state is a class of operationally equivalent preparation procedures, i.e., procedures that cannot be distinguished experimentally. It is mathematically represented by a vector noted ω in a vector space. Any measurement procedure that can be performed on the system can be decomposed into a set of 1-bit measurements. Those are called effects and noted e[2]. They are mathematically represented by vectors in a second vector space (the "dual" space, or "effect space"). A vector (state or effect) that cannot be expressed as a convex combination of other vectors is said to be extremal, or pure, while it is mixed otherwise. Performing a specific 1-bit measurement (corresponding to a specific effect) on a system in a given state yields the outcome 0 or 1, usually not in a reproducible way, but rather according to a certain probability distribution. Knowing that distribution for all the combinations of extremal states and effects is sufficient to calculate the outcome probability for any arbitrary pair of state and effect. For composite systems made of different single systems, an appropriate composition rule between the single system's state spaces on the one hand, and between the single system's effect spaces on the other hand, will determine how those sub-systems will interact with each other.

At this stage, no specific structure is imposed for the state and effect spaces, yielding a formalism that is general enough to allow a large range of theories to be formulated. This formalism is model-independent, meaning that no specific machinery, tool or apparatus is involved, and the experimental setup is reduced to a black box fed with some inputs and returning some output.

By contrast, a specific theory will correspond to a specific structure for

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[1] The present description of this framework is based on the work of Janotta and Hinrichsen (2014), Myrvold (2010), Timpson and Maroney (2013), D'Ariano, Chiribella, and Perinotti, 2017 and D'Ariano, 2010.

[2] More general transformations can also be considered in OPTs. Intermediate transformations of the system can be considered as being part of the preparation or measurement procedures.

the state and effect spaces [3]. Indeed, upon postulating a new probability distribution, symmetries of the physical law to be reproduced, or some dynamical aspects of the system to be modelled, might influence the resulting structure of the state and effect spaces. Importantly, those state and effect spaces have to satisfy consistency rules ensuring that the combination of any effect with any state yields a scalar compatible with a probabilistic interpretation, i.e., comprised between 0 and 1. Finally, the definition of the composition rule for composite systems will be crucial in determining whether the resulting probability distribution will display classical, quantum or post-quantum features, the latter being neither classical nor quantum.

A theory can be constructed by selecting an ensemble of probability distributions satisfying a set of basic axioms. Many toy theories have been constructed so far, displaying state spaces of various dimensions and shapes (Janotta et al., 2011; Janotta, 2012). When a made-up theory allows to recover classical or quantum mechanics, it provides a reconstruction, also called axiomatisation, of that theory. The OPT framework is sufficiently general to include a large number of possible probability theories, with classical and quantum theories as special cases.

Importantly, these basic axioms rely entirely on physical processes such as preparation and measurement procedures in an experimental setup. These principles are therefore based on operational processes, which explains why we talk of operational formulations of theories. Some of these basic operational axioms might be referred to as information-theoretic principles. Such principles involve processes and rules governing them that are at the core of quantum information theory, which studies how quantum systems can be used to process and communicate information (see Timpson and Maroney, 2013 for a review).

A pioneer axiomatisation of quantum mechanics from 5 axioms, some of which are inspired from quantum information theory, was made by Hardy, 2001. Subsequent work provided further operational axiomatisations (Clifton, Bub, and Halvorson, 2003; Chiribella, D'Ariano, and Perinotti, 2010; Chiribella, D'Ariano, and Perinotti, 2011; Dakic and Brukner, 2011; Hardy, 2011;

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[3] In standard quantum mechanics, for example, the states are density matrices acting on a complex Hilbert space and the effects are probabilistic mixtures of projection operators acting on that same Hilbert space. In that context, the description is model-dependent. If one focuses instead only on the probabilities distributions allowed by quantum mechanics (e.g. in order to analyse their properties such as nonlocality), one retrieves the model-independent approach provided by the generality of the operational probabilistic formalism.

Masanes and Muller, 2011; Fivel, 2012; Wilce, 2012; Masanes et al., 2013; Barnum, Muller, and Ududec, 2014; Chiribella and Spekkens, 2015).

3.3 A particular OPT: The process matrix formalism

3.3.1 Motivations

The development of the process matrix formalism was motivated by the desire to provide a more general formalism for quantum mechanics in which no global predefined causal order is assumed between different quantum events, which are basically a pair of input and output physical systems[4] connected via some quantum operation. Within such a formalism, one can investigate whether more general causal structures than the definite (yet possibly dynamical) ones are compatible with quantum mechanics.

The formalism in which a theory is to be formulated without any reference to a global causal order needs to be *causally neutral*. This means that the formalism should express the relations among systems using the same mathematical objects, irrespectively of whether the systems are causally connected or disconnected. However, the standard formalism of quantum theory does not feature such a neutrality (Brukner, 2014): while the correlations among subsystems S_1 and S_2 (described by Hilbert spaces $\mathcal{H}_1$ and $\mathcal{H}_2$ respectively, and localised implicitly[5] in the same temporal regions (Horsman et al., 2017), but in possibly different spatial regions) are generated by a joint state ρ_{1-2} acting on the tensor product of the sub-systems' Hilbert spaces ($\rho_{1-2} \in \mathcal{L}(\mathcal{H}_1) \otimes \mathcal{L}(\mathcal{H}_2)$), the correlations among the states of a single system at different times are represented by a linear map $\mathcal{M}$ transforming the initial states ($\rho_i \in \mathcal{L}(\mathcal{H}_i)$) into final states ($\rho_f = \mathcal{M}(\rho_i) \in \mathcal{L}(\mathcal{H}_f)$) in accordance with the specific outcome of the evolution. A unified formulation for correlations among systems related by spatial and/or temporal relations is achievable by using the Choi-Jamiolkowski (CJ) isomorphism (Jamiołkowski, 1972), which transforms a linear map $\mathcal{M} : \mathcal{L}(\mathcal{H}_i) \to \mathcal{L}(\mathcal{H}_f)$ between two density matrices ρ_i and ρ_f acting on two Hilbert spaces $(\mathcal{H}_i)$ and $(\mathcal{H}_f)$ into a single matrix $M \in \mathcal{L}(\mathcal{H}_i) \otimes \mathcal{L}(\mathcal{H}_f)$

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[4] A distinction needs to be made between the *classical* inputs (i.e. measurement settings) and outputs (i.e. measurements outcome) of a given quantum measurement, and the *quantum* inputs (i.e. input quantum system) and outputs (i.e. output physical system) of a given quantum operation.

[5] Unless explicitly specified.

acting on the tensor product of these Hilbert spaces. M is called the Choi matrix of $\mathcal{M}$.

At this point, a mere causal neutrality is achieved. The process matrix formalism gives up on the assumption of a definite causal structure by postulating that the local experiments performed on quantum systems by different parties obey the rules of quantum mechanics, but it makes no assumption regarding the spatio-temporal locations of these parties (Oreshkov, Costa, and Brukner, 2012). In standard quantum mechanics, the local quantum experiment of each party are described either operationally by probability distributions, or in a model-dependent form in terms of a density matrix being acted on by a linear map describing how input states are transformed into final states upon a measurement. The correspondence between the operational and Hilbertian notions is thoroughly presented in Janotta and Hinrichsen (2014): the equation describing the link between the probability distribution of an experiment and the Hilbert space formalism in the context of a fixed causal structure is given by the Born rule:

$$P(\mathcal{M}_i) = Tr[E_i \rho] \tag{3.1}$$

where $P(\mathcal{M}_i)$ is the probability to obtain the classical outcome labelled i, $\mathcal{M}_i$ is the completely positive trace non-increasing bilinear map transforming any input system S into a given final state S' in agreement with the obtained measurement's outcome i, and E_i is the operator describing the corresponding measurement performed on the system, with $E_i^T = Tr_{S'}[M_i]$, in which M_i is the Choi matrix of the map $\mathcal{M}_i$. $Tr_s[X]$ indicates the operation calculating the partial trace over the Hilbert space of a system s of a matrix X. If two parties (A and B) perform a joint measurement on a shared composite system ρ_{1-2}, Eq. (3.1) becomes:

$$P^{AB}(\mathcal{M}_i^A, \mathcal{M}_j^B) = Tr[E_i \otimes E_j \, \rho_{1-2}] \tag{3.2}$$

where $P^{AB}(\mathcal{M}_i^A, \mathcal{M}_j^B)$ is the probability to obtain the joint outcome labelled i and j, $\mathcal{M}_i^A$ ($\mathcal{M}_j^B$) is the completely positive trace non-increasing bilinear map transforming any input system of party A (B) into a given final state in agreement with the obtained measurement's outcome i (j), and $E_i \otimes E_j$ is the joint operator describing the corresponding measurement performed on the shared system. If the two parties are instead subsequently experimenting on the same system ρ, Eq. (3.1) becomes:

$$P^{AB}(\mathcal{M}_i^A, \mathcal{M}_j^B) = Tr[E_i . \mathcal{M}_j^B(\rho)] \tag{3.3}$$

where E_i is the operator describing the measurement performed by party A on the system after party B performed a measurement described by the linear map $\mathcal{M}_j^B$ applied on the system.

We see that Eq. (3.2) and (3.3) don't have the same mathematical structure. It can be shown that if we appeal to the CJ isomorphism presented earlier to transform the linear maps $\mathcal{M}_i^A$ and $\mathcal{M}_j^B$ into matrices acting on the tensor product of the Hilbert spaces describing the system before and after the corresponding operation, we obtain the following expression for the joint probability distribution of having the outcomes labelled i for the party A, and j for the party B (the generalisation to multiple parties is straightforward), independently of the kind of relation existing between the quantum states studied by the two parties:

$$P(\mathcal{M}_i^A, \mathcal{M}_j^B) = Tr[W^{A_1A_2B_1B_2}(M_i^{A_1A_2} \otimes M_j^{B_1B_2})] \tag{3.4}$$

where $M_i^{A_1A_2}$ is the (CJ) matrix acting on the tensor product of the Hilbert spaces in which the system is described before and after the measurement, $\mathcal{H}^{A_1} \otimes \mathcal{H}^{A_2}$, obtained by applying the CJ isomorphism on $\mathcal{M}_i^A$, and similarly for $M_j^{B_1B_2}$. W is a matrix acting on the tensor product $\mathcal{H}^{A_1} \otimes \mathcal{H}^{A_2} \otimes \mathcal{H}^{B_1} \otimes \mathcal{H}^{B_2}$, with $\mathcal{H}^{X_m}$ being the Hilbert space of the system of party X before the measurement ($m = 1$) or after the measurement ($m = 2$). W satisfies a set of conditions ensuring its consistency with a probabilistic interpretation of Eq. (3.4) (Oreshkov, Costa, and Brukner, 2012).

Eq. (3.4) is very similar to, and can be seen as a generalisation of, Eq. (3.1), (3.2) and (3.3) describing the link between the probability distribution of a (multipartite) experiment and the Hilbert space formalism in the context of a fixed causal structure. Indeed, it has those equations as particular cases, and encompasses even more general situations, as we will see in later sections. Meanwhile, a first intuition about the meaning of W, called process matrix, can be suggested by comparing Eq. (3.4) with Eq. (3.2) from which W appears to be a generalisation of the quantum state ρ, allowing to represent a joint quantum resource shared by multiple parties without mentioning their spatio-temporal locations.

The formal comparison between the density and process matrices will be developed in more details in chapter 4.

3.3.2 Causal nonseparability

In analogy with the definition of *separable quantum states*, the notion of *separable processes* can be defined. While the former notion describes *quantum* relations among separate degrees of freedom, to which different Hilbert spaces are attached, by referring to quantum states of systems, the latter notion describes *causal* relations among quantum events by referring to quantum processes. For those reasons, while we speak of *quantum* separability of *quantum states*, we speak of *causal* separability of *processes*.

Let be two parties A and B performing local quantum operations on a quantum system. The associated bipartite process $W^{A,B}$ describing the way those local operations are combined to form a global structure is said to be *causally separable* if it can be decomposed as a probabilistic mixture of one-way (or no-) signalling causal processes (Oreshkov, Costa, and Brukner, 2012; Oreshkov and Giarmatzi, 2016):

$$W^{A,B} = qW^{A \prec B} + (1-q)W^{B \prec A} \tag{3.5}$$

where q is a number between 0 and 1 and $W^{X \prec Y}$ represents a process for which signalling is only possible from X to Y. Although we will focus exclusively on the bipartite case here, it is worth mentioning that a generalisation of Eq. (3.5) for multipartite processes has been developed in Oreshkov and Giarmatzi (2016) and Wechs, Abbott, and Branciard (2018).

A process compatible with an underlying given, fixed causal structure is necessarily a one-way (or no-) signalling process, and *vice versa*. Indeed, if a process is compatible with an underlying fixed causal structure, it means that we are able to provide the following narrative: either the events generated by the process are causally disconnected and no signalling occurs, or they are causally connected according to a definite time ordering, in such a way that signalling is in principle possible only from the temporally anterior events to the temporally posterior ones. Reciprocally, if a process is one-way (or no-) signalling, this trivially means that it is compatible with an underlying fixed causal structure. The definition formalised in Eq (3.5) therefore means that a causally separable process is a convex combination of processes compatible with a given, fixed causal structures. We say that a causally separable process is therefore a process that is compatible with an underlying *definite* causal structure[6]. Even for a probabilistic preparation procedure that yields a given causal order with a certain probability, it is still the case that the time ordering between the events in party A and party B is definite. This means that the events measured in party A are either preceding those measured in party B (this is denoted $A \prec B$) or succeeding them (this is denoted $B \prec A$).

Chiribella et al. (2013) have imagined a circuit, called the quantum switch (QS), of which the process matrix has been proved to be causally nonseparable (Oreshkov and Giarmatzi, 2016; Araújo et al., 2015). The QS maps two local operations (noted A and B) on a global one. Two parties, Alice and Bob, perform an operation in their respective closed local laboratory, on a shared system called the *target system*. This system evolves jointly with a *control qubit*, of which the state determines the temporal order between Alice's and Bob's operations. More precisely, if the control qubit is in the state $|0\rangle$, the target system undergoes Alice's operation (noted A) before undergoing Bob's one (noted B), and *vice versa* if the qubit's state is in $|1\rangle$. A third party, Fiona [7], will perform an operation on the control qubit after Alice and Bob made their operations on the target, erasing the information about the causal order between Alice and Bob. If the control qubit is initially prepared in a superposition of states $|0\rangle$ and $|1\rangle$, then the process's causal structure becomes *entangled* with the control qubit's state. However, it is common to use a slight misuse of language to describe that situation, and say that the QS is in a superposition of causal orders between Alice and Bob. Indeed, since there are only two operations, there are

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[6] More precisely, the global causal structure among the events described by that particular process is definite. Of course, a causally separable process (therefore being a probabilistic mixture of processes with a fixed causal structure) can itself be part of a wider causally nonseparable process, which would be incompatible with a definite causal structure. Hence, the origin of the probabilistic mixture can be either a classical ignorance regarding the actual global structure of the process (referred to as a "proper mixture"), or an indefinite causal structure at a broader scale (referred to as an "improper mixture"). This situation is similar to that of quantum nonseparability (Nielsen and Chuang, 2010, p. 110).

[7] This third party, Fiona, is actually crucial for the QS to be causally nonseparable. Since Fiona (F) receives the control qubit after the target system has left both laboratories A and B, and performs some operation on it, the quantum switch is strictly speaking a tripartite process. Tracing out its process matrix over the third party would lead to an improper mixture of fixed causal orders, i.e. a causally separable process matrix. However, it can be shown that a tripartite process W in which one party has no outcome system for his/her operation (we throw it away) is causally separable if and only if it can be expressed as $W = q\, W^{A \prec B \prec F} + (1-q)\, W^{B \prec A \prec F}$. Hence, as long as Fiona has no output system for her operation (it is the case within the quantum switch), we find ourselves in a situation in which the causal order is indeterminate among two operations. For that reason, the tripartite causal nonseparability of the quantum switch amounts, to a certain extent, to a bipartite case of causal nonseparability.

two possible fixed causal structures linking them[8]: either only operation A can influence B (noted $A \prec B$), or reciprocally (noted $B \prec A$). When the control qubit is in a superposition of states $|0\rangle$ and $|1\rangle$, the global structure combining operations A and B is in a superposition of definite structures $A \prec B$ and $B \prec A$. As it will be emphasised further in chapter 4, a quantum process allows to mathematically represent relations among quantum events, independently of the system itself [9] and of the local operations that are performed in closed laboratories. These relations are underpinned by an actual causal structure. In the present case, the process describing the quantum switch is such that the relations linking the inputs and outputs of the two operations are nonseparable, in the sense that the operations cannot be said to be performed in a definite causal order. In other words, the process is incompatible with any fixed causal order. We say that the underlying causal structure is *indefinite*, and displays an *indefinite causal order*.

3.3.3 Noncausal correlations

A quantum process generates specific correlations depending on the experiment that is performed. As a result, if one focuses on those correlations instead of on the process itself, it is possible to provide an operational characterisation of the corresponding causal structures featured by the system. Let's consider a joint measurement performed by two observers, Alice and Bob, with a given set of inputs x and y corresponding to Alice's and Bob's input choices, respectively. The corresponding joint probability to obtain the outcomes a for Alice and b for Bob is noted $P^{AB}(a,b|x,y)$. A given correlation $P^{AB}(a,b|x,y)$ is causal if it satisfies a decomposition similar to Eq.(3.5):

$$P^{AB}(a,b|x,y) = qP^{A\prec B}(a,b|x,y) + (1-q)P^{B\prec A}(a,b|x,y) \tag{3.6}$$

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[8] The process matrix formalism presupposes that each of the operations forming the set that is mapped over some global operation by the quantum process is performed once and only once. An extension of the formalism allowing multiple rounds of information exchange for each party has been developed in (Hoffreumon and Oreshkov, 2021).

[9] When the preparation procedure for the input system of the process is not fixed by the process itself, and is rather left as an event occurring in an additional party.

where $q \in [0{,}1]$, and $P^{A \prec B}(a,b|x,y)$ and $P^{B \prec A}(a,b|x,y)$ are valid probability distributions compatible with the fixed causal order indicated by their superscript (Oreshkov, Costa, and Brukner, 2012; Branciard et al., 2015). More precisely, a correlation $P^{A \prec B}(a,b|x,y)$ means that no signalling can occur from Bob to Alice, i.e. the correlation satisfies the following constraints:

$$\forall\, x,y,y',a, \quad p^{A \prec B}(a|x,y) = p^{A \prec B}(a|x,y'), \tag{3.7}$$

with $p^{A \prec B}(a|x,y^{(\prime)}) = \sum_b p^{A \prec B}(a,b|x,y^{(\prime)})$.

Reciprocally, a correlation $P^{B \prec A}(a,b|x,y)$ satisfies the following constraints:

$$\forall\, y,x,x',b, \quad p^{B \prec A}(b|x,y) = p^{B \prec A}(b|x',y). \tag{3.8}$$

Correlations $P^{AB}(a,b|x,y)$ can be geometrically represented as vectors in a multi-dimensional space, the dimension depending on the number of parties, measurements settings and outcomes. It follows from Eq. (3.6) that any convex combination of causal correlations is still a causal correlation. The vectors corresponding to causal correlations form a (convex) polytope, and all the correlations in that causal polytope satisfy trivial constraints ensuring a probabilistic interpretation. They also satisfy non-trivial constraints originating from the definition of causal correlation expressed in Eq. (3.6). These constraints can be formulated as algebraic inequalities. The correlations satisfying all constraints but reaching the upper-bound value for a given inequality are still part of the causal polytope and constitute the various facets of the latter. Each facet corresponds to correlations reaching the upper-bound value of a specific inequality. As a result, it is said that non-trivial facets correspond to causal inequalities. Any valid correlation outside the causal polytope violates at least one causal inequality, and is therefore qualified as noncausal. Hence, by construction, such inequalities are used to test whether a correlation is causal.

A concrete example of causal inequalities and corresponding noncausal correlations violating them has been provided by Oreshkov, Costa, and Brukner (2012), and also by Branciard et al. (2015) in the simplest bipartite configuration with two different measurement's settings and outcomes for each party. Further work needs to be done in order to establish the causal inequalities of more complex causal polytopes.

Although causal nonseparability is necessary for noncausal correlations (Oreshkov, Costa, and Brukner, 2012; Wechs, Abbott, and Branciard, 2018), previous work showed that a causally non-separable process will not necessarily generate correlations that will violate a causal inequality (Oreshkov and Giarmatzi, 2016; Araújo et al., 2015). Its non-sufficiency can in particular be demonstrated by the example of the quantum switch, which is causally non-separable but does not lead to any noncausal correlations. So far, no physical protocol that generates noncausal correlations has yet been found (Wechs et al., 2021; Purves and Short, 2021)[10].

3.3.4 Open questions

So far, the notion of causal nonseparability presented in section 3.3.2 is purely formal. From there, one can first wonder whether this concept characterises any real physical process. Secondly, if the previous question is answered positively, one can ask about the meaning of such a concept. In particular, within the framework of a realist attitude towards quantum processes, one can explore the metaphysical implications of the notion of causal nonseparability. While the last issue will be explored in chapter 5, we will now discuss the first question.

Distinguishing the process matrices referring to physical quantum processes from process matrices that are just mathematical artefacts of the formalism, without any reference in the world itself, is a difficult task that is currently still under investigation (see e.g. (Araújo et al., 2017; Wechs et al., 2021))[11]. Yet, there are at least some causally nonseparable processes that have a rather straightforward physical implementation (Wechs et al., 2021). Among those,

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[10] Noncausal correlations have been observed in the literature, but those do not correspond strictly speaking to the notion presented in this work. Ho et al. (2018) showed that it was possible to observe noncausal correlations by relying to a specific protocol in which the parties perform operations within a spatially localised laboratory, but within an extended interval of time. Such laboratories are therefore not strictly closed, and causal cycles become allowed between the parties. Other works showed that one could obtain noncausal correlations by use of post-selection (Oreshkov and Cerf, 2016; Silva et al., 2017; Araújo, Guérin, and Baumeler, 2017; Milz et al., 2018).

[11] The question of the physical status of causally nonseparable processes has pragmatic implications, since causal nonseparability leads to computational advantages when implemented in circuits performing certain tasks (Chiribella, 2012; Colnaghi et al., 2012; Araújo, Costa, and Brukner, 2014; Facchini and Perdrix, 2015; Feix, Araújo, and Brukner, 2015; Guérin et al., 2016; Ebler, Salek, and Chiribella, 2018; Salek, Ebler, and Chiribella, 2018; Chiribella et al., 2021; Mukhopadhyay, Gupta, and Pati, 2018; Procopio et al., 2019). Yet, because the validity of such implementations is still debated, the question remains open regarding whether such a computational advantage is genuine or simulated.

the particular process called the quantum switch (see section 3.3.2) has been physically implemented in a variety of ways (Procopio et al., 2015; Rubino et al., 2017; Goswami et al., 2018; Wei et al., 2019; Guo et al., 2020).

Yet, objections have been raised against the idea that those implementations are physical realisations of a genuine indefinite causal order. The process matrix formalism assumes that each party is perfectly isolated from the rest of the world, and performs its operation once and only once. The reason is that quantum processes need to satisfy these features in order to be valid processes. Moreover, the violation of these criteria would weaken the meaningfulness of the concept of indefinite causal order as a purely quantum phenomenon. As an example, in a bipartite scenario in which parties are not isolated, the possible causal relations between A and B would not be exhausted anymore by the set {"A causes B", "B causes A"}. Indeed, one could imagine that A and B occurs simultaneously and influence each other, forming a causal cycle. Such a configuration could not be expressed as a probabilistic mixture of definite causal orders "A causes B" and "B causes A", yet would have a classical understanding (MacLean et al., 2017).

For these reasons, it is therefore important for any implementation of the quantum switch to ensure that each party is well isolated, and that its operation is performed once and only once. Oreshkov (2019) showed that local operations in a particular class of processes (including the quantum switch) are assigned time-delocalized Hilbert spaces with respect to which the causal structure is definite and involves causal cycles. This provides a clear mathematical argument for saying that the assumption that local operations are performed once and only once can be verified in practice trough quantum process tomography.

Another objection to existing implementations of the quantum switch points out that only a gravitational quantum switch (i.e. involving an actual superposition of spacetime metrics) would constitute a proper implementation of an indefinite causal order (Zych et al., 2019; Paunković and Vojinović, 2020). Indeed, it can be argued that the quantum switch, since it is defined within a classical non-relativistic spacetime, cannot guarantee a faithful description of spacetime at quantum scales. The present work is sympathetic to such an objection, and the attitude that is defended here is that the quantum switch, and more generally indefinite causal orders, point towards a tension between quantum features and classical spacetime. As such, their (metaphysical) investigation can potentially lead to fruitful conceptual tools that could be of use in more advanced quantum theories of spacetime (see chapter 5 for a development of this point).

From the above discussions, it is then rather reasonable to consider, or at least assume, that indefinite causal orders can be found in physically implementable processes. The question now will be to elucidate to what it could correspond to in the world (see chapter 5). Since that question will be discussed under the assumption of scientific realism, the next section will discuss the validity of such a stance in the context of quantum foundations.

3.4 Realist attitude towards OPTs

The debate between the two opposite stances that are scientific realism and antirealism is a central one in philosophy of science since the development of modern science (Psillos, 2005). Scientific realism holds the view that our best scientific theories provide approximately true descriptions of the objective, external world. Such a philosophical attitude relies on three assumptions (Chakravartty, 2017a): (i) a metaphysical proposition according to which there exists a mind-independent, objective world, (ii) a semantic proposition according to which whether our theories are true or false is determined by the composition of nature and (iii) an epistemic proposition according to which science can provide access to objective knowledge of the external world. Denying any of those propositions leads to a form of antirealism about scientific theories. The most common form of scientific antirealism is epistemic, as it denies the epistemologically warranted and direct access to objective reality through science.

In the context of this section, we will focus on the scientific realist and antirealist views as applied in the more specific context of quantum mechanics. A variety of antirealist approaches towards that theory have been developed. Those approaches range from a mere agnosticism regarding whether the ontology and dynamics postulated by the theory are approximately true (rejection of the epistemic proposition defined above), to a more radical stance claiming that quantum mechanics is not *about* objective reality. The underlying premises for the latter stance are either the rejection of the metaphysical and/or semantic propositions defined above, or the rejection of the epistemic proposition on the grounds that quantum mechanics is about a *subjective* reality.

The study of the foundations of quantum mechanics often appeals to the operational formalism within which the theory can be recovered from a few basic physical principles. Such a framework is sometimes used to support an antirealist view towards quantum mechanics. I intend in this section to defend the point that no formalism alone will ever provide evidence or support for either a realist or an antirealist stance, and that extra-assumptions are always needed in order to motivate a particular approach.

As such, this section is therefore not against antirealist readings of operational formalisms. Neither is it in favour of realist readings. Its goal and scope have a very specific focus: this discussion aims at providing a clarificatory emphasis on the fact that the way realist or antirealist approaches are philosophically motivated is no different in the context of operational formalisms than it is in the context of any other framework. Yet, I believe that these points deserve an explicit and clear formulation, not because they are necessarily polemical, but because they allow to highlight important non-explicit premises underlying recent antirealist approaches towards operational frameworks. Putting those under the spotlight will allow reclaiming a central fact: no new argument in favour of antirealism can be extracted from formal aspects in the foundations of quantum physics.

Through the discussion of particular antirealist arguments based on operational formalisms found in the literature, I will highlight the following points:

- The arguments attempting to support an antirealist view of quantum mechanics based solely on the operational formalism for physical theories alone are not convincing arguments (section 3.4.1).
- Similarly, there is no convincing argument for realist approaches to quantum mechanics grounded in the operational formalism in itself. Overall, both realist and antirealist approaches towards quantum mechanics are in principle equally compatible within the operational framework for physical theories (section 3.4.2).
- The reason for this *in principle* compatibility of both realist and antirealist views of quantum mechanics with operational formulations of the theory is that the realist/antirealist debate is purely epistemic and does not involve any feature of any formalism. The arguments involved are not tied to a specific theory (section 3.4.3).
- In particular, and this is the important point, the operational framework for physical theories is, in itself, epistemologically neutral, and, within a given epistemic framework, the interpretation of the theory (its ontological content) is postulated. The operational framework for physical

theories *in itself* [12] does not favour a particular epistemic or ontological stance.

This discussion will relocate the realist/antirealist debate about quantum mechanics where it should be, i.e., at the epistemological level [13]. As a result, one cannot draw a realist or antirealist argument solely based on the features of some chosen formalism. The operational physics *in itself*, not only does not favour antirealist views over realist ones, but is also neutral epistemologically *and* ontologically.

3.4.1 Main antirealist arguments based on operational physics

This section will discuss three arguments to be found in the literature defending an antirealist view of quantum mechanics based on some features of the operational framework for physical theories. For each argument, a quick presentation is followed by objections.

While the second and third discussions are, to the best of my knowledge, new in the literature, the first argument's discussion is reviewed from previously existing work (Timpson and Maroney, 2013). It seems that this first argument remains somehow present in some informal conversations in the context of conferences in foundations of quantum mechanics, or in some work posterior to that of Timpson (see below). It seems therefore interesting to recall Timpson's objections here. Moreover, including a review of Timpson's work

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[12] I.e. the purely formal content of the theory, without any additional interpretative considerations. A remark is worth mentioning here: it is true that the formal content of a theory can constrain any assigned ontology (e.g., the quantum ontologies designed within realist approaches to quantum mechanics are highly constrained by the formal aspects of the theory), and a particular ontology taken as a premise can influence the formal aspects of a theory in development (e.g. Bohmian mechanics was developed around a particular primitive ontology). Yet, such a mutual influence does not imply that a particular formalism needs to be interpreted in one particular way. There is not one unique ontology that can be postulated for any given formalism (see, e.g., a discussion in Chen (2019)). A given formalism, even designed with a specific interpretative framework in mind, can still be read along different lines by other thinkers. To sum up, even if in the practice of developing and interpreting a theory, there is a mutual dependence between the formal aspects and the postulated meaning of a theory, a given interpretation for a given formalism (whether it is a realist or an antirealist one) is never necessary.

[13] This point is actually a very general one, but we focus here on quantum mechanics (and generalisations thereof) in particular because the philosophical implications related to those theories seem to be more pressing.

in this discussion allows to show a richer variety of different strategies supporting antirealism towards quantum mechanics based solely on features of some formalism. It will be interesting to see that all of these three arguments involve interpretational steps or assumptions that need to be considered in addition to the features of the operational formalism itself. These additional assumptions either boil down to more general considerations belonging to the realism/antirealism debate, or need further justifications to overcome the objections that are raised against them.

3.4.1.1 "Quantum mechanics is about ‘quantum information’, i.e., the wavefunction is mere information”

3.4.1.1.1 Summary of the argument

As announced above, the argument discussed in this section has been reviewed and objected to by Timpson and Maroney (2013, Chap.7). It was recalled in section 3.2 that quantum mechanics can be reconstructed (axiomatised) from a limited number of basic axioms that select the set of probability distributions satisfying each of these principles. This results in a particular operational probability theory that recovers standard quantum mechanics in the sense that the axioms select the distributions allowed by quantum mechanics, while excluding all the distributions that are not. In particular, quantum mechanics can be reconstructed (axiomatised) from informational principles, i.e., from axioms governing how quantum systems can be used to process and communicate quantum information. As expressed in the work of Timpson and Maroney (2013, p. 150), such particular OPTs are sometimes taken as the sign that quantum mechanics is only “about information”, hence, should be interpreted epistemically. In other words, quantum mechanics would not be directly about an external objective world, but about the empirical knowledge, representation, or even beliefs that we have of the world, which do not necessarily coincide with the actual objective world underlying those knowledge, representation or beliefs. This therefore amounts to a form of scientific antirealism as regards to quantum mechanics (we reject the epistemic proposition for scientific realism).

Specific examples of such a move can be found in (Zurek, 1990, p. viii) or, more recently, in (Koberinski and Müller, 2018). The latter acknowledges the interpretative neutrality of the operational (information-theoretic) formalism itself:

“Our arguments [...] support the hypothesis that quantum theory is a principle theory of information, with continuously-reversible evolution in time as a characteristic property. Any further insights into an underlying "quantum reality" (if it exists), or into the question "information about what" (if it has an answer) should not be expected to arise directly from these principles, or from quantum theory itself, but from a novel, yet-to- be-found constructive theory with additional beyond-quantum predictive power (if it exists)." (Koberinski and Müller, 2018, p. 11)

However, it still mentions the reasoning described above:

"Interpretations that treat the quantum state as a state of knowledge or belief are conceptually more closely related to the view of quantum theory as a principle theory of information, which has led some physicists (e.g. Brukner) to argue that the success of the latter is evidence for the validity of the former." (Koberinski and Müller, 2018, p. 5)

While it would not be fair to claim that they embrace fully and entirely such a view (as the first quote clearly shows), it is still interesting to see that this reasoning remains shared at least informally. Whether such an informal claim is meant to be taken literally or not, it remains a good illustration of the above-mentioned reasoning, which is problematic when taken at face value.

3.4.1.1.2 Objections

An objection to the above reasoning was articulated by Timpson and Maroney (2013, Chap.7) as follows. First of all, the notion of "information" can take different meanings. We should differentiate a technical notion of information, which can be expressed in purely physical terms, from an "everyday sense" of information, associated with some elements of knowledge and language that are not expressible in purely physical terms. Then, Timpson makes clear that the notion of quantum information (appearing in information-theoretic reconstructions of quantum mechanics) is a "technical" notion of information. On the one hand, quantum information can be quantified using the quantum analogue of the classical Shannon information theory (see the work of Timpson and Maroney (2013, Chap. 3) for a review). On the other hand, Timpson identifies a piece of quantum information as being merely a sequence of quantum states in a particular order. Hence, both the content of a particular piece and the quantification of quantum information can be expressed in purely physical terms, which shows the technical nature of quantum information. As a result, viewing quantum information as a kind of epistemic notion of information is, in itself, an unwarranted jump from a technical to an everyday notion of information. Such a jump needs further justification, which is not found in the formalism of operational physics alone.

We can expand a bit on this argument by Timpson. The formalism of a physical theory is, by itself, of technical nature. Interpreting the formalism is giving meaning to its technical elements. This implies to connect those elements to a wider conceptual context. More concretely, we can see that an epistemic view of the wavefunction refers to notions (such as observer, knowledge, belief, subjectivity, representation, ...) that need a language and background to be expressed and understood, that lies far beyond the scope of physics alone. Similarly, an ontic view towards the wavefunction involves concepts (such as objectivity, reality, laws, properties, ...) that cannot be captured by technical language within a physical theory alone.

To sum up, jumping from technical (formal) notions to epistemic ones (e.g. the everyday notion of information) or ontological ones (e.g. objective entities) amounts to interpreting the notions under consideration. This jump requires philosophical justification that cannot be derived from any formalism alone. This point will be reiterated in section 3.4.3.

A final remark regarding this discussion concerns famous antirealist views built on, or developed along, information-theoretic formulations of quantum mechanics, namely the "it from bit" approach of Wheeler (1999) and Qbism (Fuchs, Mermin, and Schack, 2014)[14]. Those approaches do not need to rely on any semantic jump between different notions of information[15]. Yet, their association with information-theoretic formulations of quantum mechanics is very natural, which might suggest that the operational framework brings

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[14] Qbism falls in the category of epistemic antirealist views. Indeed, it rejects the view that quantum mechanics provides an objective picture of the world as it is. While this does not imply a form of metaphysical antirealism or instrumentalism, standard quantum mechanics is said to provide an irreducibly subjective picture of reality (Fuchs and Schack, 2014; Fuchs, 2017). In the case of Wheeler's "it from bit" approach, it is often taken as an antirealist approach (see e.g. (Cabello, 2017)). Standard quantum mechanics is seen as providing a "mere continuum idealization" (Wheeler, 1999, p. 1). However, the precise way in which Wheeler's view is to be interpreted is debated, see e.g. (Fuchs and Schack, 2014).

[15] Within the Qbist interpretation, the quantum probabilities are given a Bayesian reading, and information-theoretic reconstructions of the theory are expected to give us knowledge about the structure of the theory, which in turn is hoped to inform us about some objective features of the world (Fuchs, 2017). The motivations behind the Bayesian interpretation of quantum probabilities do not come from those information-theoretic reconstructions. Instead, the main motivations for Qbism find their root in the conceptual strength of the Bayesian interpretation of probabilities, as well as in the dissolution of the measurement problem and that of the issue of nonlocality (see (Timpson and Maroney, 2013, Chap. 9)). Wheeler's "it from bit" approach does not rely on a semantic jump between the technical and everyday senses of information since, within that view, quantum information is assimilated to none of these notions. Instead, quantum information has an ontological status as a fundamental entity from which every physical entity derives its existence (Wheeler, 1999).

some evidence for an antirealist reading of quantum physics. However, as argued in (Timpson and Maroney, 2013) and (Felline, 2018), it is very clear that Qbism, like any interpretation of quantum mechanics, is backed-up by a whole apparatus of philosophical premises that are not deduced from any formal content of quantum theory. In particular, Qbism is not deduced from operational physics, and its natural affinity with this formalism comes from its programme of relying on information-theoretic axiomatisations of quantum mechanics to identify the objective content of the theory (Fuchs, 2004). Similarly, it is argued in (Timpson and Maroney, 2013, Chap. 3)[16] that the "it from bit" approach arguably needs specific philosophical background assumptions to be fully articulated and describe how our material world can be derived from a fundamental notion of quantum information. Those assumptions are not deduced from the operational formalism, which therefore needs to be supplemented with this additional philosophical machinery. Hence, the "it from bit" approach builds on information-theoretic formulations of quantum mechanics, but is not deduced from it, leaving room for alternative readings (see, e.g., section 3.4.2). With that being said, and as will be reminded in section 3.4.3, the convenience of the operational framework relative to a specific antirealist interpretation is not evidence for the likelihood of that interpretation, given that (i) equally convenient realist readings can be attributed to operational physics (see section 3.4.2), and (ii), even if the last point was not available, the empirical equivalence of a variety of formulations of quantum theory ensures that none has a privileged status when it comes to the interpretation of quantum mechanics.

3.4.1.2 "Operational physics strongly suggests an epistemic interpretation for quantum probabilities"

3.4.1.2.1 Summary of the argument

The argument discussed in this section was neutrally reviewed[17] by Leifer (2014, section 2.3) and can be summarised as follows: there exists an operational probability theory which generalises the classical probability theory in such a way that it has both quantum theory and classical probability theory

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[16] Felline (2018, p. 5) also provides an assessment of the philosophical premises behind an ontic reading of informational-theoretic formulations of quantum mechanics (which is a (weak) form of realism towards quantum mechanics in which the theory is about some new objective stuff that is quantum information), that might be relevant to the "it from bit" approach.

[17] I do not claim anything about the actual stance of Leifer regarding that argument.

as special cases. Hence, in this particular OPT, the quantum probability distributions "play the same role" as the classical ones (Leifer, 2014, p. 76). As a result, it seems natural to give them the same interpretation. Since classical probabilities are epistemic (as they originate from some ignorance of the observer regarding the exact physical state of the studied system), quantum probabilities should be seen as epistemic as well.

Does this argument support an antirealist view towards quantum mechanics? Interpreting the quantum probabilities as being of an epistemic nature means that the quantum state from which those probabilities are computed represents "a description of what an observer currently knows about a physical system. It is something that exists in the mind of the observer rather than in the external physical world. [...] The key property that this implies is that a given ontic state is deemed possible in more than one epistemic state. [...] [The quantum state] is simply a mathematical tool for determining probabilities, existing only in the minds and calculations of quantum theorists." (Leifer, 2014, p. 69-71)[18]. While this antirealist view towards the quantum state does not necessarily entail an antirealist view of quantum mechanics itself, it rules out the main realist accounts of quantum mechanics (namely Bohmian mechanics, the GRW theory and Everettian mechanics) as those are committed to an ontic conception of the wavefunction (Cabello, 2017), which is then considered to refer to objective elements of reality[19].

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[18] This epistemic notion of the wavefunction found in (Leifer, 2014, p. 69-71) is referred to as the "ψ-epistemic" reading of the wavefunction, as opposed to a "ψ-ontic" reading, in which the wavefunction refers to something objective in the world (Leifer, 2014). To avoid any confusion, it is worth mentioning that those terms "ψ-epistemic" and "ψ-ontic" have been used and defined quite differently by some authors posteriorly to Leifer's work, e.g. in (Cabello, 2017), where "ψ-ontic" has the same meaning as in (Leifer, 2014, p. 69-71) but "ψ-epistemic" is a more restrictive term that is actually applicable to the realist views in which the wavefunction is "representing knowledge about an underlying objective reality". In the present context, it is Leifer's broader notion of the ψ-epistemic view that is considered.

[19] The question of whether the wavefunction is indeed ontic (i.e. refers to something objective in the world (Leifer, 2014)) within the main realist accounts of quantum mechanics arguably depends on the particular ontology given to the theory. Indeed, if one subscribes to a nomological reading of the wavefunction, while not assigning any objectivity to laws of nature, strictly speaking it cannot be said that the wavefunction has an ontic status (Gao, 2019). However, this scenario is a very specific one among a variety of existing accounts.

3.4.1.2.2 Objection

A given interpretation of probabilities in the context of a given theory (in our case a particular OPT) is about the meaning of the probabilities (e.g. some agent's ignorance (epistemic origin) or the existence of some fundamental randomness in nature (ontic origin)), and this meaning depends (among other things) on the kind of objects and dynamics that are involved in the theory.

Yet, although what is meant by "quantum mechanics and classical probability theory are special cases of one more general OPT" is that we can use the same mathematical tools to express both classical and quantum theories, those mathematical tools do not necessarily carry the same ontological meaning in each case. In this particular OPT, a set of basic operational axioms allows for a wide range of correlations. Those correlations involve particular objects with particular dynamics, and all display the same operational features, namely those formulated in the axioms. These operational characteristics happen to encompass both classical and quantum correlations. Yet, there is no necessity for the objects and dynamics involved in the correlations to be of the same nature in the classical and quantum cases[20]. A straightforward example is the spin of an electron, a property possibly involved in a quantum correlation that has no classical counterpart.

Another reason to doubt that an OPT encompassing the quantum and classical theories describes objects and dynamics of the same nature is that it can be evidenced within the operational framework that there exists a fundamental distinction between quantum and classical operational theories, due to the presence of entanglement. In the review of Janotta and Hinrichsen (2014), it is explained how any non-classical probability distribution for a single system formulated in the framework of OPT can be expressed as a classical distribution by increasing the dimension of the state space. Conversely, any classical single system's distribution can transform into a distribution with non-classical features by imposing additional restrictions on its effect vectors, which lowers the dimensions of its state space. This result questions the fundamental necessity of the classical/quantum distinction, since non-classical

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[20] It may seem here that the argument presupposes realism. However, this reasoning holds whether the meaning we assign to the theory is of a realist or antirealist nature. Whether the theory is considered to be about objective entities or not, we need to specify what it refers to in order to take a stance about the meaning of its probabilities. The notion of "object" considered here is taken very broadly, encompassing any posits that could correspond to a realist or an antirealist view.

theories seem to be always reducible to classical ones by resorting to higher-dimensional vector spaces. However, Janotta and Hinrichsen (2014) also explain that this situation is no longer possible for composite systems' distributions, which cannot be expressed classically, no matter the dimensionality of its vector spaces. We can therefore conclude that there exist genuinely non-classical systems, fundamentally distinct from classical ones. The existence of quantum entanglement can possibly mean that there are different dynamics at play between the classical and quantum realms. Hence, quantum mechanisms among quantum objects different from the classical ones might be taking place, which would mean a possibly different origin (and therefore interpretation) for the measured probabilities.

In summary, since the OPTs encompassing both quantum and classical theories do not necessarily involve a unique kind of objects and dynamics, then OPTs do not necessarily involve a unique interpretation for the probability distributions. This point is overlooked in the antirealist argument because of the implicit character of operational physics regarding objects, properties and underlying dynamical laws.

As a concluding remark, we can quote the statement of Timpson and Maroney (2013, p. 187) regarding the ontological significance of operational theories:

> "The form of a theory convenient for placing it within a space of theories need not be the fundamental, ontologically revealing, form of the theory: identifying properties of the theory (the former setting) and saying how the world is (the latter) are different tasks; and it is not at all surprising that we might expect different formal representations of the theory to be more or less useful for these distinct tasks. An operationalist black-box formulation of a theory might be most appropriate for the former task, but there is no reason at all why that should be taken to be the final story, the end of discussion about the theory: there is still an ontological story to be told too—the underlying dynamics giving rise to the results schematised within the story of black boxes."

The operational formulation of quantum mechanics is an alternative way of expressing the same physics as the standard (model-dependent) formulation, emphasising correlations between inputs and outputs of experiments while the underlying mechanisms involving objects and their dynamics is left unspecified. Yet, it is this underlying story that is of interest when wondering about the meaning of quantum probabilities. The existence of an OPT encompassing both classical and quantum correlations merely means that one can find a set of common axioms (operational properties) satisfied by these correlations. It therefore gives information about the similarities that exist between the structures of the two theories. However, it does not entail that the same mechanisms underlay both classical and quantum correlations. As a result, it does not imply a same ontological story for classical and quantum theories.

3.4.1.3 "Device-independent physics evacuates the notion of systems, therefore physics is not about systems"

3.4.1.3.1 Summary of the argument

The argument that will be discussed is found in (Grinbaum, 2017), in which two claims are made:

Claim 1: "Incompatible with the old explanatory mode, device-independent models [21] typically do not meet the conditions for the emergence of robust theoretical constituents corresponding to real objects. By allowing no room for systems, they inaugurate the obsolescence of this elementary building block [...]. " (Grinbaum, 2017, p. 3)

Claim 2: "[...] physical theory is about languages: it is defined by a choice of alphabets for the inputs and the outputs [of a given experiment] and by the conditions imposed on this algebraic structure. Strings, or words in such alphabets, form a common mathematical background of device-independent approaches. [...] If strings are not 'about' some elements of reality, they can be said to be 'about' languages from which they are formed." (Grinbaum, 2017, p. 14 & 16)

The first claim is defended by appealing to two further statements: the notion of systems in a particular operational formulation of quantum physics introduces difficulties (statement A). Yet, the notion of physical systems is not a necessary ingredient in the process of interpreting a theory (statement B). Since it is auxiliary and problematic, we should get rid of it.

Statement B is stated in Grinbaum's work on the grounds that what is really necessary to describe an experiment are the notions of input and output linked to a given party (which can be seen (in spite of their different philosophical flavour) as the counterparts of the notions of preparation procedure, measurement result and experimental setup seen in section 3.2, respectively), while spatiotemporally defined notions such as physical systems are mere interpretative devices (Grinbaum, 2017, p. 7-8).

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[21] I.e. model-independent approaches to physical theories presented in section 3.2.

Statement A is, for its part, defended as follows: Grinbaum refers to the particular operational theory presented in section 3.3, called process matrix formalism (Oreshkov, Costa, and Brukner, 2012). As explained earlier, that theory generalises quantum mechanics by dropping the assumption of a global causal structure relating the inputs and outputs among different parties. In other words, while there is a local temporal ordering in a given party allowing to claim that some output temporally succeeds some input of that party, we make no claim regarding the ordering of inputs and outputs pertaining to different parties. Operationally, a theory for which no global causal order among parties is assumed may allow for the prediction of joint probability distributions for which there is no definite global causal order. We speak in that case of indefinite causal order, i.e that this distribution is incompatible with any definite causal ordering among parties, as expressed within the formalism[22]. Grinbaum claims that the notion of physical systems runs into three difficulties in the context of this particular theory (Grinbaum, 2017, section 3):

- The absence of global causal structure makes it impossible to make sense of the spatiotemporally defined notion of physical system (as being spatially delimited at all times from the rest of the environment) that would endure and evolve across the multipartite experiment.
- The absence of global causal structure allows for strange situations where "'systems' in the process matrix framework may 'enter' the same local laboratory twice", which Grinbaum considers to be a situation that never happens to a physical object.
- A framework allowing for an absence of global causal structure predicts indefinite causal orders not solely for some quantum correlations, but also for certain multipartite "classical"[23] correlations, which shows that the difficulties faced by the notion of physical systems do not arise exclusively in a quantum context.

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[22] As will be discussed later, the question of whether causal relations are indeed definite or not (ontologically speaking) is not straightforward. The answer will depend on the particular reading (interpretation) that will be given to the theory.

[23] A word of caution is needed regarding the use of the term "classical". Some non-causal correlations can indeed be said to be "classical" in the sense that their corresponding process matrices are diagonal, which means that the probabilities can be interpreted classically. However, in a more stringent way, these correlations are undoubtedly nonclassical. Indeed, any non-causal correlation lies outside the polytope of classical correlations, for which everything is well-defined.

Grinbaum concludes from these three points that the notion of physical system is problematic in the case of indefinite causal orders.

Now, regarding the second claim of the main argument, Grinbaum postulates that physics is about languages, on the grounds that such a proposition provides a common philosophical background for all physical theories, and that this background ontology is minimal, yet sufficient to answer the question "What are physical theories about?". Such a view can be read as a form of scientific antirealism, since according to that position, science ceases to speak about an objective external reality independently of the language used to describe it. Instead, science is about language itself. In particular, Grinbaum explicitly mentions that the kind of language that he considers is of a formal nature, rather than of the everyday kind (Grinbaum, 2017, p.16).

3.4.1.3.2 Objections

The point is not to criticise the ontology proposed by Grinbaum (claim 2), nor its overall project (Grinbaum, 2007), but to object to a very specific part of his argument, namely the idea that considerations from the operational formalism alone preclude realist accounts of quantum physics appealing to physical systems (claim 1).

In the previous sub-section, we recalled Grinbaum's list of the various difficulties that the notion of system introduces when we deal with the notion of indefinite causal order. Yet, the core problem faced by the notion of physical system in the context of indefinite causal orders is stated in the first element of that list, and this difficulty points towards the fact that a realist account of quantum physics necessarily needs to revise our initial conception of how causal correlations are to be understood/conceived.

However, to this day, this challenge has not proved to be insurmountable, and future work will tell us what are the realist implications about quantum causality. Similar challenges are already present in standard quantum mechanics, in which a realist attitude towards nonlocality forces us to choose among a range of deep metaphysical implications such as retrocausation, the existence of a preferred frame in the universe or non-intuitive fundamental ontologies for the world (Maudlin, 2011). Such metaphysical questions only become more pressing in more general operational theories, which predict new phenomena such as indefinite causal orders.

In that new context, any realist account needs to address the following question: are indefinite causal orders objective, or are they purely theoretical notions with no objective counterpart? As seen in section 3.3.3, indefinite causal orders have not yet been observed purely operationally in terms of physical correlations violating the causal equivalent of a Bell inequality (Oreshkov, Costa, and Brukner, 2012; Branciard et al., 2015). For the purpose of his argument, Grinbaum takes their existence as a presupposition. Yet, to this day, their objective existence or non-existence remain to be proven.

If we do take indefinite causal order to exist in nature, the very question of whether spatiotemporal notions indeed become ontologically indefinite in such a situation remains open[24]. Grinbaum's claim about the absence of spacetime background (with respect to which enduring and evolving spatiotemporally localised objects can be defined) is actually not a necessity, but an interpretational possibility. As a counter-example to this scenario, it seems possible to make sense of spatiotemporally localised objects in the context of indefinite causal orders by appealing to some forms of holism of the dynamical properties of physical objects (see section 5.5 for more details).

Alternatively, one could also investigate the possibility to conceive the notion of physical system in a less stringent way than what is considered by Grinbaum (which promotes the idea of a spatially well-delimited object that endures through time). There exist ontologies that do not rely on objects enduring through time (such as the flash ontology developed in the context of the GRW theory (Ghirardi, 2018; Allori et al., 2008)), or having an intrinsic identity (such as some ontologies involving a relational notion of objects as in some ontic structural realist views (Ladyman, 2020, section 4)). Those particular ontologies of objects could possibly make sense of a notion of physical system that would not be problematic in the context of an indefinite causal order.

Finally, even in the scenario of a non-fundamental spacetime background (which is actually a possibility that is seriously contemplated in certain research programmes in quantum gravity (Huggett and Wüthrich, 2013)), this would not necessarily mean that the notion of physical system should be given up altogether (in the sense described by Grinbaum). Indeed, there are different ways to articulate fundamental ontologies not based on spatiotemporal notions, from which our familiar notion of physical (in a standard spatiotemporal sense) system would emerge (Lam and Wüthrich, 2018; Huggett and Wüthrich, 2013; Le Bihan, 2018). More precisely, as argued in (Lam and Wüthrich, 2018), the most pressing challenge to meet when faced with a theory

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[24] The following discussion will be further developed in chapter 5

in which spatiotemporal concepts would be emergent (instead of fundamental), is to guarantee a formal and philosophical connection between the fundamental non-spatiotemporal entities postulated by the theory and the empirical realm (which seems to unavoidably involve spatiotemporal notions), so that the theory can actually be tested. In other words, it requires a formal consistency between the theory and that of general relativity (our current best theory of spacetime), but it also requires a metaphysical account of how spatiotemporal notions can emerge from non-spatiotemporal ones. This twofold requirement can be referred to as a requirement of empirical coherence (Huggett and Wüthrich, 2013). Multiple strategies have been offered to reach (at least part of) that objective. Huggett and Wüthrich, 2013 have argued that, given a formal theory in which spacetime would be emergent (and provided that the theory is true, which presupposes scientific realism), a formal derivation of the spatiotemporal notions from the non-spatiotemporal ones would secure in itself theoretical and conceptual empirical coherence. Still, spelling out this emergence metaphysically remains a challenge[25]. To undertake this task, as examples, a compositional (mereological) and functional approaches have been proposed by Le Bihan, 2018 and Lam and Wüthrich, 2018, respectively.

In conclusion, the new notion of indefinite causal orders allowed by a particular operational probability theory generalising quantum mechanics does not force us to abandon scientific realist accounts of quantum physics. Grinbaum's argument is a proposal among other possible ones, rather than a conclusive argument ruling out specific realist accounts of quantum physics.

Overall, the three arguments discussed above for an antirealist view of quantum mechanics based on the operational framework for physical theories alone are not conclusive arguments. At best, they are interpretative possibilities, or "antirealist readings" of the operational formalism (which is not "already containing" hints for such an antirealist view). We should acknowledge the specific philosophical assumptions that are presupposed for such anti-realist readings of operational quantum mechanics.

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[25] This point is discussed in (Ney, 2015) in the context of wavefunction realism, where the standard 3-dimensional spacetime emerges from the high-dimensional space in which the wavefunction lives.

3.4.2 Realist approaches in the context of operational quantum mechanics

We saw in section 3.4.1 that the meaning of the probabilities in the operational framework are left "untouched/uninterpreted", and for that reason, the various arguments for antirealist views are not convincing when grounded in the operational formalism alone. Indeed, it was emphasised in section 3.1 that the notion of quantum information used in information-theoretic approaches is of a technical nature, and extra assumptions need to be added if one wants to give it an antirealist flavour. It was defended in section 3.2 that the meaning of probabilities was tied to a certain extent to the theory's ontology: whether we postulate that the theory is about objective entities or not, and whether those entities are all of the same nature with the same dynamics and properties or not, it is still an extra layer that we apply on top of the bare formalism. Finally, section 3.3 highlighted the fact that, in order to make sense of the formal notion of indefinite causal order, we need to postulate a meaning for the theory, whether realist or antirealist. It was argued that there is no necessity to follow either of those two attitudes. In summary, extra-assumptions were needed in all these three cases to jump from the operational formalism to an antirealist reading.

This section will now emphasise that the situation is similar for realist approaches. Indeed, any realist reading of the notion of quantum information needs an extra-assumption containing a postulated ontology for the theory under consideration. Interpreting the meaning of probabilities in a specific OPT requires to postulate an ontology (extra-assumption) for the objects involved in the described correlations. Explaining the notion of indefinite causal order in a realist fashion implies to postulate an appropriate ontology as well. Yet, a bare formalism does not impose any particular ontology, *a fortiori* a realist one [26]. To be sure, when adopting a scientific realist attitude towards a particular theory, the formalism in which it is expressed might suggest certain metaphysical claims (e.g., the objective nature of the wavefunction in certain formulations of quantum mechanics). Yet, those claims are incomplete (what is the exact nature of the wavefunction ? Is it a law of nature, a field, a property ?), and their completion requires to postulate additional content that is constrained, *but not uniquely prescribed*, by the formalism. To sum up, no matter the approach (realist or antirealist), we must postulate a meaning which is

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[26] One could argue that the idea of a theory's ontology makes sense only for realists. I use the expression "theory's ontology" in a rather liberal way, to refer to the meaning assigned to a theory's formalism. That meaning can coincide with a postulated world's ontology, or with subjective of fictitious entities.

not prescribed by the formal characteristics of the theory.

Hence, for similar reasons to the ones discussed in section 3.4.1, there is no convincing argument favouring realist approaches to quantum mechanics grounded in the operational formalism in itself. Yet, such realist approaches are not only possible ((Grinbaum, 2007), for that matter, recommends that realist approaches take certain axiomatic reconstructions as a starting point), but possibly well accommodated by that formalism. As discussed in section 3.4.1.2, the operational framework puts the emphasis on correlations. While some antirealist views interpret these correlations as describing either possible objective (yet unwarranted epistemologically) phenomena affecting real entities, or irreducibly subjective notions based on the observer's experience of reality, realist approaches can view these correlations as describing real objective (epistemologically warranted) phenomena among different objects. Various interpretations specify different kinds of underlying mechanisms explaining those phenomena.

As an example, a proposition in the realist camp, called ontic structural realism (see chapter 2) (Ladyman, 1998; Lam, 2017; Ladyman, 2020, and references therein), claims a particular status for structures (as a set of physical relations) in the fundamental ontology of the world. Such a proposal applied to quantum mechanics, while not constituting a complete interpretation in itself[27], can be seen as an interpretative tool to articulate the metaphysical implications of quantum mechanics in a coherent way. Because the operational framework puts the emphasis on correlations, hence on physical relations connecting different systems, ontic structural realism seems to be a particularly well suited/convenient way to interpret the operational framework for physical theories. Such a strategy was also considered in (Koberinski and Müller, 2018).

In conclusion, each realist/antirealist camp can read the operational formulation of quantum mechanics according to their own view. The operational formalism in itself, not only does not explicitly favour a particular view, but also does not present any particular difficulty for any of them either. That point is further discussed in the last section of this chapter.

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[27] An ontic structural realist reading of quantum mechanics alone does not provide a solution to the measurement problem, and can be used in the context of different realist interpretations of the theory (Lam, 2017).

3.4.3 The realist/antirealist debate is formalism-independent

We have seen in previous sections that extra-assumptions were needed to go from a bare formalism to an interpretation of the theory. These additional assumptions provide a meaning to the theory, which agrees with either a realist or an antirealist view. Both possibilities are equally well accommodated by operational formalisms because those extra-assumptions are motivated by purely philosophical considerations, that are postulated on top of sole considerations from specific formulations of theories.

The idea that arguments in favour of epistemic antirealism are formalism-independent is a very old and uncontroversial view: they are based on general considerations about the dynamics and methods of science and experimental success of scientific theories, and are independent from the particular form of a theory. Anti-realism about quantum mechanics discards realist proposals postulating a direct link between the theory's formalism and the content of the objective world on the grounds of more general considerations such as the "pessimistic meta-induction" argument (Chakravartty, 2017c, section 3.3), or on the grounds that their implications for reality are unavoidably implausible or too underdetermined (Van Fraassen, 1980), or on the grounds that an antirealist approach is ontologically/metaphysically more cautious (Grinbaum, 2009). The formalism-independent nature of the argument is obvious in the case of the pessimistic induction argument, and in the case of the underdetermination of interpretations of quantum mechanics. The argument favouring antirealist approaches on the grounds that they do not need to postulate any ontological content for the world is again an argument of an epistemic nature (Grinbaum, 2009). The fact that this argument pairs up well with operational formulations of quantum mechanics that do not appeal to any particular model for the described experiments does not change its epistemic status.

Yet, it is interesting to make this point explicit in the case of the antirealist argument based on various kinds of (what are sometimes called) 'no-go theorems'. This argument, as it is well-known, contemplates the implications for the nature of reality when adopting a realist approach in the light of various theorems such as Bell's theorem (Bell, 1964), the Kochen–Specker theorem (see Kunjwal and Spekkens (2015) for the operational version), or more recently Shrapnel-Costa's theorem (Shrapnel and Costa, 2018). The implications of those theorems for realist models are considered to be too implausible by some antirealists, and form "no-go results" that should be taken as a hint that the theory is to be interpreted on different grounds, namely by rejecting the idea that the theory is about an objective reality (see Wiseman and Cavalcanti (2017) for a discussion). Because those theorems are/can be expressed within the operational framework, it is useful to illustrate explicitly how such an antirealist argument relies on assumptions independent of the formalism itself.

Let's reconsider the case of Bell's theorem, while the narrative is the same for the other theorems. The field of application of this theorem is broader than quantum mechanics, and it applies to abstract models satisfying certain kinds of criteria. As famously known, in a nutshell, Bell's theorem states the following: "local models" imply certain empirically testable predictions (Bell, 1964). Experiments show that those predictions are violated. Therefore, there is no local model that can account for experience. As a result, successful realist models of quantum mechanics are necessarily nonlocal. The demonstration of Bell's theorem does not appeal to the particular formalism of the models, what this formalism represents and what kind of dynamics is described. The reasoning only appeals to an abstract notion of models, i.e. an unspecified formalism that generates probability distributions as predictions. Among the assumptions required to demonstrate the theorem, the ones constraining the models themselves are their empirical success, their use of a variable λ that describes entirely the observed system, and the requirement of local causality. In that context, it does not matter whether we speak of the standard ("Hilbert space") formulation of quantum mechanics, or of a particular operational axiomatisation of the theory. It is mainly experimental considerations (modulo the extra-assumption often referred to as the "free choice" of the measurement settings[28]) that imply nonlocal features of theoretical models accounting for quantum experiments. Realists found various ways to develop a world's ontology in which nonlocality is an objective feature of nature (e.g. quantum ontologies of Bohmian mechanics, GRW theory or Everettian mechanics, ...). Those who claim that the idea of nonlocality as an objective feature of nature is not plausible will reject any realist model, irrespectively of the particular formalism used to express that model.

This reasoning holds as well for the Kochen–Specker (in its operational form) and Shrapnel- Costa's theorems. The operational formulation of the Kochen–Specker theorem (and the more general version of Shrapnel and Costa) shows that any model empirically successful is necessarily contextual. Their results therefore set particular constraints on any ontological model that would agree with the predictions of quantum physics. To reach those conclusions, they rely on a conjunction of general physical principles and empirical predictions. Formally speaking, they appeal to abstract models, which makes them

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[28] This assumption is usually taken for granted, but has been critically discussed in the literature, see e.g. a review of past discussions in ((Berkovitz, 2016, and references therein)), or more recent discussions in (Hall, 2010; Friedman et al., 2019).

independent from any formalism. These results can either be taken as information about the objective world or not. Such a realist or antirealist stance is motivated by purely epistemic considerations.

To conclude, in the same way that antirealist arguments based on no-go theorems expressed in operational frameworks crucially appeal to certain premises that are not derived from (nor constrained by) the formal features of the formalism (namely the implausible character of objective nonlocal features in nature), the antirealist arguments reviewed in this section appeal to (hidden) extra-assumptions that are formalism-independent, whether they fall under the scope of metaphysics, semantics or epistemology. Operational frameworks are epistemologically and ontologically (in the sense presented in section 3.4.2) neutral in themselves.

3.5 Conclusion

This chapter presented the operational framework for physical theories and, in particular, a recent development of quantum mechanics called the process matrix formalism. Because this formalism will be investigated in a scientific realist framework in the next chapters, it was then argued that, contrary to a certain antirealist tendency in the field of quantum foundations, a realist attitude was just as suited to interpret operational theories as antirealist approaches.

In more details, we reviewed three arguments according to which operational formulations of quantum mechanics contain hints supporting an antirealist reading of the theory. Objections were provided, allowing to conclude that those arguments were not convincing. It was also argued that the operational framework was not providing any arguments for favouring a realist reading over antirealist ones. It was recalled that such results are expected, given the fact that the scientific realist/antirealist debate in the context of quantum mechanics is located at an epistemic level, and is not concerned by the specific form of the theory.

This whole discussion leads us to an important main claim, namely that the operational framework for physical theories is both epistemologically and ontologically neutral in itself. First, the operational framework is epistemologically neutral since the arguments in defence of an epistemological stance towards quantum physics do not appeal to any formalism in particular; their success is not reinforced or lessened in the operational formulation of quantum mechanics compared to the situation in the standard formulation. Second, the operational framework alone is ontologically neutral since going from operational postulates to a proposal about the theory's ontology implies specifying the status of the correlations at the centre of the formalism, this status being postulated on top of the formal aspects of the theory.

In the lights of these reflections, we can proceed to the next step of this work, which is the analysis, within a realist framework, of the central feature of the process matrix formalism.

Chapter 4

A formal analysis of causal nonseparability

As announced in the previous chapter, causal nonseparability will be analysed while adopting a realist approach. Before doing so, this chapter will prepare the discussion by clarifying the formal similarities and differences that exist between causal and quantum nonseparability. This will allow to emphasise in what way causal nonseparability is conceptually different than the standard notion of quantum entanglement. The discussion will then shift from a model-dependent approach based on the Hilbert space formalism to a model-independent approach based on an analysis of correlations existing between simple measurement results. In that context, causal nonseparability connects with the notion of noncausality, just like quantum nonseparability connects with Bell nonlocality. A discussion of the relation between nonlocality and noncausality is made, again with the objective of emphasising on the conceptual differences between these notions.

4.1 Comparison between quantum and causal nonseparability

Having presented the notion of causal nonseparability and noncausality in the previous chapter, we will now discuss the formal connection between the notions of quantum and causal nonseparability. Because those two notions are based on the density matrix and the process matrix, respectively, we will start by contrasting those two mathematical objects.

As mentioned in chapter 3, the process matrix W can be seen as a generalisation of the density matrix ρ. Yet, those two concepts are two distinct mathematical objects of a different nature.

First of all, these two objects describe different notions:

- The **density matrix** describes the *quantum state* of a given system.
- The **process matrix** describes the *process that mathematically describes certain relations between* quantum events.

Mathematically speaking, the process matrix does not encode relations among *quantum states*, as its purpose is to connect quantum *operations*. While a quantum state is, mathematically, a vector in a Hilbert space or a density matrix acting on that space, a quantum operation is a map describing how such vectors or density matrices are transformed into other vectors or matrices. Because a process does not encode quantum states, but relations among certain events, it describes how certain inputs and outputs of quantum operations are connected to each other [1].

As a result, as mentioned in chapter 3, the density and process matrices are different objects acting on Hilbert spaces having different structures: while the density matrix of a composite system (e.g. a bipartite system made of sub-systems A and B) acts on the tensor product of the Hilbert spaces associated to each sub-system ($\mathcal{H}^A \otimes \mathcal{H}^B$, with $\mathcal{H}^X$ being the Hilbert space of the system X), the process matrix of a process (e.g. relating the quantum events tied to parties A and B, each of them performing some linear operation on a physical system) acts on the Hilbert space $\mathcal{H}^{A_1} \otimes \mathcal{H}^{A_2} \otimes \mathcal{H}^{B_1} \otimes \mathcal{H}^{B_2}$, with $\mathcal{H}^{X_m}$ being the Hilbert space of the system of party X before the transformation ($m = 1$) or after the transformation ($m = 2$).

This last point is better illustrated by two particular process matrices relating the laboratories of two parties labelled A and B, each performing a local quantum operation on some quantum system. For simplicity, we make the hypothesis that the studied systems are not correlated with their environment.

Let's consider two particular cases:

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[1] It might be the case that an initial *preparation procedure* is encoded in a given process, with the effect of imposing a particular input state for the process. Yet, in a general situation, this preparation procedure can be left as an unfixed operation performed by some additional party. In that case, that preparation procedure is an operational *event* (i.e. a quantum operation) to be connected to other events in the process, and does not imply the encoding of a fixed quantum state *per se*.

1. TWO DISTINCT SYSTEMS UNDERGOING OPERATIONS OF WHICH THE OUTCOMES ARE THROWN AWAY

   We consider two distinct systems (e.g. not correlated), one entering and leaving the laboratory A and the other entering and leaving the laboratory B. If, in this specific case, we ignore whatever leaves the laboratories, it can be shown that the process matrix describing such a situation has the following form:

   $$W = \rho_{H_{A_1}} \otimes \rho_{H_{B_1}} \otimes \mathbb{1}_{H_{A_2}} \otimes \mathbb{1}_{H_{B_2}} \tag{4.1}$$

   where notations for the indexes of Hilbert spaces are similar as in section 3.3. $\rho_{H_{X_1}}$ is the density matrix describing the quantum state of the system entering the laboratory X. $\mathbb{1}_{H_{A_2}}$ is the identity operator. The mathematical expression for W can be intuitively understood as follows: a preparation procedure initially specifies the kind of quantum state that enters laboratory A (namely, $\rho_{H_{A_1}}$) and that entering laboratory B (namely $\rho_{H_{B_1}}$). $\rho_{H_{A_1}} \otimes \rho_{H_{B_1}}$ therefore specifies the joint inputs for laboratories A and B. The outputs of laboratories A and B are not sent to any other party, and are therefore ignored. $\mathbb{1}_{H_{A_2}} \otimes \mathbb{1}_{H_{B_2}}$ describes just that. Since we ignore whatever leaves the laboratories, we can even consider trivial Hilbert spaces (i.e. of dimension 1, which does not relate to any physical degree of freedom) for H_{A_2} and H_{B_2}. The process matrix, describing the causal relation (or lack thereof in this case) among uncorrelated events, which are themselves reduced to input quantum states, therefore amounts to the density matrix $\rho_{H_{A_1}} \otimes \rho_{H_{B_1}}$.

2. ONE AND THE SAME SYSTEM UNDERGOING SUCCESSIVE OPERATIONS IN LABORATORIES A AND B

   Let's consider that the system first goes through laboratory A, and is then sent through a quantum channel to laboratory B. It can be shown that the corresponding process matrix has the following form:

   $$W = \rho_{H_{A_1}} \otimes C_{H_{A_2} H_{B_1}} \otimes \mathbb{1}_{H_{B_2}} \tag{4.2}$$

   where notations are similar as above, and $C_{H_{A_2} H_{B_1}}$ is the Choi-Jamiolkowski matrix obtained by applying the CJ isomorphism on the linear map sending the quantum state of the system after it has left laboratory A on the quantum state of the system before it enters laboratory B. $C_{H_{A_2} H_{B_1}}$ therefore describes the temporal evolution of the system between the exit of laboratory A and the entrance of laboratory B. $\mathbb{1}_{H_{B_2}}$ is the identity operator, expressing the idea that we throw away the outcomes of laboratory B.

> This process matrix encodes a temporal evolution of quantum states. Such a description cannot be encoded within the density matrix alone. This fact illustrates in what sense the process matrix is a generalisation of the density matrix.

As a conclusion of this comparison, we see clearly that the process matrix W and the density matrix ρ are distinct mathematical objects, with different inner structures. A process matrix W does not represent a quantum state, but a *process* that *relates* different quantum events involving physical systems. The sense in which the process matrix is generalising the concept of density matrix is that it can, in certain cases, reduce to (possibly composite) quantum states, but can also allow representing temporal evolution thereof. More globally, process matrices allow describing and study how local quantum operations can be combined to form a single global operation.

At this stage, it is important to insist on an important nuance. While the process matrix can be seen, in the specific sense presented above, as a generalisation of the density matrix, and allows thereby to encode transformations across time, it *does not* mean that process matrices are generalisations of density matrices in the sense of relating quantum states at different spatial *and temporal* locations. As discussed previously, a process matrix does not relate quantum states, but quantum events. As a result, the process matrix does not merely add the temporal dimension to the standard picture provided by quantum mechanics. Instead, it allows to study quantum experiments from a more profoundly distinct perspective. Indeed, it allows representing relations among events, independently of (i) the systems involved, and (ii) of the quantum operations. The first claim is true, as mentioned in section 3.3.2, when the preparation procedure for the input system of the process is not fixed by the process itself, and is rather left as an event occurring in an additional party. In that case, the process does not encode a particular quantum state, hence, a particular system. The second claim is true because while the process describes how the operations' inputs and outputs are connected together, the operations themselves are not specified. A process matrix is therefore profoundly different than a mathematical tool that would merely connect quantum states at possibly different times and locations. The temporal dimension enters the picture because we shift from a picture in which the variables are the quantum states to a picture in which the variables are quantum operations.

To sum up, process matrices shift the focus from relations between quantum states to relations between quantum events, and involve both spatial and temporal dimensions. They describe the global structure (possibly indefinite) underlying different quantum events. Hence, by asking what are the kind of underlying causal structures compatible with valid[2] process matrices, one investigates the causal structures possibly compatible with quantum mechanics.

Based on the above distinction between density and process matrix, we can now emphasise the formal difference between quantum and causal nonseparability. The former expresses that for some composite system, the global quantum state is nonseparable, the quantum states of the sub-systems being indefinite. The latter expresses that for some processes, the corresponding process matrix is nonseparable, and the order among the events within that process is indefinite. Hence, the two kinds of nonseparability are conceptually very different, as they describe quantum correlations among very different notions, namely quantum states in the case of quantum nonseparability and quantum events in the case of causal nonseparability.

As a remark, we can recall that in the case of a nonseparable quantum state of some composite system, it is sometimes said that there is an *interdependence* between the quantum states of the sub-systems. That is, certain quantum states can't be expressed independently from the others. In the case of causal nonseparability, however, one cannot say, in all generality, that the causal relations within a causally nonseparable process are "interdependent". Exceptions include specific cases such as the quantum switch. Indeed, as explained in section 3.3.2, the quantum switch can be interpreted as the entanglement of a causal structure with some control system[3], and this entanglement relation yields certain correlations among causal relations. In general, however, a causally nonseparable process cannot be straightforwardly interpreted as involving some entanglement with an ancillary system, and the global structure is said to be indefinite without this claim involving that causal relations forming this structure display particular correlations.

4.2 A model-independent analysis

As explained in chapter 3, the notion of causal nonseparability is "model-dependent", meaning that it is defined through the use of a specific formal language of which the aim is to represent a restricted class of physical systems, namely those obeying quantum mechanics locally while no definite causal

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[2] A valid process matrix satisfies certain constraints in order to ensure that only valid probability distributions (i.e. non-negative and normalised) are generated when applying the generalised Born rule. Those constraints are detailed in (Oreshkov, Costa, and Brukner, 2012).

[3] See (Costa, 2020) for a discussion of that particular point.

structure is assumed between different parties. By contrast, a model-independent approach does not take any representational stance towards the studied systems. Instead, these systems and whatever is performed on them are "unrepresented" by black boxes in which no further description is provided. We work only with the so-called "inputs" (i.e. measurement settings) and "outputs" (i.e. measurement outcomes) of such black boxes, i.e. with classical labels corresponding to some choice of operation and measurement result.

Noncausality, the model-independent counterpart of causal nonseparability, was presented in section 3.3.3. This section will explore the possible meanings of noncausality and its relation with Bell nonlocality. Such a discussion will strengthen the previous model-dependent statement (namely that causal nonseparability is, in some sense, more general than quantum nonseparability) from a model-independent point of view. The following analysis will also serve as a preliminary study that will be further developed in chapter 5.

4.2.1 Meaning of noncausality

As seen in chapter 3, in a *causal* theory, any event B that causally succeeds a given event A cannot signal to that event A, in the sense that any choice of a classical input for B cannot affect the probability distribution of party A's outcomes. By definition, noncausality (in the bipartite case) would be evidenced by a probability distribution incompatible with a one-way signalling between a pair of operations.

Importantly, this two-way signalling between a pair of quantum events would be *operationally* detected, which means that an experimentally measured probability distribution would be shown to be mathematically incompatible with a one-way signalling between the two events. No additional assumption would be made to reach that conclusion, which would be then obtained on purely empirical grounds. Yet, this two-way signalling between events would still need a physical explanation in terms of underlying mechanisms, and this account might be more subtle than an appeal to direct influences between the involved parties. To illustrate this, an analogy can be drawn with the situation with Bell nonlocal correlations.

The violation of a Bell inequality indicates the presence of Bell nonlocality as formally defined in Bell's theorem. The origin of these nonlocal correlations is explained differently depending on the specific account of quantum mechanics. A prominent example takes nonlocality as originating from a holistic dynamics ruling the evolution of the universe as a whole (Egg and Esfeld, 2015). There are other models that refute implicit assumptions in Bell's theorem, namely that influences travel time in a single direction, or that reality is metaphysically determinate. As a result, one can appeal to a form of retrocausality (e.g., the two-state vector formalism (Watanabe, 1955; Aharonov, Bergmann, and Lebowitz, 1964; Aharonov and Vaidman, 2008)), or embrace a form of metaphysical indeterminacy of some aspects of reality (Calosi and Wilson, 2019) to provide a (possibly partial) underlying explanation to the observation of nonlocal correlations.

In the present case of noncausality, we are in a similar situation. If we were to observe noncausal relations, those could themselves be physically understood in a variety of ways. However, there is an important difference between Bell and causal inequalities, namely the fact that *no spatiotemporal notion* is assumed when we deal with the definitions of causal and noncausal correlations.

Indeed, by construction of the process matrix formalism, the different parties involved in causal and noncausal correlations are not embedded in a presupposed spacetime. When assigning a physical interpretation to noncausal correlations, this spatiotemporal embedding needs to be done (or more exactly, *postulated*, see chapter 5). Once a spacetime embedding has been assigned to the bipartite experiment, the physical mechanisms accounting for noncausality can be postulated in a variety of ways as well.

Let's review the possible spacetime embeddings for a bipartite experiment (in which the parties are labelled A and B) displaying noncausal correlations (see Annexe B for a recall of causal structures as mathematically defined in the context of relativity):

1. A **is embedded in a region that strictly**[4] **causally precedes the region of** B
2. B **is embedded in a region that strictly causally precedes the region of** A
3. **The regions of** A **and** B **are spacelike separated**
4. **The causal structure between the regions of** A **and** B **is indefinite**

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[4] We note that A and B are necessarily embedded in distinct spacetime regions, in virtue of the fact that one cannot perform two distinct operations at the same spacetime points.

Options 1, 2 and 3 correspond to a definite causal structure among parties A and B. They show either a retrocausal signalling between timelike separated parties or signalling between spacelike separated parties (since noncausality indicates that we have neither no-signalling from A to B nor no-signalling from B to A.).

These phenomena could possibly be explained by the following strategies (see section 2.2.2):

1. **Specific ontological features:**

   - **Holism**: It is conceivable that the operational signalling from future to past, or between spacelike separated parties, that would be observed in a noncausal correlation could be the apparent manifestation of an underlying world's ontology of quantum events that is holistic across space and time.

     In other words, the process matrix formalism would allow describing a new kind of physical relation between quantum events, namely causal nonseparability, that does not supervene on the intrinsic properties of its relata (e.g. the spatiotemporal locations of the different parties). Accordingly, the ontology of the world would contain a holistic block of quantum events defined against a definite spacetime background.

     This would correspond to the existence, within the ontology of the world, of global dependencies across time and space between these events. Could this lead to the observation of signalling between spacelike separated quantum events or retro-signalling between timelike separated quantum events ?

     In order to obtain this result, we would need the following scenario. Let be a causally nonseparable quantum process generating noncausal correlations. It should encode holistic relations between quantum events in such a way that when a given party involved in the process performs an operation A on a closed system, the measurement outcome involved in this quantum event A holistically depends on the measurement setting of another future (or spacelike separated) quantum event involved in the process.

     Now, a given quantum event links one input quantum system with one output quantum system, i.e. it connects what is called *quantum* inputs and outputs. By contrast, the measurement setting and outcome of a given party (corresponding to a given quantum event) are *classical* inputs and outputs. Those are not encoded *per se* in the corresponding quantum event. Yet, those classical inputs and outputs are not independent of their quantum counterparts. Indeed, the underlying reason why a given input quantum system is connected to a specific output quantum system is that a specific operation has been performed on that input system and transformed it accordingly (hence, possibly a specific measurement setting has been chosen).

To summarise, a possible way to account for noncausality is to appeal to a form of holistic relations among quantum events that would encode symmetric dependencies among measurements settings and outcomes of different events. Such correlations cannot be ensured by causal nonseparability alone. Indeed, this feature does not allow to discriminate between certain quantum events based on their classical inputs and outputs. This fact is in agreement with section 3.3.3, where it is emphasised that causal nonseparability is necessary but not sufficient for noncausal correlations.

- **Structuralism:** An ontic structural realist approach towards process matrices can be adopted, so that the causal structure encoded in quantum processes would be considered real and fundamental. The quantum events (relata of the causal orders) would be derivative from, or on a par with, the fundamental relations encoded in the process matrices. The former option would be obtained by adopting a radical approach to OSR (see (Ladyman, 1998)) in which only relations are part of the fundamental ontology, while the latter option corresponds to a moderate view of OSR (see (Esfeld and Lam, 2008)) in which relations and relata are ontologically on a par.

  In this specific case, because the structure (connecting quantum events within a causally nonseparable process) that is considered real and fundamental is also such that it does not supervene on the intrinsic properties of these individual quantum events, a structuralist reading of the process matrices would coincide with a form of holism as described above.

  The obtainment of noncausal correlations from an ontic structuralist ontology of quantum events would then be possible if it allows for the following scenario: either (in the radical OSR view) the measurement setting involved in one quantum event holistically influences the measurement outcome of a past (or spacelike separated) quantum event, or (in the moderate OSR view) it yields a quantum event that is itself ontologically on a par with the relation connecting it to a past (or spacelike separated) quantum event corresponding to a specific measurement outcome.

Again, similarly to the discussion about holism, causal nonseparability alone would not be sufficient to ensure that a quantum process yields noncausal correlations, since it relates only quantum events defined by a connections between a quantum input and output, instead of distinguishing quantum events via their classical inputs and outputs. The fundamental relations among (fundamental or derivative) quantum events therefore needs to encode stronger correlations than what is described in causal nonseparability.

2. **Specific dynamical features:**

   - **Holistic dynamics**: One could imagine accounting for noncausality by appeal to dynamical laws with a holistic nature, such that a choice of measurement setting of one party would affect (via holistic dynamical influences) the state of a past (or spacelike separated) system, which itself would condition the measurement outcome of the party operating on it. As a result, signalling between spacelike separated quantum events or retro-signalling between timelike quantum events could be observed. A similar (less constraining) scenario will be further discussed in section 5.5 in the context of accounting for causal nonseparability alone (i.e. without the requirement of noncausality).
   - **Retrocausal influences**: Alternatively, one can imagine that, in a definite spacetime structure with no holistic dynamics, the dynamics ruling the world could allow influences to travel both forward and backward in time. Again, these influences should engage with the classical inputs and outputs of quantum operations, and not merely describe influences between quantum events indiscriminately of the specific quantum transformation taking place in the quantum event. Moreover, the possibility of signalling between spacelike quantum events should also be allowed by this dynamics, which should not be merely about retrocausal influences among timelike separated events alone.

Finally, one way to account for **option 4** is to postulate that there is no determinate spacetime underlying the noncausal correlations. The motivations and developments of that strategy will be presented in chapter 5. In a nutshell, noncausal correlations in that scenario would come from the fact that the underlying spatiotemporal structure would be indeterminate, which, under certain (possibly not physical) conditions yet to be discovered, could affect the correlations between classical inputs and outputs of different parties. In other words, noncausal correlations would appear when the classical inputs and outputs of different parties are connected to their spatiotemporal locations, while these are indeterminate. This scenario would render the probability distributions incompatible with any definite relativistic spacetime.

4.2.2 Comparison between nonlocality and noncausality

We saw in section 4.1 that causal nonseparability, while generalising in some sense quantum nonseparability, cannot be seen as a mere extension of quantum nonseparability (describing correlations among quantum states at a given time) to a description of correlations between quantum states at different times. The differences between causal nonseparability and quantum nonseparability are of a deeper nature. It will be shown in this section that their model-independent counterparts (namely noncausality and Bell nonlocality), reflect these conclusions as well.

First, noncausality possibly characterises a wider range of scenarios than Bell nonlocality. Indeed, as recalled in chapter 2, Bell nonlocality concerns a pair of spacelike separated parties. By contrast, noncausal correlations can be found among either spacelike or timelike separated parties. Second, when considering spacelike separated parties, Bell local correlations, being no-signalling, are necessarily causal (see implication 8 in Fig. 4.3). On the contrary, a Bell nonlocal correlation does not necessarily imply signalling (see implication 7 in Fig. 4.3). This shows that, in the spacelike separated scenario, noncausality points towards a more specific phenomenon.

Yet, the underlying physical mechanisms of noncausal correlations can be similar to that of Bell nonlocal correlations. For example, retrocausal models have been suggested to account for quantum mechanics and its nonlocal correlations (Sutherland, 2008; Vaidman, 2009; Friederich and Evans, 2019). As seen in the previous section, retrocausal dynamical influences could plausibly account for noncausal correlations, although they would need to be implemented differently than in the context of standard quantum mechanics in order to account for the observed two-way signalling.

Alternatively, we have seen that dynamical holism can be applied to quantum mechanics and Bell nonlocality (see section 2.2.2.2) as well as for noncausality. For example, in the context of Bohmian mechanics or variants of the GRW theory, some realist interpretations of the universal quantum state as a nomological entity yield a holistic dynamics ruling the world (Esfeld et al., 2014; Egg and Esfeld, 2015; Esfeld et al., 2020). This holistic dynamics allows explaining the existence of Bell nonlocality. However, as it stands, this kind of dynamical holism would not provide an explanation for noncausal correlations, as it does not yield two-way signalling. A different implementation would be needed in the case of noncausality.

Similarly, a holistic (and structuralist) ontology in standard quantum mechanics (e.g. the result of a radical OSR approach towards the relation of entanglement) would not be able to account for noncausality since it yields mere dependencies between quantum states across space. In the previous section, we suggested that the kind of holism (and structuralism) needed to account for noncausality should concern quantum events and involve the spatial *and* temporal dimensions.

Another possible route is the appeal to quantum metaphysical indeterminacy, which can account for standard quantum nonseparability (which is at the core of Bell nonlocality in quantum mechanics) as well as for causal nonseparability (at the core of noncausality). Since the indeterminacy in standard quantum nonseparability concerns only the quantum state, it would not be sufficient to account for noncausal correlations. By contrast, the possibility of an indeterminate spacetime mentioned in the previous section (and developed in chapter 5) could possibly fill that task.

In conclusion, in the same way that causal nonseparability generalises in some sense quantum nonseparability while still carrying a deeper conceptual difference with that notion, we have that noncausality possibly concerns a larger range of physical configurations than Bell nonlocality, and points towards a more specific phenomenon than Bell nonlocality, in which the temporal dimension plays a more central role than in any account of quantum mechanics. Nevertheless, similar interpretative strategies (namely retrocausality, holism or structuralism) may be able to capture the underlying physics behind Bell nonlocality and noncausality.

These claims will be developed in more details in section 4.2.4. In order to have a better intuition of what noncausality means in physical terms, and in order to understand better the similarities and disanalogies between Bell nonlocality and noncausality, it is interesting to look for a principle (constraining physical models) that is violated by noncausal correlations. Just like Bell nonlocal correlations violate the principle of local causality, what is the counterpart in the context of noncausality? On the one hand, the derivation of Bell inequalities has been obtained by assuming the principle of local causality (see section 2.2.2.1). On the other hand, causal inequalities have been derived by assuming the existence of a definite causal structure among parties and the fact that two-ways signalling among them is prohibited. How these different premises connect with each others ?

A natural intuition is guided by the assessment that the temporal dimension seems to more forcefully enter the picture when it comes to noncausality. Indeed, noncausal correlations can exist between *timelike* separated parties, and causal nonseparability characterises some quantum process, which encode relations between quantum events that are possibly distant across space *and time*. We also saw that the candidate physical mechanisms underlying noncausality were involving the temporal dimension in a stronger way compared to the same kind of mechanisms in standard quantum mechanics. Hence, following that intuition that noncausality might involve similar physical mechanisms yet in a more temporal manner, we will explore a potential candidate principle underlying causality that would consist in the temporal counterpart of local causality (the main assumption behind the second version of Bell's theorem). This principle is presented in the next section.

4.2.3 Temporal nonlocality

Different authors have suggested a notion of "temporal locality" that would play, in some sense, the role of the temporal counterpart of locality appearing in Bell's theorem. In particular, three such propositions can be recalled below:

1. **Locality in time** (Brukner et al., 2004)

   Fig. 4.1 shows how a temporal version of the standard CHSH inequality (see Fig 2.1) can be derived by replacing the assumption of *local causality* by those of *temporal locality* and *realism*:

   locality in time:"the results of a measurement performed at time T2 are independent of any measurement performed at some earlier or later time T1"

   Realism:"The measurement results are determined by "hidden" properties the particles carry prior to and independent of observation".

If we start from that set of premises and apply it to the scenario expressed in Fig. 4.1, we obtain a lightcone diagram which itself corresponds to a specific factorizability condition. From there, the rest of the demonstration is exactly similar to that in Bell's theorem (see Fig. 2.1), and we obtain the temporal version of the CHSH inequality.

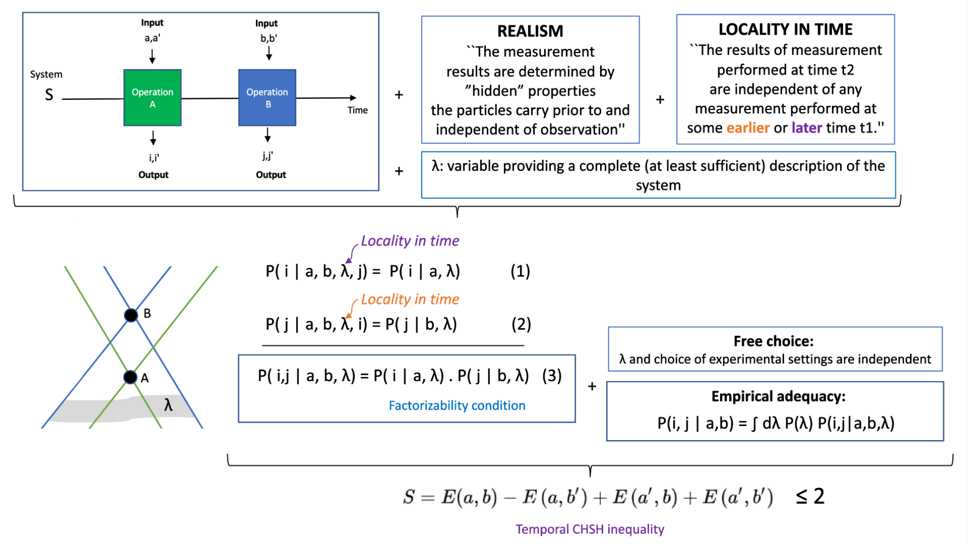

FIGURE 4.1: Temporal nonlocality: lightcones representation

2. **Noninvasive measurability** (Leggett and Garg, 1985)

   Another type of inequality has been derived, of which the violation indicates an incompatibility of observations with a set of two premises:

   Macrorealism: "A macroscopic object, which has available to it two or more macroscopically distinct states, is at any given time in a definite one of those states."

   Noninvasive measurability: "It is possible in principle to determine which of these states the system is in without any effect on the state itself, or on the subsequent system dynamics."

   The Leggett-Garg inequalities derived from these principle aim at testing whether our observations comply with the view called *macroscopic realism*, stating that macroscopic objects must always be in a determinate macroscopic state at any given time.

Leggett-Garg inequalities are sometimes called *temporal Bell inequalities*, because the experimental setup used to test the violation of these inequalities consists in a sequence of measurements of the same observable at different times. The setup involves then timelike separated parties, instead of the spacelike separated parties found in Bell's theorem.

Noninvasive measurability has, in addition, some connection with the premise of locality in time used by (Brukner et al., 2004), as it involves the idea that a measurement performed at a certain time does not disrupt the state of the system, hence does not influence measurement's outcomes at later times.

3. **Temporal nonlocality** (Adlam, 2018)

According to Adlam (2018), a definition of temporal nonlocality should correspond to the temporal counterpart of local causality. In the latter notion, the central idea is that whatever happens in a *spatial* region must have been mediated by causes *spatially* located nearby, and this mediation travels continuously through *space* at a subluminal speed. When defining temporal nonlocality, we keep that idea, well captured in the definition provided by (Adlam, 2018, p. 2):

Suppose that two observers, Alice and Bob, perform measurements on a shared physical system. At some time t_a, Alice performs a measurement with measurement setting a and at some time $t_a + \delta$ she obtains a measurement outcome i; likewise, at some time t_b, Bob performs a measurement with measurement setting b and at some time $t_b + \delta$ he obtains a measurement outcome j. Let $\lambda(t_a)$ be the state of the world at time t_a and let $\lambda(t_b)$ be the state of the world at time t_b. Then, temporal locality means that the following equalities hold:

$$p(i|a,b,\lambda(t_a),\lambda(t_b),j) = p(i|a,\lambda(t_a)) \tag{4.3}$$

$$p(j|a,b,\lambda(t_a),\lambda(t_b),i) = p(j|b,\lambda(t_b)) \tag{4.4}$$

Eq. (4.3) and (4.4) can then be combined to yield:

$$p(i,j|a,b,\lambda(t_a),\lambda(t_b)) \;=\; p(i|a,\lambda(t_a)).P(j|b,\lambda(t_b)) \qquad (4.5)$$

The above situation is pictured in Fig. 4.2.

Put simply, locality (whether spatial or temporal) means that whatever happens at a spacetime point, the outcome depends only on the information stored in the system's state at that particular time and place. This is summarised by Adlam as follows:

> " [...] all influences on a measurement outcome would be mediated by the state of the world immediately prior to the measurement." (Adlam, 2018, p. 2)

As Adlam points out, a precise meaning of temporal locality requires to specify the meaning and nature of the variable λ, hence, requires a full-fledged interpretation of the theory.

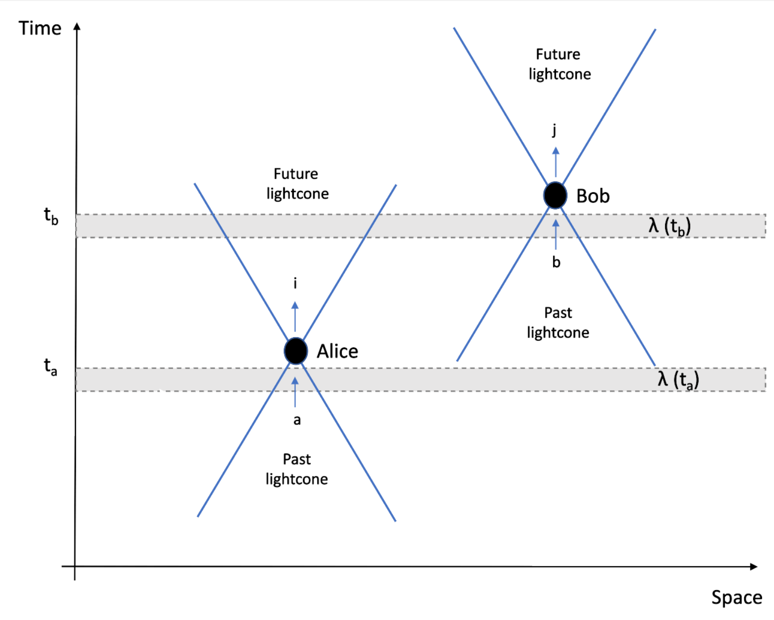

FIGURE 4.2: Schematic representation of a bipartite scenario characterised by the classical inputs a and b, the classical outputs i and j, and the variables $\lambda(t_a)$ and $\lambda(t_b)$. *Remark:* the parties Alice and Bob are here spacelike separated, but can be timelike separated as well.

Temporal locality as defined by Adlam seems more appropriate for our discussion, as it provides a temporal counterpart to the local causality principle used in the second version of Bell's theorem. As section 4.2.4 will show, this will make the analysis of the similitudes and disanalogies between nonlocality and noncausality easier. By contrast, it is less clear how the notion of locality in time of Brukner et al. (2004) should be connected to locality as defined in Bell's theorem (either in the first or second version). Regarding the Leggett-Garg inequalities, while these can be seen as temporal counterparts of Bell inequalities, this very assumption has been critically put into question in Timpson and Maroney (2013). In their work, Maroney and Timpson recall the existing debate regarding the exact significance of the violation of a Leggett-Garg inequality, and insist on the lack of a proper model-independent formulation of the *noninvasive measurability* principle.

4.2.4 Connection between noncausality and temporal nonlocality

To begin with, it can be showed that temporal locality (i.e. whatever happens at a specific time, the outcome depends only on the information stored in the system's state at that particular time) implies causality (i.e. a correlation is not two-way signalling) (see implication 5 in Fig. 4.3). Indeed, if temporal locality holds, then the statistical independence between the outcomes of one party and the settings of the other party (i.e. the no-signalling condition) ensures that the conditions for causal correlations are satisfied.

The converse is not true (see implication 6 in Fig. 4.3). Causality forbids two-way signalling between parties, but this condition is not enough to guarantee temporal locality. One might indeed conceive the existence of causal correlations violating temporal locality, e.g. (Bell) nonlocal correlations (i.e. violating a Bell inequality) as we will emphasise below.

The principle of temporal locality is indeed encompassing that of local causality (which means that the direct causes (and effects) of events are nearby, and even the indirect causes (and effects) are no further away than permitted by the velocity of light (Bell, 1995)) (see implication 1 in Fig. 4.3). Actually, an important point that Adlam makes (although without demonstrating it explicitly) is that temporal nonlocality as previously defined can be considered as a plausible implication of Bell's theorem. Indeed, it is rather easy to see that temporal locality implies the condition of local causality involved in Bell's theorem (see Fig. 2.1 which recalls Bell's theorem as expressed in the second version of 1976). Indeed, the experimental setup displayed in Bell's theorem corresponds to the situation in Fig. 4.2 in which Alice and Bob are spacelike separated. It is straightforward to see that Eq. (4.3), (4.4) and (4.5) lead to Eq.(1), (2) and (3) in Fig. 2.1 for spacelike separated parties, since there exists an inertial frame of reference in which $t_a = t_b$. As a result, the violation of local causality can be seen as originating from a violation of temporal locality. The converse is not true (see implication 2 in Fig. 4.3), i.e., local causality does not imply temporal locality, since the latter imposes constraints on either correlations among spacelike or timelike separated parties, while the former is exclusively about correlations among spacelike separated parties.

In conclusion, temporal locality as defined in section 4.2.3 implies both causality as defined in section 3.3.3, and local causality (hence, Bell locality). Fig. 4.3, in addition to summarising the previous discussion, also emphasises an important point. Bell's theorem demonstrates implication 3 (for spacelike separated parties), i.e. the assumption of local causality (along with additional side hypotheses) implies Bell inequalities. The implication 4 means that for any Bell local correlation (satisfying its corresponding Bell inequality), there exists a local causal model that can account for it. We see that the link between temporal locality and causality is not as strong, in the sense that it is not necessarily the case that there exist a temporally local model to account for any causal correlation.

This result was to be expected given the analysis of both sections 4.1 and 4.2: while the discussions of the possible underlying meaning of noncausality was pointing towards a key role of the temporal dimension compared to the situation with Bell nonlocality, it was emphasised earlier that causal nonseparability (the model-dependent counterpart of noncausality) was not to be understood as a mere extension of quantum nonseparability (the model-dependent counterpart of Bell nonlocality) across time. Instead, causal nonseparability describes a deeply distinct phenomenon by describing quantum correlations among quantum events. This disanalogy is found also when comparing the model-independent notions of causality and Bell locality: their difference runs deeper and noncausality cannot be reduced to a form of temporal version of Bell nonlocality.

Yet, this discussion has helped us understand why the temporal dimension plays a more central role in noncausality than in Bell nonlocality: Fig. 4.3 shows that a more constraining principle with respect to temporal mechanisms is violated by noncausal relations.

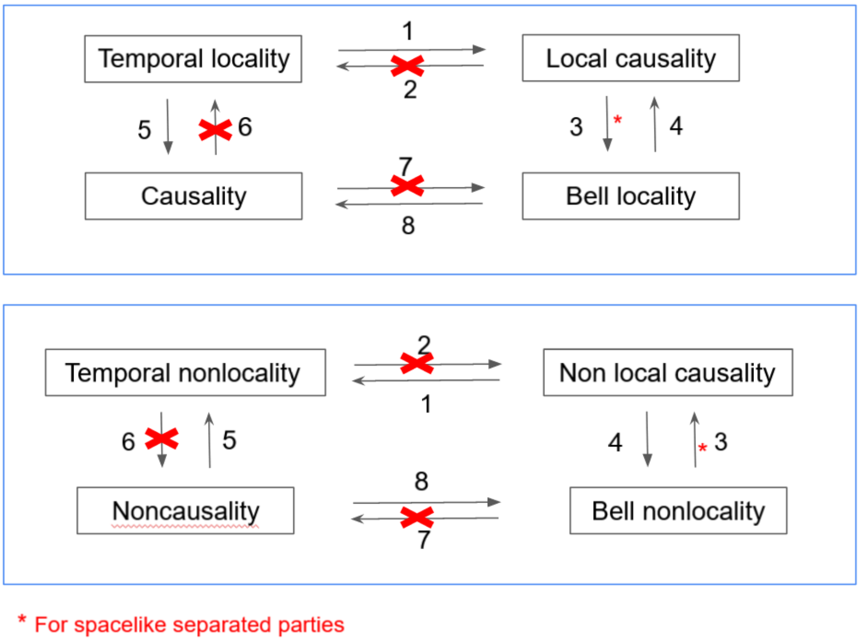

FIGURE 4.3: Summary of the various implications between the notions of temporal locality, local causality, causality and Bell locality.

4.3 Conclusion

The first part of this chapter clarified the nature of the notions of process matrices and causal nonseparability, and how they differ from the notions of density matrices and quantum nonseparability. Causal nonseparability cannot be seen as a mere extension of quantum nonseparability (which can describe correlations between quantum states at a given time) to a description of correlations among quantum states at different times. While quantum nonseparability expresses that for some composite system, the quantum states of the sub-systems are indefinite, causal nonseparability expresses that for some processes, the order among the events within that process is indefinite.

From there, it was argued that the model-independent counterpart of causal nonseparability, namely noncausality, could be possibly accounted for by a variety of views such as retrocausality, holism, structuralism and metaphysical indeterminacy of spacetime. (This last option will be discussed at length in chapter 5).

When compared to Bell nonlocality, noncausality can concern a larger range of scenarios, involving both spacelike and timelike separated parties. When considering spacelike separated events, noncausality is a more specific phenomenon than Bell nonlocality, as the former implies the latter but not reciprocally. Noncausality would also require to modify the available strategies to interpret quantum mechanics (e.g. retrocausal models, holism, structuralism, quantum indeterminacy) in order to involve more explicitly the temporal dimension and allow for two-way signalling.

These assessments motivated the search for a candidate principle violated by causal correlations that could possibly enlighten these considerations. The notion of temporal locality, as defined by Adlam (2018), was presented and it was shown that it implies both causality and local causality. The violation of a causal inequality indicates (while not being necessary to) an instance of "temporal" nonlocality (as defined in section 4.2.3), which is a more constraining premise than that of local causality used to derive Bell inequalities.

In the same way as each interpretation of quantum mechanics will spell out a particular meaning to the operationally defined Bell nonlocality, we need to specify the ontology and dynamics of the underlying theory violating a causal inequality in order to specify (within a realist framework) the details behind noncausality.

While a few possible routes were sketched in this chapter, the remainder of this work will try to motivate and develop in more details one route in particular[5], namely the idea that the causal structure of causally nonseparable processes is metaphysically indeterminate.

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[5] Although not in the specific context of noncausality, but rather in the less constraining situation of causally nonseparable processes.

Chapter 5

A metaphysical analysis of causal nonseparability

The previous chapter emphasised how noncausality, the model-independent counterpart of causal nonseparability, is linked to a form of temporal nonlocality. This chapter will explore this idea in more details, by investigating the kind of implications that one would face when adopting a realist attitude towards causal nonseparability. In particular, we will focus on their potential link with spacetime.

5.1 Scientific realism and the process matrix formalism

First, a word on the realist framework guiding this discussion. Since all currently known causally nonseparable processes can be described in standard quantum mechanics[1], it would be a perfectly coherent attitude to look at their characterisation within the process matrix formalism (hence, at process matrices and causal nonseparability) as purely formal, i.e. not capturing any novel physical features. Yet, it remains an open question whether there exist causally non-separable processes that are instantiated in nature, without being describable within standard quantum mechanics. Because such a scenario may well bring new physical and metaphysical insights, it is the option developed further in this chapter.

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[1] See Annexe A for a development of this claim.

Other motivations behind the realist attitude adopted here will be developed below as the discussion goes (see section 5.5): On the one hand, we might find potential theoretical virtues ensuing from ontologies in which processes play a central role. On the other hand, as suggested in section 2.1.4, a realist stance could help emphasising interpretative tensions between a classical spacetime background and quantum behaviours of matter.

A realist account of the process matrix formalism agrees with the view that its central object, the process matrix, refers to some objective features of the world. In that context, one has to articulate the exact meaning of a process matrix and the new idea that, for a causally nonseparable process, there is no well-defined causal structure among the events related by the process.

This task is vast and can be undertaken in a variety of ways. In this chapter, we will adopt a broader viewpoint for exploring the general consequences of causal nonseparability, which does not commit to a particular ontology for the theory or a particular solution to the measurement problem. Namely, we will focus on the notion of indefinite causal order, and motivate a natural connection to the idea of indefinite spatiotemporal structures. The role of spacetime in non-relativistic quantum mechanics is to provide a fixed background stage with a Galilean geometry for events to take place. This status still holds within the process matrix formalism[2]. Yet, in spite of spacetime's supposedly passive role in this theory, adopting a realist attitude towards causal non-separability can have philosophical implications for spatiotemporal relations, as the notion of indeterminate spatiotemporal relations suggests. From there, we will explore different readings of that indeterminacy (namely the epistemic and metaphysical ones), and the way they might impact the notion of spacetime itself. It is worth noting that although one particular reading of indeterminacy will not be compatible with all quantum ontologies and solutions to the measurement problem, the reflections presented below can be developed independently of such considerations. Finally, it will be interesting to see to what extent such implications are already suggested within standard quantum mechanics.

5.2 The quantum switch as a case study

Let's consider the particular causally nonseparable quantum process presented in section 3.3.2, called the quantum switch. For the sake of this argument, it is

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[2] See Paunković and Vojinović (2020) for a discussion of that particular point.

useful to recall the structure of this process (see Fig. 5.1).

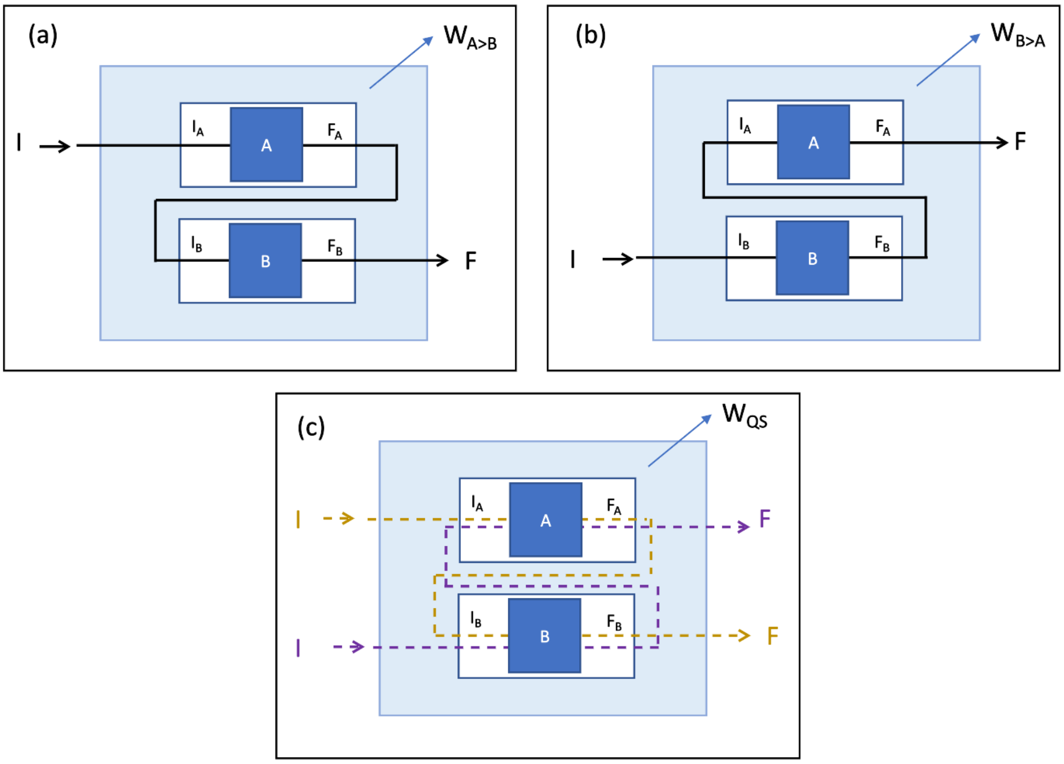

FIGURE 5.1: (a) Schematic diagram of a process describing the definite causal structure $A \prec B$. (b) Schematic diagram of a process describing the definite causal structure $B \prec A$. (c) Schematic diagram of a process describing the quantum switch. The labels I and F represent the input and output of the process, respectively. I_X and F_X represent the input and output of party X, respectively.

The quantum switch maps two local operations (noted A and B) on a global one. The map is mathematically described by a causally nonseparable process matrix noted W_{QS}, encoding a causal structure that is said to be *indefinite*. More precisely, the global structure combining operations A and B is *entangled* with the state of a control qubit. As explained in section 3.3.2, it is common, for the sake of clarity, to ignore the presence of this control qubit (as well as that of the third party, Fiona, acting on that control qubit) in the description of the QS. Instead, it is simply said that the QS as is in a *superposition* of the definite structures $A \prec B$ and $B \prec A$.

At this stage, causal nonseparability and indefinite causal orders are purely formal concepts. Assigning them a meaning is the next step, which will be discussed in sections 5.3 and 5.4

5.3 Indefinite causal orders

As the appellation "indefinite causal orders" suggests, such a phenomenon seems to indicate some indefiniteness of causal relations. Recall that the notion of causal order here is purely operational, being defined by the relations among inputs and outputs of different quantum operations (see section 3.3). In that sense, we are working with a minimalist notion of causal relations, that matches our common intuition according to which a particular event in a simple, linear, chain of events influences, hence 'causes', to a certain extent, the event that directly follows[3]. The causal relations in the indefinite causal orders considered here are therefore neutral concepts with respect to any account of causation[4]. In particular, it is neutral regarding whether causal relations are reducible to non-causal entities or not, articulated within a Humean view of the world or not, objective or not.

With that being said, it is widely held in philosophy of causation that causal relations are definite. This claim can be expressed through the following principle:

Determinacy of Causation (DetC). If c causes e, then *it is determinate* that c causes e.

Yet, while we will discuss in a while how the quantum switch challenges this principle, it is important to mention that there already exist different counterexamples to this principle in the literature (Sartorio, 2006; Ballarin, 2014; Bernstein, 2014; Swanson, 2017). One such example, called the *Battlefield*, goes as follows (Sartorio, 2006). A soldier S is on a battlefield, with four allies and four enemies. Each of those enemies is about to kill one of the allies. The soldier S has only one bullet left. Not knowing which enemy to kill, she didn't fire the bullet. All the four allies were then killed. This scenario displays a case of indefinite causal relation: did soldier S cause the death of ally 1? What about ally 2, 3 and 4? It seems wrong to say that soldier S caused the death of all of her allies. It seems equally wrong to claim that she caused the death of

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[3] Indeed, in a causally separable process, the causal structure is definite, and describes a simple chain of events ordered in a certain way. For example, in the bipartite case with two parties $\mathcal{A}$ and $\mathcal{B}$, a causally separable process will encode the causal structure $\mathcal{A} \prec \mathcal{B}$, or the causal structure $\mathcal{B} \prec \mathcal{A}$, or a probabilistic mixture of the two.

[4] See Beebee, Hitchcock, and Menzies (2009) for a comprehensive review of the field of philosophy of causation.

none of them. Finally, it also seems wrong to say that she killed one particular ally instead of another one. Her causal role in any particular death appears to be simply indeterminate.

Several strategies have been developed to analyse the *Battlefield* scenario in a way that assigns a determinate status to the causal relations among the protagonists:

1. **Denying the causal role of omissions**[5]. If one does not consider that omissions can be causal relata in a causal relation, the Battlefield would not be analysed in terms of counterfactuals with omissions leading to indeterminacy. The fact that soldier S didn't fire her one bullet would have no causal role in the scenario, and the question "did soldier S cause the death of ally X?" would receive a definite negative answer.

   However, if one wishes to remain within a Humean framework, in which omissions can be causal relata, the analysis of the Battlefield in terms of counterfactuals involving omissions holds and a specific strategy needs to be offered to save DetC, such as those described below.

2. **Dropping the principle of *causal additivity*** (Sartorio, 2006), which states the following:

   > **Causal Additivity** (CA). If c causes e, and c causes f, then c causes e and f.

   In that case, the question "did soldier S cause the death of ally X?" can receive a positive answer without implying the false claim that she killed all her four allies.

3. **Endorsing *disjunctive effects***, which states that an event $\mathcal{C}$ can be the cause of a disjunctive effect $\mathcal{E} \vee \mathcal{F}$, while $\mathcal{C}$ is not a cause of $\mathcal{E}$ nor $\mathcal{F}$ (Ballarin, 2014).

   In that situation, the question "did soldier S cause the death of ally X?" can receive a negative answer without implying the false claim that she killed none of them.

4. **Adopting a contrastive approach to causation** (Hitchcock, 1996; Schaffer, 2005), in which one analyses any causal relation by adding a contrastive clause to the cause and effect involved in that relation. A causal relation

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[5] This has been defended namely by (Dowe, 2000; Beebee, 2004; Kistler, 2007)

will then be expressed in the form "*$\mathcal{C}$ rather than $\mathcal{C}*$ caused $\mathcal{E}$ rather than $\mathcal{E}*$*".

In the case of **Battlefield**, adding contrastive clauses would arguably allow us to eliminate any indeterminacy. Take the following four propositions:

(a) Your not shooting *rather than* shooting enemy1 causes soldier1 (but not soldier2, soldier3, or soldier4) to die *rather than* survive.

(b) Your not shooting *rather than* shooting enemy2 causes soldier2 (but not soldier1, soldier3, or soldier4) to die *rather than* survive.

(c) Your not shooting *rather than* shooting enemy3 causes soldier3 (but not soldier1, soldier2, or soldier4) to die *rather than* survive.

(d) Your not shooting *rather than* shooting enemy4 causes soldier4 (but not soldier1, soldier2, or soldier3) to die *rather than* survive.

Each of those strategies requires to modify the standard analysis of causation. Those who would refuse to accommodate these strategies will likely have to accept the possibility of indeterminate causation. Whether the abovementioned approaches are satisfactory will not be debated here, as this particular discussion lies out of the scope of this thesis. Instead, we will now discuss our new scenario of indeterminate causal relations, and show that it is unaffected by the previous strategies, closing these routes to save DetC.

Indefinite causal orders, or ICOs, in the process matrix formalism, as we will show, present a different structure than the Battlefield counterexample to DetC. Yet, this is not the only difference. While *Battlefield* is a scenario that appeals to the ordinary/everyday sense of causation, ICO, as defined in quantum physics, is a theoretical concept resulting from the development of recent science. We therefore bring about a methodological shift by considering science, instead of common sense, as the guide to explore the properties of causation[6]. Such a move is consistent with the scientific realist stance adopted throughout this work.

In order to further analyse ICOs, we will rely on their most famous instance, namely the quantum switch (section 3.3.2 and section 5.2). We will represent it in a schematic way using nodes (representing individual events)

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[6] Since the scientific theory used for studying causation is in that case considered part of fundamental physics, this methodological choice is affected by the debate regarding the relevance of the notion of causation within fundamental physical theories (see (Blanchard, 2016) for a recent review). We'll see in section 5.4 that this work remains outside of the controversy.

and arrows (indicating the causal relations from the causes to the effects). This can be found in Fig. 5.2. The node I represents the initial preparation procedure on the target system, while the node F represents the final output of the operations performed on it[7]. A and B represent the events occurring in parties $\mathcal{A}$ and $\mathcal{B}$, respectively. As discussed in section 5.2, the quantum switch can be understood roughly[8] as a superposition of causal orders, where the notion of superposition is a purely formal one. The green arrows represent the definite structure $\mathcal{A} \prec \mathcal{B}$, while the blue ones represent the structure $\mathcal{B} \prec \mathcal{A}$. Obviously, such a diagram is never encountered in any traditional analysis of causation, since the notion of superposition is purely quantum and pertains to the field of fundamental physics. There are different ways to assign a meaning to the notion of superposition, as discussed in chapter 2.

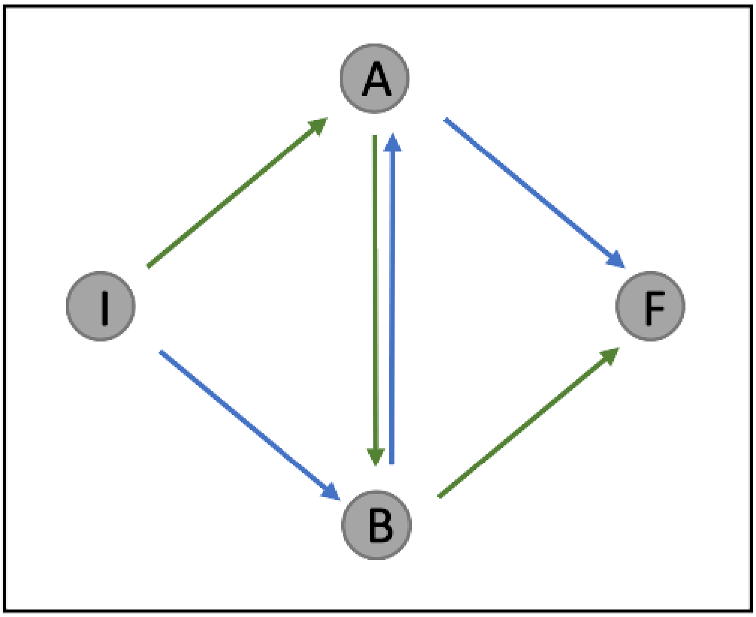

FIGURE 5.2: Schematic diagram of the causal structure of the quantum switch.

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[7] Strictly speaking, F represents the identity operation performed on the target system after it visited parties $\mathcal{A}$ and $\mathcal{B}$

[8] When simplifying the QS description to the mere operations performed on the target system, the causal structure describing what happens to that target system can be expressed as a mere superposition of the definite structures $\mathcal{A} \prec \mathcal{B}$ and $\mathcal{B} \prec \mathcal{A}$. While this is, strictly speaking, a truncated view of the QS, this remains conceptually meaningful since the quantum superposition in the control system's state along with the linear dynamics of quantum mechanics induces the entanglement of the causal structure affecting the target system with the control system's state. The causal structure behaves as if it was itself in a superposition of definite structures.

Yet, it is not needed to engage with a specific reading of quantum superpositions to see how indeterminacy affects the causal structure. Indeed, it seems important to hold to the intuition that it is determinate that I causes F, by transitivity. It seems also intuitive to not deny that a causal influence exists between A and B. As a result, by transitivity, no matter the actual causal structure, it must be determinate that I causes A, I causes B, A causes F and B causes F. What remains indeterminate are the relations "A causes B" and "B causes A".

Because the causal structure of the QS is structurally different from that of the Battlefield case, the previous strategies to save DetC in the face of counterexamples such as the *Battlefield* will not be helpful when applied to the QS. This can be seen quite straightforwardly. First, the QS's causal structure contains no omission, which makes this case challenging even for non-Humean approaches to causation. Second, it contains no problematic conjunction of causal statements. Third, including disjunctive causes or effects will not help dissipate the indeterminacy of the relations "A causes B" and "B causes A". Finally, a contrastive approach to causation does not seem to dissolve the indeterminacy of the causal relations between the events A and B. Indeed, suppose we include *A* and *B* themselves as contrastive clauses. One gets two possible causal relations among A and B:

- A rather than $\neg A$ causes B rather than $\neg B$;
- B rather than $\neg B$ causes A rather than $\neg A$.

Once again, within the QS, there simply is no way to assess which one among these two statements is true. Thus, adding a contrastive clause does not eliminate the indeterminacy.

Unlike the *Battlefield* case, we therefore see that the QS displays a kind of indefiniteness that is structurally different, and that cannot be dissolved by existing strategies found in the literature. In particular, the QS case poses a challenge even for non-Humean approaches to causation (for which, contrary to Humean views of causation, there exist necessary connections in the world that exist over and above the Humean mosaic of actual facts). This key structural difference between the Battlefield case and the QS is easily identified as the presence of a quantum superposition of causal order in the latter case. As it is well known, quantum superpositions cannot be understood without inconsistencies while remaining in a classical framework, which consists in the following assumptions: (i) interactions take place locally, and (ii) there is definiteness of physical properties or definite obtainment of states of affairs at all times. Within that framework of premises, the superposition of causal structures in the QS cannot be understood as either:

- representing a causal structure that is simultaneously both $A \prec B$ and $B \prec A$.
- representing a causal structure that is either $A \prec B$ or $B \prec A$
- representing a causal structure that is neither $A \prec B$ nor $B \prec A$.

All the available logical options fail to make sense of quantum superpositions when we reason within a purely classical setting. Of course, giving up on a least one of the above-mentioned basic classical assumptions opens a range of possibilities to account for superpositions and entanglement[9], as we will discuss shortly. This can be done either by considering a full-fledged account of quantum mechanics, or by adopting a specific reading of entanglement without being tied to a solution to the measurement problem. We will rely on the latter option, as it allows to keep a more focused attention to the meaning of entanglement itself without taking into account other constraints not relevant to this specific discussion (such as the measurement problem). We have seen in chapter 2 that entanglement can be seen as an objective feature of nature, or as an effective phenomenon reflecting imperfect knowledge of the observer.

In the latter epistemic case, the entangled causal relations displayed by the QS would correspond to what we *know* of causal relations, without this knowledge having any objective status. The indefiniteness would originate from an imperfect description of the experiment, while its actual causal structure is definite [10]. Unless such an indefiniteness would be dispelled in a more complete theory, this would mean that, at a certain level, certain causal relations among events are unknowable, not because our science is not developed enough, but because the world is such that the exact causal structure between

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[9] Recall that, rigorously, the QS displays formally a causal structure entangled with the state of an ancillary system.

[10] This approach might appeal to a different framework to describe the causal structure of the quantum switch, e.g. one that formally describes the process as having a definite causal structure, as it is done in Barrett, Lorenz, and Oreshkov (2021). This causal model explicitly admits the cyclicity of causal relations, yielding an alternative causal description of the QS. Such a cyclicity, once embedded in spacetime and if taken at face value as the correct description of reality, could correspond to a form of retrocausality, as discussed in section 4.2.4.

events is *in principle* out of reach. Such a conclusion, although potentially disheartening for some, may be accepted on the grounds that an epistemic account of ICOs arguably presents other theoretical virtues. An analogy can be made with Bohmian mechanics, which displays both an epistemic status for entanglement, while endorsing physical principles such as determinism, or definiteness for all properties of systems at all times. Similarly, it is possible that the principles underlying a potential account of the PMF in which the ICOs are epistemic, might be considered as worth the cost of an *in principle* unknowability of causal relations.

Yet, an epistemic account of ICO must still provide an explanation for the way the non-actual causal orders seem to have a physical influence within the global process due to the presence of quantum interferences between different causal orders. Indeed, just like the existence of non-actual, yet possible, electron trajectories have an influence on the interference pattern of electrons in the two-slits experiment of standard quantum mechanics, we have that the process matrix of the quantum switch is expressed as the following sum of terms:

$$W_{QS} = 1/2(W^{A\prec B} + W^{B\prec A} + |W\rangle^{A\prec B}\langle W|^{B\prec A} + |W\rangle^{B\prec A}\langle W|^{A\prec B}) \qquad (5.1)$$

in which $W^{X\prec Y} = |W\rangle^{X\prec Y}\langle W|^{X\prec Y}$, with $|W\rangle^{X\prec Y}$ representing a process vector with the causal structure "$X \prec Y$". The two first terms of Eq. (5.1) of the form $W^{X\prec Y}$ are process matrices corresponding to the definite causal structure "$X \prec Y$", while the two last terms of the form $|W\rangle^{X\prec Y}\langle W|^{Y\prec X}$ correspond to interference terms. It is their very presence that makes W_{QS} causally nonseparable by preventing its formulation as a probabilistic mixture of processes with a definite causal structure. In the same way that accounts of standard quantum mechanics in which quantum nonseparability reflects an incomplete description of reality (e.g. Bohmian mechanics) still needs to account for the effective presence of interference terms, an epistemic account of ICO needs to explain the underlying physical reason for their presence, despite the fact that the actual causal structure is definite [11]. This question meets the discussion in

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[11] Otherwise, it might be said that an epistemic reading of ICO would lack a certain explanatory appeal regarding the presence of indeterminate causal orders, since the origin of the *in principle* lack of knowledge about causal relations would ultimately remain as a brute posit about causation.

section 4.2.1 which sketched various possible scenarios underlying noncausality (hence, *a fortiori*, causal nonseparability) with the constraint that the underlying causal structure is definite. We saw, e.g., that two possible strategies would be to appeal to a form of dynamical holism or retrocausal influences. Whether such routes are found appealing is arguably a question of personal sensibility.

For the above-mentioned reasons, it is therefore interesting to explore another option, briefly mentioned in section 4.2.1, that regards those ICO more seriously.

If we embrace a metaphysical reading for the indeterminate causal relations, the entangled causal relations featured in the QS would be considered as describing objective features of nature. Those would be indefinite in a metaphysical sense, i.e. that the world itself (and not our knowledge thereof) would be such that causal relations can sometimes be ontologically indefinite. We saw earlier that the causal structure of the QS could not be analysed in a classical setting, because no logical options can make sense of quantum superpositions in that context. At least one of the basic classical assumptions needs to be dropped, namely (i) locality of interactions and (ii) definiteness of all physical properties/obtainment of states of affairs. If one embraces the ontic reading for the indeterminacy of causation, it means that one gives up on assumption (ii). From there, there exist conceptual tools allowing to make sense of the indeterminacy while not running into contradictions. The notion of ontological indefiniteness (more often called *metaphysical indeterminacy* (MI) in the literature) has been studied and formalised in previous work (Wilson, 2013; Barnes and Williams, 2011). Although already introduced in chapter 2, it is useful to expose here a brief presentation of these accounts in order to see how they might apply to the present case of indefinite causal orders.

- **The unsettledness view** (Barnes and Williams, 2011). According to that view, every state of affairs in the world is fully determinate, but it is indeterminate *which* state of affairs obtains. This approach does not provide any reduction of metaphysical indefiniteness to other notions. Instead, it offers a logic and semantic to articulate MI, which is left as a primitive.

- **The determinable-based account of metaphysical indeterminacy** (Wilson, 2013). According to that view, it is always determinate whether a given state of affair obtains. Yet, not every state of affair is determinate itself. An indeterminate state of affair involves an object with a determinable property with no unique determinate.

  Determinable properties are "*unspecific* properties which admit of specification by determinate properties, e.g., the determinable *being coloured*

> may be determined by the determinate *being scarlet*. [...] Determinables are *irreducibly* imprecise – in particular, they are not reducible to any complex combinations of precise determinates." (Wilson, 2013, p. 107)
>
> There are two ways for a determinable to lack a unique determinate. Either it possesses no determinate (which is called the *Gappy account* in Wilson (2013, p. 108-109)), or it simultaneously possesses multiple ones (which is called the *Glutty account* in Wilson (2013, p. 108-109)).

Interestingly, metaphysical indeterminacy has already been discussed in the context of standard quantum mechanics (see (Calosi and Mariani, 2021) for a review), in which it is argued that there are different sources of quantum indeterminacy which differ both mathematically and metaphysically (Calosi and Wilson, 2019). Those sources are quantum superposition, incompatible observables and entanglement. A lack of value determinacy (LVD) in those cases is read as a form of metaphysical indeterminacy. It was argued in Calosi and Wilson (2019) and Calosi and Wilson (2021) that the determinable-based account for MI is better suited than the "unsettledness view" (also called *supervaluationism*) to account for these scenarios found in various realist accounts of quantum mechanics. The reason for this is specific to each of these accounts, namely the presence of residual indeterminacy in the GRW theory after a collapse of the wavefunction, the presence of indeterminacy in the position of macroscopic objects in the GRW$_m$ theory, or the presence of incompatible observables in the Everettian interpretation. These features render impossible the task to formulate maximally precise worlds of which the obtaining is indeterminate in a supervaluationist view. By contrast, by applying a determinable-based account of MI to the various sources of quantum indeterminacy, they defend that this application is successful, proving that the determinable-based account of MI allows to accommodate the whole range of quantum indeterminacies, whether on a Gappy or Glutty variant. These results are further reinforced in (Calosi and Wilson, 2021)[12].

Now, could we apply the same strategy as the one reviewed in (Calosi and Mariani, 2021) to defend metaphysical indeterminacy in the context of ICO ? Interestingly, the causally nonseparable processes that are physically implementable so far always involve entanglement of the causal order between quantum operations with the quantum state of an ancillary system (Wechs et al., 2021; Costa, 2020). Hence, while causally and quantum nonseparability are conceptually different notions (see section 4.1), the indefiniteness in those

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[12] See also Torza (2020) for an argument promoting a modified "Unsettledness View" of (Barnes and Williams, 2011) to account for quantum indeterminacy.

physically implementable causally nonseparable processes can be analysed, at a purely formal level, in terms of standard entanglement, although between notions that are, for their part, different than in the standard case (namely the notion of causal structure). As a result, in the context of ICOs, the concept of indefiniteness can apply not for the properties of a given system, but for the causal structure of a given process.

In the case of ICO read as metaphysically indeterminate, the unsettledness view (if ever appropriate) would express the situation as follows: it is unsettled which of the causal structure $A \prec B$ or $B \prec A$ obtains. The determinable-based account of MI would analyse the situation differently. It would require the identification of a determinable with no unique determinate. "A is causally connected to B" can naturally be considered as a determinable [13], with determinates "A causally precedes B" and "A causally succeeds B". The question is now whether this determinable should be seen as possessing none or both determinates, yielding indefinite causal orders.

Clarifying the exact meaning/nature of the determinable "being in a specific causal order with regards to other events" would require a particular theory of causation. We said earlier, in the beginning of section 5.1, that the notion of causal relation in the case of the QS is left unanalysed, and that one does not take a stance on its (un)reducibility or underlying ontology. In that sense, we remain neutral with respect to existing theories of causation. Yet, we will not engage in a further discussion of a determinable-based account of metaphysically indeterminate causation. The reason is that the relevance of the notion of causation within fundamental physical theories is subject to a large debate (see (Blanchard, 2016) for a recent review), while shifting from the notion of causal structures to that of spatiotemporal ones will allow us to explore instead the implications of ICO on space and time. This is attractive as space and time *unambiguously*, in contrast to causation, constitute a central concept in the ontology of the world, and a pivotal element to connect this more formal theorising of nature with our empirical experiences thereof. Additionally, this move will allow us to get closer to the exploration of a pressing question in fundamental physics (at least within our scientific realist framework), namely the fate of spacetime in a quantum world. As discussed in section 5.1, spacetime is classical in standard quantum mechanics. Yet, this status is expected to change in a more general theory of quantum gravity. It is interesting to investigate whether there already exist tensions between this assumption of a classical spacetime and certain quantum features of non-relativistic, non-gravitational

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[13] This suggestion would require a more careful analysis, yet seems sufficiently reasonable to be assumed in the context of this dissertation.

quantum theory.

In conclusion of this section, the principle of *determinate causation*, DetC, was introduced as well as a central counter-example found in the literature (The Battlefield case). It was shown that the quantum switch could play the role of another counter-example to DetC, structurally different from the Battlefield case. The causal structure of the quantum switch does not allow to save DetC by using previous strategies to account for the Battlefield case while satisfying DetC. From there, possible routes to interpret this causal structure were discussed. On the one hand, an epistemic understanding of the indeterminacy found in the causal structure of the QS can be developed, yet, at a certain cost (namely a potential lack of explanatory power). On the other hand, a metaphysical account of the indeterminacy of causal orders can be both motivated and articulated based on the existing literature in quantum indeterminacy. Finally, we motivated a shift from the notion of causal order to that of spatiotemporal relations. This is this shift that will be discussed in the next section.

5.4 From causal to spacetime structures

The link between causation and spacetime has been extensively discussed in several theories, both in metaphysics and in philosophy of science. To make but one prominent example, consider that in the standard Lewisian supervenience picture, causation emerges from the mosaic of actual events and the spatiotemporal relations among them. In a way, Lewis starts from spacetime to get to causation. Yet, in principle, the converse can be done too. For instance, in the context of QG, causal set theory[14] aims to recover spacetime, as described by (sectors of) general relativity, from fundamental, non (fully) spatio-temporal causal structures. No matter what the priority relation between spacetime and causation is (or whether there is such a relation, or whether they are best conceived as being metaphysically on a par), nobody would deny that there is an intimate connection between spacetime and causation.

For the purpose of this argument, we will focus on the way causal relations and spacetime geometry constrain each other in physical theories. Indeed, a strong motivation for causal set theory is provided by a series of important results in general relativity basically showing that, assuming the condition

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[14] See Huggett and Wüthrich, forthcoming, ch. 2 for a recent philosophical discussion of causal set theory, as well as references therein.

of past and future distinguishability of the spacetime manifold, the geometry of spacetime is fixed by its causal structure up to a conformal factor[15]. Spacetime manifolds have to satisfy a number of properties in order to be considered as "physical" (see e.g. (Minguzzi and Sánchez, 2008)). It has been shown that for particular spacetime manifolds (namely those that are *past and future distinguishing*), a given causal structure will select, up to a conformal factor, a corresponding spacetime metric (Hawking, King, and McCarthy, 1976; Malament, 1977). In other words, there is an isomorphism between the spacetime structure (the metric) and the causal structure for spacetime manifolds that possess certain properties allowing to consider them as physical. That notion of causal structure (hereafter named "relativistic causal structure") is a mathematically defined structure describing the type of causal relation (or lack thereof) that can obtain between any two pair of spacetime points.

We see from those considerations that drawing a connection between the concepts of causality (via the mathematically defined "relativistic causal structure") and spatiotemporality is rather natural in the context of relativity. This connection is arguably present in the case of quantum processes. Of course, connecting the causal structures encoded in quantum processes to spatiotemporal relations is not as straightforward as in the case of relativistic causality and spacetime. The reason is that causal orders within process matrices are operationally defined, and independent, by construction, from spatiotemporal notions. A given process matrix encodes the relations among inputs and outputs of various quantum events. If the output O of an event E_1 is sent to the input I of another event E_2, we can say that "E_1 causally precedes E_2". In the process matrix formalism, the network of such relations is called the "global causal structure" (hereafter named "operational causal structure") underlying the process under consideration. By contrast, relativistic causal structures are defined on the basis of spacetime as constrained by relativity. Another way to put it is to emphasise that events connected in an operational causal structure correspond to quantum operations, while events connected within a relativistic causal structure are spacetime events. While, by definition, relativistic causal structures are connected to spacetime, it is not the case for operational causal structures since those, by definition (see section 3.3), are not assuming any spatiotemporal features.

Yet, a connection between the "operational causal structure" and the spacetime manifold can be defended. We will postulate that the following claim (called *claim X*) is true:

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[15] See Annexe B for a presentation of this result, along with a definition of spacetime manifolds and their properties, and their mathematical connection with causal structures.

> **Claim X**. The "operational causal structure" described by a given process agrees with the "relativistic causal structure" of the entire universe.

The motivation behind this postulate is the following. If one takes relativity and the process matrices as *both* faithfully describing the world, then the operational causal ordering encoded between quantum events in a process needs to match the causal structure among their spatiotemporal locations allowed by relativity. The idea is that what theoretically grounds a valid operationally defined causal structure is a relativistic causal structure as constrained by relativity. In other words, operational causal structures are theoretically founded on relativistic causal structures.

From there, if this postulate is true, any indeterminacy in the operational causal structure is a faithful description of the world and will be carried out in the relativistic causal structure of the universe. Then, in virtue of the isomorphism between the relativistic causal structure and the spacetime manifold, indefinite causal orders are equivalent to indefinite spatiotemporal relations.

However, if **Claim X** is false, then the operational causal structure does not necessarily describe the world faithfully[16], and indeterminate causal orders can correspond to "effective" causal structures not carried out in the relativistic causal structure.

To sum up, as long as we consider that the spacetime structure involved in quantum physics possesses the various properties we expect from a physical spacetime, the above-mentioned isomorphism holds[17], in virtue of what the indefiniteness of causal order as defined within the process matrix formalism can be understood as the indefiniteness of a corresponding spatiotemporal structure, whether objective or not. It will be objective if the operational causal structure provides a true description of the world (i.e. claim X is true), and merely epistemic in the opposite case. With that being said, we will work, from now on, with the notion of indeterminacy of *spatiotemporal relations*. The

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[16] Indeed, due to the empirical success of general relativity and the more speculative nature of the process matrix formalism, it seems more plausible to reject the faithfulness of process matrices' descriptions rather than that of relativity.

[17] Some words of caution are in order here, as Earman (1972) provided an objection to the idea of building philosophical reasoning based on the isomorphism between causal and spatiotemporal structures. He pointed out that there exist "exotic" spacetime structures not obeying the "pas and future distinguishing" constraint, but still licensed by general relativity, and there is no reason to discard such spacetime structures as valid candidates for modelling our spacetime.

next section will further explore the two above-mentioned scenarios, with an emphasis on the one bearing metaphysical implications for spacetime.

5.5 Indefinite spatiotemporal relations

Let's now discuss the meaning behind indefinite spatiotemporal relations. As explained in section 5.4, the indeterminacy of a spatiotemporal structure can have an epistemic or a metaphysical (ontic) origin. An epistemic indeterminacy of spatiotemporal relations would originate from an imperfect *knowledge* of what those relations actually are. The notion of spatiotemporal relation would be semantically adequate, so that it is, in principle, capable of referring to the objective spatiotemporal features of the world. However, instead of referring directly to those features, it refers to the (imperfect) *knowledge* we have of such matters[18]. On the other hand, a metaphysical indeterminacy of spatiotemporal relations would not come from a lack of knowledge regarding those notions, but rather from objective features of the world itself. In other words, the world would have certain objective properties leading to spatiotemporal relations being metaphysically indeterminate.

We said earlier that the kind of ontology and dynamics that can be associated to shape a full-fledged account of the theory would constrain the nature of the indeterminacy. An epistemic kind of indeterminacy is present in accounts where relevant theoretical elements (here, the process matrix associated to the experimental setting under consideration) would be considered as not objective.

An analogy with particular realist approaches towards standard quantum mechanics can be drawn: quantum indeterminacy has an epistemic reading when, e.g., entanglement is read as some effective phenomenon. As an example, an account of quantum mechanics can assign a nomological status to the universal wavefunction (Dürr, Goldstein, et al., 1997; Esfeld et al., 2014), while the quantum states assigned to sub-systems of the universe are mere mathematical tools that do not refer to something in the world. In that picture, spacetime remains fundamental.

The counterpart of that strategy, in the context of the PMF, would be to

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[18] This ignorance being not classical, in the sense that the global situation cannot be described by a probabilistic mixture of corresponding possible spatiotemporal structures.

read the universal process matrix[19] as a law of nature[20]. The corresponding constraint would then have a holistic nature. As a result, a quantum process taking place in a sub-region of the universe would provide an approximate description of what happens in that region: it would ignore the holistic character of the world's constraints and represent what happens in that sub-regions *as if* it were isolated from the rest of the world. We could then say that quantum processes assigned to sub-regions of the universe wouldn't refer to objective features of the world. To sum up, the universal process matrix would be objective, but the processes associated to any particular experimental setting would not be real. Instead, they would describe merely effective phenomena.

Instead of developing further such interpretative possibilities, it is interesting to discuss the potential motivations that would incite us to adopt them. Recall that the process matrix can be seen as a generalisation of the density matrix ρ, which can itself be interpreted as a real and fundamental entity. This interpretative approach, called ρ-realism, arguably benefits from at least as much support as an ontology in which the universal wavefunction is real and fundamental, if not more (Chen, 2020; Aharonov, Anandan, and Vaidman, 1993). If one assumes that quantum mechanics could be expressed entirely in terms of process matrices while providing a theory empirically equivalent to the standard formulation[21], one could therefore consider readings of the universal process matrix as real and fundamental as *in principle* viable [22]. Yet, since the (currently known) causally nonseparable processes that are physically implementable can be described in standard quantum mechanics, it is not clear what could be the incentive to shift from an ontology including a universal density matrix to an ontology including a universal process. One possible line of reasoning could be that process matrices are mathematical objects allowing to describe correlations among quantum events across time and space, while

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[19] The universal quantum process would describe the relations existing between all the quantum events of the universe.

[20] There is already a debate regarding whether the universal wavefunction, or the universal density matrix, are acceptable candidates as laws of nature. As suggested by Chen (2020), a law of nature is reasonably expected to be simple, fixed by the theory, generating motion and not referring to things in the ontology of the world. The universal quantum process might violate some of these criteria, e.g. as it does not generate motion. Whether this invalidates a possible nomological status for the universal process matrix is left as an open question at this stage. For example, one could be satisfied with laws of nature imposing global constraints on the ontology rather than ruling a proper dynamics.

[21] Such a question remains an open question at this stage, and would require an entirely separated work.

[22] Again, these proposals should be adequately articulated in order to ensure that matter in four-dimensional spacetime is suitably accounted for.

density matrix concern merely spatial correlations between quantum states. For the sake of treating space and time as similarly as formally possible[23], the move to a theory based on process matrix would be advantageous.

By contrast, the scenario of a metaphysical indeterminacy for ICOs would indicate that the spatiotemporal relations themselves (and not our knowledge of them) are sometimes indefinite. Metaphysical indeterminacy can be obtained within accounts in which the process matrix of a given causally nonseparable process would be taken as objective.

An analogy can be drawn with the situation in standard quantum mechanics. In that context, one can see the universal wavefunction as a real entity located at the fundamental level of the world's ontology. This view is often called ψ-realism (Albert, 2013). A similar approach in the context of the PMF would be to see the universal process matrix as a real and fundamental entity living in a high-dimensional Hilbert space (see section 3.3.1) which would correspond to the real fundamental space in which the world's fundamental ontology is located. Spacetime would then be emergent from that fundamental space, and quantum processes describing what happens in a given sub-region of our familiar spacetime would refer to objective, yet derivative, entities.

More generally, the idea of metaphysical indeterminacy within realist accounts of quantum mechanics has been discussed in the literature independently of any solution to the measurement problem, as discussed in section 5.3 (see (Calosi and Mariani, 2021) for a review). By applying the concept of metaphysical indeterminacy to causal nonseparability rather than to entanglement, and in a rather similar way than in section 5.3, one can shape the following ideas. Spatio-temporal relations can be metaphysically indefinite in the sense that the truth-value of propositions involving those relations is indeterminate (Barnes and Williams, 2011). Indeterminacy would then be a primitive, and constitutes a brute, unanalysable fact of nature. Alternatively, and as advocated in influential accounts of quantum indeterminacy, these relations can be indefinite in the sense that a spatiotemporal determinable would fail to have a unique determinate, i.e. it would either simultaneously present more than one determinate, or would lack any determinate at all (Wilson, 2013). The precise identification of such a determinable might require to engage with the philosophical accounts of spacetime in the context of Galilean relativity (namely the substantivalist and relationalist accounts), which are recalled in Annexe C. Yet, in both cases, the metrical properties can be analysed *prima*

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[23] The idea that space and time are treated differently in quantum mechanics is much debated, with existing work arguing against such a claim (Hilgevoord, 2002).

facie as a determinable that would lack a unique determinate in ICOs, while the exact signification of the metric will depend on the specific philosophical account of spacetime. The idea of an indeterminate metric is arguably better discussed in a more fundamental context, as explained in the next section.

In summary, there exist different ways to shape a realist account for indeterminacy within the quantum switch. Although of a possibly very different nature, they all necessarily involve philosophical (epistemic or metaphysical) implications for spatiotemporal relations. The preference for epistemic or metaphysical accounts will depend on the favoured interpretation of the theory. Now, as discussed earlier, causal nonseparability is a model-dependent notion, relying on the Hilbert space formalism of quantum theory. Noncausal relations (see section 3.3.3), if ever observed, would make the implications for spacetime unavoidable in virtue of their model-independent nature. Any realist ontology would have no choice but having to take them into account, and such ontologies would need to specify the meaning of the indeterminacy of spacetime relations. As the physical status of noncausal correlations remains currently under debate (Araújo et al., 2017; Wechs et al., 2021; Purves and Short, 2021), future results will hopefully tell us more on that question.

5.6 Perspectives from quantum gravity

It is interesting to note, at this stage, some possible connections with existing work in the field of quantum gravity, in which spacetime's nature and characteristics can undergo tremendous changes.

On the one hand, a connection can be made between indefinite causal orders in the PMF and the notion of superposed gravitational fields in quantum gravity. The latter are allowed in any model of quantum gravity (Paunković and Vojinović, 2020), which keeps the discussion very general. As discussed in section 3.3.4, Zych et al. (2019) have proposed a thought experiment, called the gravitational quantum switch, based on the basic principles of quantum mechanics and general relativity (namely the quantum superposition principle and gravitational time dilation[24]). The thought experiment shows that those principles naturally lead to an indefiniteness of causal orders arising from an

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[24] In general relativity, time dilation refers to a difference in the measured temporal duration between two events as measured by two different clocks located at different spatiotemporal locations between which there exists a difference in gravitational potential (Einstein and Press, 2016).

actual superposition of spacetime regions in a quantum gravity context. More precisely, indefinite causal orders originate from the entanglement of temporal orders between time-like events. While this thought experiment shares some similarities with the indefinite causal orders discussed in this work (the quantum switch displays entanglement of the temporal order between two quantum events with a control system's state), Zych et al. (2019) argue that there is a fundamental difference between the two settings. Indeed, the indefinite causal orders described by the process matrix formalism are embedded in a classical spacetime, and only two specific quantum events are described by a non-classical order as the result of a temporal order entangled with some ancillary system. On the contrary, in the thought experiment involving gravitational effects, it is a whole spacetime region that is concerned with entangled temporal orders. Indefinite temporal orders are the result of a superposition of a mass distribution, itself (as prescribed by general relativity) linked to spacetime geometry. That way, an explicit superposition of spacetime itself is obtained (Paunković and Vojinović, 2020). It is in that context only that the authors would qualify spacetime as *non-classical*. This thought experiment clearly resonates with the idea, present in many approaches to QG, that spacetime itself could be in a quantum superposition. It also shows that, while indefinite causal orders point towards a tension between a classical background spacetime and quantum features such as superposition and entanglement, the implications of indefinite causal orders should be considered within the context of a fully gravitational theory. In that context, the conceptual tools developed to account for indeterminate causal orders within the process matrix formalism might prove useful when transposed to a gravitational context.

On the other hand, some research programmes in quantum gravity suggest that spacetime is an emergent entity ontologically dependent on non-spatiotemporal notions (Huggett and Wüthrich, 2013; Lam and Wüthrich, 2018). Indeed, it is widely argued that spacetime is not fundamental in quantum gravity, on the grounds that quantum superpositions of (spacetime-like) structures at the fundamental level cannot be understood in any spatio-temporal sense. For instance, quantum superpositions of spin networks (or spin foams) are generic in loop quantum gravity, and this is often taken as indicating that spacetime vanishes at this level, and so is not fundamental (see e.g. Rovelli 2004, § 6.7.1 in the physics literature, and Huggett and Wüthrich 2013, § 2.3 in the philosophy of physics literature). In (Lam, Letertre, and Mariani, forthcoming), it is argued that allowing for indeterminacy in the fundamental spacetime structure of QG may also provide a novel, rather provocative perspective on the most debated philosophical issue in the QG context. Up to now, and to the best of our knowledge, the interpretative and metaphysical strategy of considering (certain) spacetime structures as being indeterminate in QG (in some appropriate sense) has not been seriously investigated (at least in the current philosophy of physics literature). In a way, spacetime fundamentality is rejected because spacetime indeterminacy is rejected. We notice that such a rejection of spacetime indeterminacy is only justified if we have good reasons to believe that this notion does not make sense, or even that it is inconsistent. However, there are now various proposals to make sense of metaphysical indeterminacy (see sections 2.2.2.2 and 5.3), and this notion has been argued to be explanatorily useful already within this present work (see sections 5.3 and 5.5), i.e. in the context of non-relativistic QM. This points to the intriguing suggestion that what is taken as the non-fundamentality of spacetime (or of certain spacetime features) in the QG context could also be understood in terms of some spacetime indeterminacy. Articulating this suggestion in details and its implications for the debates around the emergence of spacetime, while lying outside the scope of this thesis, constitutes a worthwhile project that may shed an interesting new light on spacetime in QG.

5.7 Back to standard quantum mechanics

Interestingly, the idea that quantum physics suggests particular implications for *space* can already be found in the context of standard quantum mechanics. Indeed, quantum states encoding the spatial position of some physical system can be indefinite, i.e. in a superposition of eigenstates of the position observable. One can also encounter entangled quantum states encoding the values of the position observable. For example, in the two-slits experiment, the position of the electrons can be seen as being entangled with the state of the slits (i.e. open or closed). The system [slits + electron] is then quantum nonseparable, and the state of the electron (describing its (observable) position) is indefinite.

Again, the nature of this indefiniteness (epistemic or metaphysical) needs to be specified within a more detailed account. While a Bohmian approach considers that the spatial locations of a quantum system are always well defined (i.e. the indeterminacy is epistemic), embracing quantum indeterminacy (see section 2.2.2.2) considers that spatial locations are metaphysically indeterminate.

It is worth noting that a notion of indefinite temporal relation would be complicated to explore in the context of standard quantum mechanics, since the very notion of a time observable in quantum mechanics is nontrivial[25] [26]. The process matrix formalism, by allowing to access the notion of causal order (hence, in some sense, that of temporal ordering), provides a way of investigating to what extent temporal relations can display quantum properties.

To conclude, it seems that, provided we relate quantum events to spacetime events (which is necessary to identify indefinite causal orders in causally nonseparable processes to indefinite spatiotemporal relations), the (formal) indefiniteness of *spatial relations* encountered in the context of quantum nonseparability of quantum states can be extended to the (formal) indefiniteness of *spatiotemporal relations* encountered in the context of causal nonseparability of process matrices. This recalls the conclusions drawn in chapter 4: whether the formal indefiniteness of causal orders is accounted for in epistemic or metaphysical terms (depending on the particular reading adopted to account for quantum or causal nonseparability), it seems that the temporal dimension is invited into the picture to play a key role in causal nonseparability compared to the situation in standard nonseparability.

We saw in section 4.1 the limits of the formal analogy that can be drawn between the density matrix and the process matrix on the one hand, and between standard and causal nonseparability on the other hand. Yet, in spite of the formal and conceptual disanalogies between those pairs of concepts, both pairs seem to suggest important implications for the way we conceive and characterise spatio(temporal) relations. Such results highlight the potentially very fruitful explorations of the implications of quantum features on the conception of spacetime, keeping in mind that quantum and spacetime theories are expected to be unified in a future theory of quantum gravity.

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[25] See Butterfield (2013) for a review of the different roles that time can take on within a theory, namely a coordinate of spacetime (i.e. an external independent variable, or parameter) versus a function of other quantities of the system *in* spacetime (i.e. a dynamical variable). The literature on time as a physical variable is vast (see, e.g., (Giovannetti, Lloyd, and Maccone, 2015; Erker et al., 2017; Brunetti, Fredenhagen, and Hoge, 2010; Hilgevoord, 2002)). While time is widely used as an external parameter in quantum mechanics, its measurement as a (definite or indefinite) variable remains non-trivial.

[26] Relatedly, earlier work suggested the development of the idea of *temporal* entanglement in standard quantum mechanics, but the task encountered technical difficulties. See (Glick, 2019) for a review. Those considerations don't affect the notion of temporal locality discussed in chapter 4, as it characterises correlations within model-independent probability distributions.

5.8 Conclusion

It was argued in this chapter that a realist attitude towards causal non-separability can, in certain cases, bear implications for spatiotemporal relations.

We rely on the quantum switch as a case study for indefinite causal orders. This notion was first analysed in terms of causal relations, conceived in a minimalist manner. It was argued that existing strategies to alleviate cases of indefiniteness in a causal structure did not work in the present case. The exact meaning and/or nature to be assigned to the indefiniteness of causal relations would depend both on the specific account that one gives to the PMF, and on the particular theory of causation under consideration. Upon shifting from a notion of causal structure to a notion of spatiotemporal structure, ICO translated into indefinite spatiotemporal relations, the exact meaning of which would rely on the account of PMF.

Yet, it was argued that there exist interesting arguments for adopting a metaphysical stance towards ICO, and in that context a parallel with the situation in quantum gravity was discussed. It was finally highlighted that important consequences for spatial relations can already be defended in standard quantum mechanics. Hence, in spite of the disanalogies between standard quantum mechanics and the process matrix formalism, both theories can support substantial implications for (the properties of) spatio-(temporal) relations.

It will be interesting to further investigate the extent to which the implications for space of realist interpretations of standard quantum nonseparability could help making sense (in a realist fashion) of causal nonseparability and its implications for spacetime, and *vice versa*. Such a study aiming at uncovering how the quantum features of physics could impact the properties of spacetime seems warranted given the fact that an important aspiration in physics is to develop a unified theory of the universe that would explain how matter and spacetime behave in the quantum realm.

Chapter 6

Conclusions

The aim of this work has been to explore the potential implications for the metaphysics of spacetime of a realist approach to the notion of causal nonseparability in quantum physics. This study took place in a context in which a naturalised approach to metaphysics is largely discussed in the literature, especially in the framework of fundamental physical theories. The overall methodology pursuing metaphysical issues upon the constraints of science has therefore been defended elsewhere in the literature. This work also took place in the global context of philosophy of physics, of which a major objective is to develop a better understanding of our current best theories by assigning an underlying ontology to their formalism. This study therefore carries on with this task by exploring recent theoretical developments in the foundations of quantum physics.

In order to study the realist implications of causal nonseparability, it was first defended that a scientific realist approach towards the process matrix formalism was as much legitimate as any antirealist reading. The reason is that operational formalisms are ontologically (in a specific sense) and epistemically neutral.

From there, the theoretical concepts of interest, namely causal nonseparability and noncausality, were analysed in more details in order to highlight in what sense they are distinct from the standard notions of quantum nonseparability and nonlocality in standard quantum mechanics. It was argued that causal nonseparability is not a mere extension of quantum nonseparability (describing correlations between quantum states at a given time) to a description of correlations between quantum states at different times. Instead, causal nonseparability encodes correlations among quantum events independently (under certain circumstances) of the systems and the operations under considerations. As a result, it describes a higher-order kind of correlations among quantum maps transforming quantum states.

The discussion then focused on noncausality, the model-independent counterpart of causal nonseparability. It was argued that noncausality has a strong connection with a notion of temporal nonlocality. In the same way that Bell nonlocality is given different underlying explanations depending on the details of the chosen quantum mechanics' account, noncausality pointing towards temporal nonlocality can be given a variety of underlying descriptions depending on the exact way to interpret the process matrix formalism.

The last chapter focused precisely on this particular point, namely on the various ways to understand process matrices and causal nonseparability under a realist attitude. As this investigation can be pursued in a large variety of ways, the approach followed here has been to focus on causal nonseparability independently of any dynamical details of the theory. In order to explore the possible implications on spacetime, we shifted from the notion of (indefinite) causal structure to (indefinite) spatiotemporal ones. This shift was allowed under a set of reasonable assumptions regarding the properties of a physical spacetime manifold. While different readings were then suggested for indefiniteness of spatiotemporal relations, we insisted in particular on an objective understanding appealing to the concept of metaphysical indeterminacy. It was argued that such an approach could prove useful in a more general theoretical context such as quantum gravity, while being already partly supported in standard quantum mechanics.

In conclusion, this work attempted to partially fill the gap between metaphysical discussions in a non-fundamental theory that is quantum mechanics, and metaphysical discussions in a more fundamental, yet not fully developed, theory of quantum gravity. The present results suggest that some metaphysical theories can prove useful across various theoretical developments. At the very least, reflecting on the metaphysical implications of non-fundamental theories emphasises certain existing tensions between inharmonious principles, such as Galilean spacetime and quantum features.

Of course, the present results have a number of limitations, inviting future discussions. First of all, while it was defended that operational formalisms were ontologically and epistemically neutral, it is still relevant to ask what could be the particular philosophical benefit of working with those formalisms. In particular, it is interesting to explore further the ability of such formalisms to study the foundations of theories, and discuss how the information extracted from these studies could constitute objective knowledge *about theories*, of which the epistemic value might go beyond the realist/antirealist debate in physics. Such questions will be investigated within the context of a forthcoming research project in the field of formal epistemology (Kvasz, Ladislav, 2021).

Second, the scientific status of causal nonseparability can be further examined. Indeed, the obtainment of proper indefinite causal orders in laboratories is debated (MacLean et al., 2017; Oreshkov, 2019; Paunković and Vojinović, 2020), and it might be questioned whether causal nonseparability is indeed scientifically supported. While it was argued here that such a claim was reasonable, further discussions might be relevant to solidify the position. In case significant support can be found against the physicality of indefinite causal orders, the present work would find value as both a catalyst for naturalised metaphysics of fundamental physics, and a conceptual junction between such a realm and the understanding of the effective theory that is standard quantum mechanics. In any case, precising the exact status of causal nonseparability from a scientific perspective remains an open question currently debated in the field of foundations of quantum physics.

Finally, the very connection we used between causal and spatiotemporal structures to explore the potential implications of causal nonseparability on spacetime can be further discussed. On the one hand, whether physical spacetime is indeed past and future distinguishing can be questioned within the field of general relativity and its philosophical significance on spacetime. On the other hand, upon accepting the presence of indefinite spatiotemporal relations as a consequence of causal nonseparability, the nature of this indefiniteness could be further discussed. In particular, the metaphysical reading of indefinite spatiotemporal relations could be made more precise by entering the details of the various possible accounts of metaphysical indeterminacy. As there exists currently a lively debate regarding what account of MI is best suited for quantum indeterminacy, it is to be expected that these discussions will prove relevant in the case of ICO, which might serve as an additional case study for the development of MI account in a quantum context.

Appendix A

Physically implementable causally nonseparable processes

Recent work has identified an entire class of causally nonseparable processes that generalise the quantum switch (of which a thorough description is provided in section 5.2) while maintaining a concrete physical interpretation (namely a superposition of causal orders) and a physical implementation (Wechs et al., 2021). More precisely, this class of processes controls the causal order between operations underwent by a target system in a coherent way, i.e. by entangling this causal order with the degree of freedom of a control system. The quantum switch features a bipartite scenario with two operations performed on the target system. Considering more operations yields a straightforward generalisation. Yet, it is possible to bring the generalisation of the quantum switch a step further, and allow for a combination of dynamical and coherent control of the causal order between operations. This is done by still relying on a control system to coherently determine the causal path of the target, albeit in a more sophisticated way to take also into account the order of past operations, as the target system navigates the different parties, to dynamically determine the order of the remaining operations.

We will now show that the quantum switch can be described in standard quantum mechanics. While the global causal structure among the operations performed on the target system cannot be represented in the standard formalism of quantum mechanics, it remains possible to express what happens in the quantum switch in terms of the evolution of the wavefunctions of the target and control systems.

Let be two parties, A and B, each performing a unitary operation (noted U_A and U_B for party A and B, respectively) on a *target system* of which the wavefunction is noted $|\psi_t\rangle$. At time t_0, this wavefunction is noted $|\psi_t(t_0)\rangle$. A qubit called the *control system*, of which the wavefunction is noted $|\psi_c\rangle$

controls the order of party A and B, as explained in section 5.2. The *control qubit* is in a superposition of states $|0\rangle_c$ and $|1\rangle_c$, i.e. $|\psi_c\rangle = \frac{1}{\sqrt{2}}|0\rangle_c + \frac{1}{\sqrt{2}}|1\rangle_c$. The global wavefunction of the composite system "*target system-control qubit*", noted $|\psi_{t+c}\rangle$ is then, at time t_0:

$$|\psi_{t+c}(t_0)\rangle = (\frac{1}{\sqrt{2}}|0\rangle_c + \frac{1}{\sqrt{2}}|1\rangle_c) \otimes |\psi_t(t_0)\rangle \tag{A.1}$$

This wavefunction evolves linearly according to the Schrödinger equation, which yields, after the *target system* visited both parties A and B (which is said to correspond to a later time t_1):

$$|\psi_{t+c}(t_1)\rangle = \frac{1}{\sqrt{2}}|0\rangle_c \otimes U_B U_A |\psi_t(t_0)\rangle + \frac{1}{\sqrt{2}}|1\rangle_c \otimes U_A U_B |\psi_t(t_0)\rangle \tag{A.2}$$

Hence, the evolution of both the *control* and *target* systems are describable in standard quantum mechanics. Such a description can be achieved for generalisations of the quantum switch, since only the number of parties or the specific inner structure of the control system's state will have to be modified[1], which is not a problem to account for within standard quantum mechanics.

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[1] Costa (2020) also highlights the fact that currently known physically implementable causally nonseparable processes all features a causal structure entangled with the state of an ancillary system.

Appendix B

Spacetime Manifolds and causal structures in spacetime physics

This section is taken from (Letertre, 2018).

B.1 Spacetime manifold

From a mathematical point of view, spacetime is a topological space with some extra structures[1]. A topological space[2] is the most general notion of mathematical space. It is a set of points organized in such a way that the relations of each point with its neighbourhoods satisfy a specific set of axioms. This set of axioms, combined with the set of points, defines a particular topological space.

Only a particular subclass of topological spaces can provide candidates to model spacetime, namely the category of 4-dimensional *topological manifolds*[3]. A 4-dimensional topological manifold $\mathcal{M}$ is a topological space that is separated[4], for which every point can be fully located by 4 coordinates, and in which every point has a neighbourhood homeomorphic to Euclidean space. A point in $\mathcal{M}$ is called *spacetime event*, and corresponds to a particular point in our physical space at a particular time. In addition to $\mathcal{M}$, extra structures are

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[1] See Carter (1971) for a thorough presentation.

[2] See Armstrong (1983) for more information.

[3] See Norton (2018) for more information.

[4] A separated space is a space for which distinct points have disjointed neighbourhoods (Willard, 2004).

needed to encode information allowing for differential calculus (differential structure $\mathcal{S}$) and for measuring spatial and temporal distances between events (conformal structure, encoded by the metric field g).

In short, the ensemble $< \mathcal{M}, \mathcal{S}, g >$ as defined above describes a 4-dimensional topological space with a specific geometry. This geometry needs to satisfy additional basic requirements in order to possibly model spacetime. In particular, the metric g has to be non-degenerate, smooth, symmetric and of signature (-,+,+,+) or (+,-,-,-). In such a case, $< \mathcal{M}, \mathcal{S}, g >$ is called a *Lorentzian manifold* (Chen, 2011).

Finally, to obtain a model of the universe, one can complete this spacetime model with a matter field $\mathcal{T}$ representing the distribution of energy and matter in spacetime.

B.2 Causal structures

The causal structure[5] of a Lorentzian manifold $< \mathcal{M}, \mathcal{S}, g >$ encodes the type of causal relation existing between any pair of events. Mathematically, a causal structure is the collection of the chronological future $I^+(x)$, chronological past $I^-(x)$, causal future $J^+(x)$ and causal past $J^-(x)$ defined for each event x in $< \mathcal{M}, \mathcal{S}, g >$. The sets $I^+(x)$, $I^-(x)$, $J^+(x)$ and $J^-(x)$ are defined as follows:

$$\begin{aligned} I^+(x) &= \{y \in \mathcal{M} | x \ll y\} \\ I^-(x) &= \{y \in \mathcal{M} | y \ll x\} \\ J^+(x) &= \{y \in \mathcal{M} | x \prec_{sp} y\} \\ J^-(x) &= \{y \in \mathcal{M} | y \prec_{sp} x\} \end{aligned} \tag{B.1}$$

where $x \ll y$ means that x chronologically precedes y, i.e. there exists a curve from x to y such that the tangent vector $\mathcal{X}$ on each point of that curve is timelike (i.e. $g(\mathcal{X}, \mathcal{X}) < 0$) and future-directed with respect to the choice of arrow of time. $x \prec_{sp} y$ means that x causally precedes y, i.e. either $x = y$, or there exists a curve from x to y such that the tangent vector $\mathcal{X}$ on each point of that curve is future-directed and either timelike (i.e. $g(\mathcal{X}, \mathcal{X}) < 0$) or null (i.e. $g(\mathcal{X}, \mathcal{X}) =$

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[5] See the references in Hawking and Ellis (1973).

0) [6].

B.3 Causality conditions

For a Lorentzian manifold to be *physically meaningful*, i.e. it can be considered as an appropriate candidate to refer to real spacetime, it has to satisfy a number of criteria imposing constrains on its conformal structure (hence on its causal structure). These criteria constitute a hierarchy, called *causal ladder*, where each element of the list is strictly more constraining than the previous one. For the most part, those requirements are related to considerations linked to the principle of causality (according to which causes always precede their effects) and prevent the existence of causal relations that would be problematic for the fulfilment of that principle. The strongest constrain imposes that the manifold should be globally hyperbolic[7]. Such manifolds are considered as "natural (generic) models for physically meaningful spacetimes" (Minguzzi and Sánchez, 2008, p. 18). The detailed definition of each causality condition of the causal ladder, namely the requirements that a physically meaningful manifold should be non-totally vicious, chronological, causal, distinguishing, strongly causal, stably causal, causally continuous, causally simple and globally hyperbolic, can be found in Minguzzi and Sánchez (2008).

B.4 Isomorphism between causal structures and spacetime manifolds

As explained in Bombelli et al. (1987), the causal structure of a manifold $< \mathcal{M}, \mathcal{S}, g >$ can be directly derived from its metric g. Reciprocally, Malament (1977) showed that, for the manifolds that are *past and future distinguishing*, the metric can be derived from the corresponding causal structure. To be past and future distinguishing is the fourth causality condition of the causal ladder presented earlier, and means that if two points of the manifold have the same chronological past I^- or future I^+, then these two points are the same point.

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[6] $\prec_{sp}$ is to be distinguished from $\prec$, for which $A \prec B$ means that signalling can occur only from A to B.

[7] A manifold $\mathcal{M}$ with a metric g is globally hyperbolic if (i) there is no closed causal curves, i.e. no closed curve for which the tangent vector $\mathcal{X}$ in every point of the curve is such that $g(\mathcal{X},\mathcal{X}) \leq 0$, and (ii) for any pair of points x and y in $\mathcal{M}$, the set $J^-(x) \cap J^+(y)$ is compact.

This implies, if one considers the entire ensemble of possible manifolds without imposing any constraints on their conformal structures, that a whole range of manifolds will have the same causal structure (see, for example, in Minguzzi and Sánchez (2008)). Imposing restrictions on the metric of manifolds will reduce the amount of acceptable manifolds, but this amounts to reducing the amount of acceptable causal structures as well. Malament (1977) showed that the condition of past and future distinguishability is enough to install a one-to-one correspondence between a metric and its corresponding causal structure.

Appendix C

Philosophical accounts of space and time

This section is taken from (Letertre, 2018).

We will briefly review the main conceptions of spacetime as conceived within the context of Galilean relativity. The reason behind this restriction is that spacetime in quantum mechanics features a Galilean geometry and is therefore the mere union of the concept of space and time, while maintaining the absolute status of the temporal dimension.

C.1 Spacetime substantivalism and relationalism

The question of the existence of an objective spacetime is the subject of an old and still ongoing debate. A relatively straightforward answer was given by Descartes in 1644, according to which space exists, and is nothing more, nothing less than matter (as discussed by Huggett and Hoefer (2006, section 3)). This view faced difficulties, namely its incompatibility with the existence of vacuum. As the existence of vacuum was established in subsequent developments of physics, this view has not been further discussed.

A very influential proposal about spacetime was then given by Newton (Huggett and Hoefer, 2006, section 4). According to Newton, space is absolute in the sense of existing both independently of our mind and independently of matter. It is rigid, three-dimensional, with an Euclidean geometry and unchanging over time. Absolute space is a *pseudo-substance*, which means that it exists in spite of being neither material nor a substance, due to its lack of causal powers. For that reason, this view of space is called *substantivalism*.

A common metaphor (yet criticised in recent debates (see e.g. Knox (2014) and Ladyman et al. (2007))) used to describe this approach assimilates space to a container for all the processes taking place in the universe.

The exact opposite view, of which Leibniz [1] was a famous proponent (Huggett and Hoefer, 2006, section 6), is called *relationalism*, and states on the contrary that space is not a concept independent of matter. Instead, it is viewed as the ensemble of geometrical relations existing among bodies, and hence, supervenes on matter. If we adopt a nominalist[2] position about mathematical constructions, those relations are not part of the ontology of the world. Space is then a purely mental creation and has no real existence outside the human mind. Accordingly, there is no "container", only material processes.

C.2 Status of the metric field

Neo-Newtonian substantivalists adapted Newton's substantivalism in order to make it compatible with Galilean relativity. The notion of absolute space, distinct from the notion of time and with an Euclidean geometry, is then replaced by the notion of absolute *spacetime*, unifying time and space into a single concept, and having a Galilean geometry (Feynman, Leighton, and Sands, 2015, chap. 17). In that configuration, the container metaphor can be formulated in terms of the topological and metrical structures defined in Annexe B. The substantivalist would see $< \mathcal{M}, \mathcal{S}, g >$ as describing the container that is spacetime. In particular, the metric encodes the geometrical aspects of spacetime and is determined completely independently of the matter and energy distributions. In other words, these distributions and the various processes taking place in spacetime are constrained by, and have no influence on, the geometry of spacetime. On the contrary, a relationalist view would see g as representing the matter and energy distributions of the world, hence being completely determined by this distribution. Therefore, one sees how, for the relationalist, the spacetime manifold $< \mathcal{M}, \mathcal{S}, g >$ is a purely mathematical object describing the geometry of the material content of the universe, without the need to postulate the existence of any container.

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[1] As discussed by Huggett and Hoefer (2006, section 6), identifying the exact view of Leibniz is a complicated topic not reviewed in this work.

[2] An introductory presentation of nominalism is found in Rodriguez-Pereyra (2016).

Appendix D

Résumé synthétique (in French)

D.1 Introduction

La mécanique quantique a suscité de nombreux débats en philosophie, touchant à la fois des questions épistémiques et métaphysiques. Cette théorie, développée au début du vingtième siècle [1], a permis de prédire avec succès le comportement de systèmes physiques appartenant au domaine sub-microscopique [2]. En raison de son succès empirique impressionnant et de sa description de la matière à très petite échelle, la mécanique quantique est considérée comme un pilier important de la physique moderne (Ismael, 2021). Ses prédictions ont conduit à des innovations importantes, telles que les supraconducteurs et les lasers, qui sont largement utilisés dans les technologies d'aujourd'hui (Jaeger, 2019). Pourtant, malgré ce succès théorique et pratique, la théorie reste déroutante sur le plan conceptuel.

La raison principale en est que la théorie, dans sa forme standard, laisse de nombreuses questions sans réponse concernant la façon dont on devrait comprendre une mesure quantique et les mécanismes qu'elle implique. Elle reste silencieuse sur la nature exacte des systèmes quantiques, leurs propriétés et dynamique non-classique, et la raison sous-jacente au fait que les systèmes quantiques semblent perdre leur comportement quantique lorsqu'ils sont observés. Cet écart explicatif est appelé le *problème de la mesure* (Maudlin, 1995). Il existe de nombreuses manières différentes de résoudre le problème de la

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[1] Voir (Cushing, 1998) pour un aperçu du développement historique de la théorie.

[2] Bien que la taille des systèmes affichant des comportements quantiques ne soit pas nécessairement petite (comme le démontrent des phénomènes tels que la supraconductivité et la superfluidité (Annett, 2004; Blundell, 2009)), la plupart des systèmes quantiques appartiennent en effet à de petites échelles.

mesure. Dans tous les cas, elles impliquent d'adopter à la fois une position épistémique particulière envers la théorie, et une interprétation particulière de son formalisme (lequel, pour compliquer les choses, existe sous différentes variantes [3]).

Plus précisément, le débat entre réalisme et antiréalisme scientifique questionne la capacité de la science à décrire avec précision le monde objectif (et, en particulier, la couche fondamentale du monde dans le cas des théories physiques fondamentales) [4]. Ce débat a pris une importance pressante dans le contexte de la physique quantique, car il conditionne fortement la stratégie adoptée pour répondre au problème de la mesure. Les antiréalistes vont *dissoudre* le problème en adoptant une approche instrumentaliste de la théorie, ou en réduisant les mesures quantiques à une manipulation des connaissances de l'observateur. Cela a l'avantage de situer les caractéristiques non-classiques de la mécanique quantique principalement au niveau de la relation entre les observateurs et les objets. Au contraire, un réaliste scientifique *résoudra* le problème de la mesure en rendant compte des mesures quantiques en termes physiques, et localisera les comportements non-classiques des systèmes quantiques dans la nature elle-même. Pour ceux qui adoptent une attitude réaliste, la science contraint, dans une certaine mesure, l'ontologie du monde et sa dynamique. En conséquence, une forme de métaphysique naturalisée est pratiquée [5]. La question se pose de la manière exacte dont l'articulation entre la métaphysique naturalisée et la science a lieu, et de savoir si cette nouvelle méthodologie devrait entrer en conflit avec d'autres façons de poursuivre les recherches métaphysiques. Le réaliste explorant la nature de la réalité telle qu'elle est contrainte par une théorie scientifique spécifique devra développer des théories métaphysiques adaptées à ces contraintes. Dans le cas de la mécanique quantique, on parlera d'ontologies quantiques pour désigner de telles images métaphysiques de la réalité fondamentale. Ces ontologies impliquent des vues non-classiques de la nature, ce qui conduit à des ontologies et/ou dynamiques non-intuitives. En particulier, les caractéristiques quantiques telles que l'intrication et la non-localité doivent être prises en compte par des théories métaphysiques innovantes [6].

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[3] Voir, par exemple, (**lewis2016quantum**; **maudlin2019philosophy**) pour un aperçu des différentes versions de la mécanique quantique.

[4] Voi (**psillos2005scientific** ; Agazzi, 2017; Lyons and Vickers, 2021).

[5] Voir, par exemple, (Ladyman et al., 2007; Morganti, 2013; Ross, Ladyman, and Kincaid, 2013) pour une discussion.

[6] Voir, par exemple, (**lewis2016quantum**) pour un aperçu.

Alors que ces questions sont discutées depuis les premiers développements de la théorie de la mécanique quantique, c'est-à-dire il y a plus d'un siècle, les physiciens ont continué à élargir l'appareil théorique de la physique quantique en développant des théories plus générales (ou des approches de celles-ci) compatibles avec la relativité (par exemple, la théorie des champs [7]), et avec la gravitation (c'est-à-dire une théorie de la gravité quantique [8]). Ces développements génèrent d'autres défis d'interprétation car ils introduisent de nouvelles caractéristiques et principes théoriques à prendre en compte.

Dans ce contexte, la philosophie de la mécanique quantique est devenue un sous-domaine important de la philosophie des sciences (et de la physique en particulier). Ce domaine fournit des études de cas utiles et de nouvelles contraintes alimentant les débats sur (i) le réalisme scientifique, sur (ii) le lien existant entre la science et la métaphysique, et sur (iii) les théories métaphysiques articulant les ontologies fondamentales. On voit que la philosophie de la physique quantique se situe à l'intersection de la physique, de l'épistémologie et (pour les réalistes scientifiques) de la métaphysique. La portée de ce travail sera principalement restreinte aux questions métaphysiques, étant donné que le cadre théorique exact utilisé pour formuler la généralisation de la mécanique quantique (appelée le *process matrix formalism* (PMF)) [9]) d'une part, et la position épistémique (à savoir le réalisme scientifique) adoptée tout au long de l'ouvrage d'autre part, seront adoptées comme hypothèses de travail.

Il n'y a actuellement aucun consensus parmi les approches réalistes de la mécanique quantique quant à quelle ontologie, ou dynamique doit être préférée. Au lieu de cela, une grande variété de théories ont été développées pour décrire le monde quantique. L'ontologie fondamentale peut, par exemple, afficher des caractéristiques holistiques ou structuralistes, voire incorporer une forme d'indétermination métaphysique [10]. L'ontologie fondamentale peut être primitive [11] (c'est-à-dire constituée d'entités localisées dans un espace-temps fondamental à 3+1 dimensions) ou non-spatio-temporelle (c'est-à-dire constituée d'entités situées dans un espace $\mathcal{S}$ autre que notre espace-temps

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[7] Voir, par exemple, (Peskin and Schroeder, 2019) pour un aperçu.

[8] Voir (Oriti, 2009) pour une présentation récente des développements actuels.

[9] Voir (**oreshkov2012quantum**) pour une présentation.

[10] Voir, par exemple, (**lewis2016quantum**) pour une présentation.

[11] Voir, par exemple, (Allori, 2013).

familier à 3+1 dimensions, lequel est plutôt dérivé de cet espace plus fondamental $\mathcal{S}$ [12]) . Le monde macroscopique émergeant du domaine quantique peut coïncider avec notre expérience du monde classique, ou peut être considéré comme dynamiquement structuré en branches causalement déconnectées [13]. En ce qui concerne la dynamique de l'ontologie fondamentale, elle peut être déterministe (e.g. la mécanique bohmienne [14]) ou stochastique (e.g. la théorie GRW [15]). Elle peut impliquer des équations linéaires (par exemple la théorie des mondes multiples [16]) ou non-linéaires (par exemple la théorie GRW). Le choix parmi ces nombreuses possibilités est basé principalement sur des préférences personnelles plutôt qu'exclusivement motivé par une supériorité philosophique décisive d'un récit sur les autres [17].

Le riche cadre théorique de la mécanique quantique et le large éventail de positions philosophiques possibles que l'on peut articuler pour lui donner un sens permettent d'explorer diverses théories métaphysiques contraintes par la physique quantique. L'approfondissement de la compréhension métaphysique d'une théorie donnée peut impacter l'approche des progrès théoriques futurs, en explicitant certains engagements métaphysiques et en leur conférant un rôle structurant au sein de l'appareil théorique global de la future théorie. La diversité des récits métaphysiques développés dans le contexte de la physique peut également s'avérer utile lorsqu'elle est appliquée à différents domaines de la nature. En tant que tel, poursuivre une analyse métaphysique de nos meilleures théories actuelles peut fournir des outils utiles aux scientifiques (Chakravartty, 2017b).

Pourtant, ces lectures métaphysiques de la physique quantique méritent encore d'être affinées et développées, car elles suscitent de nombreux débats liés à leurs diverses implications. De plus, le statut inachevé de la physique fondamentale crée une possible tension dans le travail métaphysique du réaliste scientifique. Alors que les ontologies attribuées à la mécanique quantique sont elles-mêmes abondantes, les différents programmes développant une théorie de la gravité quantique sont peut-être sous-tendus par différentes positions philosophiques sur le monde. Il est légitime de se demander si les vues métaphysiques de la mécanique quantique standard peuvent survivre à

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[12] Voir, par exemple, (Albert, 2013).

[13] Voir, par exemple, (Wallace, 2012).

[14] Voir (Bohm, 1952).

[15] Voir (Ghirardi, Rimini, and Weber, 1986).

[16] Voir (Wallace, 2012).

[17] Voir (Chakravartty, 2017b) pour une discussion.

la transition vers la gravité quantique, et, si oui, dans quelle mesure [18]. Il est également intéressant de voir si les caractéristiques métaphysiques suggérées dans le contexte de la gravité quantique pourraient éclairer les problèmes conceptuels de la mécanique quantique.

Le présent travail vise à explorer d'un peu plus près cet écart conceptuel entre la mécanique quantique et une future théorie de la gravité quantique. La principale différence entre ces deux théories est que cette dernière unifie la description quantique de la matière avec la description relativiste de l'espace-temps, produisant une description quantique de la gravité. En tant que tel, on s'attend à ce qu'une description précise de l'espace-temps au niveau fondamental ne puisse être atteinte que dans le cadre d'une théorie de la gravité quantique. Pourtant, nous avons l'intuition que la manière dont l'espace-temps est contraint dans une théorie non-gravitationnelle, mais quantique, est susceptible d'exposer les tensions existantes entre les caractéristiques potentiellement incompatibles de l'espace-temps classique avec la physique quantique. Pour cette raison, notre méthodologie sera d'utiliser la mécanique quantique comme point de départ, et d'explorer dans quelle mesure cette théorie pose des contraintes possibles sur la façon dont l'espace-temps est conçu. Plus précisément, nous nous concentrerons sur l'extension de la mécanique quantique, appelée process matrix formalism, mentionnée ci-dessus et dans laquelle les corrélations entre plusieurs laboratoires peuvent être décrites sans spécifier *a priori* leurs emplacements spatio-temporels. En tant que tel, ce cadre permet d'explorer la manière dont les caractéristiques théoriques quantiques peuvent impacter, dans une certaine mesure, les relations spatio-temporelles entre les parties en interaction, tout en imposant des contraintes minimales sur les caractéristiques de l'espace-temps lui-même (il est littéralement non spécifié au niveau formel). La question reste ouverte de savoir si les réflexions menées dans le cadre du process matrix formalism resteraient pertinentes une fois la gravité prise en compte [19]. Pourtant, cette recherche permettra d'avoir un premier aperçu de l'impact possible des caractéristiques quantiques sur l'espace-temps, indépendamment de la façon dont la gravité est décrite au niveau quantique. Cela pourrait servir de base pour des réflexions futures et plus avancées en gravité quantique. Rétrospectivement, cela pourrait également apporter un nouvel éclairage sur la façon dont la mécanique quantique standard est interprétée. En d'autres termes, les problèmes conceptuels découlant des généralisations non-gravitationnelles de la mécanique quantique ont le potentiel d'agir comme le chaînon manquant reliant les études métaphysiques en mécanique

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[18] Voir (McKenzie, 2020).

[19] Voir (Zych et al., 2019; Paunković and Vojinović, 2020).

quantique standard et la théorie encore inachevée de la gravité quantique.

Plus précisément, ce travail se concentrera sur une caractéristique théorique centrale du process matrix formalism, appelée *nonséparabilité causale* [20]. Elle est définie, dans une certaine mesure, par analogie avec la nonséparabilité quantique, qui caractérise l'état quantique d'un système composite qui ne peut être exprimé par un mélange probabiliste de produits tensoriels des états quantiques des sous-systèmes. La non-séparabilité causale, en revanche, caractérise les processus quantiques (reliant les entrées et les sorties de différentes opérations quantiques locales) qui sont incompatibles avec toute structure causale définie entre les parties en interaction. On parle d' *ordres causaux indéfinis*. Un exemple célèbre de processus causalement nonséparable est appelé le *quantum switch* (QS). Il est abondamment étudié dans la littérature en raison de son architecture simple et de ses diverses implémentations en laboratoire. Le présent travail discutera des interprétations possibles de la non-séparabilité causale sous les hypothèses suivantes : (i) une approche scientifiquement réaliste des processus quantiques est adoptée, et (ii) la physicalité de la non-séparabilité causale pour au moins certains processus (y compris le quantum switch) est assumée, c'est-à-dire que la non-séparabilité causale est considérée comme pointant vers de nouvelles caractéristiques objectives de la nature. Les objectifs de ce travail seront alors de donner un aperçu des attitudes réalistes possibles vis-à-vis de la nonséparabilité causale, et de discuter des liens que cette caractéristique établit avec l'espace-temps. Nous réfléchirons à la mesure dans laquelle ces points de vue pourraient rester pertinents dans différents contextes théoriques, à savoir la mécanique quantique standard et la gravité quantique. Les résultats viseront à souligner une tension existante entre des caractéristiques théoriques quantiques telles que la non-séparabilité causale et l'idée d'un espace-temps classique.

Dans ce contexte, et suivant la méthodologie mentionnée ci-dessus, plusieurs résultats ont été obtenus, dont le résumé est fourni dans la section suivante.

D.2 Résultats

Le **deuxième chapitre** de la thèse originale donne un aperçu de la discipline appelée *métaphysique naturalisée*, qui considère que la science est le meilleur guide pour la recherche métaphysique, et explore diverses questions connexes sur la façon dont cette connexion entre science et métaphysique (devrait) avoir

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[20] Voir (Oreshkov, Costa, and Brukner, 2012).

lieu. Le chapitre décrit également le domaine spécifique de la métaphysique naturalisée appliquée à la mécanique quantique. Cela dresse une esquisse des ontologies quantiques existantes par rapport auxquelles les implications de nouveaux concepts tels que la non-séparabilité causale seront présentées.

La première section passe en revue les discussions concernant la relation entre la science et la métaphysique, en mettant en évidence les nombreux arguments en faveur d'un dialogue étroit entre les deux domaines. Bien que ce travail ne vise pas à aborder des questions spécifiques relatives au domaine de la métaphysique naturalisée, il est important de garder à l'esprit les diverses questions ouvertes soulevées dans cette littérature qui pourraient affecter les motivations même et la légitimité de la présente recherche. Pour cette raison, ces questions particulières seront ré-examinées en temps voulu dans le reste de cette thèse.

La deuxième section fournit un large aperçu des représentations métaphysiques de la réalité qui ont été discutées dans le contexte de la mécanique quantique, soit dans le cadre d'une solution complète au problème de mesure, soit en tant que lecture cohérente d'une caractéristique théorique particulière (et centrale) de la théorie. Cet aperçu global constitue une boîte à outils métaphysiques qui pourra guider le développement du présent travail.

Le **troisième chapitre** de la thèse introduit ensuite la nouvelle caractéristique théorique qu'est la nonséparabilité causale, et le cadre global dans lequel elle est définie, à savoir le *process matrix formalism*. Parce que ce formalisme est dit "opérationnel" et sera étudié dans un cadre scientifique réaliste dans les prochains chapitres, il est ensuite avancé que, contrairement à une certaine tendance antiréaliste dans le domaine des fondations quantiques, une attitude réaliste est tout aussi adaptée pour interpréter les théories opérationnelles que les approches antiréalistes.

Plus en détail, nous passons en revue trois arguments selon lesquels les formulations opérationnelles de la mécanique quantique contiennent des indices soutenant une lecture antiréaliste de la théorie. Des objections ont été formulées, permettant de conclure que ces arguments n'étaient pas convaincants. Il a également été avancé que le cadre opérationnel ne fournissait aucun argument en faveur d'une lecture réaliste plutôt qu'antiréaliste. Il a été rappelé que de tels résultats sont attendus, étant donné que le débat scientifique réaliste/antiréaliste dans le contexte de la mécanique quantique se situe à un niveau épistémique, et n'est pas concerné par la forme spécifique de la théorie.

Toute cette discussion nous conduit à une affirmation principale importante, à savoir que le cadre opérationnel des théories physiques est à la fois épistémologiquement et ontologiquement neutre en soi. Premièrement, le cadre opérationnel est épistémologiquement neutre puisque les arguments en faveur d'une position épistémologique envers la physique quantique ne font appel à aucun formalisme en particulier ; leur succès n'est ni renforcé ni amoindri dans la formulation opérationnelle de la mécanique quantique par rapport à la situation dans la formulation standard. Deuxièmement, le cadre opérationnel seul est ontologiquement neutre puisque passer de postulats opérationnels à une proposition sur l'ontologie de la théorie implique de préciser le statut des corrélations au centre du formalisme, ce statut étant postulé audessus des aspects formels de la théorie.

Le **quatrième chapitre** de la thèse originale analyse ensuite la non-séparabilité causale à un niveau purement formel, soulignant en quoi elle est différente de la non-séparabilité quantique standard. La non-séparabilité causale ne peut pas être considérée comme une simple extension de la non-séparabilité quantique (qui décrit les corrélations entre les états quantiques à un moment donné) à une description des corrélations entre les états quantiques à des moments différents. Au lieu de cela, la non-séparabilité causale encode des corrélations entre les événements quantiques, indépendamment (sous certaines conditions) des systèmes et des opérations considérés. En conséquence, il décrit un type de corrélations d'ordre supérieur entre les fonctions transformant les états quantiques. Cette clarification est une première étape importante pour éviter les raccourcis interprétatifs potentiels qui pourraient affecter les discussions d'ordre métaphysique.

A partir de là, un point de vue indépendant de tout modèle est adopté, c'est-à-dire que la non-séparabilité causale est remplacée par la notion de noncausalité [21], qui caractérise (dans le cas où seuls deux expérimentateurs seraient présents) les corrélations expérimentales entre les résultats de mesure de ces expérimentateurs pour lesquelles du signalling (qui signifie que le choix de mesure chez l'un est statistiquement corrélé aux résultats de mesure chez l'autre) bidirectionnel est observé. Tout comme les processus quantiques causalement nonséparables, les corrélations noncausales sont incompatibles avec une structure causale définie entre les expérimentateurs impliqués.

La possible signification physique sous-jacente aux corrélations noncausales est discutée. En particulier, une ontologie holistique ou ontique structuraliste est envisagée pour expliquer une éventuelle noncausalité de certaines

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[21] Voir (Oreshkov, Costa, and Brukner, 2012).

corrélations. Alternativement, une dynamique holistique ou retro-causale appropriée peut également rendre compte des corrélations noncausales. Les différences entre la non-causalité et la non-localité quantique standard sont ensuite discutées. Alors que les corrélations nonlocales violent le principe dit de causalité locale (i.e. les causes (et effets) directs des évènements sont proches, et même les causes (et effets) indirects ne sont pas plus éloignés que ce qui est permis par la vitesse de la lumière), il est proposé que la noncausalité viole une forme de non-localité *temporelle*, définie comme la contre-partie temporalle de la causalité locale. La violation d'une inégalité causale implique donc une forme de non-localité temporelle. Il est mis en évidence que la localité temporelle est un principe plus contraignant que la causalité locale, car la première implique la seconde mais pas réciproquement.

Enfin, les principaux résultats de cette thèse sont présentés dans le **cinquième chapitre** de la thèse, dans laquelle la non-séparabilité causale est discutée métaphysiquement selon diverses positions possibles. Il est soutenu dans ce chapitre qu'une attitude réaliste envers la non-séparabilité causale peut avoir des implications pour les relations spatio-temporelles. Nous nous appuyons sur le quantum switch comme étude de cas pour la nonséparabilité causale (menant à des ordres causaux indéfinis). Il est mis en évidence que les stratégies pré-existantes dans la litérature en philosophie de la causalité pour dissoudre l'indétermination de certaines structures causales ne fonctionnent pas dans le cas du quantum switch. Le sens et/ou la nature exacts à attribuer à l'indétermination des relations causales dans le cas du quantum switch dépendraient à la fois de l'explication spécifique que l'on donne au PMF, et de la théorie particulière de la causalité considérée. En passant d'une notion de structure causale à une notion de structure spatio-temporelle, l'indétermination des ordres causaux peut (sous certaines conditions) être transposée aux relations spatio-temporelles. La signification exacte de cette indétermination dépendrait alors de l'interprétation donnée au PMF.

Pourtant, il est soutenu qu'il existe des arguments intéressants (e.g., en termes de pouvoir explicatif) pour considérer les relations spatio-temporelles comme étant métaphysiquement indéfinies. Dans ce contexte, un parallèle avec la situation de la gravité quantique, au sein de laquelle l'idée d'un espace-temps indéterminé a déjà été avancée, est discutée. Il est aussi mis en évidence que de telles conséquences pour les relations spatiales peuvent déjà être défendues en mécanique quantique standard. Par conséquent, malgré les différences entre la mécanique quantique standard et le process matrix formalism, les deux théories peuvent soutenir des implications substantielles pour les propriétés des relations spatio-(temporelles).

D.3 Conclusions et perspectives

En conclusion, ce travail a tenté de combler en partie le fossé existant entre les discussions métaphysiques au sein de la théorie non-fondamentale qu'est la mécanique quantique et les discussions métaphysiques au sein de la théorie plus générale mais non encore complètement développée qu'est la gravité quantique. Les présents résultats suggèrent que certaines théories métaphysiques peuvent s'avérer utiles dans divers développements théoriques. À tout le moins, la réflexion sur les implications métaphysiques des théories non-fondamentales met l'accent sur certaines tensions existant entre des principes inharmonieux, tels que l'espace-temps galiléen et les caractéristiques quantiques.

Bien entendu, les présents résultats présentent un certain nombre de limites, invitant à de futures discussions. Tout d'abord, alors qu'il a été défendu que les formalismes opérationnels étaient ontologiquement et épistémiquement neutres, il est toujours pertinent de se demander quel pourrait être l'avantage philosophique particulier de travailler avec ces formalismes. En particulier, il est intéressant d'explorer plus avant la capacité de tels formalismes à étudier les fondements des théories, et de discuter comment les informations extraites de ces études pourraient constituer des connaissances objectives *sur les théories*, dont la valeur épistémique pourrait aller au-delà du débat réaliste/antiréaliste en physique. Ces questions seront prochainement étudiées dans le cadre d'un projet d'épistémologie formelle (Kvasz, Ladislav, 2021).

Deuxièmement, le statut scientifique de la non-séparabilité causale peut être examiné plus avant. En effet, l'obtention de véritables ordres causaux indéfinis dans les laboratoires est débattue, et on peut se demander si la non-séparabilité causale est en effet scientifiquement étayée. Bien qu'il ait été soutenu ici qu'une telle affirmation était raisonnable, d'autres discussions pourraient être pertinentes pour solidifier la position. Dans le cas où des arguments significatifs à l'encontre de la physicalité des ordres causaux indéfinis pourraient être développés, le présent travail trouverait de la valeur à la fois comme catalyseur pour la métaphysique naturalisée de la physique fondamentale et comme jonction conceptuelle entre la compréhension des théories plus fondamentales et celle des théories effectives telles que la mécanique quantique. En tout état de cause, préciser le statut exact de la nonséparabilité causale d'un point de vue scientifique reste une question ouverte actuellement débattue dans le domaine des fondements de la physique quantique.

Enfin, le lien même que nous avons présenté entre les structures causales et spatio-temporelles afin d'explorer les implications potentielles de la non-séparabilité causale sur l'espace-temps peut être développé plus en détail. D'une part, les hypothèses nécessaires à l'établissement de ce lien peuvent être questionnées et leur motivation développée. D'autre part, en acceptant la présence de relations spatio-temporelles indéfinies comme conséquence de la nonséparabilité causale, la nature de cette indétermination pourrait être davantage discutée. En particulier, la lecture métaphysique des relations spatio-temporelles indéfinies pourrait être rendue plus précise en entrant dans le détail des diverses articulations possibles de l'indétermination métaphysique. Comme il existe actuellement un débat sur la meilleure manière d'articuler l'indétermination métaphysique dans le cadre de l'indétermination quantique, il faut s'attendre à ce que ces discussions se révèlent pertinentes dans le cas des ordres causaux indéfinis, lesquels pourraient servir d'étude de cas supplémentaire pour le développement de l'indétermination métaphysique dans un contexte quantique.

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