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QUINE'S INTERPRETATION PROBLEM AND THE EARLY DEVELOPMENT OF POSSIBLE WORLDS SEMANTICS*

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In this paper, I shall consider the challenge that Quine posed in 1947 to the advocates of quantified modal logic to provide an explanation, or interpretation, of modal notions that is intuitively clear, allows "quantifying in", and does not presuppose, mysterious, intensional entities. The modal concepts that Quine and his contemporaries, e.g. Carnap and Ruth Barcan Marcus, were primarily concerned with in the 1940's were the notions of (broadly) logical, or analytical, necessity and possibility, rather than the metaphysical modalities that have since become popular, largely due to the influence of Kripke. In the 1950's modal logicians responded to Quine's challenge by providing quantified modal logic with model-theoretic semantics of various types. In doing so they also, explicitly or implicitly addressed Quine's interpretation problem. Here I shall consider the approaches developed by Carnap in the late 1940's, and by Kanger, Hintikka, Montague, and Kripke in the 1950's, and discuss to what extent these approaches were successful in meeting Quine's doubts about the intelligibility of quantified modal logic.

1. Background: The search for the intended interpretation

Starting with the work of C. I. Lewis, an immense number of formal systems of modal logic have been constructed based on classical propositional or predicate logic. The originators of modern modal logic, however, were not very clear about the intuitive meaning of the symbols $\Box$ and $\Diamond$, except to say that these should stand for some kind of necessity and possibility, respectively. For instance, in *Symbolic Logic* (1932), Lewis and Langford write:

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> It should be noted that the words "possible", "impossible" and "necessary" are highly ambiguous in ordinary discourse. The meaning here assigned to ◇p is a *wide* meaning of "possibility" – namely, logical conceivability or the absence of self-contradiction. (pp. 160-61)

This situation early on led to a search for more rigorous interpretations of modal notions. Gödel (1933) suggested interpreting the necessity operator □ as standing for provability (*informal provability* or, alternatively, *formal provability* in a fixed formal system), a suggestion that subsequently led to the modern *provability interpretations* of Solovay, Boolos and others.

After Tarski (1936a, b) had developed rigorous notions of satisfaction, truth and logical consequence for classical extensional languages, the question arose whether the same methods could be applied to the languages of modal logic and related systems. One natural idea, that occurred to Carnap in the 1940's, was to let □φ be true of precisely those formulas φ that are *logically valid* (or logically true) according to the standard semantic definition of logical validity. This idea led him to the following semantic clause for the operator of logical necessity:

□φ is true in an interpretation I iff φ is true in every interpretation I'.

This type of approach, which we may call the *validity interpretation*, was pursued by Carnap, using so-called state descriptions, and subsequently also by Kanger (1957a, b) and Montague (1960), using Tarski-style model-theoretic interpretations rather than state descriptions. In Hintikka's and Kanger's early work on modal semantics other interpretations of □ were also considered, especially, epistemic ('It is known that φ') and deontic ones ('It ought to be the case that φ'). In order to study these and other non-logical modalities, the introduction by Hintikka and Kanger of *accessibility relations* between possible worlds (models, domains) was crucial.

2. Carnap's formal semantics for quantified modal logic

Carnap's project was not only to develop a semantics (in the sense of Tarski) for intensional languages, but also to use metalinguistic notions from formal semantics to throw light on the modal ones. In 'Modalities and Quantification' from 1946 he writes:

> It seems to me ... that it is not possible to construct a satisfactory system before the meaning of the modalities are sufficiently clarified. I further believe that this clarification can best be achieved by correlating each of the modal concepts with a corresponding semantical concept (for example, necessity with L-truth).

In (1946, 1947) Carnap presented a formal semantics for logical necessity based on Leibniz's old idea that a proposition is necessarily true if and only if it

is true in all possible worlds. Suppose that we are considering a first-order predicate language L with predicate symbols and individual constants, but no function symbols. In addition to Boolean connectives, quantifiers and the identity symbol = (considered as a logical symbol), the language L also contains the modal operator $\Box$ for logical necessity. We assume that L comes with a *domain of individuals* D and that there is a one-to-one correspondence between the individual constants of L and the individuals in D. Intuitively speaking, each individual in D has exactly one individual constant as its (canonical) name. A *state description* S for L is simply a set of (closed) atomic sentences of the form $P(a_1,..., a_n)$, where P is an n-ary predicate in L and $a_1,..., a_n$ are individual constants in L. Intuitively speaking, an n-ary predicate symbol ($n \geq 1$) represents an n-ary relation among individuals in D. An atomic sentence of the form $P(a_1,..., a_n)$ represents the state-of-affairs of the relation represented by P holding between the individuals denoted by $a_1,..., a_n$ (in that order). State descriptions represent logically possible worlds. $P(a_1,..., a_n) \in S$ represents that the relation P obtains between $a_1,..., a_n$ in the world represented by S. $P(a_1,..., a_n) \notin S$ represents the fact that the relation P doesn't hold between $a_1,..., a_n$ in the corresponding world.

In order to interpret quantification, Carnap introduced the notion of an *individual concept* (relative to L): An individual concept is simply a function f which assigns to every state description S an individual constant f(S) (representing an individual in D). Intuitively speaking, individual concepts are functions from possible worlds to individuals. According to Carnap's semantics, individual variables are assigned values *relative to* state descriptions. An *assignment* is a function g which to every state description S and every individual variable x assigns an individual constant g(x, S). Intuitively, g(x, S) represents the individual which is the value of x under the assignment g in the possible world represented by S. We may speak of g(x, S) as the *value extension* of x in S relative to g. Analogously, the individual concept $(\lambda S)g(x, S)$ which assigns to every state description S the value extension of x in S relative to g, we call the *value intension* of x relative to g. Thus, according to Carnap's semantics a variable is assigned both a *value intension* and a *value extension*. The value extension assigned to a variable in a state description S is simply the value intension assigned to the variable applied to S.

With these notions in place, we can define what it means for a formula φ of L to be *true* in a state description relative to an assignment g (in symbols, $S \vDash \varphi[g]$).

For atomic formulas of the form $P(t_1,..., t_n)$, where $t_1,..., t_n$ are individual terms, i.e., variables or individual constants, we have:

(1) $S \models P(t,..., t_n)[g]$ iff $P(S(t_1, g)..., S(t_n, g)) \in S$.

Here, $S(t_i, g)$ is the extension of the term t_i in the state description S relative to the assignment g. Thus, if t_i is an individual constant, then $S(t_i, g)$ is t_i itself; and if t_i is a variable, then $S(t_i, g) = g(t_i, S)$.

The semantic clause for the identity symbol is the following:

(2) $S \models t_1 = t_2[g]$ iff $S(t_1, g) = S(t_2, g)$.

That is, the identity statement $t_1 = t_2$ is true in a state description S relative to an assignment g if and only if the terms t_1 and t_2 have the same extensions in S relative to g.

The clauses for the Boolean connectives are the usual ones:

(3) $S \models \neg\varphi[g]$ iff $S \varphi[g]$,

(4) $S \models (\varphi \rightarrow \psi)[g]$ iff $S \varphi[g]$ or $S \models \psi[g]$.

Carnap's clause for the universal quantifier is the following:

(5) $S \models \forall x\varphi[g]$ iff for every assignment g' such that $g =_x g'$, $S \models \varphi[g']$.

Explanation: g' is like g except possibly at x (also written, $g =_x g'$) if and only if, for each state description S' and each variable y other than x, g'(y, S') = g(y, S'). Intuitively, $g =_x g'$ means that the assignments g and g' assign the same value intensions to all the variables that are distinct from x and possibly assign different value intensions to x. Intuitively, then $\forall x\varphi(x)$ may be read: "for every assignment of an individual concept to x, $\varphi(x)$".

Finally, the semantic clause for the necessity operator is the expected one:

(6) $S \models \Box\varphi[g]$ iff, for every state-description S', $S' \models \varphi[g]$.

That is, the modal formula 'it is (logically) necessary that φ' is true in a state-description S (relative to an assignment g) if and only if φ is true in every state-description S' (relative to g).

A formula φ is *true in a state description* S (in symbols, $S \models \varphi$) if it is true in S relative to every assignment. *Logical truth* (logical validity) is defined as truth in all state-descriptions. We write $\models \varphi$ for φ being logically true.

It is easy to verify that Carnap's semantics satisfies the following principles:

(7) All truth-functional tautologies are logically true.

(8) The set of logical truths is closed under modus ponens.

(9) The standard principles of quantification theory (without identity) are valid. In particular, *universal instantiation*
$\forall x\varphi(x) \rightarrow \varphi(t/x)$ (where t is substitutable for x in φ)
holds without restrictions.

We also have, as expected:

(10) $\models \Box\varphi$ iff $\models \varphi$,
(11) $\models \neg\Box\varphi$ iff φ.

The operator $\Box$, of course, satisfies the usual laws of the system S5, together with the so-called Barcan formula and its converse, as well as the rule of necessitation.

For identity, we have:

(LI) $\models t = t$.

However, the unrestricted principle of *indiscernibility of identicals* is not valid in Carnap's semantics. In other words, the following principle does not hold for all formulas φ:

(I=) $\models \forall x\forall y[x = y \rightarrow (\varphi(x/z) \rightarrow \varphi(y/z))]$.

Instead, we have a restricted version of (I=).

($I=_{restr}$) $\models \forall x\forall y[x = y \rightarrow (\varphi(x/z) \rightarrow \varphi(y/z))]$, provided φ does not contain any occurrences of $\Box$.

For the unrestricted case, we only have:

($I\Box=$) $\models \forall x\forall y[\Box(x = y) \rightarrow (\varphi(x/z) \rightarrow \varphi(y/z))]$.

The following principle is of course not valid according to Carnap's semantics:

($\Box=$) $\forall x\forall y(x = y \rightarrow \Box(x = y))$. (*Necessity of identity*)

In the presence of the other principles, it is equivalent to the unrestricted principle of indiscernibility of identicals. Nor do we have:

($\Box\neq$) $\forall x\forall y(x \neq y \rightarrow \Box(x \neq y))$. (*Necessity of non-identity*)

Carnap introduced the notion of a *meaning postulate* to account for analytic connections between the non-logical symbols of a predicate language. Thus, suppose that MP is the set of all the meaning postulates of a given language L. MP is then a set of sentences in the non-modal fragment of L. We say that a state description S is *admissible* if MP ∪ S is consistent. Then, we can interpret $\Box$ as 'analytic necessity' by modifying clause (6) above to:

(6') $S \models \Box\varphi$ iff, for every admissible state-description S', $S' \models \varphi$.

We also say that φ is *analytically true* iff φ is true in all admissible state descriptions. In the modified semantics, we have:

$S \models \Box\varphi$ iff φ is analytically true,
$S \models \neg\Box\varphi$ iff φ is not analytically true.

Carnap's semantics for the quantifiers can be understood in two ways. The most straightforward interpretation is to say that the quantifiers simply range over individual concepts. Sometimes Carnap himself characterizes his interpretation of the quantifiers in this way and this is how Quine describes it. There is, however, another more subtle interpretation according to which every individual term, including the (free) variables, has a double semantic role given by its extension and its intension, respectively. Each variable has a value extension as well as a value intension. According to this interpretation – which I think is the one that Carnap really had in mind – it is simply wrong to ask for *the* range of the individual variables. In ordinary extensional contexts the variables can be thought of as ranging over ordinary individuals. However, in intensional contexts the intensions associated with the variables come into play. This is what explains why the following principle fails:

$(\Box=)\quad \forall x\forall y(x = y \rightarrow \Box(x = y))$.

Carnap's interpretation of the quantifiers can still be criticized for being unintuitive. The problem is that he lacks a way of discriminating between those individual concepts that, intuitively speaking, pick out one and the same individual in all possible worlds and those that don't. Suppose that we have assigned to the variable x as its value intension the individual concept: *the number of planets*. Relative to this assignment it is true that:

(1) $\quad x = 9 \wedge \neg\Box(x = 9)$.

However, there is no *object* that has the property of being identical with 9 but doesn't have this property necessarily. So from (1) it should not follow that:

(2) $\quad \exists x(x = 9 \wedge \neg\Box(x = 9))$.

But of, course, on Carnap's interpretation of the quantifiers, (2) is a logical consequence of (1). Intuitively, one should be able to make the inference from (1) to (2) only if the concept assigned to x in (1) is a *logically rigid concept*, i.e. a concept that picks out the same individual relative to every state description.

Thus, it seems that one could turn Carnap's semantics into a more intuitively satisfactory one, by requiring that all variable assignments g be (logically) rigid, i.e., satisfy the condition:

$$g(S, x) = g(S', x),$$

for all variables x and all state descriptions S. In the language, one could then have individual constants of two kinds: logically rigid one's ("logically proper names") and constants that are not logically rigid. For the logically rigid constants, universal instantiation and existential generalization would hold, but not, of course, for the others. Presumably, most ordinary proper names are not rigid in this strong sense.

3. Quine's interpretational challenge

Quine's criticism of quantified modal logic comes in different strands. First, there is the simple observation that classical quantification theory with identity cannot be applied to a language in which substitutivity of identicals for singular terms fails. It seems that either universal generalization (and its mirror image: existential specification) or indiscernibility of identicals:

$$\forall x \forall y[x = y \rightarrow (\varphi(x/z) \rightarrow \varphi(y/z))],$$

has to be given up. This observation gives rise to the following weak, and I take it, uncontroversial, Quinean claim: Quantification theory (with identity) cannot be combined with non-extensional operators (i.e., operators for which substitutivity of identicals for singular terms fail) without being modified in some way. This weak claim already gives rise to the challenge of extending quantification theory in a consistent way to languages with non-extensional operators.

In addition to the weak claim, there is the much stronger claim, that one sometimes can find in Quine's early works, that objectual quantification into non-extensional (so called "opaque") constructions simply does not make sense.[1] The argument for this claim is based on the idea that occurrences of variables inside of opaque constructions do not have purely referential occurrences, i.e., they do not serve simply to refer to their objects, and cannot therefore be bound by quantifiers outside of the opaque construction. Thus quantifying into contexts governed by non-extensional operators would be like trying to quantify into quotations. This claim is hardly credible in the face of the multitude of quantified intensional logics that have been developed since it was first made. I take it to be refuted by the analysis and criticism of Quine's argument by, among others, David Kaplan (1969, 1986) and Kit Fine (1986, 1991). Then, there is Quine's claim that quantified modal logic is committed to *Aristotelian essentialism*, i.e., the view that it makes sense to say of an object,

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[1] Cf. Quine (1943, 1953a, b).

quite independently of how it is described, that it has certain of its traits necessarily, and others only contingently. Aristotelian essentialism, however, comes in stronger and weaker forms. Kripke's "metaphysical necessity" of *Naming and Necessity* represents a strong form of essentialism, while there are weaker forms according to which only logical properties that are shared by all individuals are essential. A quantified modal logic needs only be committed to this weak relatively benign form of essentialism.

Here I shall only consider the specific criticism that Quine directed in 1947 toward quantification into contexts of logical or analytical necessity. In his paper 'The problem of interpreting modal logic' from 1947, Quine formulates what one might call *Quine's challenge* to the advocates of quantified modal logic:

> There are logicians, myself among them, to whom the ideas of modal logic (e. g. Lewis's) are not intuitively clear until explained in non-modal terms. But so long as modal logic stops short of quantification theory, it is possible ... to provide somewhat the type of explanation required. When modal logic is extended (as by Miss Barcan) to include quantification theory, on the other hand, serious obstacles to interpretation are encountered – particularly if one cares to avoid a curiously idealistic ontology which repudiates material objects.

What Quine demands of the modal logicians is nothing less than an explanation of the notions of quantified modal logic in non-modal terms. Such an explanation should satisfy the following requirements:

(i) It should be expressed in an extensional language. Hence, it cannot use any non-extensional constructions.

(ii) The explanation should be allowed to use concepts from the 'theory of meaning' like analyticity and synonymy applied to expressions of the metalanguage. Quine is, of course, quite skeptical about the intelligibility of these notions as well. But he considers it to be progress of a kind, if modal notions could be explained in these terms.

(iii) The explanation should make sense of sentences like:

$$\exists x[x \text{ is red} \wedge \Diamond(x \text{ is round})],$$

in which a quantifier outside a modal operator binds a variable within the scope of the operator and the quantifier ranges over ordinary physical objects (in distinction from Frege's "Sinne" or Carnap's "individual concepts"). In other words, the explanation should make sense of 'quantifying in' in modal contexts.

Quine (1947) – like Carnap before him – starts out from a metalinguistic interpretation of the necessity operator $\Box$ in terms of the predicate '... is analytically true'. Disregarding possible complications in connection with the

interpretation of iterated modalities, we have for sentences φ of the object language:

'□φ' is true iff φ is analytically true.

Now Quine argues for the thesis that it is impossible to combine analytical necessity with a standard theory of quantification (over physical objects). The argument (a variation of "the Morning Star Paradox") goes as follows:

The paradox arises from the following premises:

(1) □(Hesperus = Hesperus)
(2) Phosphorus = Hesperus
(3) ¬□(Phosphorus = Hesperus),

where 'Phosphorus' and 'Hesperus' are two proper names (individual constants) and □ is to be read 'It is analytically necessary that'. We assume that 'Phosphorus' is used by the language community as a name for a certain bright heavenly object visible in the morning and that 'Hesperus' is used for some bright heavenly object visible in the evening. Unbeknown to the community, however, these objects are one and the same, namely, the planet Venus.

'Hesperus = Hesperus' being an instance of the Law of Identity is clearly an analytic truth. It follows that the premise (1) is true. (2) is true, as a matter of fact. 'Phosphorus = Hesperus' is obviously not an analytic truth, 'Phosphorus' and 'Hesperus' being two different names with quite distinct uses. So, (3) is true.

From (1), (2), (3) and the Law of Identity, we infer by sentential logic:

(4) Phosphorus = Hesperus ∧ ¬□(Phosphorus = Hesperus),
(5) Hesperus = Hesperus ∧ □(Hesperus = Hesperus).

Applying:

(EG) φ(t/x) → ∃xφ, *(Existential Generalization)*

to (4) and (5), we get:

(6) ∃x(x = Hesperus ∧ ¬□(x = Hesperus)),
(7) ∃x(x = Hesperus ∧ □(x = Hesperus)).

As Quine (1947) points out, however, (6) and (7) seem to be incompatible with interpreting ∀x and ∃x as objectual quantifiers meaning "for all objects x (in the domain D)" and "for at least one object x (in D)" and letting the identity sign stand for genuine identity between objects (in D). Because, under this interpretation, (6) and (7) have the readings:

(6') There is an object x (in the actual domain D) which is identical with Hesperus and which is not necessarily identical with Hesperus.

(7') There is an object x (in the actual domain D) which is identical with Hesperus and which is necessarily identical with Hesperus.

meaning that one and the same object, Hesperus, both is and is not necessarily identical with Hesperus, which seems absurd.

The following are classical proposals for solving Quine's interpretational challenge:

(i) *Russell-Smullyan*. According to this proposal, all singular terms except variables are treated as *Russellian terms*, i.e., as "abbreviations" of definite descriptions that are eliminated from the language by means of contextual definition à la Russell. If we let 'Hesperus' and 'Phosphorus' be Russellian terms having minimal scope everywhere – which clearly corresponds to the intended reading – then the inference will not go through (i.e., once the Russell terms have been contextually eliminated): the (EG)-steps above will not correspond to valid steps in primitive notation. With this treatment of singular terms, the paradox is avoided. One has the feeling, however, that the problem has been circumvented rather than solved.

(ii) *Carnap* (at least the way Quine reads him): The individual variables are not taken to range over physical objects, but instead over individual concepts. According to this reading, the names 'Phosphorus' and 'Hesperus' stand for different but coextensive individual concepts. The identity sign is interpreted not as a genuine identity between physical objects but as coextensionality between individual concepts. That is, an identity statement 'u = v' is true if and only if the terms 'u' and 'v' stand for coextensive individual concepts. According to this interpretation, (6) and (7) mean:

(6'') There is an individual concept x which actually coincides with the individual concept Hesperus but does not do so by analytical necessity.

(7'') There is an individual concept x which not only happens to coincide with the individual concept Hesperus but does so by analytic necessity.

No contradiction ensues from these two statements. The price for this interpretation, however, seems to be as Quine expresses it: "a curiously idealistic ontology which repudiates material objects".[2]

4. Semantics for quantified modal logic in 1957: Hintikka and Kanger

1957 was a pivotal year in the history of modal logic. In that year Stig Kanger published his dissertation *Provability in Logic* and a number of other papers where he outlined a new model-theoretic semantics for quantified modal logic. In the same year, Jaakko Hintikka published two papers on the semantics of quantified modal logic: 'Modality as referential multiplicity' and 'Quantifiers in deontic logic' (Hintikka 1957a, b). There are some striking parallels between these works by Hintikka and Kanger, but there are also important differences.

Hintikka and Kanger had both made important and closely similar work in non-modal predicate logic. Using so-called model sets (nowadays often called "Hintikka sets") for predicate logic, Hintikka (1955) had developed a new complete and effective proof procedure for predicate logic.

Let L be a language of predicate logic with identity and let U be a non-empty set of individual constants that do not belong to L. A *model set* (over U) is a set m of sentences of the expanded language L_U satisfying the following conditions:[3]

(C.¬) if $\neg\varphi \in$ m, then $\varphi \notin$ m,
(C.¬¬) if $\neg\neg\varphi \in$ m, then $\varphi \in$ m,
(C.∧) if $\varphi \wedge \psi \in$ m, then $\varphi \in$ m and $\psi \in$ m,
(C.¬∧) if $\neg(\varphi \wedge \psi) \in$ m, then $\neg\varphi \in$ m or $\neg\psi \in$ m,
(C.∀) if $\forall x\varphi \in$ m, then for every constant a in U, $\varphi(a/x) \in$ m,
(C.¬∀) if $\neg\forall x\varphi \in$ m, then for some constant a in U, $\neg\varphi(a/x) \in$ m,
(C.=) for no individual constant a in L_U, $\neg(a = a) \in$ m,
(C.Ind) if $\varphi(a/x) \in$ m, where φ is atomic, and $a = b \in$ m, then $\varphi(b/x) \in$ m.

A set Γ of sentences of a first-order language L is satisfiable iff there is a Tarski-style model satisfying the sentences of Γ. Or equivalently, Γ is satisfiable

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[2] Notice, however, that there is an alternative way of understanding Carnap's quantifiers that may circumvent Quine's objection (cf. section 2, above).

[3] Here we have assumed that ¬, ∧ and ∀ are primitive and that ∨, → and ∃ are introduced as abbreviations in the usual way. For other choices of primitive logical constants, the definition of a model set has to be adjusted accordingly.

iff Γ can be embedded in a maximal model set (over some non-empty set U of new constants).

Hintikka showed, what nowadays goes under the name *Hintikka's lemma*, namely, that a set Γ of sentences is satisfiable (true in some Tarski-style model) iff it can be imbedded in a model set over some non-empty set U of (new) individual constants. Furthermore, he provided an effective proof procedure for classical predicate logic. The proof procedure is by *reductio*: In order to prove that a sentence φ is valid, one attempts, using the closure conditions on model sets, to embed ¬φ in a model set m. If ¬φ is not satisfiable, then every branch in the search tree for a model set containing ¬φ will terminate after finitely many steps with an explicitly inconsistent set. Thus, if ¬φ is not satisfiable, then the effective search of a model set for ¬φ will end with a finite tree that can be viewed as a proof of the fact that no counter-model of φ exists. Such a tree is of course a proof of the fact that φ is valid, i.e., true in all models. In other words, Hintikka's (1955) method yields an effective proof procedure for predicate logic that is complete in the sense of producing a proof of any valid formula. The method is very similar to the nowadays more familiar semantic tableaux method of Beth (1955).[4]

Hintikka (1955, p. 47) points out that there is a close connection between his proof procedure and proofs in Gentzen's sequent calculus. The systematic search for a counterexample of a formula φ corresponds to the backward application of the rules of Gentzen's cut-free calculus for predicate logic. As a matter of fact, Kanger in *Provability in Logic* (1957a) provided an elegant effective proof procedure for classical predicate logic based on the sequent calculus that is equivalent to Hintikka's.

Hintikka's formal semantics for modal logic. When studying classical predicate logic, Hintikka and Kanger used strikingly similar techniques and obtained similar results. However, their approaches to modal logic were different. Kanger started out from the work of Tarski and set himself the task of extending the method of Tarski-style truth-definitions to predicate languages with modal operators. Hintikka, on the other hand, generalized his method of model sets to the case of modal logic. In doing so he invented the notion of a *model system*. Roughly speaking, a model system consists of a set Ω of model sets and a binary relation R defined between the members of Ω. Different versions

[4] Hintikka's proof procedure does not yield a decision method for predicate logic, since it does not give us an effective method of finding a model set for a satisfiable formula. If φ is in fact satisfiable, the attempted search of a counterexample of φ may go on forever without terminating.

of Hintikka's semantics impose different conditions on model sets, but in order simplify the exposition, we can say that a model system is an ordered pair S = <Ω, R>, such that:

(a) Ω is a non-empty set of model sets for L,
(b) R is a binary relation between the members of Ω (the accessibility relation),
(c) for all m ∈ Ω, if □φ ∈ m, then for all n ∈ Ω such that mRn, φ ∈ n,
(d) for all m ∈ Ω, if ¬□φ ∈ m, then ¬φ ∈ n, for some n ∈ Ω such that mRn.

Hintikka thought of the members of Ω as partial descriptions of possible worlds. A set Γ of sentences is *satisfiable* (in the sense of Hintikka) iff there exists a model system M = <Ω, R> and a model set m ∈ Ω such that Γ ⊆ m. A sentence φ is valid iff the set {¬φ} is not satisfiable.

Hintikka (1957b) sketched a tableaux-style method of proving completeness theorems in modal logic. The idea is a generalization of his proof procedure for first order logic. In order to prove that a formula φ is logically true, one attempts to show that there is no model system satisfying the formula ¬φ. One starts out from the formula ¬φ and tries to construct a model system S = <Ω, R> satisfying it, by building better and better approximations to such a system. Whenever one of the conditions defining a model system is violated by an approximation S' = <Ω', R'>, one tries to remedy this by adding a new formula to one of the members of Ω' or by adding a new alternative set to one of the members of Ω'. If, however, φ is valid, every branch in the search tree for a model system satisfying ¬φ will end in failure. The failed attempt of constructing a model system satisfying ¬φ can then be thought of as a proof that φ is valid. Thus, Hintikka was the first to outline how the tableaux method for proving completeness could be extended to modal logic. Hintikka (1961) states (without formal proofs) that the systems T, B, S4, S5 for sentential logic are sound and complete with respect to the Hintikka-style semantics where R is assumed to be reflexive, symmetric, reflexive and transitive and an equivalence relation, respectively. Rigorous completeness proofs using the tableaux method were published by Kripke, (1959a), for the case of quantified S5, and for numerous systems of propositional modal logic in (Kripke 1963b and 1965).[5]

[5] In (1959b), Kripke announces a great number of completeness results in modal propositional logic. He also notes "For systems based on S4, S5, and M, similar work has been done independently and at an earlier date by K. J. J. Hintikka".

An important difference between Hintikka's semantics for modal logic, on the one hand, and the ones developed by Carnap, Kanger and Montague (1960), on the other, is that Hintikka allows the space of possibilities Ω to vary from one system to another. The only requirement is that Ω is a non-empty set satisfying the constraints (b), (c) and (d) above. In the formal semantics of Carnap, Kanger and Montague, on the other hand, the space of possibilities is fixed once and for all to be the set of all state descriptions (Carnap), the class of all systems (or alternatively, domains) (Kanger), or all first-order models over a given domain (Montague). One could say that Carnap, Kanger and Montague only allow interpretations of modalities that are in a sense *standard* and disallow *non-standard interpretations*. Thus, the relationship between Hintikka's semantics (and the one later developed by Kripke) and the ones developed by Carnap, Kanger and Montague is analogous to that between *standard* and *non-standard* semantics for higher-order predicate logic. This distinction between the various approaches has been emphasized by Cocchiarella (1975) and Hintikka (1981). Allowing non-standard interpretations for modal logics, of course, facilitated the proofs of completeness results, since the logics for logical or analytical necessity corresponding to the standard semantics are in general not recursively enumerable.

Kanger's Tarski-style semantics for quantified modal logic. Kanger's ambition was to provide a language of quantified modal logic with a model-theoretic semantics à la Tarski.[6]

A Tarski-style interpretation for a first-order predicate language L consists of a non-empty domain D and an assignment of appropriate extensions in D to every non-logical symbol and variable of L. Kanger's basic idea was to relativize the notion of extension to various possible domains. In other words, he thought of an interpretation for a given language L as a *function* that *simultaneously* assigns extensions to the non-logical symbols and variables of L for *every* possible domain. Such a function Kanger called a (*primary*) *valuation*. Formally, a valuation for a language L of quantified modal logic is a function v which for *every* non-empty domain D assigns an appropriate extension in D to every individual constant, individual variable, and predicate constant in L. Kanger also introduced the notion of a *system* S = <D, v> consisting of a designated domain D and a valuation v. Notice that v does not only assign extensions to symbols relative to the designated domain D, but relative to *all* domains simultaneously.

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[6] Cf. Kanger (1957a, b, c, d, e). See also Lindström (1998) for a more extensive discussion of Kanger's approach to quantified modal logic.

Kanger then defined the notion of a formula φ being *true in a system* S = <D, v> (in symbols, S ⊨ φ) in the following way:

(1) S ⊨ $(t_1 = t_2)$ iff $v(D, t_1) = v(D, t_2)$,
(2) S ⊨ $P(t_1,..., t_n)$ iff $<v(D, t_1),...,v(D, t_n)> \in v(D, P)$,
(3) S ⊥,
(4) S ⊨ $(\varphi \rightarrow \psi)$ iff S φ or S ⊨ ψ,
(5) <D, v> ⊨ ∀xφ iff <D, v'> ⊨ φ, for each v' such that $v' =_x v$,
(6) for every operator □, S ⊨ □φ iff ∀S', if $SR_{\Box}S'$, then S' ⊨ φ.

Explanation: v' is like v except possibly at x (also written, $v' =_x v$) if and only if, for each domain D and each variable y other than x, v'(D, y) = v(D, y). In the above definition, $R_{\Box}$ is a binary relation between systems that is associated with the modal operator □. $R_{\Box}$ is the *accessibility relation* associated with the operator □.

Among the modal operators in L, Kanger introduced two designated ones **N** ("analytic necessity") and **L** ("logical necessity") with the following semantic clauses:

<D, v> ⊨ **N**φ iff for every domain D', <D', v> ⊨ φ,
<D, v> ⊨ **L**φ iff for every system S, S ⊨ φ.

A formula φ is *true* in a system <D, v> iff <D, v> ⊨ φ. A formula φ is said to be *valid (logically true)* if it is true in every system <D, v>. A formula φ is a *logical consequence* of a set Γ of formulas (in symbols, Γ ⊨ φ) if φ is true in every system in which all the formulas in Γ are true.

In order to get a clearer understanding of Kanger's treatment of quantification, I shall speak of selection functions that pick out from each domain an element of that domain as *individual concepts.* To be more precise, an individual concept, in this sense, is a function f, with the collection of all domains as its range, such that for every domain D, f(D) ∈ D. We can think of a system S = <D, v> as assigning to each individual constant c the individual concept {<D, v(D, c)>: D is a domain} and to each variable x the individual concept {<D, v(D, x)>: D is a domain}. The formula $P(t_1,..., t_n)$ is true in S = <D, v> if and only if the individual concepts designated by $t_1,..., t_n$ pick out objects in the domain D that stand in the relation v(D, P) to each other. The identity symbol designates the relation of *coincidence* between individual concepts (at the "actual" domain D). That is. $t_1 = t_2$ is true in a system S = <D, v> if and only if the individual concepts designated by t_1 and t_2, respectively, pick out one and the same object in the domain D of S.

The universal quantifier ∀x can now be thought of as an objectual quantifier that ranges not over the "individuals" in the "actual" domain D, but over the (constant) domain of all individual concepts. That is, ∀xφ is true in a system <D, v> if and only if φ is true in every system which is exactly like <D, v> except, possibly, for the individual concept that it assigns to the variable x. Note that, interpreted in this way, the range of the quantifiers ∀x and ∃x is independent not only of the domain D, but also of the system S: The range of the quantifiers ∀x and ∃x is fixed, once and for all, to be the collection of absolutely all individual concepts. While formulas of the form $t_1 = t_2$ express coincidence, identity between individual concepts is expressed by formulas of the form $\mathbf{N}(t_1 = t_2)$.

Kanger's solution to Quine's paradox of identity is essentially the same as Carnap's. Quine's objection to Kanger would therefore be the same as to Carnap: Kanger's quantifiers do not range over ordinary individuals but over individual concepts instead. The paradox is solved at the prize of an ontology of arbitrary individual concepts (selection functions). As in the case of Carnap, the identity symbol is not interpreted as genuine identity but as coincidence. Kanger's response to these (imagined) objections could be along the following lines: Firstly, the individual concepts are not mysterious intensional entities, but perfectly respectable functions. Secondly, it is misleading to say that the quantifiers range over individual concepts. If one should speak of *the* domain of quantification, that should be the actual domain. But rather than speaking of *one* domain for the quantifiers, it is more appropriate to speak of a *multiplicity* of such domains. The quantifiers do not range over a single domain but are instead associated with a multiplicity of such domains.[7]

One could still object to Kanger's treatment of quantification in modal contexts that it does not provide any means of *identifying* individuals from one domain to another. Hence there is no way of saying in Kanger's modal language that *one and the same* individual has a property P and possibly could have lacked P. That is, neither Carnap's nor Kanger's semantics can account modality *de re*. This is the same objection that we leveled against Carnap's semantics.

5. Hintikka's response to Quine's challenge: referential multiplicity

Quine's interpretational challenge seemed to place the advocates of quantified modal logic in a dilemma. They would either have to accept standard quantification theory (with the usual laws of universal instantiation, existential generaliza-

[7] Cf. Hintikka's notion of referential multiplicity below.

tion and indiscernibility of identicals) and reject quantified modal logic, or accept a quantified modal logic, where the quantifiers were interpreted in a non-standard way à la Carnap as ranging over intensional entities (individual concepts), rather than over robust extensional entities as Quine would demand.

Hintikka (1957a, b), however, rejected the terms in which Quine's interpretational challenge was stated. First of all he broadened the discussion by not only considering the logical modalities and Quine's metalinguistic interpretation of these, but also epistemic modalities ('It is known that φ') and deontic ones ('It is obligatory that φ'). He then introduced the idea of *referential multiplicity*. In answer to Quine's question whether a certain occurrence of a singular term in a modal context is purely referential, and thus open to substitution and existential generalization or non-referential, in which case substitution and existential generalization would fail according to Quine, Hintikka (1957a) pointed to a third possibility. According to the classical Fregean approach singular terms would in non-extensional contexts not have their standard reference but instead refer to intensional entities, their ordinary senses. Hintikka saw no need to postulate special intensional entities for the singular terms to refer to in non-extensional contexts. The failure of substitutivity was instead explained by the referential multiplicity of the singular terms and by the fact that in intensional contexts the reference of the terms in various alternative courses of events ("possible worlds") is considered simultaneously.

Informally Hintikka (1957a) expressed the basic ideas behind the possible worlds interpretation of modal logic in the following words:

> ...we often find it extremely useful to try to chart the different courses the events may take even if we don't know which one of the different charts we are ultimately going to make use of. ... This analogy is worth elaborating. The concern of a general staff is not limited to what there will actually be. Its business is not just to predict the course of a planned campaign, but rather to be prepared for all the contingencies that may crop up during it. ... Most of the maps prepared by the general staff represent situations that will never take place. ... There are for the most parts some actual units for which the marks on the map stand, and the mutual positions of the units are such that the situation could conceivably arise. ... But the location of the units on the maps may be different from the locations the units have or ever will have. Some of the marks may stand for units which have not yet been formed; other maps may be prepared for situations in which some of the existing units have been destroyed. All these features have their analogues in modal logic.

In this example Hintikka informally speaks of the same units as occurring in different situations ("cross-world identification of individuals") and of individuals coming into existence or disappearing as one goes from one situation to another ("varying domains").

Hintikka goes on to explain the bearing of the above example on referential opacity.

> We may perhaps say that when we are doing modal logic, we are doing more than one thing at one and the same time. We use certain symbols – constants and variables – to refer to the actually existing objects of our domain of discourse. But we are also using them to refer to the elements of certain other states of affairs that need not be realized. Or, which amounts to the same, we are employing these symbols to build up 'maps' or models for the purpose of sketching certain situations that will perhaps never take place. If we could confine our attention to one of these possible states of affairs at a time, the occurrences of our symbols would be purely referential. The interconnections between the different models interfere with this. But since the symbols are purely referential within each particular model, the deviation from pure referentiality is not strong enough to destroy the possibility of employing quantifiers with pretty much the same rules as in the ordinary quantification theory. If I had to characterize the situation briefly, I should say that the occurrences of our terms in modal contexts are not usually *purely referential*, but rather that they are *multiply referential*.

This idea of referential multiplicity is perhaps the basic intuitive idea behind the possible worlds interpretation of modal notions and of indexical semantics in general. It seems that Hintikka here gives one of the earliest, or perhaps the earliest, clear expression of the idea.

Hintikka's semantics for quantified modal logic is informally interpreted in such a way that the quantifiers range over genuine individuals. Thus, Hintikka has a notion of cross-world identification: one and the same individual may occur in different worlds. However, the semantics allows individuals to *split* from one world to another, i.e., the individuals a and b may be identical in one world w_0 but they may fail to be identical in some alternative world to w_0. Thus, the principle:

$$\forall x \forall y(x = y \rightarrow \Box(x = y)),$$

is not valid in Hintikka's semantics. As a consequence, the unrestricted principle of indiscernibility of identicals does not hold in modal contexts according to Hintikka (c.f., Hintikka (1961) and later writings).

Hintikka's solution to Quine's paradox of identity. There are two cases to consider:

(1) One or the other of the singular terms under consideration ('Hesperus' or 'Phosphorus') is not a "rigid designator", that is it does not designate the same individual in every possible world (or "scenario") under consideration. Then, existential generalization fails and Quine's paradoxical argument does not go through.

(2) Each of the two names picks out "the same" individual in every world under consideration. However, some scenario w under consideration is such that the individual Hesperus in w is distinct from the individual Phosphorus in w. In this case, Quine's argument goes through, but Hintikka has to argue that the conclusion:

(6) $\exists x(x = \text{Hesperus} \wedge \neg\Box(x = \text{Hesperus})$

(7) $\exists x(x = \text{Hesperus} \wedge \Box(x = \text{Hesperus}))$,

contrary to appearance, is not absurd, since an individual can "split" when we go from one possible scenario to one of its alternatives. Consider for example:

> Superman and Clark Kent are in fact identical, but Lois Lane doesn't believe that they are identical.

Hintikka may explain the apparent truth (according to the story) of this sentence by the fact that some scenarios (possible worlds) in which Superman and Clark Kent are different individuals are among Lois Lane's doxastic alternatives in the actual world (where they are identical).

6. Montague's early semantics for quantified modal logic

A semantic approach to first-order modal predicate logic that has a certain resemblance to Kanger's was developed by Montague (1960).[8] Like Kanger, Montague starts out from the standard model-theoretic semantics for non-modal first-order languages and extends it to languages with modal operators. He defines an *interpretation* for an ordinary first-order predicate language L to be a triple S = <D, I, g>, where (i) D is a non-empty set (the *domain*); (ii) I is a function that assigns appropriate denotations in D to the non-logical constants (predicate symbols and individual constants) of L; and (iii) a function g that assigns values in D to the individual variables of L. For each non-logical constant or variable X, let S(X) be the *semantic value* (i.e., *denotation* for non-logical constants and *value* for variables) of X in the interpretation S. Then the notion of *truth* relative S is defined as follows:

(1) $S \vDash P(t_1,..., t_n)$ iff $<S(t_1),..., S(t_n)> \in S(P)$,

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[8] Montague (1960) writes: "The present paper was delivered before the Annual Spring Conference in Philosophy at the University of California, Los Angeles, in May, 1955. It contains no results of any great technical interest; I therefore did not initially plan to publish it. But some closely analogous, though not identical, ideas have recently been announced by Kanger [(1957b)], [(1957c)] and by Kripke in [(1959a)]. In view of this fact, together with the possibility of stimulating further research, it now seems not wholly inappropriate to publish my early contribution."

(2) $S \models (t_1 = t_2)$ iff $S(t_1) = S(t_2)$,
(3) $S \models \neg\varphi$ iff $S \not\models \varphi$,
(4) $S \models (\varphi \rightarrow \psi)$ iff $S \not\models \varphi$ or $S \models \psi$,
(5) $S \models \forall x\varphi$ iff for every object $a \in D$, $S(a/x) \models \varphi$.

Here, S(a/x) is the interpretation which is exactly like S, except for assigning the object a to the variable x as its value.

Montague now asks the same question as Kanger: How can this definition of the truth-relation $\models$ be generalized to first-order languages with modal operators? As we recall, Kanger solved the problem by modifying the notion of an interpretation: a Kanger-type interpretation (what he called 'a system') assigns denotations to the non-logical constants and values to the variables not only for one single domain (the 'actual' one) but for all domains in one fell swoop. Montague's approach is simpler than Kanger's: he keeps the notion of an interpretation S of first-order logic intact, and just adds semantic evaluation clauses for the modal operators. As in the Kanger semantics, each modal operator $\Box$ is associated with an accessibility relation $R_\Box$. Now, however accessibility relations are relations between interpretations $S = \langle D, I, g\rangle$ of the underlying non-modal first-order language. The semantic clause corresponding to the operator $\Box$, with associated accessibility relation $R_\Box$, is:

(6) $S \models \Box\varphi$ iff for every interpretation S' such that $SR_\Box S'$, $S' \models \varphi$.

Montague associates with the operator **L** of *logical necessity* the accessibility relation $R_{\mathbf{L}}$ defined by:

$\langle D, I, g\rangle R_{\mathbf{L}} \langle D', I', g'\rangle$ iff $D = D'$ and $g = g'$.

Thus, his semantic clause for **L** becomes:

(1) $\langle D, I, g\rangle \models \mathbf{L}\varphi$ iff for every I' defined over D, $\langle D, I', g\rangle \models \varphi$.

This semantic clause should be compared with Kanger's stricter condition:

(2) $S \models \mathbf{L}\varphi$ iff for every system S', $S' \models \varphi$.

The difference between (1) and (2) corresponds to a difference between two conceptions of logical truth: Tarski's (1936b) conception and the modern model-theoretic one. According to Tarski (1936b), truth and logical truth are properties that primarily apply to *interpreted* formal languages. An *interpreted* first-order language L comes with a *domain of discourse* D and an interpretation function I that gives the denotations in D of the non-logical constants of L. Hence, the (absolute) notion of *truth* is well-defined for such a language. A formula φ of L is *true relative to an assignment* g of values in D to the variables of

L if <D, I, g> ⊨ φ. φ is *true*, simpliciter, if it is true relative to every assignment, that is if it is true in the intended model <D, I>. Now, according to Tarski (1936b), a sentence (closed formula) φ of L is *logically true* if it is true and its truth is invariant with respect to all reinterpretations of its non-logical constants relative to the given domain of discourse D. That is, φ is logically true if and only if, for every interpretation <D, I', g'> with the given domain D, <D, I', g'> ⊨ φ. If φ is a sentence (closed formula), then its truth-value is independent of the assignment of values to the variables. Hence, Tarski's (1936b) definition yields for sentences φ of L:

φ is logically true iff for every I' defined over D, <D, I', g> ⊨ φ,

where D is the domain of discourse of L and g is any assignment of values in D to the variables. Hence, for closed formulas of the underlying first-order language, Montague's truth-clause (1) above coincides with Tarski's (1936b) definition of logical truth. It also accords well with Carnap's definition of logical truth as truth in all state descriptions, provided that the language contains names for all the objects in a fixed domain of discourse.

In conclusion, Montague's and Kanger's respective truth clauses for **L** yield, for modality-free sentences φ,

Lφ is true iff φ is logically true in the sense of Tarski (1936b),
(Montague)

Lφ is true iff φ is logically true in the model-theoretic sense.
(Kanger)

Montague's solution to Quine's paradox of identity. According to Montague's interpretation, **L**φ is logically equivalent with a *formula of second-order predicate logic* ()φ, where () stands for a string of universal quantifiers that bind all non-logical symbols in φ. In other words, Montague's semantics induces a translation from first-order modal logic to extensional second-order predicate logic. According to Montague's semantics from 1960, the quantifier ∀x is interpreted as a genuine quantifier over individuals. Free variables are "directly referential", i.e., a free variable is interpreted uniformly inside a formula as standing for one and the same individual regardless of where in the formula it occurs. Individual constants, on the other hand, are reinterpreted freely from one interpretation to another.

Montague's semantics validates the following principles without restrictions:

(LI) $\forall x(x = x)$, *(Law of Identity)*

(I=) $\forall x \forall y(x = y \rightarrow (\varphi(x/z) \rightarrow \varphi(y/z)))$.
(Indiscernibility of Identicals)

In addition, we have: $\forall x\mathbf{L}(x = x)$. Therefore, the following principle is valid:

(NI) $\forall x\forall y(x = y \rightarrow \mathbf{L}(x = y))$. (*Necessity of Identity)*

But the following is not valid:

Phosphorus = Hesperus $\rightarrow$ **L**(Phosphorus = Hesperus).

It follows that the principle:

(UI) $\forall x\varphi \rightarrow \varphi(t/x)$, *(Universal Instantiation)*

is not valid. Neither is:

(EG) $\varphi(t/x) \rightarrow \exists x\varphi$. *(Existential Generalization)*

It follows that Quine's paradoxical argument cannot be carried through within Montague's logic.

As far as I can see, Montague's semantical interpretation satisfies all the requirements imposed by Quine (1947) on an interpretation of quantified modal logic for the logical modalities.

7. Kripke's 1959 semantics for quantified modal logic

Here I present Kripke's (1959a) semantics with one modification. Kripke's paper from 1959 did not contain a semantics for individual constants. Here, I have taken some liberty of interpretation and extended Kripke's formal language with individual constants and extended his semantic treatment of free individual variables also to individual constants. With this change, Kripke's (1959a) formal semantics takes the following form. Let D be a non-empty domain. We define a *valuation* over D to be a function V which to every individual term (variable or individual constant) t assigns a value V(t) in D and to every n-ary predicate symbol P assigns a value $V(P) \subseteq D^n$.

A *model* over D is an ordered couple $M = <\mathbf{K}, V_0>$, such that (i) **K** is a set of valuations over D; (ii) V_0 is a member of **K**; and (iii) all valuations in **K** agree in their assignments to individual terms.

We say that a model $M = <\mathbf{K}, V_0>$ is a *standard model* over D if **K** contains all possible valuations that agree with V_0 in their assignments of values to individual terms.

Let $M = <\mathbf{K}, V_0>$ be a model over some non-empty domain D and let $V \in \mathbf{K}$. Truth in V (relative to M) is then defined recursively as follows:

(1) $V \vDash P(t_1,..., t_n)$ iff $<V(t_1),..., V(t_n)> \in V(P)$,
(2) $V \vDash (t_1 = t_2)$ iff $V(t_1) = V(t_2)$,
(3) $V \vDash \neg\varphi$ iff $V \varphi$,

(4) $V \vDash (\varphi \rightarrow \psi)$ iff $V \varphi$ or $V \vDash \psi$,
(5) $V \vDash \forall x\varphi$ iff for every object $a \in D$, $V(a/x) \vDash \varphi$,
(6) $V \vDash \Box\varphi$ iff for every valuation V' in **K**, $V' \vDash \varphi$.

We say that φ is *true* in M relative to V iff $V \vDash \varphi$. φ is *true* in M iff $V_0 \vDash \varphi$. φ is valid in the domain D iff φ is true in all models over D. φ is *logically true* (alternatively; *universally valid*) iff φ is valid in every domain (i.e., iff φ is true in every model). Kripke shows that the system $S5^{*=}$ for quantified modal logic is sound and complete for the given semantics.

Quine's paradox of identity: Kripke's semantics validates all of the following principles:

(i) classical sentential logic,

together with the principles:

(ii) $\forall x\varphi(x) \rightarrow \varphi(t)$, where c individual term (i.e., e variable or an individual constant).
(iii) $\forall x\forall y(x = y \rightarrow (\varphi(x/z) \rightarrow \varphi(y/z)))$.

Moreover the following is true in every model:

(1) $\Box$(Phosphorus = Phosphorus).

Thus, we can infer:

(2) Phosphorus = Hesperus $\rightarrow \Box$(Phosphorus = Hesperus).

Provided it is true that Phosphorus = Hesperus, it follows according our modified Kripke semantics that $\Box$(Phosphorus = Hesperus) is also true. This is of course contrary to one of the premises of Quine's paradoxical argument. But on the other hand, is it not a paradoxical result in its own right?

The answer depends on what the intended interpretation of Kripke's (1959a) necessity operator is. If it is some kind of logical or analytical necessity, then the result (2) is hard to swallow. It is difficult, but perhaps not impossible, to argue that all true identities between proper names are logically or analytically true. If, on the other hand, the intended interpretation of $\Box$ is "metaphysical necessity" à la *Naming and Necessity*, then (2) is perhaps what one would expect to hold.

One reason for arguing that Kripke's notion of necessity in 1959 is not logical necessity is Kripke's use of non-standard models. Instead of working with all models or valuations over D, like Montague, or with all possible systems as Kanger, Kripke is considering an arbitrary subset of all possible valuations. This feature suggests that Kripke's intended interpretation of the necessity operator is not strict logical necessity, but perhaps some kind of

metaphysical necessity instead. This conclusion is however, not unavoidable: Kripke's *intended interpretations* in (1959a) could still have been some or all of the *standard models*. Kripke's reason for allowing non-standard models, in addition to standard ones, when defining validity, could have been logical rather then philosophical. If Kripke, like Kanger and Montague, had chosen to work only with standard models, the set of valid sentences would not have been recursively enumerable and there would be no completeness theorem to be proved. Kripke's intended model could, for instance, be a standard model over some infinite set D, where the individual constants, unlike the variables, were treated as non-rigid designators. Interpreted in this way, Kripke's 1959 approach would be very close to Montague's of 1960. The only essential difference would be Kripke's use of non-standard models in addition to the standard ones for the purpose of defining logical validity.[9]

8. Conclusion

In 1959 Kripke wrote:

> It is noteworthy that the theorems of this paper can be formalized in a metalanguage (such as Zermelo set theory) which is "extensional," both in the sense of possessing set-theoretic axioms of extensionality *and* in the sense of postulating no sentential connectives other than the truth-functions. Thus it is seen that at least a certain non-trivial portion of the semantics of modality is available to an extensionalist logician.

Perhaps, Kripke meant that he had refuted Quine's skepticism about quantified modal logic. Had he not after all done for quantified modal logic what Tarski and others had done for non-modal predicate logic: provided it with an extensional set-theoretic semantics? In addition he had axiomatized the logic and proved it complete for the given semantics. What else could one require of the interpretation of a logic?

Quine, however, was not satisfied. In 1972 he writes in a review of Kripke's paper 'Identity and Necessity':

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[9] In (1959a), p. 3, Kripke speaks of **K** as representing the set of all "conceivable" worlds. He writes "...a proposition □B is evaluated as true when and only when B holds in all conceivable worlds". This seems to indicate that Kripke's operator □ of 1959 should not be interpreted as strict logical necessity. It is very likely that the set of valuations representing all "conceivable" worlds is a proper subset of the set of absolutely all valuations. Thus Kripke presumably had philosophical reasons, perhaps in addition to formal ones, for favoring a non-standard semantics to a standard one. Cf., however, Almog (1986), p. 217, who writes about Kripke (1959a): "...Kripke had at the time nothing more than "complete assignments," and the modality he worked with was definitely *logical* possibility".

> The notion of possible world did indeed contribute to the semantics of modal logic, and it behooves us to recognize the nature of its contribution: it led to Kripke's precocious and significant theory of models of modal logic. Models afford consistency proofs; also they have heuristic value; but they do not constitute explication. Models, however clear in themselves, may leave us still at a loss for the primary, intended interpretation.[10]

Here it seems that Quine has overstated his case. Sometimes models do more than provide us with consistency or completeness proofs. Sometimes they may also provide us with the intended interpretation. Montague's (1960) model theory, for instance, gives us more than a collection of models; it also gives us a translation of first-order modal logic into extensional second-order predicate logic. Thereby it provides us with an intuitive interpretation of quantified modal logic that seems to satisfy the demands once imposed by Quine. In this interpretation, the notion of a possible world plays no role.

Whatever was his aim in 1959, in his later work Kripke's project is not to give an explanation of modal concepts in non-modal terms. In the Preface to *Naming and Necessity*, 1980 he writes:

> I do not think of 'possible worlds' as providing a *reductive* analysis in any philosophically significant sense, that is, as uncovering the ultimate nature, from either an epistemological or a metaphysical point of view, of modal operators, propositions, etc., or as 'explicating' them.

Clearly, Kripke's essentialist concept of necessity ("metaphysical necessity") simply cannot be reductively explained in non-modal terms. On the other hand, it seems that Montague in his early semantics *did* give an explanation of a concept of *logical necessity* that satisfies the requirements that Quine once formulated.

In his early work, Kanger provided a type of semantics for quantified modal logic that differs in interesting ways from the Kripke-type semantics that has since become standard.[11] Hintikka emphasized epistemic and deontic modalities, rather than the logical modalities that had dominated the early discussions of Quine's challenge. He also stressed the importance of the notions of referential multiplicity and cross-world identification for the interpretation of quantified modal logic. Hintikka's approach to the problem of "quantifying in" appears to be – at least for the purposes of doxastic and epistemic logic – more natural and flexible than the approaches of Montague and Kripke that do not admit the splitting or merging of individuals from one possible world to another.

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[10] Quine (1972). I found this quote in Ballarin (1999).
[11] Cf. Kripke (1963a).

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