NSS

University of New Mexico

A new method to construct entropy of interval-valued Neutrosophic Set

Chunfang Liu[1,2], YueSheng Luo[2,3]

[1] College of Science, Northeast Forestry University, Hexing Road 26, Harbin 150040, China. E-mail: liuchunfang1112@163.com
[2] College of Automation, Harbin Engineering University, Nantong Road 145, Harbin 150001, China
[3] College of Science, Harbin Engineering University, Nantong Road 145, Harbin 150001, China. E-mail: luoyuesheng@hrbeu.edu.cn

Abstract Interval-valued neutrosophic set (INS) is a generalization of fuzzy set (FS) that is designed for some practical situations in which each element has different truth membership function, indeterminacy membership function and falsity membership function and permits the membership degrees to be expressed by interval values. In this paper, we first introduce the similarity measure between single valued neutrosophic sets, then propose a new method to construct entropy of interval-valued neutrosophic sets based on the similarity measure between the two single valued neutrosophic sets, finally we give an example to show that our method is effective and reasonable.

Keywords: Interval-valued neutrosophic set (INS), Entropy, Similarity measure

1.Introduction

In 1965, Zadeh first introduced Fuzzy set, which has been widely used in decision making, artificial intelligence, pattern recognition, information fusion, etc [1,2]. Later, several high-order fuzzy sets have been proposed as an extension of fuzzy sets, including interval-valued fuzzy set, type-2 fuzzy set, type-n fuzzy set, soft set, rough set, intuitionistic fuzzy set, interval-valued intuitionistic fuzzy set, hesitant fuzzy set and neutrosophic set (NS) [2,3,4,5].

As a generalization of fuzzy set, the NS was proposed by Smarandache [5] not only to deal with the decision information which is often incomplete, indeterminate and inconsistent but also include the truth membership degree, the falsity membership degree and the indeterminacy membership degree. Since NS contains both non-standard and standard intervals in its theory and related operations which restricts its application in many fields. For simplicity and practical application, Wang proposed the interval NS (INS) and the single valued NS (SVNS) which are the instances of NS and gave some operations on these sets [9,10]. Ye proposed the similarity measure of interval valued neutrosophic set and applied them to decision making [11], he also proposed the vector similarity measures of simplified neutrosophic sets [12]. Ali proposed the entropy and similarity measure of interval valued neutrosophic set [13]. Zhang proposed the cross-entropy of interval neutrosophic set and applied it to multi-criteria decision making [14]. All these papers have enriched the theory of neutrosophic set.

Consistently with axiomatic definition of entropy of INS, we introduce the similarity measure between single valued neutrosophic sets, and propose a new method to construct entropy of interval-valued neutrosophic sets based on the similarity measure between single valued neutrosophic sets, then we give an example to show that our method is effective and reasonable.

The structure of this paper is organized as follows. Section 2 introduces some basic definitions of the interval-valued neutrosophic sets and the single valued neutrosophic sets (SVNSs). Section 3 presents a new similarity measure of SVNSs. Section 4 gives entropy of INS. Section 5 concludes our work.

2. Preliminaries

Definition1 [9] Let X be a space of points (objects), and its element is denoted by x. A NS A in X, if the functions $T_A(x)$, $I_A(x)$, $F_A(x)$ are singleton subsets in the real standard [0,1]. Then, a single valued NS A is denoted by

$$A=\{\langle x;T_A(x),I_A(x),F_A(x)\rangle | x\in X\}$$

which is called a single valued neutrosophic set (SVNS).

Definition2 [9] For two SVNSs A and B, A is contained in B, if and only if

$$T_A(x)\le T_B(x),\, I_A(x)\ge I_B(x),\, F_A(x)\ge F_B(x)$$

for every x in X.

Definition3 [9] The complement of SVNS A is defined by

$$A^C=\{\langle x;F_A(x),1-I_A(x),T_A(x)\rangle | x\in X\}$$

Interval-valued neutrosophic set (INS) improves the ability of NS expressing the uncertainty of information whose membership functions take the form of interval values.

Definition 4 [10] Assume *X* be a universe of discourse, with a generic element in *X* denoted by *x*. An interval-valued neutrosophic set *A* in *X* is

$$A=\left\{\left\langle x,T_A(x),I_A(x),F_A(x)\right\rangle \middle| x\in X\right\}$$

where $T_A(x)$, $I_A(x)$ and $F_A(x)$ are the truth membership function, indeterminacy membership function and falsity membership function, respectively. For each point x in *X*, we have

$T_A(x)=[\inf T_A(x),\sup T_A(x)]\subseteq[0,1]$,

$I_A(x)=[\inf I_A(x),\sup I_A(x)]\subseteq[0,1]$,

$F_A(x)=[\inf F_A(x),\sup F_A(x)]\subseteq[0,1]$

and $0\le \mathrm{Sup}T_A(x)+\mathrm{Sup}I_A(x)+\mathrm{Sup}F_A(x)\le 3$.

Definition5 [10] For two INSs A and B, A is contained in B, if and only if

$\inf T_A(x)\le\inf T_B(x)$, $\sup T_A(x)\le\sup T_B(x)$,

$\inf I_A(x)\ge\inf I_B(x)$, $\sup I_A(x)\ge\sup I_B(x)$,

$\inf F_A(x)\ge\inf F_B(x)$, $\sup F_A(x)\ge\sup F_B(x)$,

for every x in X.

Definition6 [10] The complement of INS A is defined by

$$A^C=\left\{\left\langle x;T_{A^C}(x),I_{A^C}(x),F_{A^C}(x)\right\rangle \middle| x\in X\right\}$$

where

$$T_{A^C}(x)=F_A(x)=[\inf F_A(x),\sup F_A(x)],$$

$$I_{A^C}(x)=[1-\sup I_A(x),1-\inf I_A(x)],$$

$$F_{A^C}(x)=T_A(x)=[\inf T_A(x),\sup T_A(x)]$$

3. Similarity measure of single valued neutrosophic sets

Definition7 [11] Let A and B be two SVNSs, a function S is the similarity measure between A and B, if S satisfies the following properties:

(N1) $S(A,A^C)=0$ if A is a crisp set;

(N2) $S(A,B)=1\Leftrightarrow A=B$;

(N3) $S(A,B)=S(B,A)$;

(N4) for all SVNSs A,B,C, if $A\subseteq B\subseteq C$, then $S(A,C)\le S(A,B)$, $S(A,C)\le S(B,C)$.

Let

$A=\{T_A(x),I_A(x),F_A(x)\}$,

$B=\{T_B(x),I_B(x),F_B(x)\}$ be two SVNSs, we will use the Hamming distance to define the similarity measure of single valued neutrosophic sets.

$$S(A,B)=1-\frac{1}{3}\sum_{j=1}^{n}\left(\begin{array}{l}\left|T_A(x_j)-T_B(x_j)\right|+\\ \left|I_A(x_j)-I_B(x_j)\right|+\\ \left|F_A(x_j)-F_B(x_j)\right|\end{array}\right) \quad (1)$$

It is easy to prove the similarity measure satisfies the Definition 7.

4. Entropy of interval-valued neutrosophic set

Based on [15], we give the definition of entropy of INS as follows:

Definition8 A real valued function *E* : INSs→ [0, 1] is called an entropy of INS, if *E* satisfies the following properties:

(P1) $E(A)=0$ if A is a crisp set;

(P2) $E(A)=1$ iff $\inf I_A(x)=\sup I_A(x)$, $[\inf T_A(x),\sup T_A(x)]=[\inf F_A(x),\sup F_A(x)]$;

(P3) $E(A)=E(A^C)$;

(P4) $E(A)\le E(B)$

if $A\subseteq B$ when

$\inf T_B\le\inf F_B$ and $\sup T_B\le\sup F_B$

$\inf I_B\ge 1-\sup I_B$;

or $B\subseteq A$ when

$\inf F_B\le\inf T_B$ and $\sup F_B\le\sup T_B$

$\inf I_B\ge 1-\sup I_B$.

Let

$$A=\left\{\begin{array}{l}[\inf T_A(x),\sup T_A(x)],[\inf I_A(x),\sup I_A(x)]\\ ,[\inf F_A(x),\sup F_A(x)]\end{array}\right\}$$

be an INS, we construct the new SVNSs based on A.

$$A_1=\{\inf T_A(x),\inf I_A(x),\inf F_A(x)\} \quad (2)$$

$$A_2=\{\sup T_A(x),\sup I_A(x),\sup F_A(x)\} \quad (3)$$

$$A_2^C = \{\sup F_A(x), 1 - \sup I_A(x), \sup T_A(x)\} \quad (4)$$

Theorem1 Suppose S is the similarity measure of SVNSs, E is the entropy of INS, $S(A_1, A_2^C)$ is the similarity measure of SVNSs A_1 and A_2^C, then $E(A) = S(A_1, A_2^C)$.

Proof. (P1) If A is a crisp set, then for every $x \in X$, we have

$\inf T_A(x) = \sup T_A(x) = 1$,

$\inf I_A(x) = \sup I_A(x) = 0$,

$\inf F_A(x) = \sup F_A(x) = 0$

or

$\inf T_A(x) = \sup T_A(x) = 0$,

$\inf I_A(x) = \sup I_A(x) = 0$,

$\inf F_A(x) = \sup F_A(x) = 1$

which means that

$A_1 = \{1,0,0\}$, $A_2 = \{1,0,0\}$, $A_2^C = \{0,1,1\}$. It is obvious that $E(A) = S(A_1, A_2^C) = 0$.

(P2) By the definition of similarity measure of fuzzy sets, we have

$$E(A) = S(A_1, A_2^C) = 1$$

$$\Leftrightarrow A_1 = A_2^C$$

$$\Leftrightarrow \inf T_A(x) = \sup F_A(x),$$
$$\inf I_A(x) = 1 - \sup I_A(x),$$
$$\inf F_A(x) = \sup T_A(x).$$

$$\Leftrightarrow A = \left\{\begin{matrix}[0.5, 0.5], [\inf I_A(x), \\ 1 - \inf I_A(x)], [0.5, 0.5]\end{matrix}\right\}$$

$$\Leftrightarrow E(A) = 1$$

(P3) Because $(A^C)_2 = A_2^C, (A^C)_1^C = A_1$,

we have

$$E(A) = S(A_1, A_2^C) = S(A_2^C, A_1)$$
$$= S\left((A^C)_1^C, (A^C)_2\right) = S\left((A^C)_2, (A^C)_1^C\right).$$
$$= E(A^C)$$

(P4) Since $A \subseteq B$ it means that

$\inf T_A(x) \le \inf T_B(x)$, $\sup T_A(x) \le \sup T_B(x)$

$\inf I_A(x) \ge \inf I_B(x)$, $\sup I_A(x) \ge \sup I_B(x)$

$\inf F_A(x) \ge \inf F_B(x)$, $\sup F_A(x) \ge \sup F_B(x)$.

when

$\inf T_B(x) \le \inf F_B(x)$, $\sup T_B(x) \le \sup F_B(x)$,

$\inf I_B(x) \ge 1 - \sup I_B(x)$

then we get

$$\inf T_A(x) \le \inf T_B(x) \le \inf F_B(x) \le \inf F_A(x)$$

$$\sup T_A(x) \le \sup T_B(x) \le \sup F_B(x) \le \sup F_A(x)$$

By computing, we can get

$$A_1 \subseteq B_1 \subseteq B_2^C \subseteq A_2^C,$$

and using the definition of similarity measure, we get

$$E(A) = S(A_1, A_2^C) \le S(A_1, B_2^C) \le S(B_1, B_2^C) = E(B)$$

With the same reason, if $B \subseteq A$ when $\inf F_B \le \inf T_B$ and $\sup F_B \le \sup T_B$, $\inf I_B \ge 1 - \sup I_B$, we conclude $E(A) \le E(B)$.

Hence, we complete the proof of Theorem 1.

We can define entropy of INS by similarity measure between two SVNSs, which constructed by A, it satisfied the definition of entropy.

Example .

Let $X = \{x_1, x_2, \cdots x_n\}$ be a universe of discourse.

Let $A = \{\langle x_i, [0.7, 0.8], [0.5, 0.7], [0.1, 0.2]\rangle | x_i \in X\}$. $B = \{\langle x_i, [0.6, 0.8], [0.4, 0.6], [0.1, 0.3]\rangle | x_i \in X\}$ be two INSs.

Now we will obtain the entropy $E(A)$, $E(B)$ as follows.

For A, from (1), (2), (3), (4), we obtain

$A_1 = \{0.7, 0.5, 0.1\}$, $A_2 = \{0.8, 0.7, 0.2\}$ and $A_2^C = \{0.2, 0.3, 0.8\}$;

$$E(A) = S(A_1, A_2^C) = 1 - \frac{1}{3}(0.5 + 0.2 + 0.7) = 0.5333$$

For B, $B_1 = \{0.6, 0.4, 0.1\}$, $B_2 = \{0.8, 0.6, 0.3\}$ and $B_2^C = \{0.3, 0.4, 0.8\}$;

$$E(B) = S(B_1, B_2^C) = 1 - \frac{1}{3}(0.3 + 0.7) = 0.6667.$$

$E(A) < E(B)$ is consistent with our intuition.

5. Conclusion

Neutrosophic set is a necessary tool to deal with the uncertain information. In this paper, we commented on the axiomatic definitions of similarity measure of SVNSs and entropy of INSs, respectively. We first introduced the similarity measure between SVNSs, and proposed a new method to construct entropy of INS based on the similarity measure between SVNSs, then we gave an example to show that our method is effective and reasonable. In the future, we want to give the entropy of INSs based on similarity measure of INSs.

References

[1] L.A. Zadeh. Fuzzy sets, Information and Control, 8 (1965), 338-353.

[2] L.A. Zadeh. In there a need for fuzzy logic?, Information Sciences, 178(2008), 2751-2779.

[3] K. Atanassov. Intuitionistic fuzzy sets, Fuzzy sets and systems, 20(1986), 87-96.

[4] V. Torra. Hesitant fuzzy sets, International Journal of Intelligent Systems, 25(2010), 529-539.

[5] F. Smarandache. A unifying field in logics.Neutrosophy: Neutrosophic probability, set and logic, Rehoboth, DE: American Research Press.1999.

[6] J. Ye. A multi-criteria decision-making method using aggregation operators for simplified neutrosophic set, Journal of Intelligent and Fuzzy Systems, 26(2014), 2459-2466.

[7] S. Broumi and F. Smarandache. Several Similarity Measures of neutrosophic set, Neutrosophic Sets and Systems, 1(2013), 54-62.

[9] H. Wang, F. Smarandache, Y.Q. Zhang, R. Sunderraman. Single valued neutrosophic sets, Multispace Multistructno, 4(2010), 410-413,.

[10] H. Wang, F. Smarandache, Y.Q. Zhang, R. Sunderraman. Interval neutrosophic sets and logic: theory and applications in computing, Hexis, Phoenix, AZ, 2005.

[11]J. Ye. Similarity measures between interval valued neutrosophic sets and their applications in multi-criteria decision making, Journal of Intelligent and Fuzzy Systems, 26(2014), 2459-2466.

[12] J. Ye. Vector similarity measures of simplified neutrosophic sets their applications in multi-criteria decision making, International Journal of Fuzzy Systems, 16(2014), 204-210.

[13]Ali Aydogdu. On entropy and similarity measure of interval valued neutrosophic sets, Neutrosophic Sets and Systems, 9(2015), 47-19.

[14]Z. Tian, H. Zhang, J. Wang, X. Chen. Multi-criteria decision making method based on a cross-entropy with interval neutrosophic sets, International Journal of Systems Science, 2015, 1-11.

[15]H.Bustince, P. Burillo, Entropy on intuitionistic fuzzy sets and on interval-valued fuzzy sets, Fuzzy sets and Systems 78(1996), 305-316.

Received: Feb. 12, 2016. Accepted: Mar. 27, 2016