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Working Paper

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Working Paper, No. 2013:7

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Suggested Citation: Liang, Che-Yuan (2013) : Optimal inequality behind the veil of ignorance, Working Paper, No. 2013:7, Uppsala University, Department of Economics, Uppsala, <https://nbn-resolving.de/urn:nbn:se:uu:diva-199754>

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Working Paper 2013:7

*Optimal Inequality behind the Veil of
Ignorance*

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Working paper 2013:7
May 2013
ISSN 1653-6975

OPTIMAL INEQUALITY BEHIND THE VEIL OF IGNORANCE

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Optimal Inequality behind the Veil of Ignorance*

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April 29, 2013

Abstract: In Rawls' (1971) influential social contract approach to distributive justice, the fair income distribution is the one that an individual would choose behind a veil of ignorance. Harsanyi (1953, 1955, 1975) treats this situation as a decision under risk and arrives at utilitarianism using expected utility theory. This paper investigates the implications of applying prospect theory instead, which better describes behavior under risk. I find that the specific type of inequality in bottom-heavy right-skewed income distributions, which includes the log-normal income distribution, could be socially desirable. The optimal inequality result contrasts the implications of other social welfare criteria.

Keywords: veil of ignorance, prospect theory, social welfare function, income inequality

JEL classification: D63, D03, D31, D81

* I thank Spencer Bastani, Mikael Elinder, Per Engström, and Eskil Forsell for valuable comments and suggestions. The Jan Wallander and Tom Hedelius Foundation and the Swedish Council for Working Life and Social Research are acknowledged for their financial support.

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1. Introduction

How to distribute income fairly is a question that has been discussed across different disciplines of social science and philosophy. Harsanyi (1953, 1955, 1975) and Rawls (1971) offer two of the most influential theories of distributive justice, both using the popular social contract approach. A central idea is that the normative question can be transformed to the descriptive question of what income distribution an individual would choose in a hypothetical original position before knowing her identity in the society. Under such a veil of ignorance, the decision maker becomes an impartial observer, internalizing the interests of *all* members of the society appropriately and therefore decides upon the fair distribution. However, the resulting principle of justice and social welfare function depends on the framing of the original position. Whereas Rawls' arrives at the maximin principle, Harsanyi favors utilitarianism.

Under the veil of ignorance, income distributions can be thought of as lotteries of birth because the decision maker *randomly* becomes somebody in her chosen distribution. The randomness is often perceived to be *unfair ex post* because it is beyond individuals' control. However, because the decision maker chooses and accepts the randomness of an income distribution *ex ante*, the randomness is what makes the income distribution *fair*. Harsanyi embraces the lottery interpretation of the original position and uses von Neumann and Morgenstern's (1944) theory of decision under risk applying expected utility theory to the problem. Since Harsanyi's seminal work, there has been plenty of new empirical evidence that expected utility theory provides a poor description of individual behavior under risk.¹ To cope with the deficiencies of expected utility theory, Kahneman and Tversky (1979) developed prospect theory. There are, by now, many empirical studies in support of this theory.² In this paper, I explore the consequences of applying prospect theory given Harsanyi's lottery interpretation of the original position. This corresponds to using actual individuals' preferences for lottery distributions to evaluate the social welfare of income distributions.

I study the problem of distributing a certain fixed amount of income in a population once. It is a decision under risk because the frequencies of different income levels are known. Production and efficiency concerns are ignored. I start out by investigating the simplest two-income-level distribution for analytical tractability and to pin down the intuition before moving on to continuous income distributions using simulations. I explore the effects of using the mean income as the reference income, which is the income all individuals would have in the even income distribution with complete equality. This exercise corresponds to the evaluation of mean-preserving spreads and fair-odds lotteries, which is a theoretical exercise that, as far as I know, never has been done systematically before using prospect theory. I also develop and use "representative aggregation of reference incomes" which takes the mean of the social welfare *evaluations* of each of the individuals in the realized income distribution, letting each realized income level serve as reference income representatively.

¹ See, e.g., Kahneman and Tversky (1979), Fishburn and Kochenberger (1979), Hershey and Schoemaker (1980), Payne et al. (1981), Wehrung (1989), Tversky and Kahneman (1992), Camerer and Ho (1994), and Wu and Gonzales (1996).

² See Kahneman and Tversky (2000) and the references in footnote 1.

Prospect theory differs from expected utility theory in four aspects. First, it is gains and losses relative to a reference level that carry utility rather than absolute levels of income. Second, not only gains but also losses exhibit decreasing marginal sensitivity. Third, losses carry more disutility than gains carry utility. Fourth, probabilities are not linearly weighted; instead, probabilities of large gains and losses are overweighted compared to probabilities of small gains and losses.

The expected utility and maximin decision makers would, in the original position, select complete equality. For the expected utility decision maker, the inequality aversion is caused by diminishing marginal utility in income implying risk aversion. A prospect decision maker has, however, two reasons to prefer an uneven income distribution. First, incurring small losses with a high probability to afford large gains with a low probability could be attractive because large gains are overweighted. Second, incurring large losses with a low probability to afford small gains with a high probability could be attractive because large losses have low marginal disutility. In a two-income-level world, this leads to two possible types of optimal uneven income distributions. The first type is a bottom-heavy right-skewed superstar distribution where few individuals have very high income and many individuals have low income. The second type is a top-heavy left-skewed scapegoat distribution where few individuals have very low income and many individuals have high income. However, loss aversion and the overweighting of large losses are components of prospect theory that work in the opposite direction towards inequality aversion.

Whether inequality is desirable depends on the exact parameterization of prospect theory. I show that the superstar distribution is optimal under some assumptions. Furthermore, the superstar type of inequality is more desirable than complete equality when using a reasonably chosen prospect theory parameterization for two-income level distributions and log-normal income distributions which many countries have (Gibrat, 1931; Aitchison & Brown, 1957; Battistin et al., 2007). The intuition is that these income distributions resemble fair odds lotteries that people do buy. Such distributions contain the American dream with an ex ante opportunity to become a superstar creating a strong psychological possibility effect.

The result that some types of inequality may be inherently socially desirable contrasts traditional social welfare functions. These criteria are typically based on diminishing marginal utility, positional concerns for the lower end of income distributions, or are directly inversely related to income inequality. They include, besides expected utility and maximin, e.g., the Cobb-Douglas welfare function, the quadratic welfare function (Epstein & Segal, 1992), Atkinson's social welfare function (Atkinson, 1970), Gini, entropy, and Boulding's principle (Boulding, 1962). In the simple income distribution problem in this paper, absent other concerns, they all favor complete equality. Of course, when production is introduced into the problem, inequality may be tolerated because there is usually an efficiency-equity trade-off.

This paper is related to a few papers in behavioral economics discussing the link between individual choice and welfare. Whereas behavioral models describe observed choice, it is disputed whether welfare should be based on decision utility, experienced utility, or remembered utility, which may differ (Kahneman et al., 1997). Bernheim and Rangel (2009) suggest a purely choice-based approach to welfare evaluations and call the approach behavioral welfare economics. An obstacle is that choice may be inconsistent (it could, e.g.,

vary with reference point, over time, and with framing), and Bernheim and Rangel suggest a procedure to remove such inconsistencies from the welfare measure.

A few papers ignore the issue of the normative attractiveness of the welfare measure when constructing a choice-based behavioral measure of welfare based on individual decision utility. Günther and Mayer (2008) and Jäntti et al. (2013) apply the hybrid model in Koszegi and Rabin (2006) combining a traditional concave utility function that depends on income levels and a prospect theory part that depends on income changes to construct new measures of poverty and welfare. Prospect theory better captures choice patterns involving income dynamics by being reference-point dependent. As Jäntti et al. (2013) argue, such a welfare measure is interesting by reflecting the *perceived* welfare of individuals experiencing income changes. Their main conclusion is that income changes generally reduce welfare because of loss aversion.

This paper does not directly take a position on what *individual* welfare should be based on. Instead, it applies an individual behavioral model in evaluating *social* welfare. Individual choice is relevant because the original position reduces the question of social welfare into one of individual choice that seems not necessarily related to welfare. Unlike the previous applications of prospect theory in welfare economics, the simplest problem set up here is static. The main feature is therefore not related to dynamics.

Another difference is that the results of this paper have full normative force – if accepting the lottery interpretation of the original position. If the optimal inequality conclusion cannot be accepted because it is an unpleasant type of justice, a possible argument is that the original position needs to be modified or rejected. Rawls (1971) offers one such modification by depriving the decision maker any personal characteristics, including risk preferences. He further argues that the decision is then one under uncertainty, where the decision maker does not know or should disregard the frequencies of different income levels.³

Another line of argument could be that individuals' normal behavior under risk should not be applied in the original position. This application may be inappropriate because prospect theory is purely descriptive and maybe there should be some normative constraints on the decision maker in the original position.

Another view is that the descriptive question of how people would choose in the original position should be answered by asking individuals or groups about what they would prefer or could agree on. There are numerous experimental studies (e.g., Frohlich et. al., 1987; Bosmans & Schokkaert, 2004; Herne & Soujanen, 2004; Johansson-Stenman et. al., 2004; Traub et al., 2005; Amiel et. al., 2009), and the outcome turns out to depend crucially on the framing of the original position. Among other things, the degree of inequality aversion depends on factors such as whether the scenario is about risk or uncertainty, whether the respondents should consider them to be external observers or involved in the realized distributions, the thickness of the veil, individual background characteristics, and the exact rules of negotiation in the case of groups agreeing on a principle of justice. The results are not clear-cut and are difficult to summarize, except that often both the maximin and utilitarian

³ The merits of this argument have been disputed by others as well as what social function it would imply (e.g., Harsanyi, 1975). In other parts of Rawls (1971), he indicates that the original position is merely one of many devices that should be used to think about distributive justice and that the different devices may lead to conflicting principles of justice.

social welfare functions perform poorly. The results of this paper, corresponds to asking individuals about their preferences in an original position that they perceive as a lottery.

The next section presents the model used. Section 3 presents the income distributions investigated. Section 4 reports some analytical results. Section 5 reports some simulation results. The final section concludes and further discusses the implications of the results.

2. Model

The problem at hand concerns how to evaluate different income distributions once. It is a purely static problem and income can be thought of as life-time income, resources, endowment, wealth, or consumption goods. Assume that each income level x carries a utility for individuals according to $u(x)$. By normalizing the population to 1, the frequencies in the income distribution can be interpreted as probabilities and they sum to 1. Let $P(x)$ be the probability distribution function and let $p(x) = \frac{dP(x)}{dx}$ be the associated probability density function.⁴

The original position transforms society's choice of the optimal income distribution into an individual decision's maker's choice of the optimal lottery, interpreting the frequencies described by $P(x)$ as probabilities of different lottery outcomes. The lottery interpretation is attractive because it forces the social welfare evaluation to account for the outcome of *all* individuals (with different incomes) in the income distribution, in the same manner as an individual's preference evaluation of a lottery where she accounts for each of the different lottery outcomes *she* could end up with. The social desirability of an income distribution can therefore be answered by asking individuals about their preferences for a lottery with the same income distribution.

Fortunately, preferences for and actual choice patterns of lottery distributions have been extensively studied theoretically and empirically before. It is therefore possible to apply a calibrated model of decision under risk that relies on the insights of this literature to investigate the problem without the need to ask individuals about their hypothetical preferences in the original position. This circumvents the issues of how to appropriately frame the original position to remove normative elements and to obtain truthful answers of behavior in a hypothetical scenario.

I now formulate a general model to evaluate income and lottery distributions that encompasses (at least) the two most popular theories: expected utility theory (von Neumann & Morgenstern, 1944) and prospect theory (Kahneman & Tversky, 1979). The decision maker attaches a weight to the probabilities of the different income levels according to the probability weighting distribution function $W(P(x))$ and the associated probability weighting

⁴ In the previous literature, prospect theory is formulated for discrete income distributions. I work in a framework that can handle continuous income distributions. The formulation and parameterizations become somewhat different and may feel unfamiliar to readers who are used to the standard formulation of cumulative prospect theory in Kahneman and Tversky (1992). The intuition behind prospect theory may be perceived as less clear. But the current formulation simplifies the construction of the objective function by treating probability weights using distribution functions and contains other theories such as expected utility theory as a special case.

density function $w(P(x)) = \frac{dW(P(x))}{dP(x)}$.⁵ We are now interested in evaluating income distributions with a fixed total and mean income x_m . Any income distribution can be obtained by starting out from an even income distribution where everyone has income x_m and then transferring income from some individuals to others. Any uneven income distribution then corresponds to a mean-preserving spread of the even income distribution.

Assume that the income distribution itself carries no value. We therefore do not care about inequality in itself. This implies that the value of an income distribution is separable in the utility of the different income levels. The optimal income distribution characterized by $P(x)$ is then the distribution that maximizes the weighted average of the utility attached to each income level, U , according to:

$$\begin{aligned} \max_{P(x)} U &= \int u(x)w(P(x))dx \\ \text{s. t. } \int p(x)dx &= 1 \text{ and } \int p(x)x dx = x_m. \end{aligned} \quad (1)$$

From the society's perspective, the objective function U represents the social welfare of an income distribution. From the individual decision maker's perspective, U represents the perceived decision utility of a lottery distribution.

The criterion in Equation (1) can be further specified by choosing functional forms for $u(\cdot)$ and $w(\cdot)$. For an expected utility decision maker, the (individual Bernoulli) utility function is concave ($u''(x) < 0$), which reflects risk aversion. Furthermore, the probability weight is linear ($W(P) = P$). Such a decision utility leads to the utilitarian social welfare function.

For a prospect theory decision maker, the utility function depends on the reference income x_0 , and it is concave for gains ($u''(x > x_0) < 0$), convex for losses ($u''(x < x_0) > 0$), and exhibits loss aversion ($u'(x_0 + a) < u'(x_0 - a), a > 0$). The probability weights fulfill subcertainty ($W(P) + W(1 - P) < 1$) and subproportionality ($\frac{1-W((1-P)q)}{1-W(1-P)} < \frac{1-W((1-P)qr)}{1-W((1-P)r)}, p, q \in (0,1)$) when comparing weights at the gain and loss sides separately, and $W(P = 0) = 0$. These properties result in the overweighting of probabilities of large gains and losses and the underweighting of probabilities of small gains and losses. Because large gains and losses usually occur with low probabilities in applications, this is often interpreted as the overweighting of low probabilities and the underweighting of high probabilities.

The modifications in prospect theory reflect the fact that people evaluate income relative to an anchoring point, that accumulated losses are better than many small losses, and that probabilities tend to be categorized as impossible, possible, probable, and certain. The theory produces a fourfold pattern of risk attitudes: risk aversion for small gains and large losses and risk seeking for large gains and small losses. The theory can explain, e.g., why some people buy both lottery tickets and insurance.

⁵ Unlike normally for distribution functions, generally, $W(P = 1) \neq 1$. The probability weighting distribution and density functions are here defined increasingly in x . The typical prospect theory formulation corresponds to defining the probability weighting distribution and density functions decreasingly in x for gains and increasingly in x for losses.

Given these properties, the utility and probability weighting functions can be parameterized in different ways, which affect the results. In the simulations, I use the standard constant relative risk-aversion (CRRA) utility function. I normalize utility to 0 at the reference income and I normalize the function so that the utility of income levels above the reference income (these income levels constitute gains when the mean income is used as the reference income) is the same in expected utility theory and prospect theory. The utility functions for expected utility theory (*EU*) and prospect theory (*PT*) are:

$$u_{EU}(x) = x^\alpha - x_0^\alpha, \quad (2)$$

$$u_{PT}(x) = \begin{cases} -\lambda[(2x_0 - x)^\alpha - x_0^\alpha] & x \leq x_0 \\ x^\alpha - x_0^\alpha & x > x_0, \end{cases} \quad (3)^6$$

where $0 < \alpha < 1$, and $\lambda > 1$. In Equation (2), risk aversion decreases when α increases. In Equation (3), marginal sensitivity of gains and losses increases when α increases. λ measures loss aversion. Because x is interpreted as income, $x > 0$. Note that reference dependence is incorporated by using a utility function that is different on the gain and loss sides. Marginal utility is also discontinuous at the reference point. The utility functions in Equations (2) and (3) are illustrated in Figure 1.

For the probability weighting function, I use the following commonly used parameterizations in the simulations:

$$W_{EU}(P) = P, \quad (4)$$

$$W_{PT}(P) = \begin{cases} \frac{P^\gamma}{(P^\gamma + (1 - P)^\gamma)^{1/\gamma}} & x \leq x_0 \\ \frac{P(x_0)^\gamma + (1 - P(x_0))^\gamma}{[P(x_0)^\gamma + (1 - P(x_0))^\gamma]^{1/\gamma}} - \frac{(1 - P)^\gamma}{(P^\gamma + (1 - P)^\gamma)^{1/\gamma}} & x > x_0, \end{cases} \quad (5)$$

where γ reflects the degree of overweighting of large gains and losses and is sometimes allowed to be different on the gain and loss sides; this collapses to linear weights when $\gamma = 1$.⁷ Like for the utility function, reference dependence produces a probability weighting distribution function that contains two pieces. The implied density function may be discontinuous at the reference income. Furthermore, mostly $W^{PT}(1) \neq 1$, although one can normalize it to 1.

⁶ Note that the formulation on the loss side implies that losses carry the same amount of disutility as λ times the utility of gains of the same size.

⁷ Normally, $\frac{P^\gamma}{(P^\gamma + (1 - P)^\gamma)^{1/\gamma}}$ is referred to as the probability weighting function. The formulation here expresses the differences between expected utility theory and prospect theory as differences in utility and probability weighting distribution functions alone.

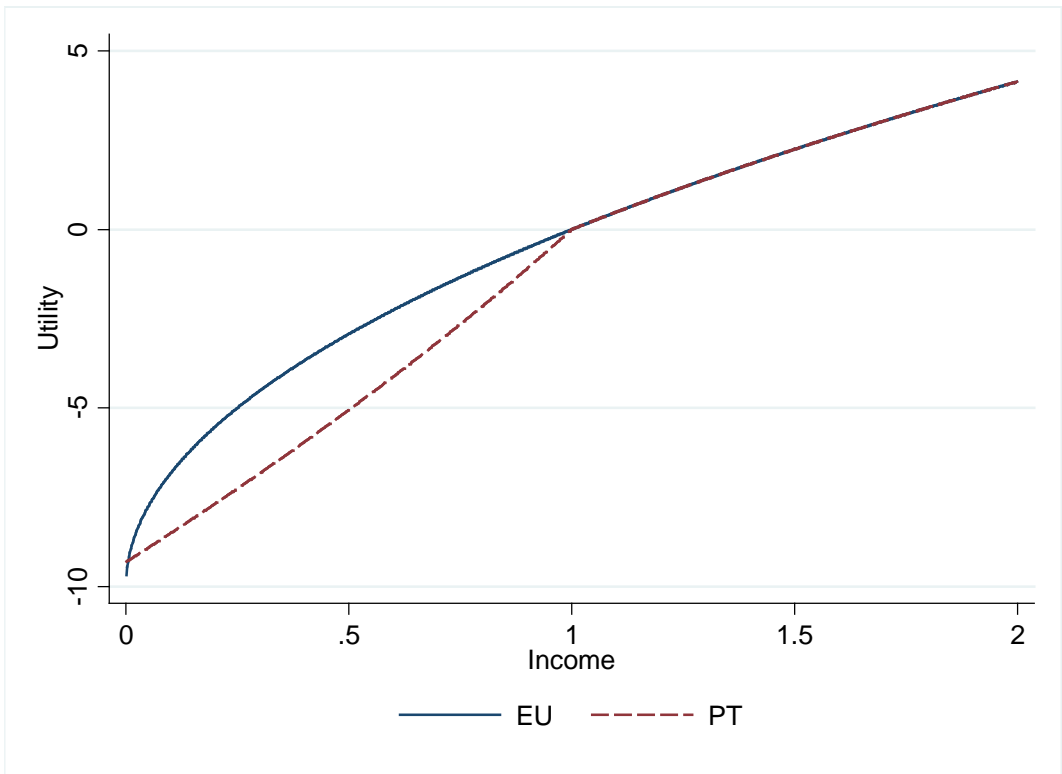


Figure 1. Individual utility function

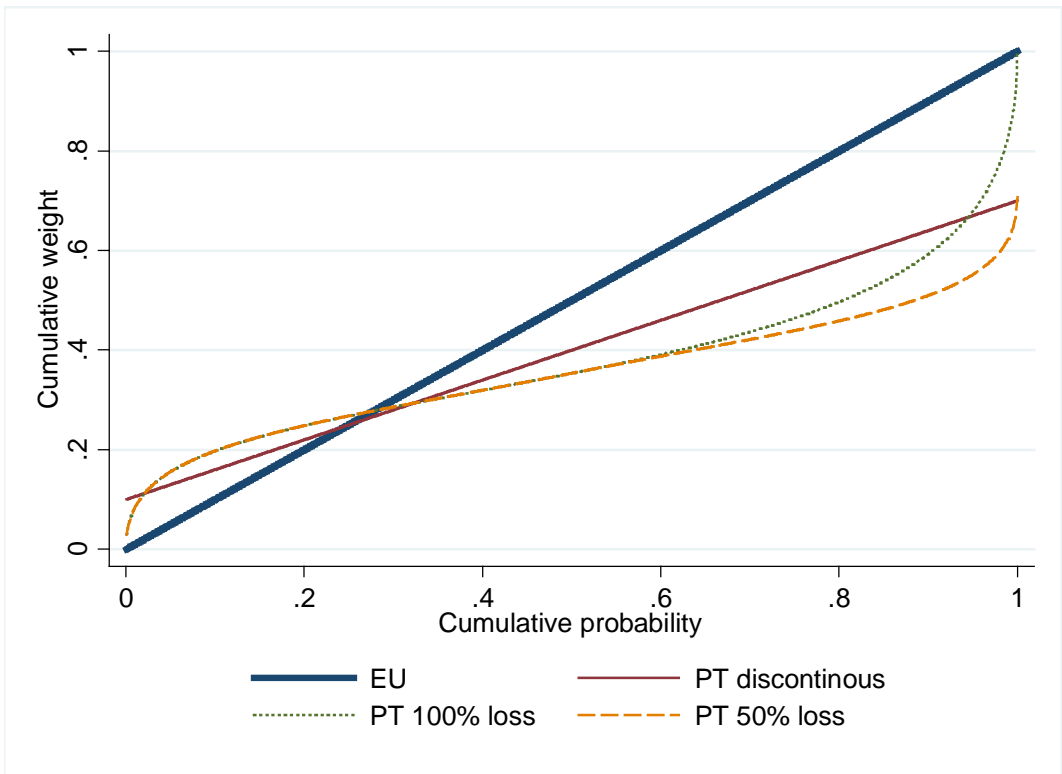


Figure 2. Probability weighting function

The probability weighting distribution function in Equations (4) and (5) are illustrated in Figure 2, where one graph displays the case with only losses and another graph the case when the gain probability is 50 percent. These two graphs are not the same because the reference income affects the weights. Figure 2 also shows a prospect theory weighting function that is discontinuous at the ends with $W(P = 0) = 0$ and $W(P = 1) = 1$, and which is close to linear in between similar to the function presented in the original prospect theory paper by Kahneman and Tversky (1979). This simple function captures the essence of prospect weighting, but the discontinuities may be difficult to work with.

Although most prospect theory applications, e.g., Tversky and Kahneman (1992), Camerer and Ho (1994), and Wu and Gonzales (1996), use the parametric form in Equations (2) to (5), there is no consensus on the parameter values. α varies between 0.32 and 0.88, and γ varies between 0.56 and 0.74 in these studies. Neilson and Stowe (2002) show, however, that none of these parameterizations can accommodate behavior according to the Allais paradox. The main lesson of the Allais paradox is that there is a certainty effect giving large weight to the probability increase of an outcome from close to 1 to 1. This is one of the observed behavioral patterns prospect theory was designed to accommodate in Kahneman and Tversky (1979). Furthermore, none of the parameter combinations in Camerer and Ho (1994) and Wu and Gonzales (1996) can accommodate gambling on unlikely gains, again an observed behavioral pattern prospect theory was designed to accommodate.

Neilson and Stowe show that given the functional form, only high values of α (> 0.5) can accommodate some gambling on unlikely gains. Furthermore, given high values of α , only low values of γ (< 0.3) can accommodate the Allais paradox. However, Neilson and Stowe's restrictions would not give the best fit to the data in the mentioned prospect theory studies. Neilson and Stowe also show that the alternative parameterization in Prelec (1998) suffers from the same issues. They do not, however, provide a functional form that can solve these issues completely, but they mention that a segmented probability weighting function with different γ that is low for large gains and losses and higher for small gains and losses can remedy some of issues and improve the fit to the data in Kahneman and Tversky (1992) at the same time.

To stay close to the literature, I continue to use the parameterization in Equations (2) to (5), despite the issues mentioned by Neilson and Stowe (2002). However, based on the discussion in Neilson and Stowe (2002), I chose parameter values that can accommodate both some gambling on unlikely gains and the Allais paradox. The numbers I use are $\alpha = 0.5$, $\gamma = 0.3$. Most result patterns are insensitive to quite large variations of the two parameters. In particular, all patterns are preserved when increasing α and when decreasing γ , which is moving in a direction that preserves the gambling on unlikely gains and Allais paradox patterns. I set $\lambda = 2.25$ like estimated in Tversky and Kahneman (1992), which is a commonly accepted value without any controversy. The simulation results of this paper are, of course, only trustworthy to the extent that the selected functional forms and parameterizations based on the literature can explain different observed phenomena and empirical data.

The desirability of income distributions for prospect theory decision makers depends on the reference income as both the utility and probability weighting functions are reference dependent. In real life applications, the reference income is typically the income level

individuals have at the decision moment and it changes over time. It may also depend on the framing of the problem. There is, however, no dynamic aspect in the original position. Also, it is unattractive to have an evaluation of distributions that depends on framing the problem in such a way that some reference income is favored over another.

I explore the effects of using the mean income in the population as the reference income, which seems to be a natural choice. This is also the income everyone has under complete equality, which is the optimal income distribution when using other social welfare criteria. However, it could be criticized for being a choice that already embodies a normative statement – that some representative individual is the standard. Besides also elaborating with the median income as the reference income, I suggest and use a procedure to overcome the arbitrariness in choosing a *specific* reference income. The procedure takes the mean of the (hypothetical) social welfare evaluations (behind the veil of ignorance) of all individuals in the realized distribution, using the realized income of each individual as her reference income. This is formally defined in Definition 1.

Definition 1. Representative aggregation of reference incomes:

$$U(P(x)) = E(U(P(x)|x_0)|P(x_0)). \quad (6)$$

$U(P(x)|x_0)$ is the objective function of an individual behind the veil of ignorance calculated using Equation (1), given her reference income x_0 . We now take the expectation over a distribution of reference incomes. The distribution function of the reference income is set to be the probability distribution function of incomes, $P(\cdot)$. This corresponds to letting every individual in the evaluated income distribution evaluating the income distribution behind the veil of ignorance *given* her realized *reference* income, and then averaging over all individuals' *evaluations*. The procedure is representative by giving each individual's evaluation the same weight in the aggregation.⁸

Using representative aggregation of reference incomes with a prospect theory utility function transforms the decision problem in Equation (1) to:

$$\max_{P(x)} U = \iint u_{PT}(x|x_0 = y)w(P(x)|x_0 = y)p(y)dx dy. \quad (7)$$

Note that for expected utility decision makers, the reference income does not affect the social welfare evaluation.

3. Income distributions

To explore the effects of different components of prospect theory and to illustrate the basic intuition, I start with the simplest problem, where income can take two different levels. Because of the revenue neutrality constraint, there are three possible types of income

⁸ It is, of course, possible to argue that giving each individual's evaluation the same weight is also a normative statement. An alternative could be to apply prospect theory weights to the different evaluations. But to do this would require choosing a higher order reference point because prospect theory weights are also reference dependent.

distributions. One type of income distribution is the bottom-heavy right-skewed distribution where a majority of individuals have less than the mean income and a minority of individuals have much more than the mean income. I call this type of distribution “the superstar distribution”. Ex ante, before its realization, it embodies the American dream providing the decision maker the opportunity to take a fair-odds long-shot gamble on becoming a superstar.

Another type of distribution is the top-heavy left-skewed distribution where a minority of individuals have much less than the mean income and a majority of individuals have more than the mean income. I call this type of distribution “the scapegoat distribution”. Ex ante, it provides the decision maker the possibility to take a fair-odds “safe bet” on not becoming the scapegoat. The two different types of distributions are displayed in Figure 3. Furthermore, there is also the type of distribution where half of the individuals have less than the mean income and half of the individuals have more than the mean income.

In the simulation part, I also investigate some continuous income distributions. I investigate some symmetric income distributions and some asymmetric superstar distributions because the two-income-level analysis will indicate that superstar distributions are particularly promising. The investigated income distributions include the uniform, normal, triangular, and log-normal income distributions. They are displayed in Figure 4. The log-normal distribution is of particular interest because the income distributions of most countries have this shape (Gibrat, 1931; Aitchison & Brown, 1957; Battistin et al., 2007).

The decision maker can, of course, choose from any positive income distribution that preserves the mean and not only the income distributions investigated here. The optimal income distribution may be one that is not investigated here. Because of the difficulties with functional form and parameterization discussed in the last section, the exact welfare numbers should not be taken too seriously.

In fact, prospect theory has not even been empirically tested for all the income distributions that I investigate here. In particular, there are few studies on behavior under risk involving extremely low income levels where individuals cannot afford basic goods and risk their lives. The reason is that there are little observational data and difficult to run experiments with such outcomes. It is not unlikely that prospect theory and extrapolation of prospect theory parameterizations are poor descriptions of the decision utility of these outcomes and income distributions involving such outcomes. Nevertheless, this does not invalidate the results for income distributions that do not have extreme variance and do not involve extremely low income levels, and the preference for some such income distributions over complete equality. The setting can also be reframed as distributing a fixed amount of non-basic goods, where zero income corresponds to individuals having access to only basic goods.

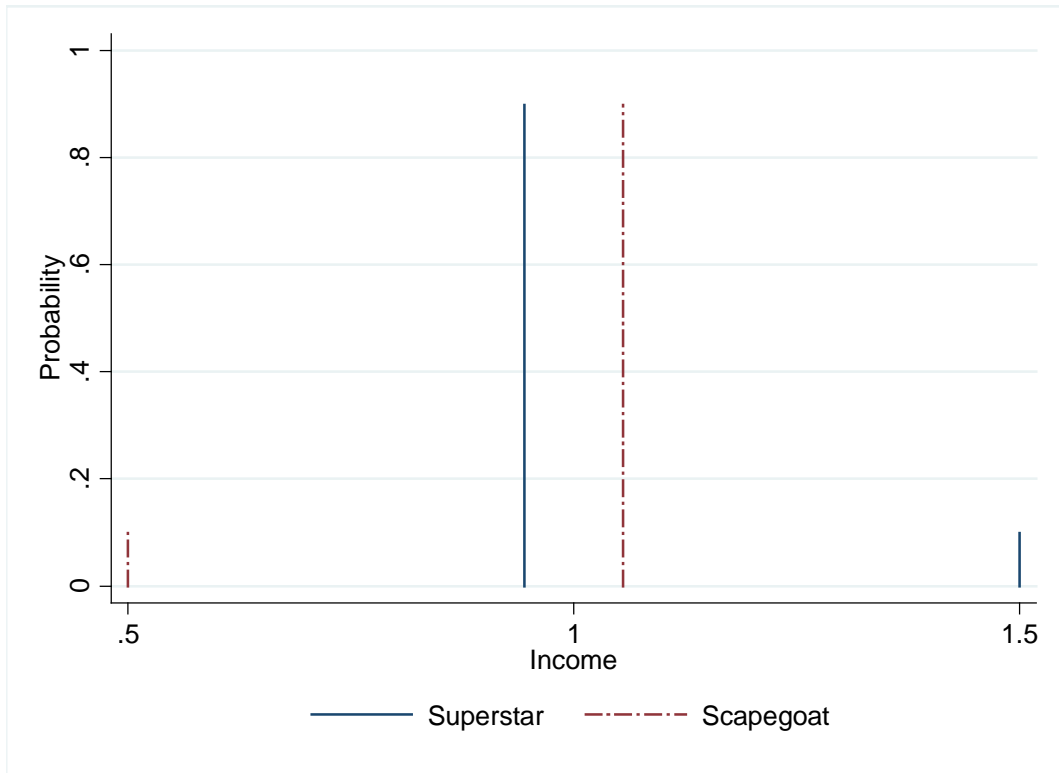


Figure 3. Two-income-level distributions

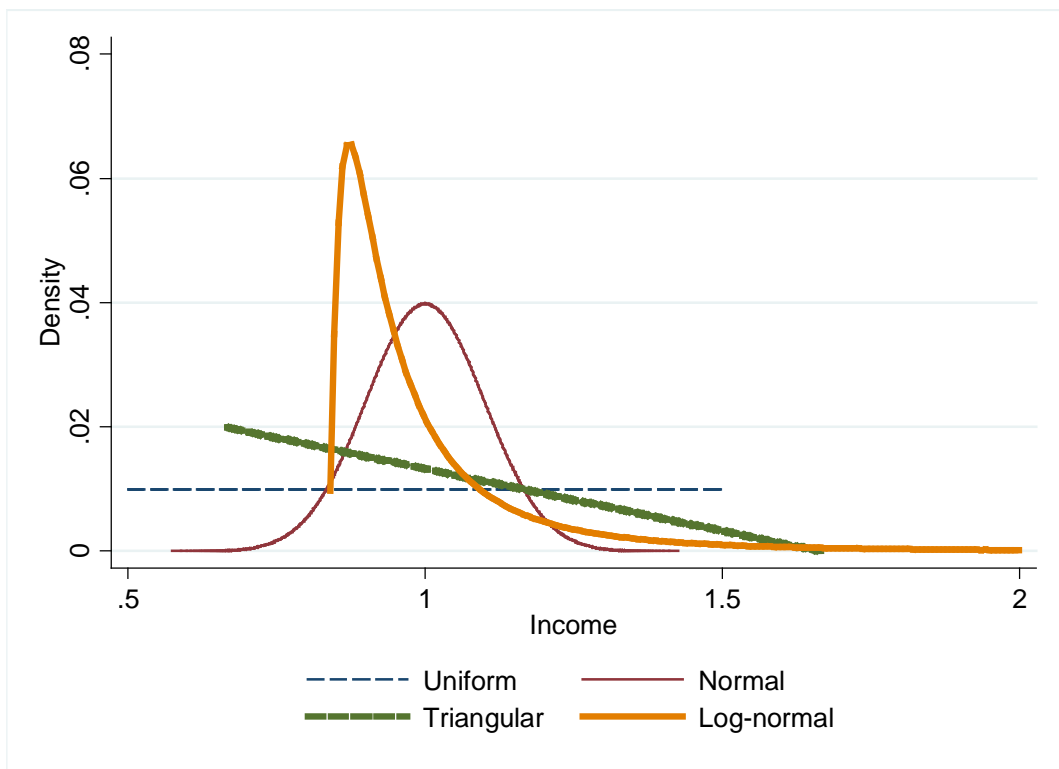


Figure 4. Continuous income distributions

For two-income level distributions, when the mean income in the population is the reference income, the decision problem in Equation (1) is reduced to:

$$\max_{x_g, p_g} w(p_g)u(x_m + x_g) + w(1 - p_g)u\left(x_m - \frac{p_g}{1 - p_g}x_g\right), \quad (8)^9$$

where x_g is the size of the gains relative to the reference income $x_0 = x_m$, p_g is the gain probability, and $x_g \geq 0$. If the number of individuals in the population is finite, $0 < \underline{p}_g < p_g < 1 - \underline{p}_g < 1$. \underline{p}_g is the lower bound corresponding to one individual with more than the mean income. I assume that the number of individuals in the population is large and that \underline{p}_g is close to 0. In the setting here, in order for some individuals to gain, some other individuals must lose.

When using representative aggregation of reference incomes, the decision problem in Equation (1) instead becomes:

$$\max_{x_g, p_g} (1 - p_g)w(p_g)u_{PT}\left(x_0 + \frac{1}{1 - p_g}x_g\right) + p_gw(1 - p_g)u_{PT}\left(x_0 - \frac{1}{1 - p_g}x_g\right). \quad (9)$$

4. Analytical Results

Let us start with the expected utility optimum for two-income level distributions. The decision problem is formulated in Equation (8). Because of diminishing marginal utility, spread cannot be desirable. This classical result of equality is stated in Proposition 1. The equality solution provides a utility of 0. This result can be extended to the case allowing for continuous income distributions for concave utility functions. See, e.g., Mas Collé et al. (1995), who show that concavity implies preferences against mean-preserving spreads.

Proposition 1. For two-income level distributions, complete equality is optimal when using expected utility, i.e., $x_g^* = 0$.

Proof. We have the following derivative of the objective function in Equation (8):

$$\frac{dU_{EU}}{dx_g} = u'_{EU}(x_m + x_g) - u'_{EU}\left(x_m - \frac{p_g}{1 - p_g}x_g\right) > 0 \text{ since } u'(x > x_m) < u'(x < x_m)$$

Hence, increasing spread from $x_g = 0$ decreases expected utility. ■

The overweighting of probabilities of large gains and losses in prospect theory creates a possibility for an uneven income distribution to be optimal by accumulating gains (incomes above the mean income) among a few individuals at the expense of smaller losses (incomes below the mean income) for a greater number of individuals. With a concave utility function, such an outcome is optimal, both when the mean income is the reference income and when using representative aggregation of reference incomes, according to Proposition 2.

⁹ Note that the cumulative probability P reduces to the plain probability p in the two-income level case.

Proposition 2. For two-income level distributions, when using a prospect theory probability weighting function and a concave utility function, we have that:

- a) A superstar distribution with $0 < x_g^* \leq \frac{1-p_g^*}{p_g^*} x_m$ and $p_g^* < 0.5$ is optimal. The crucial condition for this result is:

$$\frac{w(p_g)}{p_g} > \frac{w(1-p_g)}{1-p_g}, \quad (10)$$

which is implied by the prospect theory probability weighting function.

- b) When the concave utility function becomes linear, a superstar distribution with $x_g^* = \frac{1-p_g^*}{p_g^*} x_m$ and $p_g^* < 0.5$ is optimal.

Proof.

- a) We have the following derivatives of the objective function in Equation (8):

$$\frac{dU}{dx_g} = w(p_g)u'_{EU}(x_m + x_g) - \frac{p_g}{1-p_g} w(1-p_g)u'_{EU}\left(x_m - \frac{p_g}{1-p_g}x_g\right).$$

For $p_g < p_g^\# < 0.5$, where $w(p_g^\#) = p_g^\#$, subproportionality implies overweighting and $w(p_g) > p_g$, which together with subcertainty implies $w(1-p_g) < 1-p_g$. The two inequalities imply Equation (10), which gives $\frac{dU}{dx_g}(x_g = 0) > 0$ and $x_g^* > 0$. $p_g < p_g^\#$ is a sufficient condition, but not a necessary condition. $p_g^* < 0.5$ is, however, a necessary condition.

- b) We have the following derivative of the objective function in Equation (8):

$$\frac{dU}{dx_g} = w(p_g) - \frac{p_g}{1-p_g} w(1-p_g).$$

The same argument as in a) leads to Equation (10) which now implies $\frac{dU}{dx_g} > 0$. We want to increase x_g^* until its maximum, Because of the lower bound of x , $x_m - \frac{p_g}{1-p_g}x_g > 0$, we get $x_g^* = \frac{1-p_g^*}{p_g^*}x_m$. ■

The superstar distribution in Proposition 2a contains at least one superstar and at most half the population as superstars, with much more than the mean income, supported by all other individuals having less than the mean income. The results depend on the parameterization. The upper bound on gains occurs where the individuals with less than the mean income have no income, and the superstars have all incomes. The key property of prospect theory probability weighting giving this result is the overweighting of low probability large gains and the underweighting of high probability small losses. When the concave utility function approaches linearity, no factor works against spread, and the optimal income of superstars approach its upper bound.

The diminishing marginal sensitivity in gains and losses in prospect theory creates another possibility for an uneven income distribution to be optimal by accumulating losses among a few individuals to allow smaller gains for a greater number of individuals. With a linear probability weighting function, such an outcome could be optimal when the mean

income is the reference income, but not when using representative aggregation of reference incomes, according to Proposition 3.

Proposition 3. For two-income level distributions, when using a prospect theory utility function and a linear probability weighting function, we have that:

- a) When the mean income is the reference income, *either* of the following two conditions is sufficient for a scapegoat distribution with $x_g^* > 0$ and $p_g^* \rightarrow 1 - \bar{p}_g$ to be optimal:

$$\lim_{x_g \rightarrow 0} \left[u'_{PT}(x_m + x_g) - u'_{PT} \left(x_m - \frac{1 - \underline{p}_g}{\underline{p}_g} x_g \right) \right] > 0 \text{ and} \quad (11)$$

$$(1 - \underline{p}_g) \left[u_{PT} \left(\frac{1}{1 - \underline{p}_g} x_m \right) - u_{PT}(x_m) \right] > \underline{p}_g [u_{PT}(x_m) - u_{PT}(0)]. \quad (12)$$

- b) If x is unrestricted and $\underline{p}_g \rightarrow 0^+$ in a), the following weaker condition is sufficient for a scapegoat distribution: there is an x such that $u'_{PT}(x > x_m) > \lim_{z \rightarrow -\infty} u'_{PT}(z)$.
- c) Complete equality is optimal when applying representative aggregation of reference incomes.

Proof.

- a) We have the following derivatives of the objective function in Equation (8):

$$\frac{d^2U}{dp_g^2} = \frac{x_g^2}{(1 - p_g)^3} u''_{PT} \left(x_m - \frac{p_g}{1 - p_g} x_g \right),$$

$$\frac{dU}{dx_g} = p_g u'_{PT}(x_m + x_g) - p_g u'_{PT} \left(x_m - \frac{p_g}{1 - p_g} x_g \right).$$

Because $\frac{d^2U}{dp_g^2} > 0$, U has an interior minimum in p_g , and it must be either that $p_g^* \rightarrow \underline{p}_g$ or $p_g^* \rightarrow 1 - \underline{p}_g$. For $p_g^* \rightarrow \underline{p}_g$, we have $u'_{PT}(x_m + x_g) < u'_{PT}(x_m - ax_g)$ with $a < 1$ because of loss aversion, which implies $\frac{dU}{dx_g} < 0$, giving $x_g^* = 0$ as the optimum. For $p_g^* \rightarrow 1 - \underline{p}_g$, Equation (11) implies $\frac{dU}{dx_g}(x_g = 0) > 0$ and $x_g^* > 0$. If $\frac{dU}{dx_g}$ is strictly decreasing in x_g , we want to increase x_g up until $\frac{dU}{dx_g} = 0$, or until its upper bound $x_g = \frac{\underline{p}_g}{1 - \underline{p}_g} x_m$. Equation (12) is sufficient for $x_g^* > 0$, because it implies $U \left(x_g = \frac{\underline{p}_g}{1 - \underline{p}_g} x_m, p_g = 1 - \underline{p}_g \right) > 0 = U(x_g = 0)$.

- b) In a), there is an x such that $u'_{PT}(x > x_m) > \lim_{z \rightarrow -\infty} u'_{PT}(z)$ implies $\lim_{x_g \rightarrow 0^+} u'_{PT}(x_m + x_g) > \lim_{z \rightarrow -\infty} u'_{PT}(z)$ and $\frac{dU}{dx} > 0$. This gives $x_g^* > 0$ when $p_g^* \rightarrow 1^-$. We want to increase x_g infinitely, i.e., $x_g^* = \infty$.
- c) We have the following derivative of the objective function in Equation (9):

$$\frac{dU}{dx_g} = \frac{1}{1 - p_g} \left[u'_{PT} \left(x_0 + \frac{1}{1 - p_g} x_g \right) - u'_{PT} \left(x_0 - \frac{1}{1 - p_g} x_g \right) \right].$$

We have $u'_{PT}(x_m + z) < u'_{PT}(x_m - z)$ for all z , because of loss aversion. Hence, $\frac{dU}{dx_g} < 0$. ■

The scapegoat distribution in Proposition 3a contains one individual scapegoat with much less than the mean income, sacrificed so that all other individuals can have more than the mean income. Whether such a distribution is optimal depends on the parameterization. The first sufficient condition in Equation (11) requires the marginal utility of gains at the mean income to be larger than the marginal disutility of losses that are larger than the gains at the mean income. Whether the condition holds depends on three factors. Loss aversion works against the condition because it leads to the marginal utility being greater for losses of the same size as gains. The number of individuals and the degree of diminishing sensitivity in losses work in favor of the condition because the losses are larger than the gains by a factor equal to the number of individuals in the society minus one and because marginal disutility decreases with losses.

The second sufficient condition in Equation (12) requires that the gain utility of a small gain weighted by the number of individuals getting the gain is greater than the loss utility of one individual losing all her income. Again, loss aversion works against the condition, whereas diminishing sensitivity in losses work in favor of it. The purpose of having this second condition is to show that we do not require a condition involving marginal utility at the mean income.

When allowing for income to be unbounded, the condition required for a scapegoat distribution to be optimal becomes weaker in Proposition 3b. We then only require the marginal disutility at the (possibly hypothetical) worst of loss to be small enough in comparison with the marginal utility at an arbitrary gain. This is fulfilled, e.g., if the marginal disutility of losses converges to zero or if the marginal utility of gains is infinite for the first dollar, an Inada condition often assumed on utility functions. When interpreting the input in the utility function as income or resources, the lower bound is natural. If the amount of resources to distribute is great, the lower bound may, however, be very low, and the condition assuming unboundedness may be a good approximation.

A lower bound larger than zero may be desirable for other reasons, e.g., if there are basic goods without which individuals experience extreme disutility. As mentioned in the last section, there is little evidence on decision under risk involving income levels where individuals risk basic goods. It may, however, be that people behave according to prospect theory under risk even in cases involving basic goods, which may differ from their experienced utility. Then, the acceptability of the conclusion of one suffering individual to let the others thrive (a tiny bit more) brings us back to the question of whether there should be normative constraints in the original position.

The optimization problem becomes much more complicated when combining prospect theory utility and probability weighting functions. In general, the solution depends on the exact parameterization of the functions. In Proposition 4, I state two sufficient conditions for a superstar distribution to increase welfare compared to complete equality.

Proposition 4. For two-income level distributions, when using prospect theory utility and probability weighting functions, we have that:

- a) When the mean income is the reference income or when using representative aggregation of reference incomes,

$$\lim_{p_g \rightarrow 0^+} w(p_g) > 0 \quad (13)$$

and $\underline{p}_g \rightarrow 0^+$ are sufficient for a superstar distribution with $x_g^* > 0$ and $p_g^* \rightarrow \underline{p}_g$ to be preferred to complete equality, i.e., there is an x_g such that $U_{PT}(x_g > 0, p_g \rightarrow \underline{p}_g) > 0 = U_{PT}(x_g = 0)$.

- b) When using representative aggregation of reference incomes, the following condition is sufficient for a superstar distribution with $x_g^* > 0$ and $p_g^* < 0.5$:

$$\frac{w(p_g)}{p_g} \frac{1 - p_g}{w(1 - p_g)} > \lim_{x_g \rightarrow 0^+} \frac{u'_{PT}(x_m - x_g)}{u'_{PT}(x_m + x_g)}. \quad (14)$$

Proof.

- a) In both cases, Equation (13) implies $\lim_{p_g \rightarrow 0^+} U_{PT}(x_g > 0, p_g) = \lim_{p_g \rightarrow 0^+} w(p_g)u_{PT}(x_m + x_g) > 0 = U_{PT}(x_g = 0)$.

- b) We have the following derivatives of the objective function in Equation (9):

$$\frac{dU_{PT}}{dx_g} = w(p_g)u'_{PT}\left(x_m + \frac{1}{1 - p_g}x_g\right) - \frac{p_g}{1 - p_g}w(1 - p_g)u'_{PT}\left(x_m - \frac{1}{1 - p_g}x_g\right).$$

Equation (14) implies $\frac{dU_{PT}}{dx_g}(x_g = 0) > 0$ and hence $x_g^* > 0$. $p_g^* < 0.5$ is needed for

$$\frac{w(p_g)}{p_g} \frac{1 - p_g}{w(1 - p_g)} > 1, \text{ see the proof of Proposition 2a. } \blacksquare$$

The superstar distributions in Proposition 4a and 4b are not necessarily the optimal income distribution, but preferred to complete equality. Because $w(p) = 0$ when $p = 0$, the condition in Equation (14) requires the weighting function to be discontinuous at $p = 0$. The uneven income distribution is better because increasing the probability of gains from 0 results in a positive discontinuous increase in welfare due to positive probability weights on the positive gain utility, whereas decreasing the accompanying losses much less results in a continuous decrease in the negative loss utility. The exact optimal gains and gain probability depend, however, on the parameterization. The condition is not very strong. In the original version of prospect theory in Kahneman and Tversky (1979), the authors had this type of weighting function in mind, which reflects that a small probability is categorized as a possibility given significant weight, even very small probabilities.

The left-hand side of the condition in Equation (14) reminds of Equation (10). It is the degree of overweighting of small probabilities of large gains or losses multiplied by the inverse of the degree of underweighting of large probabilities of small gains or losses. As discussed in connection to Proposition 2, this factor is larger than 1. The right hand side of Equation (14) is the quotient between the marginal disutility of losses and the marginal utility of gains around the reference income. Hence, the condition requires the departure from linear probability weights to be larger than the degree of loss aversion.

Thus far, we have dealt with two-income-level distributions. Real income distributions, however, certainly allow for more income levels, and often have more income levels. Can the results be extended to such income distributions? The optimization problem increases in dimensionality by twice the additional number of income levels, increasing the difficulty of obtaining analytical solutions. Nevertheless, the shapes of the prospect theory utility and probability weighting functions still have similar impacts. A three-income-level distribution can be created from a two-income distribution by applying a mean-preserving spread on one of the income levels in a two-income level distribution. In Proposition 5, I show that such a spread can increase utility when the mean income is the reference income. The argument in Proposition 5 can be iterated to show that much more complicated multi-income-level distributions can be preferred to the optimal three-income-level distribution. The results depend, however, crucially on the parameterization.

Proposition 5. When using prospect theory utility and probability weighting functions with the mean income as the reference income, three-income-level distributions can be preferred to two-income-level distributions.

Proof. Start out from the optimal two-income-level distribution. Can we improve on it by dividing one of the income levels into two levels using a mean-preserving spread? Start with the income level below the mean income. Assume the mean-preserving spread increases income by a small $z < \frac{p_g}{1-p_g} x_g$ (so that the new income levels are still below the mean income) in half the cases and $-z$ in the other half. Then, the convexity of the prospect theory utility function implies that such a spread increases utility if linear probability weights or prospect theory probability weights that are close enough to linear probability weights are used. Can the income level above the mean income be improved by such a spread? Assume the same type of mean-preserving spread with $z < x_g$. Then, if the probability weighting function is convex around p , which it may be in prospect theory, the spread increases utility if a linear utility function or a prospect theory utility function that is close enough to the linear function is used. We have thus established at least two situations where a three-income-level distribution created from a two-income-level distribution can be preferred to the optimal two-income-level distribution. ■

As already discussed, a crucial component of prospect theory is the selection of a reference point. We have so far taken this reference point to be the mean income or used representative aggregation of reference incomes. Another option is the median income. Unlike the mean, the median can change discontinuously when altering the income distribution continuously. The preference for an income distribution increases when the reference income decreases, leaving room for manipulation of the reference point. A systematic way to increase the preference for an income distribution is presented in Proposition 6.

Proposition 6. When the median income is used as the reference income, decreasing the income levels in the lower end of the income distribution including the median income is desirable if it decreases the distance between the median income and every lower income level.

Proof. Obviously, decreasing the distance between the median income and every lower income levels decreases the loss utility of those lower income levels. The decrease in the median income also increases the gain utility of the higher income levels. ■

Note that if we decrease the income levels according to Proposition 6, keeping the mean income fixed, the decrease in income at low income levels provides income that can be redistributed to individuals at high income levels, which additionally increases the preference for the income distribution. An adverse implication of Proposition 6 is that income distributions that first-order stochastically dominate other income distributions may be less preferred when using the median income as the reference income. This result implies an opposite version of Rawls' difference principle stating that unilateral improvements in outcomes should only be tolerated if it increases the situation of the worst-off. Finally, note that Proposition 6 does not say anything about the effect of spreading incomes for those above the median income.

5. Simulation Results

The analytical complexity involved in obtaining results when combining prospect theory utility and probability weighting functions can be avoided by using simulations. Continuous income distributions can also be investigated approximately numerically by using discrete income distributions with many data points. Furthermore, we can quantify the welfare effects. On the down side, some results are driven by the parameterization on which there is no consensus.

When reporting the social welfare of an income distribution, I report certainty equivalents rather than social welfare. The certainty equivalent is the additional percent of income that the individuals need when each of them have the mean income to reach the social welfare of an income distribution. In constructing the certainty equivalent of social welfare, I use the expected utility social welfare formula to enable straightforward comparison of certainty equivalents independent of which theory is used to calculate social welfare. The simulations are performed using 1,000 individuals. At this sample size, the results are insensitive, but computational time is very sensitive, to varying sample size.

The certainty equivalents of different two-income-level superstar distributions are reported in Table 1. I vary the size of income gains relative to the mean income in percent and the gain probability in percent (keeping the mean income constant in all income distributions). I report the certainty equivalents in percent of the mean income for an expected utility (EU) decision maker, and for prospect theory (PT) decision makers when the mean income (mean) is the reference income and when using representative aggregation of reference incomes (representative).

Table 1. Certainty equivalents of some two-income-level superstar distributions

Gains	Gain probability	EU	PT mean	PT representative
0.5	1	-0.000006	0.05	0.26
5	1	-0.0006	0.54	2.55
50	1	-0.05	4.91	24.36
5	0.1	-0.00006	0.41	1.61
5	1	-0.0006	0.54	2.55
5	10	-0.007	0.35	3.31

Notes: Gain probability is expressed in percent and other numbers are expressed in percent of the mean income.

We observe that the superstar distributions provide negative social welfare for the expected utility decision maker. The social welfare loss relative to complete equality increases with the size of the income gains and the gain probability. However, the income distributions provide positive social welfare for the prospect theory decision makers and are hence preferred to complete equality for them. The social welfare gain is much larger when using representative aggregation of reference incomes than when the mean income is the reference income. It increases with the size of the income gains. It also increases with the gain probability, albeit only up to a gain probability of 1 percent when the mean income is the reference income.

The size of the social welfare gains for the prospect theory decision makers is large, up to a certainty equivalent of 24.36 percent of the mean income when using representative aggregation of reference incomes. The magnitude of the effects is larger for the prospect theory decision makers than for the expected utility decision maker. This is because the gain probabilities are small and hence carry small weight for the expected utility decision maker, whereas the prospect theory decision makers overweight those probabilities. The patterns found are in line with Propositions 1, 2, and 4.

The certainty equivalents of different two-income-level scapegoat distributions are reported in Table 2, which is similarly organized as Table 1. The expected utility decision maker again prefers income distributions that are the closest to complete equality. Unlike superstar distributions, social welfare is also negative for prospect theory decision makers. The social welfare loss increases with the size of the income losses and the loss probability. Furthermore, the size of the social welfare losses is much larger for the prospect theory decision makers. This reflects the impact of loss aversion and overweighting of large losses.

Table 2. Certainty equivalents of some two-income-level scapegoat distributions

Losses	Loss probability	EU	PT mean	PT representative
0.5	1	-0.000006	-0.13	-0.13
5	1	-0.0006	-1.30	-1.29
50	1	-0.09	-11.54	-11.38
5	0.1	-0.00006	-0.94	-0.94
5	1	-0.0006	-1.30	-1.29
5	10	-0.007	-1.39	-1.06

Notes: Loss probability is expressed in percent and other numbers are expressed in percent of the mean income.

However, as the income losses or loss probability increase, the additional social welfare loss is less in relative terms for the prospect theory decision makers than for the expected utility decision maker. For instance, increasing the income losses 10 times (from, e.g., 5 to 50)

increases the certainty equivalent of the social welfare loss more than 100 times (from -0.0006 to -0.09) for the expected utility decision maker, but less than 10 times (from around -1.3 to around -11.5) for the prospect theory decision makers.

The certainty equivalents of the symmetric uniform and normal continuous income distributions are reported in Table 3. We observe that social welfare is negative for both distributions independent of the theory applied. It also decreases as spread and variance increase. More inequality is therefore, in general, also undesirable for the prospect theory decision makers. Like for the scapegoat distributions, the social welfare loss is larger for the prospect theory decision makers because of loss aversion. The additional relative negative effect of additional spread on social welfare is, however, again relatively smaller for the prospect theory decision makers.

Table 3. Certainty equivalents of some symmetric continuous income distributions

Spread	Uniform		
	EU	PT mean	PT representative
1	-0.0002	-0.09	-0.10
10	-0.02	-0.89	-0.95
100	-2.18	-7.99	-8.31
Variance	Normal		
	EU	PT mean	PT representative
1	-0.002	-0.50	-0.53
10	-0.02	-1.55	-1.63
100	-0.25	-4.66	-4.89

Notes: Spread is top income minus bottom income in percent of the mean income, Variance is the distribution variance in percent of the mean income, and other numbers are expressed in percent of the mean income.

Because of the desirability of two-income level superstar distributions, I also investigate some asymmetric continuous superstar distributions. The certainty equivalents of some triangular and log-normal distributions are reported in Table 4. They are all negative for the triangular distributions. The pattern is very similar to the one of uniform distributions. A difference is that the social welfare loss is smaller for a triangular distribution with the same spread. Furthermore, the welfare loss for the prospect theory decision makers relative to that of the expected utility decision maker is smaller for the triangular distribution (for a spread of 100 when the mean income is the reference income, we have the certainty equivalent comparison -2.42 versus -1.35) than for the uniform distribution (for a spread of 100 when the mean income is the reference income, we have the certainty equivalent comparison -7.99 versus -2.18). This is the effect of prospect theory decision makers liking superstar distributions, although not enough to make them preferring triangular distributions to complete equality.

Table 4. Certainty equivalents of some asymmetric continuous superstar distributions

Spread	Triangular		
	EU	PT mean	PT representative
1	-0.0001	-0.02	-0.03
10	-0.01	-0.22	-0.30
100	-1.35	-2.42	-3.02
Variance	Log-normal		
	EU	PT mean	PT representative
1	-0.01	1.65	1.52
10	-0.09	4.76	4.35
100	-0.77	12.19	11.09

Notes: Spread is top income minus bottom income in percent of average income, Variance is the distribution variance in percent of average income, and other numbers are expressed in percent of the mean income.

The log-normal distribution is also right-skewed like the triangular distribution. However, the skewness is larger. This skewness manages to turn social welfare positive for prospect theory decision makers. Using the mean income as the reference income or representative aggregation of reference incomes has small effects on the results. Furthermore, social welfare increases as variance increases. The social welfare gains are large with a certainty equivalent of 12 percent of the mean income. However, they are not as large as in the most preferred two-income level superstar distribution which had a social welfare gain corresponding to a certainty equivalent of 25 percent of the mean income (see Table 1).

6. Concluding Discussion

Harsanyi (1953, 1955, 1975) and Rawls (1971) offer two of the most influential theories of distributive justice. Both use the popular social contract approach starting from an original position where the decision maker does not know her identity in the society. Under such a veil of ignorance, her choice of income distribution is to be considered the fair distribution. This paper asked how a decision maker applying prospect theory would choose. Applying prospect theory is appealing because it better describes behavior under risk than expected utility theory, which Harsanyi's decision maker uses.

I found that the desirability of different income distributions depends on the parameterization of prospect theory. Two properties of prospect theory work in the direction of increasing the desirability of uneven income distributions: the overweighting of large gains and diminishing sensitivity in losses. Two properties work in the opposite direction: the overweighting of large losses and loss aversion. For a reasonably chosen parameterization, prospect theory decision makers are in general *more* inequality averse than an expected utility decision maker. However, inequality is socially desirable when it comes to a *specific type* of income distribution that is bottom-heavy and right-skewed where few superstars have very high income and many individuals have low income, such as the log-normal income distribution.

The normative conclusion to take away from this exercise depends on the viability of the original position and how this position *should* be framed. The starting point of this paper was that the original position is a valid way to transform the normative question of fairness

into a purely descriptive question. Furthermore, like in Harsanyi (1953, 1955, 1975), the original position was interpreted as a lottery from the decision maker's perspective. The key modification here was the application of a theory that better describes individuals' behavior under risk in other situations. Because individuals do gamble on some types of lotteries, I obtained the result that some types of inequality are inherently socially desirable.

If the optimal inequality conclusion cannot be accepted because it is an unpleasant type of justice, then a possible argument is that the original position needs to be modified or rejected. An issue is what type of risk preferences the decision maker should have, if any, in the original position. It could be argued that some risk preferences such as the one implied by prospect theory are inappropriate in the original position. This line of argument, however, amounts to rejecting that the original position transforms a normative question into a descriptive question; it really implies that the difficult normative question (about the fair income distribution) is replaced by another (equally difficult?) normative question (about what risk preferences a decision maker in the original position ought to have). If resorting to this argument, it is possible to argue that expected utility theory is a better normative theory than prospect theory and that the best normative theory should be applied in the original position. An alternative it to be agnostic on what risk preferences to apply (explicitly or implicitly), but then nothing can really be said about the decision makers' preferences for different lottery and income distributions.

Another issue is whether the problem in the original position is one about decision under risk. Rawls (1971) argues that it is a decision under uncertainty where the decision maker should disregard the frequencies of different income levels. In comparing distributions with known frequencies, an additional argument is, however, needed to motivate why these known frequencies should be disregarded. A possible answer is that the frequencies should be morally irrelevant in the original position. Again, this replaces a difficult normative question (about the fair income distribution) by another (equally difficult?) normative question (about what the decision maker ought to account for in the income distribution in the original position). A possible interpretation of Rawls (1971) is that the original position is not meant to be used to evaluate income distributions but rather to arrive at the moral principle that should be used to evaluate income distributions and that the problem at hand in deriving the moral principle is about evaluating uncertain distributions without frequency information.

Even if we would acknowledge that the decision problem is one under uncertainty, it is unclear which theory of behavior under uncertainty is the appropriate one. Is it how actual individuals behave under uncertainty or is it how the decision maker ought to behave under this framing of the original position (which again, would be another difficult normative question)? Rawls argues that the maximin principle should be used, whereas Harsanyi can be interpreted as advocating putting equal probability on each income level when frequencies are unknown or disregarded (principle of insufficient reasons). If the approach to assign probabilities is taken, the second step issue of which evaluation rule to use still needs to be decided. If this is seen as a descriptive question, then prospect theory could again be more suitable than expected utility theory.

Another alternative approach to applying prospect theory in the original position that treats the decision problem in the original position as a descriptive question is to ask real individuals or groups about what they would prefer or could agree on in the original position.

A problem could be that individuals may not fully interpret the original position the way they are supposed to, that they may not know how they actually would choose in the original position, or that they may not truthfully reveal how they would choose (but instead reveal how they think they ought to choose). As mentioned in the introduction, the outcome crucially depends on the framing of the original position.

If the original position is successfully framed as a decision under risk, where the income distributions are interpreted as lotteries, inducing people to reveal how they really would choose when facing lotteries, the outcome would be the pattern predicted by prospect theory. The remaining question is whether individuals' normal behavior under risk is suitable for the social welfare evaluation of an income distribution. The lottery interpretation of the original position has a central place in most discussions on this topic, although not undisputed. This paper spelled out the implications of accepting this framing. It also provided an additional argument for why some uneven income distributions may be socially desirable – such distributions resemble outcomes of lotteries that most people would choose *ex ante*.

References

- Aitchison, J., Brown, J. (1957). *The Lognormal Distribution*. Cambridge University Press, Cambridge.
- Amiel, Y., Cowell, F., Gaertner, W. (2009). To be or not to be involved: a questionnaire-experimental view on Harsanyi's utilitarian ethics. *Social Choice and Welfare* 32, 299–316.
- Atkinson, A. (1970). On the measurement of inequality. *Journal of Economic Theory* 2, 244–263.
- Battistin, E., Blundell, R., Lewbel, A. (2007). Why is consumption more log normal than income? Gibrat's law revisited. Manuscript.
- Bernheim, D., Rangel, A. (2009). Beyond revealed preference: choice-theoretic foundations for behavioral welfare economics. *Quarterly Journal of Economics* 124, 51–104.
- Bosmans, K., Schokkaert, E. (2004). Social welfare, the veil of ignorance and purely individual risk: an empirical examination. *Research on Economic Inequality* 11, 8–114.
- Boulding, K., (1962). Social justice in social dynamics. In: Brandt, R. (ed), *Social Justice*. Prentice Hall, Englewood Cliffs, New Jersey.
- Camerer, C., Ho., T. (1994). Violations of the betweenness axiom and nonlinearity in probability. *Journal of Risk and Uncertainty* 8, 167–196.
- Epstein L., Segal, U. (1992). Quadratic social welfare functions. *Journal of Political Economy* 100, 691–712
- Fishburn, P., Kochenberger, G. (1979). Two-piece von Neumann-Morgenstern utility functions. *Decision Science* 10, 503–518.
- Frohlich, N., Oppenheimer, J., Eavey, C. (1987). Choices of principles of distributive justice in experimental groups. *American Journal of Political Science* 31, 606–36.
- Gibrat, R. (1931). *Les Inegalites Economiques*. Librairie du Recueil Sirey, Paris.
- Günther, I., and Maier, J. (2008). Poverty, vulnerability and loss aversion. Manuscript, University of Göttingen.
- Harsanyi, J. (1953). Cardinal utility in welfare economics and in the theory of risk-taking. *Journal of Political Economy* 61, 434–435.
- Harsanyi, J. (1955). Cardinal welfare, individualistic ethics and interpersonal comparisons of utility. *Journal of Political Economy* 63, 309–32.
- Harsanyi, J. (1975). Can the maximin principle serve as a basis for morality? A critique of John Rawls' theory. *American Political Science Review* 69, 594–606.
- Herne K, Suojanen, M., (2004). The role of information in choices over income distributions. *Journal of Conflict Resolution* 48, 173–193.
- Hershey J., Schoemaker, P. (1980). Prospect theory's reflection hypothesis: a critical examination. *Organizational Behavior and Human Decision Processes* 25, 395–418.
- Johansson-Stenman, O., Carlsson, F., Gupta., G. (2002). Measuring future grandparents' preferences for equality and relative standing. *Economic Journal* 112, 362–83.
- Jäntti, M., Kabur, R., Nyssölä, M., Pirttilä, J. (2013). Poverty and welfare measurement on the basis of prospect theory. Working Paper 2013-07, Charles H. Dyson School of Applied Economics and Management Cornell University, Ithaca, New York.

- Kahneman, D., Tversky, A. (1979). Prospect theory: an analysis of decision under risk. *Econometrica*, 47, 263–91.
- Kahneman, D., Tversky, A. (2000). *Choices, Values, and Frames*. Cambridge University Press, Cambridge.
- Kahneman, D., Wakker, P., and Sarin, R. (1997). Back to Bentham? Explorations of experienced utility. *Quarterly Journal of Economics*, 112, 375–406.
- Koszegi, B., Rabin, M. (2006). A model of reference-dependent preferences. *Quarterly Journal of Economics* 121, 1133–1165.
- Mas-Collel, A., Whinston, M., and Green, J. (1995). *Microeconomic Theory*. Oxford University Press, Oxford.
- Neilson, W., Stowe, J. (2002). A further examination of cumulative prospect theory parameterizations. *Journal of Risk and Uncertainty* 24, 31–46.
- Payne, J., Laughhunn, D., Crum, R. (1981). Aspiration level effects in risky behavior. *Management Science* 27, 953–958.
- Prelec, D. (1998). The probability weighting function. *Econometrica* 66, 497–527.
- Rawls J. (1971). *A Theory of Justice*. Harvard University Press, Cambridge, Massachusetts.
- Traub, S., Seidl, C., Schmidt, U., Levati, M., (2005). Friedman, Harsanyi, Rawls, Boulding - or somebody else? An experimental investigation of distributive justice. *Social Choice and Welfare* 24, 283–309.
- Tversky, A., Kahneman, D. (1992). Advances in prospect theory: cumulative representation of uncertainty. *Journal of Risk and Uncertainty* 5, 297–323.
- Von Neumann, J., Morgenstern, O. (1944). *Theory of Games and Economic Behavior*. Princeton University Press, Princeton.
- Wehrung, D. (1989). Risk taking over gains and losses: a study of oil executives. *Annals of Operations Research* 19, 115–139.
- Wu, G., Gonzalez, R. (1996). Curvature of the probability weighting function. *Management Science* 42, 1676–1690.

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