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How Do I Know That I Know Nothing? The Axiom of Selection and the Arithmetic of Infinity

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Abstract

We show that the statement 'I only know that I know nothing,' attributed to the Greek philosopher Socrates, contains, at its core, Zermelo's Axiom of Selection and the arithmetic of the infinite cardinal $\aleph_0$ (aleph-0).

keywords: Socratic paradox, epistemology, axiom of selection, cardinal number

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Introduction

1. The proposition "I only know that I know nothing," famously attributed to the Greek philosopher Socrates [1], encapsulates the essence of epistemic humility.

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2. Socrates is known for his dialectical approach to knowledge, often questioning conventional wisdom and recognizing his own limitations in terms of knowledge.

3. This attitude of intellectual humility and constant search for truth is a hallmark of the Socratic method.

4. The dialectical approach, also known as the dialectical method, is a form of argumentation and philosophical inquiry that involves the exploration of contradictions and oppositions to arrive at a deeper understanding of truth.

5. Historically, this method was not only central to the philosophies of Socrates and his disciple Plato, but also significantly influenced later thinkers like Hegel, who integrated it into his own philosophical system.

Interpretation

6. The proposition "I only know that I know nothing" can be interpreted as "I know nothing, except for the knowledge that I know nothing."

7. This recursive statement serves as an acknowledgment of one's own epistemic limitations, encapsulating the philosophical pursuit of wisdom through the recognition of one's ignorance.

Apparent Paradox

8. *"How can one know something and nothing at the same time?"*

Russell's Paradox

9. [2–6]

10. Let $R = \{x : x \notin x\}$ be the set of all sets that are not elements of themselves.

11. If $R \in R$, then from $R = \{x : x \notin x\}$, we conclude that $R \notin R$.

12. If $R \notin R$, then from $R = \{x : x \notin x\}$, we conclude that $R \in R$.

13. (11) and (12) lead to $R \in R \leftrightarrow R \notin R$, which is a contradiction.

14. Therefore, $\nexists \{x : x \notin x\}$.

Axioms of Foundation and Antifoundation

15. [6]

16. The axiom of foundation ensures that there are no infinitely descending chains of sets, where each set is an element of the preceding set, such as in

$$F = \{\{\{f\}\}\}.$$

17. The axiom of antifoundation allows an infinite number of sets contained within a set; for example,

$$\overline{F} = ...\{\{\{g\}\}\}...$$

The axiom of selection

18. [4,5]

19. The idea is to modify the set $R = \{x : x \notin x\}$, assuming $x \in A$.

20. (19) solves Russell's paradox, which we will explore further.

21. Let $S = \{x \in A : x \notin x\}$ be the set of all elements of A which are not elements of themselves.

22. If $S \in S$, then from $S = \{x \in A : x \notin x\}$, we conclude that $S \notin S$.

23. (22) is a contradiction, so $S \in S$ is impossible.

24. If $S \notin S$, then from $S = \{x \in A : x \notin x\}$, we have two cases:

    (i) $S \notin S$ and $S \notin A$,

    (ii) $S \notin S$ and $S \in A$.

25. Case $(24.ii)$ leads to $S \in S$, which is a contradiction; therefore, $S \notin A$.

26. Thus, (24.i), $S \notin S$ and $S \notin A$ is a valid case.

27. In summary, we have the following three scenarios:
    (a) $S \in S$ leads to a contradiction, (22)-(23);
    (b) $S \notin S \rightarrow S \notin A$, (24.$i$);
    (c) $(S \notin S \wedge S \in A) \rightarrow S \in S$, which is a contradiction, (24.ii).

28. Therefore, $\exists S : S = \{x \in A : x \notin x\}$ when $S \notin S$ and $S \notin A$.

Socrates' Selection

29. From (21), we have that $S = \{x \in A : x \notin x\}$.

30. We define A as the set comprising everything that can be known.

31. S is called the 'set of knowledge' because it contains the set A.

32. Let $x = \{1\}$ be the set containing what Socrates knows.

33. The element '1' represents the only thing that Socrates claims to know.

34. The braces { }, which delimit the set x, indicate each instance of knowledge.

35. Thus,
$$\{1\} := \text{I know that I know nothing.}$$

36. And,
$$\{\{1\}\} := \text{I know that I know that I know nothing.}$$

37. It follows that $x \in A$, since
$$A = \{\{1\}, \{2\}, \{3\}, \ldots\},$$
where the numbers $2, 3, 4, \ldots$ denote other potential knowledge beyond Socrates' admission.

38. As Socrates only knows that he knows nothing, the axiom of foundation is established in $x = \{1\}$.

39. If Socrates were aware of an infinite cycle of recognizing his own ignorance (that is, ... I know that I know that I know nothing), then we would be considering the axiom of antifoundation, as we would have

$$x' = \ldots \{\{\{1\}\}\} \ldots$$

40. In this case, $x' = \{x'\}$, which implies $x' \in x'$.

41. Therefore, Socrates' selection is in S with the aid of the selector set A.

Arithmetic of Transfinite Ordinals

42. [7–9]

43. Cantor defined ω as the smallest transfinite ordinal, which is represented by the set of all natural numbers.

44. This set is characterized by having cardinality $\aleph_0$, denoted as:

$$\omega = \{0, 1, 2, 3, \ldots\}.$$

45. ω-left subtraction can be defined in terms of set difference as follows:

$$\omega - 1 = \{0, 1, 2, 3, \ldots\} \backslash \{\omega\} = \{0, 1, 2, 3, \ldots\} = \omega,$$

since $\omega \notin \omega$.

46. This indicates that subtracting an element not in the set ω (considered as the natural numbers under ordinal notation) yields the set itself.

47. Alternatively, if ω-left subtraction is defined by removing an element that is part of the set, such as:

$$\omega - 1 = \{0, 1, 2, 3, \ldots\} \backslash \{1\} = \{0, 2, 3, \ldots\},$$

then the resulting set is not equal to ω.

48. However, it still has the same cardinality as ω, demonstrating that the cardinality of infinite sets remains unchanged under the removal of finitely many elements.

49. In either definition, subtracting an element from an infinite set with cardinality $\aleph_0$ results in a set that preserves the same cardinality.

Socrates' Arithmetic

50. Let `S'` := *I only know that I know nothing.*

51. We define ω as the set of everything that can be known.

52. The number 1 represents the only thing Socrates claims to know, that is,

$$1 := \text{I know nothing.}$$

53. Thus, we have that

$$\texttt{S'} = \omega - 1 = \omega \quad \text{or} \quad \texttt{S'} = \omega - 1 = \omega \backslash \{1\}.$$

54. Both outcomes of `S'` yield the same interpretation: subtracting one piece of knowledge from a set that encompasses all possible knowledge does not alter the cardinality (size) of the set.

55. An individual who knows nothing, and hence is unaware of their own ignorance, possesses the following fraction of total knowledge:

$$c_0 = \lim_{n \to |\omega|} \frac{0}{n} = \lim_{n \to \infty} \frac{0}{n} = 0.$$

56. Socrates, who is aware that he knows nothing, has the fraction of total knowledge given by:

$$c_1 = \lim_{n \to |\omega|} \frac{1}{n} = \lim_{n \to \infty} \frac{1}{n} = 0.$$

Final Remarks

57. From the interpretation provided in this white paper, the axiom of selection is present in Socrates' proposition "I only know that I know nothing."

58. Knowing that one knows nothing does not change the size of the set that contains all that is known, when this set is the smallest mathematical infinity.

59. The set ω, encompassing everything that can be known, remains infinite despite the paradoxical removal of a single known or unknown element.

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Supplementary Files

[12]

How to Cite this White Paper

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Agreement

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References

[1] Platão. *Apologia de Sócrates.*

[2] Velleman, Daniel J. *How to prove it: A structured approach*. Cambridge University Press, 2006.

[3] Warner, Steve. *Pure Mathematics for Beginners*. GET 800, 2018.

[4] Pinter, Charles C. *A book of set theory*. Courier Corporation, 2014.

[5] Lobo, Matheus P. "The Axiom of Selection Resolves Russell's Paradox." *OSF Preprints*, 19 Aug. 2019. https://doi.org/10.31219/osf.io/pt8ax

[6] Warner, Steve. *Set Theory for Beginners*. GET 800, 2019.

[7] Stillwell, John. *Roads to Infinity: the mathematics of truth and proof*. CRC Press, 2010.

[8] Lobo, Matheus P. "Aritmética Dos Ordinais Transfinitos." *OSF Preprints*, 10 Sept. 2020. https://doi.org/10.31219/osf.io/h3t5f

[9] Lobo, Matheus P. "Subtraction of Transfinite Ordinals." *OSF Preprints*, 8 Mar. 2020. https://doi.org/10.31219/osf.io/yvrf3

[10] Lobo, Matheus P. "Microarticles." *OSF Preprints*, 28 Oct. 2019. https://doi.org/10.31219/osf.io/ejrct

[11] Lobo, Matheus P. "Simple Guidelines for Authors: Open Journal of Mathematics and Physics." *OSF Preprints*, 15 Nov. 2019. https://doi.org/10.31219/osf.io/fk836

[12] https://doi.org/10.5281/zenodo.12511400

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