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Research Article

A Novel SHLNN Based Robust Control and Tracking Method for Hypersonic Vehicle under Parameter Uncertainty

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Hypersonic vehicle is a typical parameter uncertain system with significant characteristics of strong coupling, nonlinearity, and external disturbance. In this paper, a combined system modeling approach is proposed to approximate the actual vehicle system. The state feedback control strategy is adopted based on the robust guaranteed cost control (RGCC) theory, where the Lyapunov function is applied to get control law for nonlinear system and the problem is transformed into a feasible solution by linear matrix inequalities (LMI) method. In addition, a nonfragile guaranteed cost controller solved by LMI optimization approach is employed to the linear error system, where a single hidden layer neural network (SHLNN) is employed as an additive gain compensator to reduce excessive performance caused by perturbations and uncertainties. Simulation results show the stability and well tracking performance for the proposed strategy in controlling the vehicle system.

1. Introduction

The long-distance unpowered glide reentry vehicle is an important hypersonic vehicle which has been of significant aerodynamic configuration with high lift-to-drag ratio. It can reach the target after long-distance gliding and fulfill a throwing mission through reentering from orbit or suborbit. Given strong ability to fulfill high-speed remote precision attack and power projection, this vehicle is of great implication function for strategic planning. However, the vehicle is a complex nonlinear object, and how to design the control strategy to ensure the stability of vehicle system has become a crucial topic [1–6].

Conventional technologies are majorly based on performing time-domain simulation and relied heavily on the results of human experience. Since the birth of modern control theory in the 1950s, control theory develops rapidly and has been successfully adopted in the aerospace application in the 1960s [7, 8]. In the recent few decades, robust control has gained remarkable attentions due to the well adaptation ability in dealing with objects in uncertain and noisy environment [9–13]. With the maturation of robust control theory, Kharitonov interval theory, $H\infty$ control theory, and structural singular value theory (μ theory) have been widely used in aircraft controller design and trajectory tracking. For instance, the refinement of the existing method by considering 16 segment plants instead of 16 Kharitonov plants provides an efficient tool for designing all robustly stabilizing PID controllers for an interval system [14]. An $H\infty$ method for designing reduced-order output-feedback controllers for linear time-invariant retarded systems was introduced to achieve a minimum bound on the H-infinity performance level [15]. The clearance of flight control law for a hypersonic gliding vehicle (HGV) and two linear clearance criteria based on structural singular value (μ) theory were proposed in [16]. However, since the aircraft is a nonlinear system, of which the mathematical model has parametric uncertainties, it is straightforward to deviate from the actual control by using direct linearization method.

Assume that the system is in instantaneous equilibrium, basic formula of linearized equations is then applied, and small deviation model of simplified equations is achieved [17]. In order to consider the effects of nonlinearity on the system, the article [18] shows the identifiability of a nonlinear delayed-differential model describing aircraft dynamics. In order to reduce the impact of model parameters perturbation on the system, a mixed $H2/H\infty$ control was proposed using fuzzy singularly perturbed model with multiple perturbation parameters [19]. A new strategy for missile attitude control using a hybridization of Linear Quadratic Gaussian (LQG), Loop Transfer Recovery (LTR), and Linear Quadratic Integral (LQI) control techniques was established [20]. However, it will result in relatively conservative results and will undermine the performance robustness of the system, due to the robust LQG control to maintain the minimum performance index. Guaranteed cost control (GCC) on uncertainty system is an effective method to solve the flaws of LQG design [21].

The GCC method can maintain the stability of the closed-loop system particularly when the controlled object has significant uncertainty. Meanwhile, it also ensures that the secondary performance index does not exceed the upper bound. A typical application of GCC method for a flexible air-breathing hypersonic vehicle (FAHV) can be found in [22]. In [23], the tracking GCC law was presented combined with the decoupling control to accommodate the parameter uncertainties without coupling. A modified GCC strategy has also been established for discrete-time uncertain systems with both state and input delays [24]. A robust guaranteed cost controller was proposed for quadrotor UAV system with uncertainties to address set-point tracking problem [25]. In order to eliminate disturbance effects and guarantee the robust stability of a quadrotor helicopter with state delay, improved guaranteed cost control and quantum adaptive control were developed [26]. A neural network (NN) based approximate optimal GCC design was developed to find a robust state feedback controller such that the closed-loop system has not only a bounded response in a finite duration of time for all admissible uncertainties but also a minimal guaranteed cost [27].

However, in order to obtain stronger robustness, robust control gains might be sensitive or fragile with respect to some errors or variations in control gains of feedback control. Therefore, a concept of nonfragile control strategy has been proposed, which gives a state feedback controller with enough regulating margin when control gains are variated. In [28], a synchronization problem for complex dynamical networks with additive time-varying coupling delays via nonfragile control was investigated. It has also been concerned with a problem of nonfragile robust optimal guaranteed cost control for a class of uncertain two-dimensional discrete state-delayed systems and the state feedback controllers are designed [29]. Robust nonfragile control of uncertain linear system and application to vehicle active suspension were described in [30]. In [31, 32], nonfragile guaranteed cost control of parametric uncertain systems was studied and the guaranteed cost nonfragile tracking control on the omnidirectional rehabilitative training walker was examined.

Though numerous research methods have been proposed in robust controller design, very limited work has been focused on the application of the hypersonic vehicles. Aiming at the complex hypersonic vehicle nonlinear system, small deviation linear equations are widely used in numerical analysis, but it may lead to the reduced model which can hardly achieve sufficient effect in the application of nonlinear system. The controller remains to be adjusted with considerable efforts before it can guarantee required control index. In this paper, we for the first time propose a linear and nonlinear combination in the course of system modeling, in order to make the expected model closer to the actual system. The Lyapunov function can be applied to get control law for nonlinear system when it satisfies certain Lipschitz conditions, and the problem is transformed into a feasible solution with linear matrix inequalities (LMI) method. Besides, adaptive SHLNN based nonfragile guaranteed cost control strategy is utilized to design the robust controller, with equivalent solution derived from LMI optimization approach. SHLNN are exploited as additive gain adjustments to eliminate the influence of results show that conservative control gains and counteract excessive upper bound of cost function are caused by uncertainties.

The rest of this article is organized as follows. In Section 2, motion model of hypersonic vehicle is formulated, where the state equations of vehicle body are established to testify the effectiveness of the proposed RGCC method. To prove the following theorems, several lemmas and assumptions are described in Section 3. Section 4 demonstrates the robust GCC law in the form of theorem under the Lipschitz conditions. In Section 5, a new adaptive nonfragile robust control strategy is presented, in which a nonfragile guaranteed cost controller solved by LMI optimization approach is applied. In Section 6, SHLNN controller design for nonfragile GCC strategy is treated as an additive gain compensator to reduce excessive performance caused by perturbations and uncertainties. Finally, simulation results of robust control and attitude tracking control are conducted and better stability and tracking performance by the proposed strategies for hypersonic vehicle model are gained.

2. Kinematics Model of Hypersonic Vehicle

According to the instantaneous equilibrium condition, the small deviation model of vehicle can be obtained based on the basic formulation of linearization equations. By analyzing motion mechanism and flight characteristics of the hypersonic vehicle, the motion equations of the vehicle body coordinate system are achieved as follows:

$$\dot{\alpha} = a_{11}F_{x1} + a_{12}F_{y1} + a_{13}\omega_{x1} + a_{14}\omega_{y1} + a_{15}\omega_{z1},$$

$$\dot{\beta} = b_{11}F_{x1} + b_{12}F_{y1} + b_{13}F_{z1} + b_{14}\omega_{x1} + b_{15}\omega_{y1},$$

$$\dot{\gamma} = c_{11}\omega_{x1} + c_{12}\omega_{y1} + c_{13}\omega_{z1},$$

$$J_{x1}\frac{d\omega_{x1}}{dt} + \left(J_{z1} - J_{y1}\right)\omega_{z1}\omega_{y1} = M_{x1},$$

$$J_{y1}\frac{d\omega_{y1}}{dt}+\left(J_{x1}-J_{z1}\right)\omega_{x1}\omega_{z1}=M_{y1},$$
$$J_{z1}\frac{d\omega_{z1}}{dt}+\left(J_{y1}-J_{x1}\right)\omega_{y1}\omega_{x1}=M_{z1}, \tag{1}$$

where α represents attack angle, β denotes sideslip angle, and γ is roll angle. F_{x1}, F_{y1}, F_{z1} represent the components acting on missile body coordinates; ω_{x1}, ω_{y1}, ω_{z1} represent the ω on x-, y-, z-axis of missile body coordinates. J_{x1}, J_{y1}, J_{z1} are vehicle's moment of inertias relative to each axis of vehicle body coordinate system; $d\omega_{x1}/dt$, $d\omega_{y1}/dt$, $d\omega_{z1}/dt$ are components of vehicle rotation angular acceleration vector on each axis, respectively.

In these equations, values of parameters a, b, and c are varied in aerodynamic model of vehicle. During the entire flight course, dramatic environmental changes will cause tens or even hundreds of times change of the aerodynamic parameters, which results in significant uncertainties on the mathematical vehicle model.

3. State Equation Description Form of Hypersonic Vehicle

Aiming at a classical nonlinear uncertain system, the state equation can be described as follows:

$$\dot{x}(t)=\left(A_1+\Delta A\right)x(t)+\left(B_1+\Delta B\right)u(t)+f(x,t),$$
$$y(t)=Cx(t). \tag{2}$$

$x(t)\in R^n$ represents state vector, and $x(0)=x_0$. $u(t)\in R^m$ is control input vector, and $f(x,t)\in R^n$ is a nonlinear part and is a state-related nonlinear function which meets the global Lipschitz condition in Assumption 5. A_1 and B_1 are matrices with the certain dimension. ΔA and ΔB represent parameter uncertainties, assuming that the uncertainties are norm-bounded, which can be expressed as follows:

$$\left[\Delta A \quad \Delta B\right]=DF(t)\left[E_1 \quad E_2\right], \tag{3}$$

where $D\in R^{n\times r}$, $E_1\in R^{q\times n}$, $E_2\in R^{q\times m}$ are known real matrices with specific dimension, which characterize the structure of uncertainty in the system, and $F(t)\in R^{r\times q}$ is an unknown time-varying matrix, but norm-bounded as follows.

$$\Omega=\left\{F(t)\mid F^T(t)F(t)\le I,\ \forall t\right\}. \tag{4}$$

The performance indicator is defined as follows.

$$J=\int_0^\infty\left[x^T(t)Qx(t)+u^T(t)Ru(t)\right]dt, \tag{5}$$

where Q and R are symmetric positive definite weighted matrices.

Lemma 1 (see [33]). *For a given symmetric matrix, $F\in R^{n\times n}$ is expressed as*

$$F=\begin{bmatrix}F_{11} & F_{12}\\ F_{21} & F_{22}\end{bmatrix}, \tag{6}$$

where $F_{11}\in R^{r\times r}$, $F_{12}\in R^{r\times(n-r)}$, $F_{21}\in R^{(n-r)\times r}$, $F_{22}\in R^{(n-r)\times(n-r)}$, and the conclusions are as follows.

(1) $F<0$;

(2) $F_{11}<0$, $F_{22}-F_{12}^TF_{11}^{-1}F_{12}<0$;

(3) $F_{22}<0$, $F_{11}-F_{12}F_{22}^{-1}F_{12}^T<0$.

Lemma 2. *For $\sigma_1(y)=y^TQ_1y\ge 0$, assuming there is $\tilde{y}\in R^m$, where $\sigma(\tilde{y})>0$, then the equivalent forms are as follows.*

(1) $y\in R^m$ *makes* $\sigma_1(y)\ge 0$, $y^TQ_0y>0$.

(2) $\tau\ge 0$ *makes* $Q_0-\tau Q_1>0$.

Corollary 3. *When $P>0$ and all $\xi\ne 0$, π satisfying $\pi^T\pi\le\xi^TC^TC\xi$ is established.*

$$\begin{bmatrix}\xi\\ \pi\end{bmatrix}^T\begin{bmatrix}A^TP+PA & PB\\ B^TP & 0\end{bmatrix}\begin{bmatrix}\xi\\ \pi\end{bmatrix}<0. \tag{7}$$

When $\tau\ge 0$ and $P>0$, then

$$\begin{bmatrix}A^TP+PA+\tau C^TC & PB\\ B^TP & -\tau I\end{bmatrix}<0. \tag{8}$$

Lemma 4. *When D, E, and $F(t)$ satisfy the certain dimension real matrices, and $F^T(t)F(t)\le I$, one can get the inequality for $\varepsilon>0$.*

$$DF(t)E+E^TF^T(t)D^T\le\varepsilon DD^T+\varepsilon^{-1}E^TE. \tag{9}$$

Assumption 5. Nonlinear function $f(x,t)$ meets global Lipschitz condition, namely,

$$\|f(x,t)\|\le\|Gx(t)\|,$$
$$\|f(x,t)-f(y,t)\|\le\|G(x(t)-y(t))\|. \tag{10}$$

4. Robust Guaranteed Cost Control of Hypersonic Vehicle

Theorem 6. *According to the parameter uncertain system (2), if $f(x,t)=0$ and it meets the performance (5), then there exists $u(t)=Kx(t)$ which satisfies the sufficient and necessary conditions for robust guaranteed cost with parameter uncertain closed-loop system: (1) there exists an appropriate constant $\varepsilon>0$, which makes inequality (11) have a positive definite solution $P>0$; (2) the robustness performance index of closed-loop system meets $J\le\mathrm{tr}(P)$ at the same time.*

$$\left(A_1+B_1K+\Delta A+\Delta BK\right)^TP$$
$$+P\left(A_1+B_1K+\Delta A+\Delta BK\right)+Q+K^TRK<0. \tag{11}$$

Corollary 7. *Given that formula (11) is satisfied, there exist P and X which establish the existence of appropriate positive* *constant, and matrices W and X satisfy allowable uncertainties [34].*

$$\begin{bmatrix} \Pi & \varepsilon D & (E_1X + E_2W)^T & X & W^T \\ * & -\varepsilon I & 0 & 0 & 0 \\ * & * & -\varepsilon I & 0 & 0 \\ * & * & * & -Q^{-1} & 0 \\ * & * & * & * & -R^{-1} \end{bmatrix} < 0, \quad (12)$$

where $\Pi = (A_1X + B_1W)^T + (A_1X + B_1W)$. *"$*$" denotes the transpose of symmetric part in equalities, and the definitions in the following matrix are the same. Furthermore, if inequality (12) has a solution* (W, X), *it can be described as follows:*

$$u^*(t) = WX^{-1}x(t). \quad (13)$$

This denotes a RGCC law of vehicle system. The performance indicator upper bound is

$$\bar{J} \leq \operatorname{Trace}(X^{-1}) = \bar{J}^*. \quad (14)$$

Proof. For system (2), order $f(x,t) = 0$, and based on Lemma 4 and (3), inequality (11) can be transformed into

$$\begin{aligned}&(A_1 + B_1K)^T P + P(A_1 + B_1K) + \varepsilon PDD^TP \\ &\quad + \varepsilon^{-1}(E_1 + E_2K)^T (E_1 + E_2K) + Q + K^TRK \\ &< 0.\end{aligned} \quad (15)$$

For inequality (15), based on Lemma 1, the following linear matrix inequalities can be obtained.

$$\begin{bmatrix} \Pi_1 & (E_1 + E_2K)^T & I & K^T \\ * & -\varepsilon I & 0 & 0 \\ * & * & -Q^{-1} & 0 \\ * & * & * & -R^{-1} \end{bmatrix} < 0. \quad (16)$$

For inequality (16), multiply it by $\operatorname{diag}\{P^{-1}, I, I, I\}$, and let $P^{-1} = X$, $KX = W$; then inequality (12) is obtained based on Lemma 1. We introduce the equation of $V(x(t)) = x^T(t)Px(t)$; then formula (17) for uncertain closed-loop system is obtained:

$$\begin{aligned}\dot{V}(x(t)) &= \dot{x}^T(t)Px(t) + x^T(t)P\dot{x}(t) = x^T(A_1 \\ &\quad + B_1K + \Delta A + \Delta BK)^T Px(t) + x^TP(A_1 + B_1K \\ &\quad + \Delta A + \Delta BK)x(t) \\ &= x^T\left[(A_1 + B_1K + \Delta A + \Delta BK)^T P \right. \\ &\quad \left. + P(A_1 + B_1K + \Delta A + \Delta BK)\right]x(t).\end{aligned} \quad (17)$$

We know from Theorem 6 that

$$\begin{aligned}&\dot{V}(x(t)) < -x^T(t)\left(Q + K^TRK\right)x(t) \\ &\int_0^\infty \dot{V}(x(t))\,dt = V(x(\infty)) - V(x(0)) \\ &\qquad < -\int_0^\infty x^T(t)\left(Q + K^TRK\right)x(t)\,dt.\end{aligned} \quad (18)$$

On the basis of the stability condition of system, we get $V(x(\infty)) = 0$; then

$$\begin{aligned}&\int_0^\infty x^T(t)\left(Q + K^TRK\right)x(t)\,dt < V(x(0)) \\ &\quad = x^T(0)Px(0).\end{aligned} \quad (19)$$

Thereupon we get in a further way

$$\bar{J} \leq E\{V(x(0))\}\operatorname{Trace}\left(X^{-1}\right) = \bar{J}^*. \quad (20)$$

Proof is over. □

Theorem 8. *Aiming at uncertain nonlinear system (2) as well as index (5), if there are matrices X, Y and quantity* $\varepsilon > 0$, $\tau > 0$, *the following inequality will hold:*

$$\begin{bmatrix} \Theta_1 & \tau I & \varepsilon D & \Theta_2 & XQ & W^TR & XG^T \\ * & -\tau I & 0 & 0 & 0 & 0 & 0 \\ * & * & -\varepsilon I & 0 & 0 & 0 & 0 \\ * & * & * & -\varepsilon I & 0 & 0 & 0 \\ * & * & * & * & -Q & 0 & 0 \\ * & * & * & * & * & -R & 0 \\ * & * & * & * & * & * & -\tau I \end{bmatrix} < 0, \quad (21)$$

where $\Theta_1 = A_1X + XA_1^T + B_1W + W^TB_1^T$, $\Theta_2 = (E_1X + E_2W)^T$.

Then $u = Kx(t)$ is RGCC control law of system (2), where $K = WX^{-1}$, and performance index is $J^* \leq x^T(0)X^{-1}x(0)$.

Proof. Considering function $V(x(t)) = x^T(t)Px(t)$, we take $u = Kx(t)$ into (2):

$$\begin{aligned}&\dot{V}(x(t)) + x^T(t)Qx(t) + x^T(t)K^TRKx(t) = \dot{x}^T(t) \\ &\quad \cdot Px(t) + x^T(t)P\dot{x}(t) + x^T(t)Qx(t) + x^T(t) \\ &\quad \cdot K^TRKx(t) = \left[x(t)(A_1 + B_1K + \Delta A + \Delta BK)\right. \\ &\quad \left. + f^T(x,t)\right]Px(t) + x^T(t) \\ &\quad \cdot P\left[(A_1 + B_1K + \Delta A + \Delta BK)x(t) + f(x,t)\right] \\ &\quad + x^T(t)Qx(t) + x^T(t)K^TRKx(t) = x^T(t) \\ &\quad \cdot \left[(A_1 + B_1K + \Delta A + \Delta BK)^T P\right.\end{aligned}$$

$$+ P\left(A_1 + B_1K + \Delta A + \Delta BK\right)\Big] x(t) + x^T(t) \cdot Pf(x,t) + f^T(x,t) Px(t) + x^T(t) Qx(t) + x^T(t) K^T RKx(t). \tag{22}$$

Order $\zeta^T(t) = \left[x^T(t) \quad f^T(x,t)\right]$; then

$$\dot{V}(x(t)) + x^T(t) Qx(t) + x^T(t) K^T RKx(t) = z^T(t) \begin{pmatrix} \Omega & P \\ P & 0 \end{pmatrix} z(t), \tag{23}$$

where $\Omega = (A_1 + B_1K + \Delta A + \Delta BK)^T P + P(A_1 + B_1K + \Delta A + \Delta BK) + Q + K^T RK$.

Based on Assumption 5 we get

$$\zeta^T(t) \begin{bmatrix} -G^TG & 0 \\ 0 & I \end{bmatrix} \zeta(t) \le 0. \tag{24}$$

Considering Lemma 2 and Corollary 3, when $\tau_0 > 0$, we can get

$$\begin{bmatrix} \Omega & P \\ P & 0 \end{bmatrix} - \tau_0 \begin{bmatrix} -G^TG & 0 \\ 0 & I \end{bmatrix} < 0. \tag{25}$$

Then,

$$\dot{V}(x(t)) + x^T(t) Qx(t) + x^T(t) K^T RKx(t) < 0. \tag{26}$$

Namely,

$$\begin{bmatrix} \Omega + \tau_0 G^TG & P \\ P & -\tau_0 I \end{bmatrix} < 0. \tag{27}$$

Multiplying $\mathrm{diag}(P^{-1} \quad I)$ on inequality (27) left and right, we have

$$\begin{bmatrix} P^{-1}\left(\Omega + \tau_0 G^TG\right) P^{-1} & I \\ I & -\tau_0 I \end{bmatrix} < 0. \tag{28}$$

Multiplying inequality (28) by $\mathrm{diag}(I \quad \tau_0^{-1} I)$ from both sides, we get

$$\begin{bmatrix} P^{-1}\left(\Omega + \tau_0 G^TG\right) P^{-1} & \tau_0^{-1} I \\ \tau_0^{-1} I & -\tau_0^{-1} I \end{bmatrix} < 0. \tag{29}$$

Order $P^{-1} = X$, $KX = W$, and $\tau_0^{-1} = \tau$, and based on Lemma 4, we get

$$\begin{bmatrix} \Xi & \tau I \\ \tau I & -\tau I \end{bmatrix} < 0, \tag{30}$$

where

$$\Xi = A_1X + XA_1^T + B_1W + W^TB_1^T + \varepsilon_1 DD^T + \varepsilon_2 DD^T + \varepsilon_1^{-1}(E_1X)^T(E_1X) + \varepsilon_2^{-1}(E_2W)^T(E_2W) + XQX + W^TRW + \tau^{-1}XG^TGX. \tag{31}$$

Based on Lemma 1 we know that (30) and (21) are equivalent. The proof is finished. Then from (26), we get

$$\dot{V}(x(t)) < -x^T(t)\left(Q + K^TRK\right)x(t) < 0. \tag{32}$$

□

Under this condition, the system is stable.

Integrating both sides of formula (32), we have

$$\int_0^\infty \dot{V}(x(t))\,dt = V(x(\infty)) - V(x(0)) < -\int_0^\infty x^T(t)\left(Q + K^TRK\right)x(t)\,dt. \tag{33}$$

According to system stability conditions, $V(x(\infty)) = 0$; then

$$\int_0^\infty x^T(t)\left(Q + K^TRK\right)x(t)\,dt < V(x(0)) = x^T(0)Px(0). \tag{34}$$

That is,

$$J^* \le E\{V(x(0))\} = E\left\{x^T(0)Px(0)\right\} = \mathrm{tr}\left(X^{-1}\right). \tag{35}$$

Theorem 9. *Towards uncertain system (2) as well as performance index (5), the following optimization problem*

$$\min_{\varepsilon,\tau,X,Y,M} \quad \mathrm{Trace}(M)$$

$$\begin{bmatrix} \Theta_1 & \tau I & \varepsilon D & \Theta_2 & XQ & W^TR & XG^T \\ * & -\tau I & 0 & 0 & 0 & 0 & 0 \\ * & * & -\varepsilon I & 0 & 0 & 0 & 0 \\ * & * & * & -\varepsilon I & 0 & 0 & 0 \\ * & * & * & * & -Q & 0 & 0 \\ * & * & * & * & * & -R & 0 \\ * & * & * & * & * & * & -\tau I \end{bmatrix} < 0, \tag{36}$$

$$\begin{bmatrix} M & I \\ * & X \end{bmatrix} > 0$$

has a solution $(\tilde{\varepsilon}, \tilde{\tau}, \widetilde{X}, \widetilde{W}, \widetilde{M})$; then $u^*(t) = \widetilde{W}\widetilde{X}^{-1}x(t)$ will be the optimal state feedback GCC law for such system. More free variables were introduced into the problem above, so that the solution to (36) was less conservative.

5. Nonfragile Guaranteed Cost Control Containing Nonlinear Perturbation

In order to reduce the system tracking error, we suppose that the output y_{dr} of system is constant vector which is a nonzero constant vector, and then the error vector is $e = y(t) - y_{dr} = x(t) - y_{dr}$. So the error system can be deduced such that

$$\dot{e} = \left(A_1 + \Delta A\right)e + \left(B_1 + \Delta B\right)u_k + \left(A_1 + \Delta A\right)y_{dr} + f_L(x,t). \tag{37}$$

For formula (37), design controller is with guaranteed cost. Consider that reference state input y_{dr} is bounded and assume that nonlinear function $f_L(e,\xi,t) = f_L(x(t))$ satisfies

$$f_L^T(e,\xi,t)\, f_L(e,\xi,t) \le e^T(t)\, G^T G e(t), \tag{38}$$

where G is a constant matrix. Meanwhile, it satisfies $G^T G > 0$.

It is clear that the regulator of system (37) is equal to design of the tracking controller for system (2). Let $u(t) = u_{ke} + v_{tr} = Ke + v_{tr}$, and (37) can be expressed as

$$\begin{aligned}\dot{e} = {} & \left(\left(A_1 + \Delta A\right) + \left(B_1 + \Delta B\right) K\right) e + \left(B_1 + \Delta B\right) v_{tr} \\ & + \left(A_1 + \Delta A\right) y_{dr} + f_L(e,\xi,t).\end{aligned} \tag{39}$$

To realize the regulation and control of system (37), it is requested that 0 is the balance point of this system; thus let

$$\left(B_1 + \Delta B\right) v_{tr} + \left(A_1 + \Delta A\right) y_{dr} + f_L(e,\xi,t) = 0. \tag{40}$$

If this system is progressively stable, then u_∞ is approximated to meet formula (41).

When $t \to \infty$, $e(t) \to 0$, $\dot{e}(t) \to 0$, $f_L(e,\xi,t) \to 0$, $u(\infty) \to u_\infty = v_{tr}$, then get

$$\left(B_1 + \Delta B\right) u_\infty + \left(A_1 + \Delta A\right) y_{dr} = 0. \tag{41}$$

That is,

$$v_{tr} = u_\infty = -B_1{}^{+} A_1 y_{dr}. \tag{42}$$

Let $K_c = -B_1{}^{+} A_1$; a feedback controller with uncertainties is given as

$$u(t) = v_k + v_{tr} = (K + \Delta K)\, e(t) + K_c y_{dr}, \tag{43}$$

where K is state feedback matrix and K_c is feedforward compensation matrix. ΔK is an uncertain matrix with corresponding dimension, which is norm-bounded in assumption and satisfies

$$\Delta K = D_K N_K(t)\, E_K, \tag{44}$$

where D_K, E_K are known matrices and $N_K(t)$ is an unknown time-varying matrix, represents a neural network control output, and satisfies

$$N_K^T(t)\, N_K(t) \le I. \tag{45}$$

Order $u_e(t) = u(t) - u_\infty$, and define quadratic performance index as the tracking performance of system; we have

$$J_e = \int_0^\infty \left(e^T(t)\, Q e(t) + u_e{}^T(t)\, R u_e(t)\right) dt. \tag{46}$$

Definition 10. For uncertain system (37) and cost function (46), K^* can be defined as a nonfragile guaranteed cost control gain matrix with the corresponding upper bound J^* of cost function, only if there exists a controller (43) satisfying inequality (45), which makes the closed-loop system with system uncertainties in (3) and nonlinear perturbation in (38) asymptotically stable, where K^* is a constant gain matrix and $J^* \ge J$ is a positive constant.

Theorem 11. *For an uncertain system (37) and cost function (46), if there exist symmetric positive definite matrices P and K, with a scalar quantity $\varepsilon_1 > 0$, satisfying the following equation*

$$M = \begin{bmatrix} Q + \widetilde{K}^T R \widetilde{K} + \Lambda_1 + \Lambda_1^T + \Lambda_2 & P \\ P & -\varepsilon_1^{-1} I \end{bmatrix} < 0, \tag{47}$$

K *is a nonfragile guaranteed cost control gain matrix and $J^* = e_0^T P e_0$ is the upper bound of cost function (46), where*

$$\begin{aligned} \Lambda_1 &= P\left(\widetilde{A} + \widetilde{B}\widetilde{K}\right), \\ \Lambda_2 &= \varepsilon_1^{-1} G^T G, \\ \widetilde{A} &= A + \Delta A, \\ \widetilde{B} &= B + \Delta B, \\ \widetilde{K} &= K + \Delta K. \end{aligned} \tag{48}$$

Proof. Select the Lyapunov function $V(t) = e^T(t) P e(t)$, where P is a positive definite matrix, and based on the control law (43), the time derivative of $V(t)$ with respect to time t yields [35]

$$\dot{V}(e) = \begin{bmatrix} e(t) \\ f_L(e,v,t) \end{bmatrix}^T \begin{bmatrix} \Lambda_1^T + \Lambda_1 & P \\ P & 0 \end{bmatrix} \begin{bmatrix} e(t) \\ f_L(e,v,t) \end{bmatrix}. \tag{49}$$

And (37) can be transformed into

$$\begin{bmatrix} e(t) \\ f_L(e,v,t) \end{bmatrix}^T \begin{bmatrix} -G^T G & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} e(t) \\ f_L(e,v,t) \end{bmatrix} < 0. \tag{50}$$

According to matrix inequality (47), it can be concluded that

$$\begin{bmatrix} e(t) \\ f_L(e,v,t) \end{bmatrix}^T M \begin{bmatrix} e(t) \\ f_L(e,v,t) \end{bmatrix} < 0. \tag{51}$$

Multiply ε_1 and add (51) to the left side of (50); it can be derived as $\dot{V}(e) < -e^T Q e - e^T \widetilde{K}^T R \widetilde{K} e = -e^T Q e - v_k^T R v_k < 0$. According to Lyapunov stability theory, system (37) is asymptotically stable.

After the integration of (51) on both sides from $t = 0$ to $t = \infty$ and equation $e(\infty) = 0$ inferred from asymptotic stability of the closed-loop system, it can be concluded that

$$\begin{aligned} J &= \int_0^\infty e^T \left(Q + \widetilde{K}^T R \widetilde{K}\right) e\, dt < V(e(0)) - V(e(\infty)) \\ &= e_0{}^T P e_0 = J^*. \end{aligned} \tag{52}$$

This completes the proof. □

In order to carry out nonfragile guaranteed cost controller for the system, the equivalent LMI expression of condition (45) is given based on Theorem 12.

Theorem 12. *For given positive definite matrices P and K, the closed-loop system (37) has a feasible solution $(\rho, \varepsilon_1, \varepsilon_2, Y, X)$, which guarantees the establishment of condition (47) for all allowable uncertainty. A nonfragile state feedback controller gain matrix $K = YX^{-1}$ and an upper bound cost function $\widehat{J} \leq \text{Trace}(X^{-1}) = \widehat{J}^*$ exist, if and only if there exist ρ, ε_1, $\varepsilon_2 > 0$, symmetric positive definite matrix X, and real matrix Y, such that the following LMI holds:*

$$\begin{bmatrix} \Lambda + \rho DD^T & * & * & * & * & * & * & * \\ E_1 X + E_2 Y & -\rho I & * & * & * & * & * & * \\ X & 0 & -Q^{-1} & * & * & * & * & * \\ Y & 0 & 0 & -R^{-1} & * & * & * & * \\ E_K X & 0 & 0 & 0 & -\varepsilon_2 I & * & * & * \\ \varepsilon_2 D_K^T B^T & \varepsilon_2 D_K^T E_2^T & 0 & \varepsilon_2 D_K^T & 0 & -\varepsilon_2 I & * & * \\ X & 0 & 0 & 0 & 0 & 0 & -\varepsilon_1 \left(G^T G\right)^{-1} & * \\ \varepsilon_1 I & 0 & 0 & 0 & 0 & 0 & 0 & -\varepsilon_1 I \end{bmatrix} < 0, \quad (53)$$

where $\Lambda = AX + BY + (AX + BY)^T$ and $*$ is the corresponding symmetric part of the matrix.

Proof. Following proof process of Theorem 11, we can transform the existence condition (47) based on Lemmas 1 and 2 into the following expression:

$$\begin{bmatrix} \Lambda_2 + \Lambda_3 + \Lambda_4 + \Lambda_5 & * & * & * \\ E_1 + E_2 \widetilde{K} & -\rho I & * & * \\ I & 0 & -Q^{-1} & * \\ \widetilde{K} & 0 & 0 & -R^{-1} \end{bmatrix} < 0, \quad (54)$$

where $\Lambda_3 = P(A + B\widetilde{K}) + (A + B\widetilde{K})^T P$, $\Lambda_4 = \rho PDD^T P$, $\Lambda_5 = \varepsilon_1 P^T P$. Substituting (44) into left side of (54), it can be decomposed as

$$Y_1 + \Sigma_1 + \Sigma_1^T < 0, \quad (55)$$

where

$$Y_1 = \begin{bmatrix} \Lambda_2 + \Lambda_4 + G^T G & * & * & * \\ E_1 + E_2 K & -\rho I & * & * \\ I & 0 & -Q^{-1} & * \\ K & 0 & 0 & -R^{-1} \end{bmatrix},$$
$$\Sigma_1 = \begin{bmatrix} PBD_K \\ E_2 D_K \\ 0 \\ D_K \end{bmatrix} N_K \begin{bmatrix} E_K & 0 & 0 & 0 \end{bmatrix}. \quad (56)$$

According to matrix inequality lemma, for all matrices N_k meeting $N_K^T N_K \leq I$ and a scalar $\varepsilon_2 > 0$, (55) is equivalent to

$$Y_1 + \varepsilon_2 \begin{bmatrix} PBD_K \\ E_2 D_K \\ 0 \\ D_K \end{bmatrix} \begin{bmatrix} PBD_K \\ E_2 D_K \\ 0 \\ D_K \end{bmatrix}^T + \varepsilon_2^{-1} \begin{bmatrix} E_K^T \\ 0 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} E_K^T \\ 0 \\ 0 \\ 0 \end{bmatrix}^T < 0. \quad (57)$$

Applying Schur complement and multiplying each side of (57) by diag matrix $\{P^{-1}, I, I, I, I, \varepsilon_2 I, I, \varepsilon_1 I\}$, the linear matrix inequality (53) including variables ρ, ε_1, ε_2 and matrices Y and X can be obtained and parametric expression of the nonfragile guarantee cost control gain is given as $K = YX^{-1}$, where $X = P^{-1}$, $Y = KP^{-1}$. Proof is then finished. □

Theorem 13. *For (37) and cost function (46), there exist an optimal nonfragile guaranteed cost control gain matrix $K^* = Y^*(X^*)^{-1}$ and a minimal upper bound* Trace(*S*) *of the cost function J^*, and the following optimization problem*

$$\min_{\rho, \varepsilon_1, \varepsilon_2, Y, X, S} \quad \text{Trace}(S) \quad (58)$$

has a solution as $(\rho^*, \varepsilon_1{}^*, \varepsilon_2{}^*, Y^*, X^*, S^*)$, satisfying both the linear matrix inequality (53) and the following equation:

$$\begin{bmatrix} S & * \\ I & X \end{bmatrix} > 0. \quad (59)$$

Proof. If $(\rho^*, \varepsilon_1{}^*, \varepsilon_2{}^*, Y^*, X^*, S^*)$ is a feasible solution of (58), it is also feasible to (53). According to Theorems 11 and 12, $K^* = Y^*(X^*)^{-1}$ is a feasible nonfragile guaranteed cost control gain matrix for the system. And using Lemma 1, (59) is equivalent to $S > X^{-1} > 0$, and the minimum of Trace(*S*) will ensure the minimization of Trace(X^{-1}), which is the minimization of cost function's upper bound. This completes the proof. □

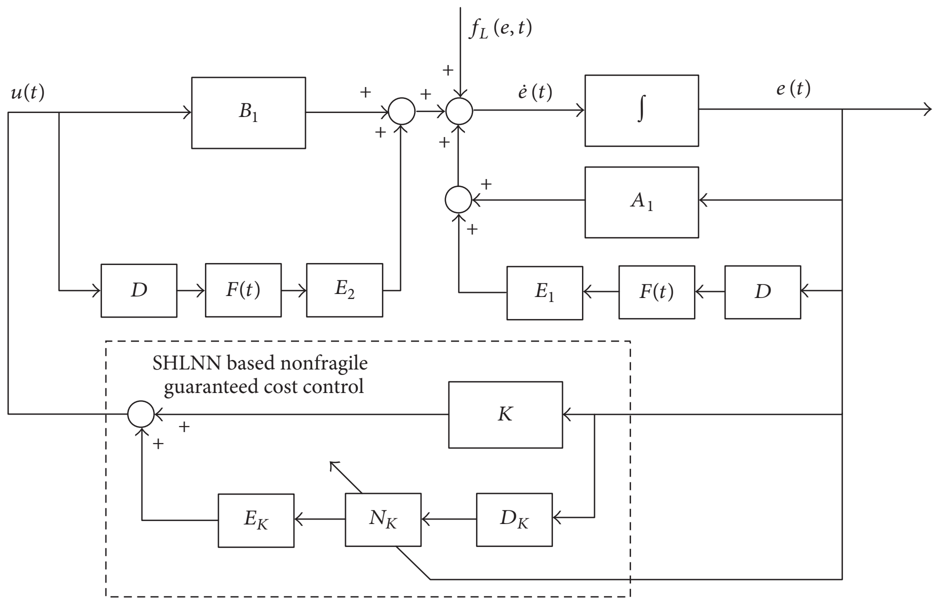


Figure 1: SHLNN based nonfragile robust control structure.

6. SHLNN Controller Design for Nonfragile Guaranteed Cost Control Strategy

SHLNN controller, with multilayer architecture, is composed of artificial neurons which simulate biologic ones. SHLNN can achieve mapping to arbitrary nonlinear function. The basic structure includes weighted summation, nonlinear function map, and linear dynamic states, and the input-output relationship is defined as follows [36, 37]:

$$y(W,V,x) = g\left(W^T f\left(V^T x\right)\right). \tag{60}$$

Take the output of SHLNN $N_K(t)$ as additive gain perturbation of the gain matrix K in (37); it will eliminate the influence of conservative nonfragile guaranteed cost control gains by online learning mechanism. As a result, the system error can quickly converge to zero and reduce the upper bound of cost function finally [38]. The stability of closed-loop system is guaranteed by Theorem 12. The cost function of NN controller is given as

$$E(t) = \frac{1}{2} e^T(t)\, e(t). \tag{61}$$

If $E(t)$ can be minimized by SHLNN, the tracking error $e(t)$ will be reduced as small as possible correspondingly, and better tracking performance can be achieved in the system. As for (37), the proposed control structure is shown in Figure 1.

If $N_K(t)$ is used as network output, the constraint condition $N_K^T(t)N_K(t) \le I$ should be satisfied, so modification of the SHLNN structure is needed. We choose hyperbolic tangent function $\sigma(\bullet)$ as activation function $g(\bullet)$ for output layer and keep the mapping relation $f(\bullet)$ unchanged in hidden layer, where

$$f(z) = g(z) = \sigma(z) = \frac{e^z - e^{-z}}{e^z + e^{-z}}. \tag{62}$$

Through the modification above, network outputs can be constrained between −1 and +1 by the hyperbolic tangent function $\sigma(\bullet)$, which satisfies $N_K^T(t)N_K(t) \le I$.

Suppose ΔK can be transformed as

$$\Delta K = \begin{bmatrix} D_{K_1} \\ M \\ D_{K_l} \end{bmatrix}_{l\times m}^T \begin{bmatrix} N_{K_1} & 0 & 0 \\ 0 & O & 0 \\ 0 & 0 & N_{K_l} \end{bmatrix}_{l\times l} \begin{bmatrix} E_{K_1} \\ M \\ E_{K_l} \end{bmatrix}_{l\times n}, \tag{63}$$

where N_K is a matrix of $l \times l$. Consider each SHLNN as an output; partial derivative of (43) can be expressed as

$$\frac{\partial u(t)}{\partial N_{K_i}(t)} = D_{K_i} E_{K_i} e(t) = p(t), \quad (i = 1, L, l). \tag{64}$$

If the system input is defined as vector π, we can describe neural network equations as $O_V = V^T \pi$ and $O_W = W^T \sigma(O_V)$, in which σ_W and σ_V represent derivatives of activation function to weighted matrices W and V, respectively. According to the chain rule of SHLNN weight, an update scheme is inferred as follows:

$$\begin{aligned} \dot{W}_i &= \kappa_{Wi} e \operatorname{sgn}\left(\frac{\partial e}{\partial u}\right) p \times \sigma \times \sigma_W^T + \lambda_i W_i, \\ \dot{V}_i &= \kappa_{Vi} e \operatorname{sgn}\left(\frac{\partial e}{\partial u}\right) p \times \pi \left(\sigma_V \times W^T \sigma_W\right)^T + \lambda_i V_i, \end{aligned} \tag{65}$$

where κ_{Wi} and κ_{Vi} stand for learning rates and λ_i represents the inertial coefficient, $i = 1, \ldots, l$.

7. Simulation Results Analysis

7.1. State Regulating Simulation of Hypersonic Vehicle. For a specific hypersonic vehicle system, the relevant parameters of the system equation are given as follows.

$$A_1 = \begin{bmatrix} -0.0299 & 1.0000 & 0 & 0 & 0 & 0 \\ -0.6345 & -0.0184 & 0 & 0 & 0 & 0 \\ 0 & 0 & -0.0058 & \begin{matrix} 0.9879 \\ -0.0063 \end{matrix} & 0 & 0.1543 \\ 0 & 0 & -1.0467 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1.0000 \\ -0.0009 & 0 & 56.5463 & 0 & 0 & -0.0167 \end{bmatrix},$$

$$B_1 = \begin{bmatrix} -0.0008 & 0 & 0 \\ -4.3885 & 0 & 0 \\ 0 & -0.0001 & 0.0001 \\ 0 & -0.3703 & 1.9365 \\ 0 & 0 & 0 \\ 0 & 0.6356 & -27.4183 \end{bmatrix},$$

$$D = \operatorname{diag}\begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}, \tag{66}$$

$$E_1 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0.0346 & 0.0001 & 0.0001 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.0001 & 0 & 0.0009 \\ 0.0001 & 0 & 0.0088 & 0 & 0 & 0 \\ 0 & 0 & 0.01 & 0 & 0 & 0 \\ 0.0012 & 0 & 0.2974 & 0 & 0 & 0.0001 \end{bmatrix},$$

$$E_2 = \begin{bmatrix} 0 & 0 & 0 \\ 0.0177 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0.0015 & 0.0103 \\ 0 & 0 & 0 \\ 0 & 0.0025 & 0.1295 \end{bmatrix},$$

$$Q = \operatorname{diag}\begin{pmatrix} 20 & 0.1 & 20 & 0.1 & 20 & 0.1 \end{pmatrix},$$

$$R = \operatorname{diag}\begin{pmatrix} 1 & 1 & 1 \end{pmatrix}.$$

According to Theorem 6, matrix K can be achieved.

$$K = \begin{bmatrix} 7.3272 & 4.1847 & 0.0280 & 0.0079 & -0.0012 & 0.0007 \\ 0.0091 & 0.0005 & 40.3491 & 19.6845 & -2.7003 & 1.0848 \\ 0.0009 & 0.0002 & 4.3130 & 0.9314 & 3.2004 & 1.7965 \end{bmatrix}. \tag{67}$$

From the above equations, we can get the performance index $J^* = 4.0560$ through Matlab simulation. Given the initial state vector $\mathbf{X}_0 = [0.2 \ \ 0.1 \ \ 0.1 \ \ 0.1 \ \ 0.1 \ \ 0.1]^T$, the simulation results of $\mathbf{x}(t)$ and $u(t)$ are shown in Figures 2 and 3, respectively. From Figure 2, six response curves of angles and angular velocities show that the regulating system is stable and controllable, with short settling time. Through the definition of $J(t) = \int_0^t [x^T(\tau)Qx(\tau) + u^T(\tau)Ru(\tau)]d\tau$, the evolution process of $J(t)$ is shown in Figure 4, which shows that the system has a given performance index upper bound.

7.2. Robust Tracking Control Simulation Result of Hypersonic Vehicle. In numerical simulation, we decompose ΔA and ΔB like Section 7.1. Suppose there exists $G^TG = 0.5I$ in (38) and number of SHLNN network outputs in the matrix N_K is three. Furthermore, in order to guarantee the ability to counteract system uncertainties, gain matrices D_K and E_K are given as

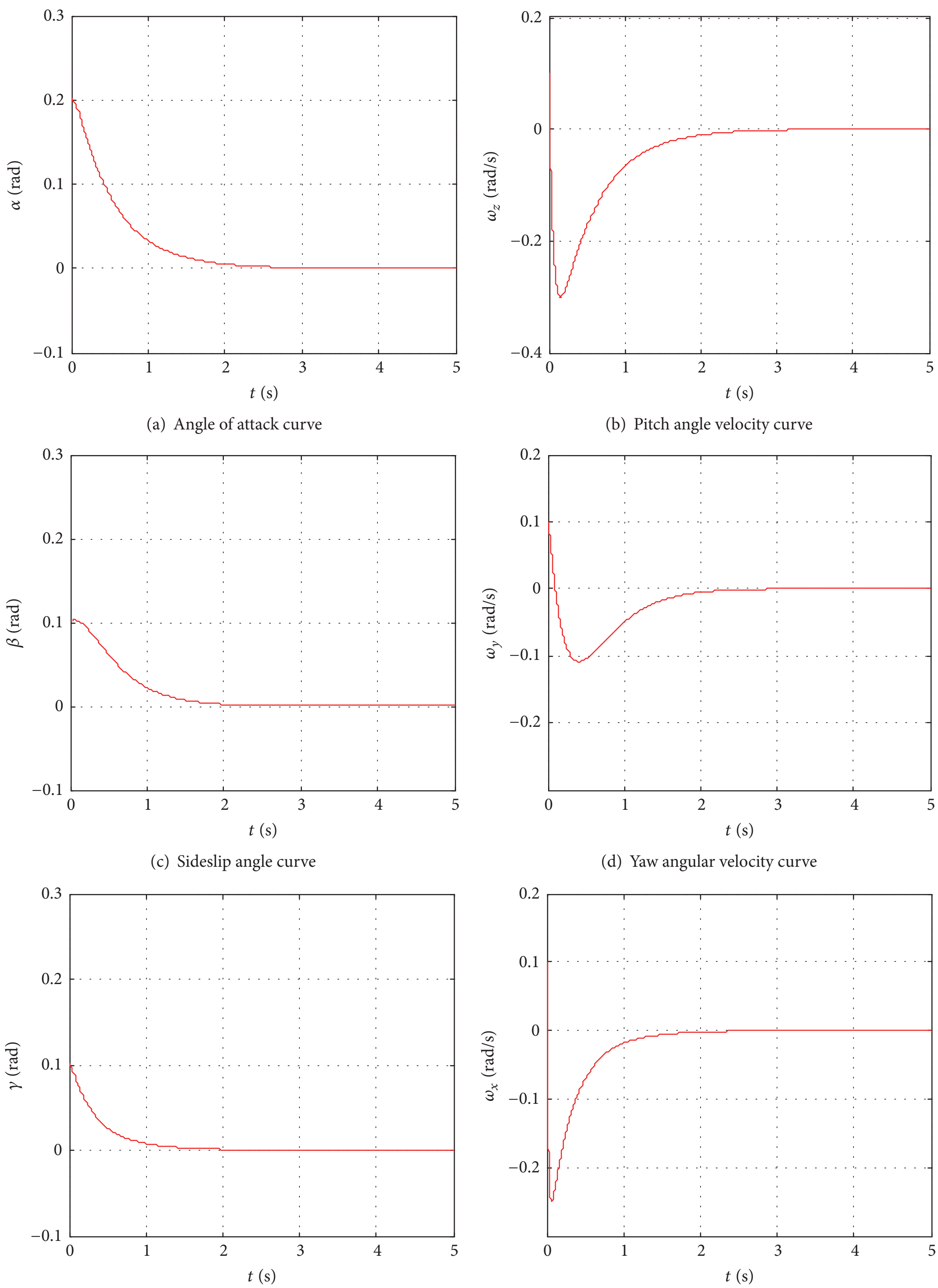
$$D_K = \begin{bmatrix} 0.5 & 0 & 0 \\ 0 & 0.5 & 0 \\ 0 & 0 & 0.5 \end{bmatrix},$$
$$E_K = \begin{bmatrix} 20 & 15 & 0 & 0 & 0 & 0 \\ 0 & 0 & 10 & 5 & 0 & 0 \\ 0 & 0 & 0 & 0 & 20 & 15 \end{bmatrix}. \tag{68}$$

So the gain matrix $\mathbf{K}$ can be obtained by LMI optimization approach according to Theorems 12 and 13.

$$\mathbf{K} = \begin{bmatrix} 29.7482 & 22.4213 & 0.0166 & 0.0044 & -0.0012 & 0.0004 \\ 0.0306 & 0.0004 & 74.1146 & 31.5391 & -8.0931 & -0.4972 \\ 0.0008 & 0.0000 & 3.9106 & 0.7396 & 11.1610 & 8.4109 \end{bmatrix}. \tag{69}$$

Three SHLNN outputs are utilized to adjust gain coefficients of pitch, yaw, and roll, respectively. The network inputs are chosen as $\pi_\alpha = [\alpha_c, \dot{\alpha}_c, e_\alpha, u_\alpha]^T$, $\pi_\beta = [e_\beta, u_\beta]^T$, $\pi_\gamma = [\gamma_c, \dot{\gamma}_c, e_\gamma, u_\gamma]^T$, with predefined network outputs N_{Ki}. Moreover, learning rates in (65) are $\kappa_{W1} = \kappa_{V1} = \kappa_{W2} = \kappa_{V2} = 0.4$, $\kappa_{W3} = \kappa_{V3} = 0.2$, with inertial coefficients $\lambda_1 = \lambda_2 = \lambda_3 = 0.1$.

To test the tracking effect of the above control law, square waves are selected as the command of pitch and roll channels with the alterative amplitudes from 2.5° to 5° and -10° to 10° correspondingly, with time period as 20 seconds. In Figure 5, tracking performance results of GCC method and the proposed scheme are presented, where dash lines stand for standard GCC method results and solid lines are the responses using nonfragile robust control strategy. Figure 6 denotes the three channel angular velocity curves and Figure 7 is the curves of elevator and rudder angles in pitch, yaw, and roll channels. Figure 8 illustrates the regulating process of the control gain matrix $K+\Delta K$ elements, where ΔK is adjusted by the three SHLNN outputs. As shown above, the tracking effect of nonfragile robust control gives a good improvement in dynamic performance and tracking errors are apparently decreased. Therefore, the nonfragile guaranteed cost control method, integrated with SHLNN controller to update gain values, is effective in improving the control performance as proposed.



(a) Angle of attack curve

(b) Pitch angle velocity curve

(c) Sideslip angle curve

(d) Yaw angular velocity curve

(e) Roll angle curve

(f) Roll angular velocity curve

Figure 2: Responses of GCC controller.

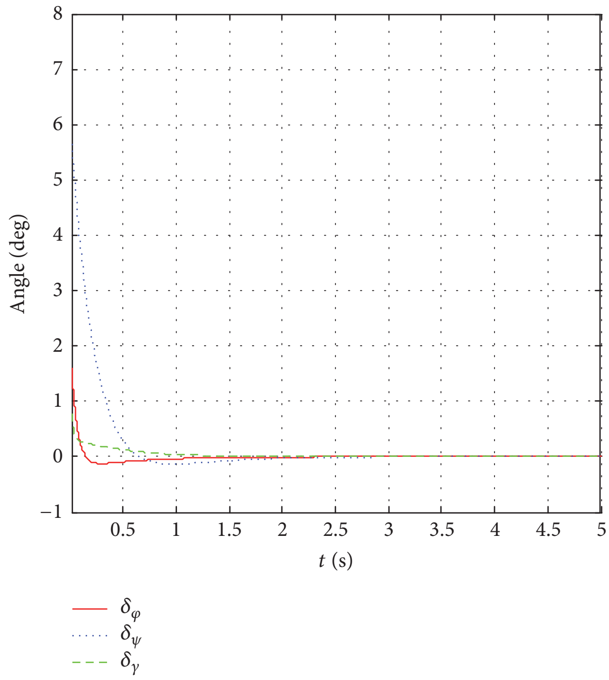


Figure 3: Regulating process of rudder angles as control inputs.

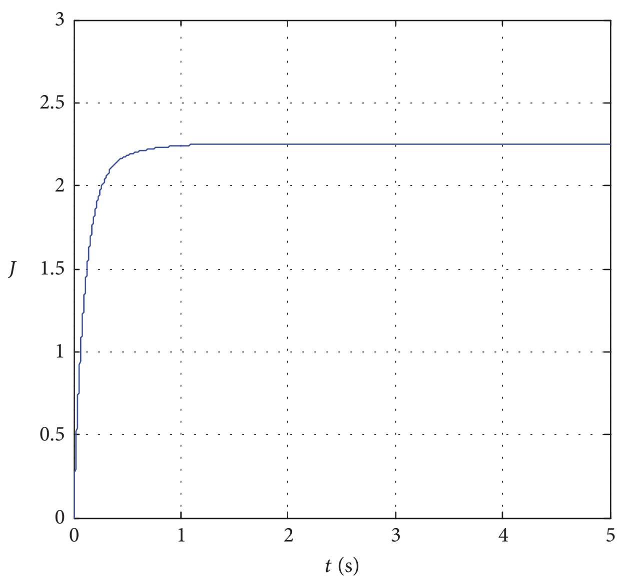
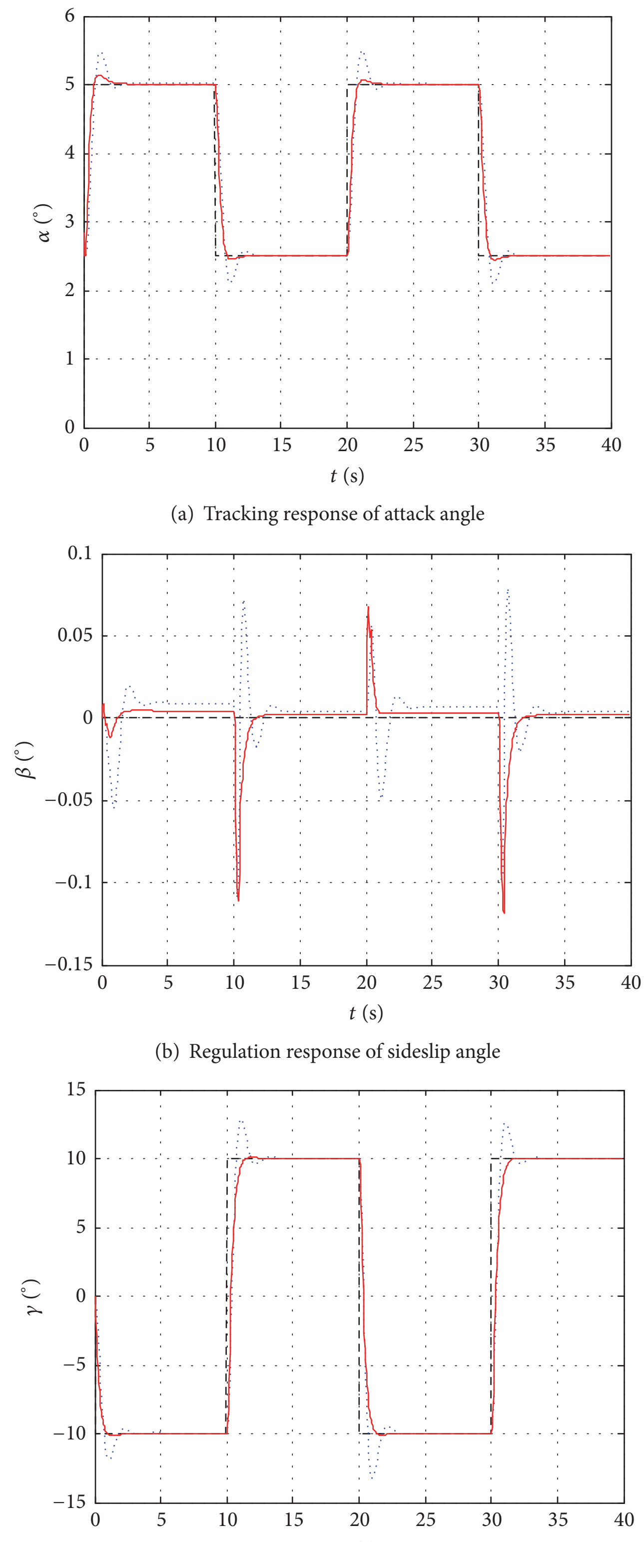


Figure 4: Performance index regulating process.



(a) Tracking response of attack angle

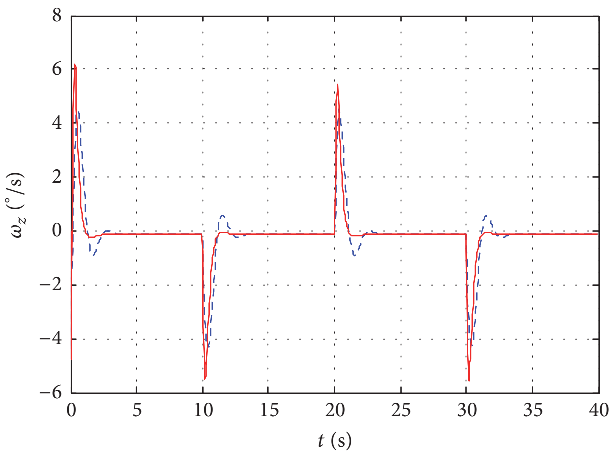
(b) Regulation response of sideslip angle

(c) Tracking response of roll angle

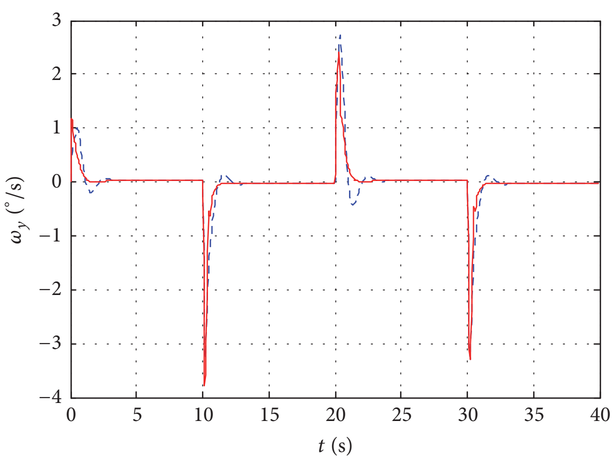
Figure 5: Tracking curves of three channel angles.

8. Conclusions

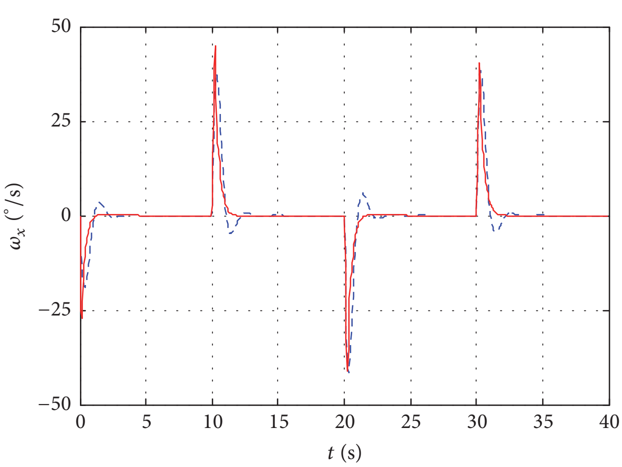
On the basis of RGCC design theory, the state feedback control law is obtained in this paper by applying Lyapunov function and satisfying overall Lipschitz condition. Specifically, the state-related nonlinear equation is built with a nonlinear part. The equations establishment formulates the process for solving the solution with LMI. Furthermore, adaptive SHLNN based nonfragile guaranteed cost control strategy is utilized to design the robust controller, with equivalent solution derived from LMI optimization approach, where SHLNN are exploited as additive gain adjustments to eliminate the influence of conservative control gains and



(a) Regulation response of pitch angular velocity

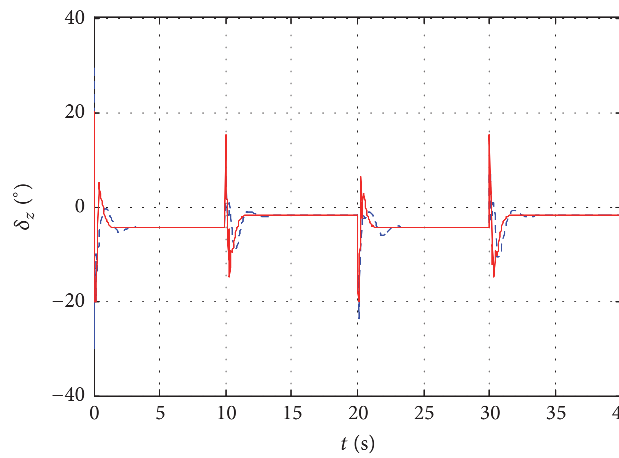


(b) Regulation response of yaw angular velocity

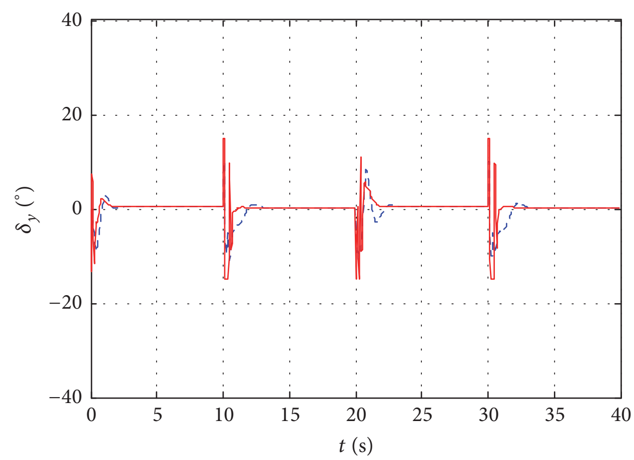


(c) Regulation response of roll angular velocity

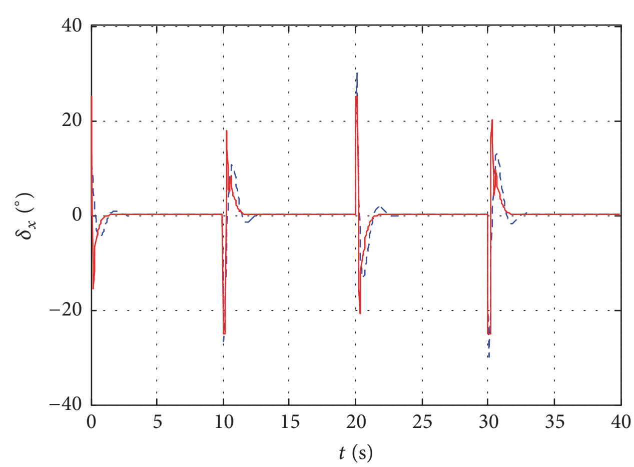
FIGURE 6: Regulation curves of angular velocities.



(a) Regulation curve of elevator angle in pitch channel



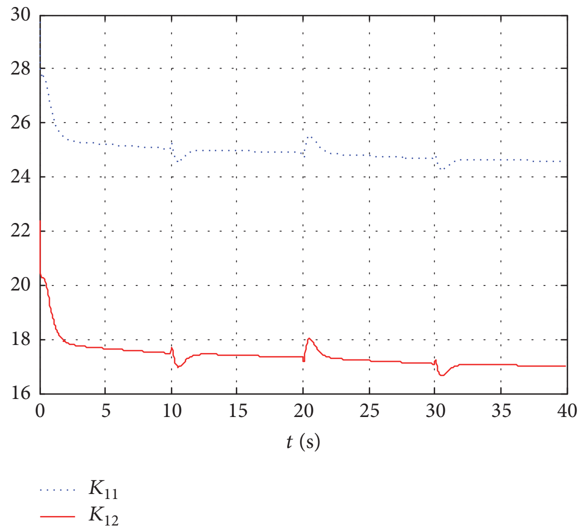
(b) Regulation curve of rudder angle in yaw channel



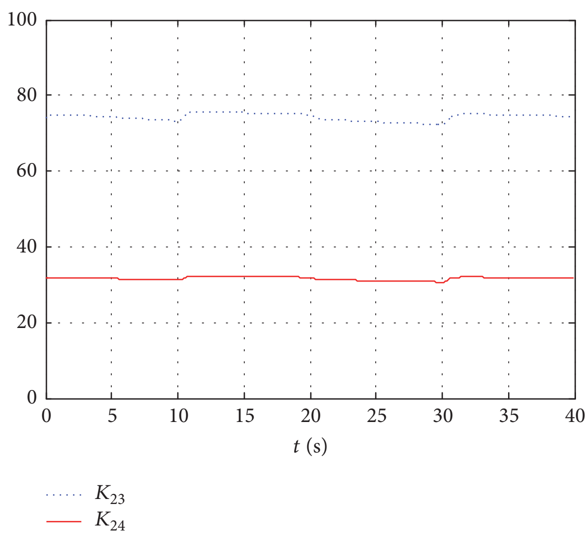
(c) Regulation curve of elevator angle in roll channel

FIGURE 7: Regulation curves of elevator and rudder angles in three channels.

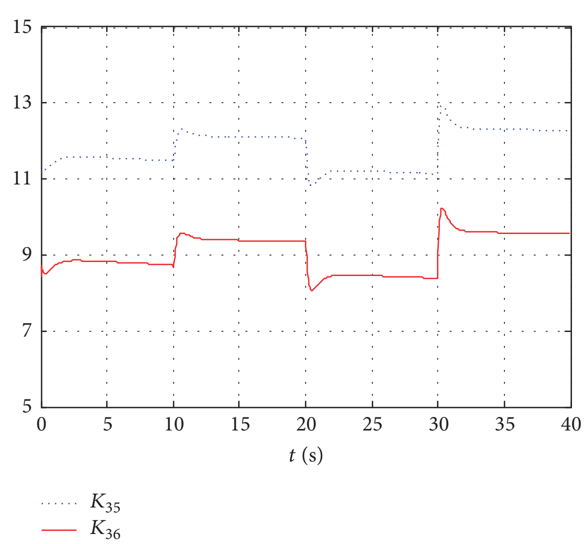
counteract excessive upper bound of cost function caused by uncertainties. Finally, simulation verifications are carried out with a specific model of hypersonic vehicle, and feasibility and adaptability of the proposed algorithm are demonstrated accordingly, where the proposed method has better tracking performance in attitude control on the vehicle.



(a) Gains variation in pitch channel



(b) Gains variation in yaw channel



(c) Gains variation in roll channel

FIGURE 8: Control gains variation in three channels.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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