The Reasoner

Volume 15, Number 1
January-February 2021

thereasoner.org
ISSN 1757-0522

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Guest Editorial

Dear Reasoners,

I am delighted to present to you, in this new year issue of *The Reasoner*, an interview with Paul Égré. Paul Égré is a Senior Researcher at the Institut Jean Nicod (CNRS) and a Professor in the Philosophy Department of École Normale Supérieure in Paris. Much of his work deals with the semantics, epistemology, and psychology of vagueness in language and in perception. Some of his more recent contributions concern the varieties of logical consequence and the nature of concepts. I had the chance to meet him during my studies in Paris, and, more recently, to have him as a member of my PhD thesis committee in Turin. The path of an analytic philosopher in continental France, the value of vagueness in communication, and what science can bring to philosophy are only some of the topics we discuss in this interview. I hope you will find our conversation as interesting and stimulating as I did.

Lina Lissia
University of Turin

Features

Interview with Paul Égré

Lina Lissia: First of all, I'm curious about your study path. I know that you went to the École normale Supérieure, a very selective French institution to which only the very best students in France are admitted. What happened before that? When did you decide you would become a philosopher? Was it before your arrival at the ENS or when you were studying there?

Paul Égré: The first time I heard about philosophy was from my elder sister. I was 8. In France philosophy is part of the curriculum in high school; in your final year you get an introduction. My sister was very enthusiastic about her philosophy teacher and classes. Around the dinner table she would talk to us about consciousness and other topics. I remember being impressed by the idea that we have consciousness, which as a child I mainly interpreted in a moral sense, but the issue was also whether there are things that we are not conscious of. I think I got pretty quickly the idea that philosophy is about discussing problems to which you don't have the answer.

In the early 90s my parents moved to Toulouse. There I was very lucky to meet Serge Dejean (1950-2020), a philosopher and teacher who was a wonderful person and mentor. We met at the public library, where I remember doing research on the

sophists, and where Serge was working on the history of deportation around Toulouse. It was a blessing to have a friend like him, he opened a world of books and philosophy to me. Funny enough, Serge was not at all into analytic philosophy. Back then, almost nobody in France knew about analytic philosophy, apart from a handful of people, whom I happened to meet later. Serge respected all classic and modern philosophers, but was particularly enthusiastic about more recent figures, who, you know, are not well looked upon by the analytic tradition. Thanks to him I discovered Heidegger, Barthes, Bergson, Levinas. I admired Bergson's scientific culture: specifically, the combined interests in metaphysics and psychophysics in his early work. I tried to read as much as I could.

After high school, I went to *classe préparatoire* and was admitted at École normale supérieure the next year. Even before my time at ENS I was aware that what I wanted to learn was a combination of mathematical methods and philosophy. In Paris, I quickly learned that there was a logic programme at Sorbonne. I had three great logic professors there in my first year: François Rivenc, Jean Mosconi, and Jacques Dubucs. Jacques Dubucs became my PhD supervisor later. Thanks to Rivenc, in particular, I discovered Frege, Russell, Husserl, and Carnap. I also took classes in undergraduate mathematics at Paris Diderot University. Then I passed the *Agrégation* in philosophy (another examination you take in France if you want to become a philosophy teacher). After that, thanks to advice given by Francis Wolff and François Rivenc, I spent a fantastic and decisive year at Princeton in the philosophy department as a visiting grad student. John Burgess was my advisor, and I took seminars with David Lewis, Bas van Fraassen, Delia Fara, Gideon Rosen... I also attended courses by John Conway and Yakov Sinai in the Maths Department. Upon my return I decided I should learn more logic and did the Master's in logic at Paris 7 University.

LL: How did you get interested in analytic philosophy in the first place? You told me that at some point you realized that you wanted to learn mathematical methods. How come? Your early philosophical education seems to be the typical, continental, French education. So what brought you to analytic philosophy?

PÉ: By the time I was leaving Toulouse to go to Paris I was very interested in the work of Claude Lévi-Strauss, the French anthropologist. Lévi-Strauss was also a philosopher, and in his writings he made very clear that he was interested in a combination of philosophy and empirical methods. He also made very clear that he had a lot of respect for mathematics (witness his collaboration with André Weil): for him, you couldn't do serious anthropology if you ignored mathematics. My role models back then where people who could bring together philosophy, with its very speculative questions, and science, with its exact methods. I felt that there was too much room for doubt in philosophy. I liked the firm grounds mathematics seemed to provide.

LL: There is something really striking about your CV, which is the centrality of vagueness as a research topic. What brought you to this topic in the first place? Did you get interested in vagueness during your PhD?

PÉ: In part, yes. My PhD was about epistemic paradoxes and propositional attitudes; so it was really about looking at knowledge and belief from various perspectives, from the perspective of attributions as well as from the perspective of modeling their content. In 2000, the year I started my PhD, Timothy Williamson published *Knowledge and its Limits*. That book had a huge impact, including in Paris, where Williamson was invited by Pascal Engel. I was particularly intrigued by the chapter on *Anti-luminosity*, where Williamson argues against positive introspection, the idea that knowing implies knowing that one knows. The way he argues against this principle is by building an epistemic Sorites. This led me into thinking about soritical arguments more broadly. The discussion of Williamson's anti-luminosity argument became a chapter of my PhD and kept me busy for several years after that.

LL: My previous question was "what brought you to vagueness?". My next question will be: what kept you there? What is it that you find so fascinating and important about vagueness? When someone skims through your CV one has the impression that no matter the topic, it's always a bit about vagueness; not all the time of course, but still...

PÉ: That's very true, actually! Beside my interest for paradoxes, what kept me there were further foundational questions. In particular, it was the gradual realization that vagueness is such a pervasive phenomenon in language and thought. Each speaker attaches a slightly different meaning to a large number of expressions in natural language, and still we manage to communicate. The fact that there is this capacity for mutual understanding, but on a backdrop of massive inter-individual variability in meaning is what fascinates me the most about vagueness. As you know, Frege has a very objectivist view of meaning; his conception of meaning, which is based on the compositionality principle, seems to rule out vagueness. What I find really interesting and not completely solved yet is the problem of combining Frege's programme for compositionality of language with the observation that linguistic meanings can vary significantly between speakers. This is one important aspect of what "kept me there".

LL: It's funny because I have the impression that your interest for vagueness reflects your early interest for analytic philosophy, as you're still trying to keep together objectivity and subjectivity: as a student you thought that there was too much place for subjectivity and doubt in philosophy, whence the need for mathematical methods. My next question originates from something which is also quite striking about your work, that is, the constant dialogue with two disciplines: psychology and linguistics. On this, I have a broader question and a more narrow one. The (relatively) narrow question is about your method in philosophy: what can psychology and linguistics bring to philosophy? And the broader question (which is perhaps too broad!) is: what is your conception of philosophy? You never get to ask philosophers this question at conferences, so I'll jump at the chance...

PÉ: During and after my PhD I learned a lot of linguistics. That was driven in large part by the fact that back then I was working on knowledge and belief as propositional attitudes, and in a way the history of formal semantics originates in the long-standing problem of giving truth conditions for propositional attitudes. That's how I came to work on questions and their interaction with attitude verbs. Then what gradually led me into psychology was my work on vagueness: it was clear to me that you couldn't understand much about vagueness without looking into how people effectively categorize and discriminate. Of course, there are various ways of doing that: you can turn to mathematical psychology, which can be very philosophical. You can also collect behavioral data and take a more naturalized approach to philosophical questions. When I started my first project on vagueness (in 2007-2008), these were also the years experimental philosophy was becoming big. Many people back then viewed experimental philosophy as a way of sweeping away the traditional methods in philosophy, especially armchair methods. But this has never been my view. To me if there is anything interesting in psychology (and in empirical sciences more generally), it is rather to get more answers out of theoretical models, or to extract more theory out of empirical data.

This leads to me to your second question, about my conception of philosophy. My conception is very simple: it really has to do with concepts and meaning. I see philosophy as the business by which we try to clarify our concepts, by which we try to get as clear conceptions as we can. Because of that, I consider that the role of science is to eliminate vagueness and confusion as much as possible. I believe in the ideal of precision, and this is also why I completely share the Carnapian and neo-Carnapian projects: explicating concepts. The purpose of philosophy is to give the clearest possible formulation of problems which are really difficult to articulate.

LL: What are you working on now and what are your projects for the near future?

PÉ: Unsurprisingly, one of my latest projects still concerns vagueness. The project aims to give an account of the function of vagueness in ordinary language. My goal is to understand the sources of vagueness: why is natural language vague? I think that there are different sources of vagueness. I believe that in part the epistemic theory is right: vagueness partly originates from imperfect discrimination. But I think that the source of vagueness is in part semantic too. In fact, I believe that the opposition between the epistemic view and the semantic view is inadequate: we should move beyond that and recognize that vagueness is a multi-source phenomenon.

Concerning communication, the view that I've been exploring is that linguistic vagueness is not a deficiency; it is a feature we pragmatically recruit to optimize accuracy in communicative exchanges. This may seem counterintuitive, especially in light of my comment about the ideal of precision in science, but here's the idea: a cooperative speaker who is not perfectly informed about the world (maybe because of imperfect discrimination), would make too much error if pressed to communicate with only precise expressions. More than that, she would misrepresent her uncertainty. In joint work with Benjamin Spector, Adèle Mortier and Steven Verheyen, we show that there exist situations in which using precise expressions is suboptimal compared to using vague expressions. This is a view that was partly sketched by some theorists of vagueness, but the precise shape we're giving it is novel.

Another project I'm working on at the moment concerns the philosophy of mind and the philosophy of perception: Jackson Graves, Vincent de Gardelle, Daniel Pressnitzer and I are investigating the phenomenon of perceptual ambiguity. We're wondering under which conditions we become aware of the ambiguity of a stimulus. This is still connected to vagueness, because ambiguity and vagueness are related phenomena.

Finally, I am keeping an active interest in logic, in particular in the many-valued and substructural approaches originating from my collaboration with Pablo Cobreros, David Ripley, and Robert van Rooij on strict-tolerant logic. With Eduardo Barrio, we are currently editing a special issue of the *Journal of Philosophical Logic* on substructural logics, and with Lorenzo Rossi I am coordinating a handbook project on trivalent logics.

LL: This all sounds very interesting, thanks. My last question will be about the situation of analytic philosophy in France. France has the reputation of not being an easy place for analytic philosophers. Are things changing? In recent years, two analytic philosophers, Claudine Tiercelin and François Récanati, have been appointed professors at the Collège de France, arguably the most important academic institution in France. Is this a good sign?

PÉ: I'm very optimistic about the situation of analytic philosophy in France. When I look back at the mid-90's, the status of analytic philosophy in France was very confidential. What I see now, more than twenty years later, is that analytic philosophy is more widely accessible to the students, and there is more demand for it too, with more international hirings (viz. Andrew Arana in Sorbonne/Nancy, Igor Douven at CNRS, or recently Denis Buehler at ENS). People who have graduated from IJN, IHPST, and other analytic-oriented programmes in France generally find positions, either at the university or in high school. Some are active on the social media, even on Youtube, such as Thibaut Giraud. Beside Claudine Tiercelin and François Récanati, very recently Luigi Rizzi was elected at the Collège de France on the chair of general linguistics. A friend and a disciple of Chomsky, he is very supportive of the circulations between philosophy and linguistics. More broadly I think the perception of analytic philosophy has changed, I see more interest from the students, and various subfields are very active in France at the moment, not just cognitive philosophy, but philosophy of biology, political philosophy, etc. What I also observe is a lot more mutual respect and interaction between various philosophical traditions. This is very palpable in the Department of Philosophy of ENS where I teach.

LL: Well, these seem to be very good signs indeed. Thanks a lot for your time!

Necessity in a Possible World

Can a proposition be necessarily true in one possible world and not in another? It would appear not, if we accept S5. For

$$S5a : PosNecp \rightarrow NecNecp$$

is a theorem of S5. And S5a expressed as:

$$S5b : (Ew)Necpw \rightarrow (w)Necpw$$

where w ranges over the domain of possible worlds, rules it out. For it states, that:

> $S5c$: If there is a possible world where p is necessary then in every possible world p is necessary.

Hence a possible world cannot be defined by the propositions which are true in that world. For if p were part of the definition of a world w, then p could not be false in w. But p will be contingent even if it is necessarily true that p is true in w. For there will be another world in which p is false.

Hence the propositions which are true in a world cannot provide a possible world its identity. In that case possible worlds have no identity. But then, following Quine's dictum of "no entity without identity" (1969: "Speaking of Objects" in his *Ontological Relativity and other Essays*. Columbia University Press, p 23.), we must say that there are no possible worlds. Hence we must then give up either S5, possible worlds or ignore Quine's dictum.

I wish to acknowledge my deep debt to Laureano Luna and to thank Yehuda Gellman, Peter Genco, Dan Wardinon and David Widerker, for discussion.

Alex Blum
Bar-Ilan University

The Reasoner Speculates

Dear fellow Reasoners,

we all learned to follow mathematical rules. We follow them, and good things tend to happen. We can calculate mortgage payments given a fixed interest rate, solutions of polynomials, the intersection of lines, and so on and so forth.

There is a certain element of shock, when we learn that some of these rules fail to apply in all cases. For example, not all multiplications are commutative (turning a body in the $x-y$-plane by 90 degrees and then reflecting it along the y-axis is different from first reflecting along the y-axis and then rotating it), negative numbers have a root ($\sqrt{-1} = \pm i$), parallel lines can intersect in the finite (parallel lines in curved space may intersect) and the function $f(x)$ which puts $f(x) = 1$ for rational numbers and $f(x) = 0$ for non-rational real numbers can be integrated over every finite interval I (using the Lebesgue integral one finds $\int_I f(x)d\mu(x) = 0$).

Some mathematical theories are first developed without any clear interpretation nor with an application in mind. Significant progress in mathematics has been made based on purely formal analysis without interpretation and without application. Interpretations and applications for some fields were later fund for some such theories. Today, non-commutative algebra, complex numbers, non-Euclidean geometry (e.g., differential and projective geometry) and Lebesgue integration do have very well-established interpretations and applications.

I now want to suggest that we can consider giving up a cherished and mathematical rule deeply engrained into us. I do not know how to properly interpret nor do I know where to apply my suggestion. I also do not know, whether this has been suggested before. My point is, my suggestion *may* be interpreted and applied at some later point in time. It may not.

We all learned in school that 'minus times minus equals plus', $(-1)\cdot(-1) = 1$. What if not? What if 'minus times minus equals minus', $(-a)\cdot(-b) = -a\cdot b$ for all positive real numbers a, b?

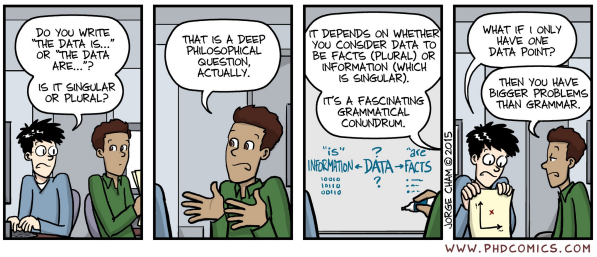
So, such a "product" of n factors is "positive", if and only if all factors are "positive". If there is one or more "negative" number, then the entire "product" becomes "negative". This might mean something like: "a single bad apple (mould) makes the entire bag of apples go to waste".

If we accept that $(-a)\cdot(-b) = -a\cdot b$ and keep multiplication unchanged otherwise, we can note some properties that look peculiar to our eyes (trained to believing that 'minus times minus equals plus'):

1. For all $a < 0$ there exists a number $m = -1 \neq 1$ such that $m\cdot a = a = a\cdot m = a\cdot 1 = 1\cdot a$ (in the lingo of group theory, there is more than one neutral element for negative a).
2. For all $a < 0$ there exists no number m such that $a\cdot m = 1$ (in the lingo of group theory, there is no inverse for negative a).
3. For all $a, b < 0$ it holds that $a < b$, if and only if $-a < -b$ (multiplication by -1 does not necessarily invert inequalities).
4. For all $a \leq 0 \leq b$ it holds that $a\cdot b = a\cdot(-b)$.
5. For all $n \in \mathbb{N}$ it holds that $-(x^n) = 1$ does not have a solution (linear functions from $\mathbb{R}$ to $\mathbb{R}$ such as $f(x) = -x - 1$ that are not parallel to the y-axis may not have a root).
6. For all $n \in \mathbb{N}$ it holds that $-(x^n) = -1$ has exactly two solutions, $x^* \in \{1, -1\}$ (linear functions from $\mathbb{R}$ to $\mathbb{R}$ such as $f(x) = -x + 1$ may have exactly two roots).
7. For all x and all $n \in \mathbb{N}$ there exists a unique $y \in \mathbb{R}$ such that $x = y^n$, for negative x we have $y = -\sqrt[n]{|x|}$ and for positive x we have $y = \sqrt[n]{x}$ (roots always exist, are in $\mathbb{R}$ and unique).

The properties look strange to our eyes. I have long and unsuccessfully thought about finding a sensible interpretation and/or an application. I speculate that there may just be. Let us find out.

Juergen Landes
MCMP, Munich



News

Responsible Life Science Policy between Private and Public Funding - Workshop Report

Life sciences receive funding from both the public and private sectors. These sectors variably emphasize commercially viable and socially responsible research. Given the COVID-19 pandemic and the fact that most medical research is privately-funded, the question of how to responsibly fund life science

becomes even more urgent. For instance, decisions about how the vaccine will be distributed will likely favor richer countries and perhaps even deepen existing global economic inequalities. One argument to justify such inequality is that the countries or corporations who pay for the science should be the ones to reap the rewards. To what extent this is convincing depends on ethical questions about the status of intellectual property rights and a host of national and international laws, as well as more general issues about fairness and justice. In November 2020, researchers gathered to discuss responsible life science funding policies. The speakers came from different backgrounds including social studies of science (Sergio Sismondo), science funding sector (Matthew Wallace), medicine (Ivor Ralph Edwards), pharmacology (Rade Injac), and philosophy of science (Manuela Fernandez Pinto and Jacob Stegenga).

The workshop started with Sergio Sismondo's (Queen's University) talk, which provided an overview of canonical works in science and technology studies that may be useful for thinking about socially responsible funding policy more broadly. These include insights that research should go into technologies whose impacts are relatively easy to undo, research on civic epistemologies that tie questions of funding policy together with a host of national decision-making considerations, and contentions that we should not engage in research where there are no problems. He goes on to claim that all, or maybe most, pharmaceutical research is best understood as a form of marketing. Publication planning, ghost-writing articles, sponsoring conferences or keynotes, and regulatory approval are, according to Sismondo, steps towards mass prescriptions rather than the development of reliable medical knowledge, as contemporary drugs are often either clinically ineffective or are only slightly more effective than previous drugs.

In her talk, Manuela Fernandez Pinto (Universidad de los Andes) focused more specifically on the impact of commercialization of biomedical research. The impacts are, more or less, the same during the COVID-19 pandemic, as Pinto argues that very little has changed. Since the end of the Cold War private funding for R&D, especially in pharmaceuticals, has steadily increased to the point where roughly 70% of research is performed and funded by the private sector. Most of this funding comes from companies profiting within the global north. Research in the global south is mostly funded through special grants, charity donations, and other mechanisms to transfer both funds and research agendas from the global north. While some journals such as Elsevier are making publications in their journals open access, research on COVID-19 vaccines and therapeutics are still driven by commercial interests. It should come as no surprise, then, that the top 10 candidates for COVID-19 vaccines all involve private firms.

Mattew Wallace (International Development Research Centre) reminded us that science is often improved when it draws on diverse sources of evidence. In the context of a global pandemic, we look for global solutions, which work best when international collaborators are able to participate in their own ways. Wallace highlighted three barriers to autonomous democratic global collaboration in science. First, many actors in the global south face systemic external pressures. Often, they are not in a position to set their own research agendas, as funding agencies from the global north dictate what is to be researched, and how. Also, they do not have the same access to research infrastructures, publishing venues, or even always to the output of their own work. Second, within the global south, national science funding bodies also face homogenizing influences, for example, from the private sector and lobby groups. Third, and perhaps most fundamentally, the notion of "research excellence" itself, which drives most science funding decisions, inherits neo-colonial legacies unfit for the global south. To move forward, more research is needed on how funding agencies in the north and south make funding decisions, and to identify more precisely the power dynamics between all the relevant institutions and organizations that influence these decisions.

Rade Injac (Sandoz International GmbH, and the University of Ljubljana) began his talk by defending the pharmaceutical industry, emphasizing that it has increased the quality of life for millions of people. According to Injac, many people criticize pharmaceutical companies without really knowing what goes on within them. On many occasions, companies receive their funding from private agencies and individuals, e.g., funding from LGBT communities financed important HIV/AIDS treatments. Also, some of the big corporations often work with smaller start-ups, as with Pfizer and BioNTech, which allows for the sharing of knowledge, methods, and resources. Ivor Ralph Edwards (Uppsala Monitoring Center) approached the topic of responsible life science from a medical perspective and emphasized the importance of good evidence in clinical trials. After revealing the worrisome fact that adverse drug reactions are the fifth-highest cause of death in the US, Edwards advocated for transparency in medical evidence and interaction with the patients during the trials. In this way the monitoring agencies can better evaluate the outcomes of the trials. Moreover, responsible science funding, according to Edwards, should include not only short term project goals, but also their long term impacts.

In the concluding talk, Jacob Stegenga (University of Cambridge) discussed optimal ways of funding pandemic science. As the pandemic spreads fast, rapid response from scientists is required. In return, the rapid increase of scientific articles makes it harder to track the research quality. In order to tame quick science, Stengega pointed out, we need controls such as randomized trials. He advocated for a proactive approach that would be organized on an international level, as diseases do not "respect" borders. Moreover, in the ideal case, this global scientific response should be independent of industry biases.

The event has been co-hosted by the Carl Fredrich von Weizsäcker Center of the University of Tübingen, Centre for Philosophy of Science of the University of Geneva, and the Forum for Advancing Science and Education through Philosophy – Advise. The videos of the talks are available at the following link.

Jamie Shaw
University of Toronto
Vlasta Sikimić
University of Tübingen
Michael T. Stuart
University of Tübingen & University of Geneva

Calls for Papers

Pursuitworthiness in Scientific Inquiry: special issue of *Studies of History and Philosophy of Science, Part A*, deadline 1 May.

Classic Methodologies in the Philosophy of Science: special issue of *Journal for General Philosophy of Science*, deadline

30 April.

What's Hot in ...

Science Policy

Science in ideal terms is often imagined as purely grounded on epistemic factors. From the perspective of sociology of science, already Max Weber famously argued that professors should not express their political, religious, and economic beliefs in front of students. Though the motivation for studying certain social phenomena is often external, a scientific approach should, according to Weber, be free of non-epistemic values. The neutral point of view should allow researchers to draw conclusions based on evidence and not based on their preconceptions. Moreover, traditional philosophy of science relied on the hypothesis that epistemic and non-epistemic values in science should be kept apart. For instance, according to Thomas Kuhn, simplicity, predictive power, and other epistemic values should guide the researchers when choosing a theory. However, contemporary authors argue that science is not and *should not be* detached from moral and social values (e.g., Douglas, *Science, Policy, and the Value-Free Ideal* 2009). Decisions about which line of research to pursue can be morally relevant, e.g., refusing to participate in the development of biological weapons is a perfectly rational decision of a scientist. Also, researchers could advise and direct responsible public polices based on their results. Finally, the impact of certain non-epistemic values on the research decisions of scientists can be empirically tested.

While some non-epistemic values might be advantageous and justified, for others one hopes that they do not affect scientists. Moreover, not every scientific field operates the same. It can be argued that economics as a field of research is strongly influenced by social views and depending on them it develops in different directions (e.g., proposing a regulated or a free market). Mathematics, on the other hand, has a quite different nature. Methods and results in natural sciences are less often questioned in comparison to the ones in social sciences.

There are certain epistemic decisions of scientists that should ideally be independent of their non-epistemic beliefs. For instance, when scientists disagree about two theories, epistemic tolerance stands for the readiness to accept that the opposing view is also scientifically relevant and part of the research practice. On the other hand, epistemic authoritarianism stands for accepting the dominant views and paradigms in a field without questioning them. Epistemic tolerance can be understood as an epistemic virtue promoting a plurality of methods and hypotheses. It should enable the inclusion of minority views, while epistemic authoritarianism makes it more difficult.

In a democratic society, political choice is guaranteed, and every scientist freely chooses her political orientation. However, do these political and social choices influence the epistemic decisions of researchers? Our empirical study (Sikimić et al., *Rev.Phil.Psych.* 2020) has shown that the political orientation (left-center-right) did not have an impact on epistemic tolerance and epistemic authoritarianism, while the correlation with social conservatism was relatively small. Scientists scored high on the epistemic tolerance scale and low on the epistemic authoritarianism scale, meaning that they are in general open towards new ideas and approaches of their peers. The expected difference has been detected between scientific fields: social scientists scored higher on the epistemic tolerance and lower on the epistemic authoritarianism scale than natural scientists. These results are encouraging. They indicate that the scientific decisions of researchers are independent of their socio-political views, while there is room for assuming that researchers take into account morally relevant considerations and can suggest policies based on their results.

Vlasta Sikimić
University of Tübingen

Mathematical Philosophy

Mathematicians are interested not only in proving things but in understanding why they're true. What should we make of this? Under what circumstances does one piece of math explain another?

Mathematical explanation is tricky. One reason why is that it resists straightforward treatment along traditional lines. The various popular approaches that link explanation to causation seem like non-starters in the mathematical realm, for example. And it's not clear that we do much better with liberalizations of causal theories that countenance a wider array of ontic dependence relations—roughly, the problem is that there are too many explanations and not enough of the right kind of metaphysical structure to underwrite them all. (Cf. D'Alessandro, "Viewing-as Explanations and Ontic Dependence", 2020, *Philosophical Studies* 177: 769-792.)

Unificationist and covering-law-type approaches face serious problems too. From among the major broadly realist theories of explanation, then, perhaps the most promising one left standing is the counterfactual account. Associated in recent times with James Woodward and his followers, the basic idea is that fact F explains fact G if G's obtaining counterfactually depends on F's obtaining, meaning that G wouldn't be the case if F weren't the case.

Before we can turn this idea loose on mathematical explanation, though, we'll first need to come up with a method for evaluating counterpossibles (or at least countermathematicals) that doesn't trivialize them all. (Recall that on the usual Stalnaker-Lewis semantics, one evaluates a counterfactual *If not-F, then not-G* by determining whether G is false in the closest possible worlds where F is false. If there are no worlds where F is false, the conditional is vacuously true. This will always be the case when F is a mathematical fact, on the standard assumption that all mathematical facts are necessary.) It's worth noting that we have good independent reasons to want a workable theory of counterpossibles, so the fact that the counterfactual approach requires such a theory needn't be seen as a strike against it. (Cf. Nolan, "Hyperintensional Metaphysics", 2014, *Philosophical Studies* 171: 149-160.)

It's not so obvious what an adequate treatment of countermathematicals should look like, though. In the closest world where, say, Fermat's Last Theorem is false, what else is different? Does the disturbance ramify all the way down to the axioms, or can it be localized in some principled way? If we keep the axioms the same but negate FLT, we'll end up with a con-

tradiction somewhere, and what should we do about that? Our intuitions about the right answers to such questions seem much less clear than our judgments about ordinary counterfactuals, so it may not even be apparent what the criteria for success should be.

Nevertheless, recent work has tackled these problems in hopes of claiming mathematical explanation as a victory for the counterfactual theory. Two major efforts in this direction are Baron, Colyvan and Ripley, "A Counterfactual Approach to Explanation in Mathematics", 2020, *Philosophia Mathematica* 28: 1-34, and Reutlinger, Colyvan and Krzyżanowska, "The Prospects for a Monist Theory of Non-causal Explanation in Science and Mathematics", 2020, *Erkenntnis*. These papers have Mark Colyvan as a common denominator and their approaches are intended to be complementary; I'll call this group of authors CBRRK.

CBRRK favor a Lewis-like approach to evaluating counterpossibles. On this view, if we want to make sense of a statement like *If Fermat's Last Theorem were false, then P*, we first identify the closest worlds to actuality where FLT is false. Here closeness is determined by sameness of general mathematical principles "upstream" from FLT, and by overall similarity in matters of mathematical fact. In turn, "upstreamness" is characterized in terms of an intuitive notion of dependence: one fact is upstream of another if the latter depends on the former in an appropriate sense. Putting all this together, our FLT counterpossible is true just in case *P* holds in the worlds identified by this procedure. (CBRRK's view involves further details which I've left out here for brevity.)

I think this approach has a lot to recommend it. CBRRK use their theory to give a clever analysis of an explanatory proof from Euclidean geometry (which evokes the analysis of the Butterfly Theorem from Frans and Weber, "Mechanistic Explanation and Explanatory Proof", 2014, *Philosophia Mathematica* 22, 231-248). CBRRK also have reasonable things to say about how to handle contradictions and how their proposal can be realized in the structural equations modeling framework. On the whole, it's certainly the best extant version of a counterfactual theory of mathematical explanation.

Nevertheless, I think some important and thorny questions remain for CBRRK's account in particular and for counterfactual theories in general. To name just one, is there an overgeneration problem here? Countermathematical dependence seems pretty easy to come by, whereas explanatory relations are relatively rare. Consider Gauss's first proof of the quadratic reciprocity theorem, for instance, which he and many others considered paradigmatically unexplanatory. The proof is a complicated case analysis that considers and rules out eight possible types of counterexample. On any sensible analysis, we'll presumably find that the truth of the theorem depends on the nonexistence of any such counterexample—if there were a counterexample, then quadratic reciprocity wouldn't hold. By the counterfactual theorist's lights, this would seem to make Gauss's proof explanatory. But this is the wrong result. (For more on this case, see D'Alessandro, "Proving Quadratic Reciprocity", 2020, *Synthese*.)

There's clearly lots of room here for discussion, and that's an exciting place for a field to be. I look forward to future work on CBRRK's approach as well as the continued growth of the subject as a whole.

William D'Alessandro
MCMP, Munich

Events

d.whiting@soton.ac.uk: H-OE, Higher-Order Evidence Online Workshop, virtual.25 January

Courses and Programmes

Courses

Programmes

MA in Reasoning, Analysis and Modelling: University of Milan, Italy.
APhil: MA/PhD in Analytic Philosophy, University of Barcelona.
Master Programme: MA in Pure and Applied Logic, University of Barcelona.
Doctoral Programme in Philosophy: Language, Mind and Practice, Department of Philosophy, University of Zurich, Switzerland.
Doctoral Programme in Philosophy: Department of Philosophy, University of Milan, Italy.
LogiCS: Joint doctoral program on Logical Methods in Computer Science, TU Wien, TU Graz, and JKU Linz, Austria.
HPSM: MA in the History and Philosophy of Science and Medicine, Durham University.
Master Programme: in Statistics, University College Dublin.
LoPhiSC: Master in Logic, Philosophy of Science and Epistemology, Pantheon-Sorbonne University (Paris 1) and Paris-Sorbonne University (Paris 4).
Master Programme: in Artificial Intelligence, Radboud University Nijmegen, the Netherlands.
Master Programme: Philosophy and Economics, Institute of Philosophy, University of Bayreuth.
MA in Cognitive Science: School of Politics, International Studies and Philosophy, Queen's University Belfast.
MA in Logic and the Philosophy of Mathematics: Department of Philosophy, University of Bristol.
MA Programmes: in Philosophy of Science, University of Leeds.
MA in Logic and Philosophy of Science: Faculty of Philosophy, Philosophy of Science and Study of Religion, LMU Munich.
MA in Logic and Theory of Science: Department of Logic of the Eotvos Lorand University, Budapest, Hungary.
MA in Metaphysics, Language, and Mind: Department of Philosophy, University of Liverpool.
MA in Mind, Brain and Learning: Westminster Institute of Education, Oxford Brookes University.
MA in Philosophy: by research, Tilburg University.
MA in Philosophy, Science and Society: TiLPS, Tilburg University.
MA in Philosophy of Biological and Cognitive Sciences: Department of Philosophy, University of Bristol.

MA in Rhetoric: School of Journalism, Media and Communication, University of Central Lancashire.
MA programmes: in Philosophy of Language and Linguistics, and Philosophy of Mind and Psychology, University of Birmingham.
MRes in Methods and Practices of Philosophical Research: Northern Institute of Philosophy, University of Aberdeen.
MSc in Applied Statistics: Department of Economics, Mathematics and Statistics, Birkbeck, University of London.
MSc in Applied Statistics and Datamining: School of Mathematics and Statistics, University of St Andrews.
MSc in Artificial Intelligence: Faculty of Engineering, University of Leeds.
MSc in Cognitive & Decision Sciences: Psychology, University College London.
MSc in Cognitive Systems: Language, Learning, and Reasoning, University of Potsdam.
MSc in Cognitive Science: University of Osnabrück, Germany.
MSc in Cognitive Psychology/Neuropsychology: School of Psychology, University of Kent.
MSc in Logic: Institute for Logic, Language and Computation, University of Amsterdam.
MSc in Mind, Language & Embodied Cognition: School of Philosophy, Psychology and Language Sciences, University of Edinburgh.
MSc in Philosophy of Science, Technology and Society: University of Twente, The Netherlands.
MRes in Cognitive Science and Humanities: Language, Communication and Organization: Institute for Logic, Cognition, Language, and Information, University of the Basque Country (Donostia San Sebastián).
Open Mind: International School of Advanced Studies in Cognitive Sciences, University of Bucharest.
Research Master in Philosophy and Economics: Erasmus University Rotterdam, The Netherlands.

Jobs and Studentships

Studentships

Doctoral Programme in Philosophy: Language, Mind and Practice, Department of Philosophy, University of Zurich, Switzerland.
LogiCS: Joint doctoral program on Logical Methods in Computer Science, TU Wien, TU Graz, and JKU Linz, Austria.

Jobs

Post doc: in Inferences Under Severe Uncertainties, University of Technology of Compiégne, open until filled.