

# *The problem of identifying the system and the environment in the phenomenon of decoherence*

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## *1. Introduction*

The term ‘decoherence’ usually refers to the quantum process that supposedly turns a pure state into a mixed state, which is diagonal in a well-defined basis. The orthodox explanation of the phenomenon is given by the so-called *environment-induced decoherence* (EID) approach (Zurek 1982, 1993, 2003; Paz and Zurek 2002), according to which decoherence results from the interaction of an open quantum system and its environment. By studying different physical models, it is proved that the reduced state of the open system rapidly diagonalizes in a basis that identifies the candidates for classical states. By contrast to non-dissipative accounts to decoherence, the EID approach is commonly understood as a dissipative approach: “*if one believes that classicality is really an emergent property of quantum open systems one may be tempted to conclude that the existence of emergent classicality will always be accompanied by other manifestations of openness such as dissipation of energy into the environment*” (Paz and Zurek 2002, p.6).

The EID approach has been extensively applied to many areas of physics with impressive practical success. Nevertheless, from a conceptual viewpoint it still faces a difficulty derived from its open-system perspective: the problem of defining the system that decoheres.

From the einselection view, the split of the Universe into the degrees of freedom which are of direct interest to the observer –the system– and the remaining degrees of freedom –the environment– is absolutely essential for decoherence. However, the EID approach offers no general criterion for deciding where to place the “cut” between system and environment: the environment may be “external” (a bath of particles interacting with the system of interest) or “internal” (such as collections of phonons or other internal excitations). This fact often leads to the need of assuming the observables that will behave classically in advance. For instance, in cosmology the usual strategy consists in splitting the Universe into some degrees of freedom representing the “system”, and the remaining

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degrees of freedom that are supposed to be non accessible and, therefore, play the role of an internal environment (see, e.g., Calzetta *et al.* 2001). Zurek recognizes this difficulty of his proposal: “*In particular, one issue which has been often taken for granted is looming big, as a foundation of the whole decoherence program. It is the question of what are the ‘systems’ which play such a crucial role in all the discussions of the emergent classicality. This issue was raised earlier, but the progress to date has been slow at best*” (Zurek 1998, p.1820; for a discussion, see Castagnino and Lombardi 2004).

The main purpose of this paper is to argue that decoherence is a *relative* phenomenon, better understood from a *closed-system perspective* according to which the split of a closed quantum system into an open subsystem and its environment is just a way of selecting a particular space of relevant observables of the whole closed system. In order to support this claim, we shall consider the results obtained in a natural generalization of the simple spin-bath model usually studied in the literature (Castagnino *et al.* 2010a). Our main thesis will lead us to two corollaries. First, the “looming big” problem of identifying the system that decoheres is actually a pseudo-problem, which vanishes as soon as one acknowledges the relative nature of decoherence. Second, the link between decoherence and energy dissipation is misguided. As previously pointed out (Schlosshauer 2007), energy dissipation and decoherence are different phenomena, and we shall argue for this difference on the basis of the relative nature of decoherence.

## *2. Open-system perspective versus closed-system perspective*

As it is well-known in the discussions about irreversibility, when a –classical or quantum– state evolves unitarily, it cannot follow an irreversible evolution. Therefore, if a non-unitary evolution is to be accounted for, the maximal information about the system must be split into a discarded irrelevant part and a relevant part that may evolve non-unitarily. This idea can be rephrased in operator language. Since the maximal information about the system is given by the space  $\mathcal{O}$  of all its possible observables, then we restrict that information to a relevant part by selecting a subspace  $\mathcal{O}_R \subset \mathcal{O}$  of *relevant observables*. The irreversible evolution is the non-unitary evolution viewed from the perspective of those relevant observables.

As emphasized by Omnès (2001, 2002), decoherence is a particular irreversible process; then, the selection of the subspace  $\mathcal{O}_R \subset \mathcal{O}$  is required. In fact, the different approaches to decoherence select a set of relevant observables in terms of which the time-behavior of the system is described: gross

observables (van Kampen 1954), macroscopic observables of the apparatus (Daneri *et al.* 1962), relevant observables (Omnès 1994, 1999), van Hove observables (Castagnino and Lombardi 2005, Castagnino 2006). In the case of the EID approach, the selection of  $\mathcal{O}_R$  requires the partition of the whole closed system  $U$  into the open system  $S$  and its environment  $E$  (see Castagnino *et al.* 2007).

Let us consider the Hilbert space  $\mathcal{H} = \mathcal{H}_S \otimes \mathcal{H}_E$  of the closed system  $U$ , where  $\mathcal{H}_S$  and  $\mathcal{H}_E$  are the Hilbert spaces of  $S$  and  $E$  respectively. In the EID approach, the relevant observables are:

$$O_R = O_S \otimes I_E \in \mathcal{O}_R \subset \mathcal{O} \quad (1)$$

where  $O_S \in \mathcal{H}_S \otimes \mathcal{H}_S$  corresponds to  $S$  and  $I_E$  is the identity operator in  $\mathcal{H}_E \otimes \mathcal{H}_E$ . The reduced density operator  $\rho_S(t)$  of  $S$  is computed by tracing over the environmental degrees of freedom,

$$\rho_S(t) = Tr_E \rho(t) \quad (2)$$

The EID approach adopts an open-system perspective: it concentrates the attention on the open subsystem  $S$  and, then, studies the time-evolution of  $\rho_S(t)$ , governed by an effective non-unitary master equation. For many physical models it is proved that, under certain definite conditions,  $\rho_S(t)$  converges to a stable state  $\rho_{S^*}$ :

$$\rho_S(t) \longrightarrow \rho_{S^*} \quad (3)$$

However, the same phenomenon can be viewed from a closed-system perspective, according to which the only univocally defined system is the whole closed system, whose physically meaningful magnitudes are the expectation values of its observables. In fact, since  $\rho_S(t)$  is defined as the density operator that yields the correct expectation values for the observables corresponding to the subsystem  $S$ ,

$$\langle O_R \rangle_\rho = \langle O_S \otimes I_E \rangle_\rho = Tr[\rho(O_S \otimes I_E)] = Tr[\rho_S O_S] = \langle O_S \rangle_{\rho_S} \quad (4)$$

the convergence of  $\rho_S(t)$  to  $\rho_{S^*}$  implies the convergence of the expectation values:

$$\langle O_R \rangle_{\rho(t)} = \langle O_S \rangle_{\rho_S(t)} \longrightarrow \langle O_S \rangle_{\rho_{S^*}} = \langle O_R \rangle_{\rho_*} \quad (5)$$

where  $\rho_*$  is a “final” diagonal state of the closed system  $U$ , such that  $\rho_{S^*} = Tr_E \rho_*$  (for details, see Castagnino *et al.* 2008). More precisely, the expectation value  $\langle O_R \rangle_{\rho(t)}$  can be computed as the sum of a term coming from the diagonal part of  $\rho(t)$  and a term coming from the non-diagonal part of  $\rho(t)$ : in the energy eigenbasis, this second term is what vanishes through the time-evolution,

$$\langle O_R \rangle_{\rho(t)} = \langle O_S \rangle_{\rho_S(t)} = \Sigma^d + \Sigma^{nd}(t) \longrightarrow \langle O_S \rangle_{\rho_{S^*}} = \langle O_R \rangle_{\rho_*} = \Sigma^d \quad (6)$$

This means that, although the off-diagonal terms of  $\rho(t)$  never vanish through the unitary evolution: the system decoheres from the observational viewpoint given by any observable belonging to the space  $\mathcal{O}_R$ .

From this closed-system perspective, the discrimination between system and environment turns out to be the selection of the relevant observables. By following Harshman and Wickramasekara (2007), we shall use the expression ‘tensor product structure’ (TPS) to call any factorization  $\mathcal{H} = \mathcal{H}_A \otimes \mathcal{H}_B$  of a Hilbert space  $\mathcal{H}$ , defined by the set of observables  $\{O_A \otimes I_B, I_A \otimes O_B\}$ , such that the eigenbases of the sets  $\{O_{A_i}\}$  and  $\{O_{B_i}\}$  are bases of  $\mathcal{H}_A$  and  $\mathcal{H}_B$  respectively. If  $\mathcal{H}$  corresponds to a closed system  $U$ , the TPS  $\mathcal{H} = \mathcal{H}_A \otimes \mathcal{H}_B$  represents the decomposition of  $U$  into two open systems  $S_A$  and  $S_B$ , corresponding to the Hilbert spaces  $\mathcal{H}_A$  and  $\mathcal{H}_B$  respectively. In turn, given the space  $\mathcal{O} = \mathcal{H} \otimes \mathcal{H}$  of the observables of  $U$ , such a decomposition identifies the spaces  $\mathcal{O}_A = \mathcal{H}_A \otimes \mathcal{H}_A$  and  $\mathcal{O}_B = \mathcal{H}_B \otimes \mathcal{H}_B$  of the observables of the open systems  $S_A$  and  $S_B$ , such that  $\mathcal{O}_A \otimes I_B \subset \mathcal{O}$  and  $I_A \otimes \mathcal{O}_B \subset \mathcal{O}$ . Once these concepts are considered, the selection of the space  $\mathcal{O}_R$  of relevant observables in the EID approach amounts to the selection of a particular TPS,  $\mathcal{H} = \mathcal{H}_S \otimes \mathcal{H}_E$ , such that  $\mathcal{O}_R = \mathcal{O}_S \otimes I_E \subset \mathcal{O} = \mathcal{H} \otimes \mathcal{H}$ .

In this paper we will consider the particular case where the closed system  $U$  is composed of  $n$  spin-1/2 particles, each represented in its Hilbert space. It is quite clear that  $U$  can be decomposed into the subsystems  $S$  and  $E$  in different ways, depending on which particles are considered as the open system  $S$ . In the following sections we will study the phenomenon of decoherence for different partitions of the whole closed system  $U$ .

### 3. The traditional spin-bath model

This is a very simple model that has been exactly solved in previous papers (Zurek 1982). Here we shall consider it from the closed-system perspective presented in the previous section.

Let us consider a closed system  $U = P \cup P_1 \cup P_2 \cup \dots \cup P_N = P \cup \left(\bigcup_{i=1}^N P_i\right)$ , where (i)  $P$  is a spin-1/2 particle represented in the Hilbert space  $\mathcal{H}_p$ , and (ii) each  $P_i$  is a spin-1/2 particle represented in its Hilbert space  $\mathcal{H}_i$ . The Hilbert space of the composite system  $U$  is, then,

$$\mathcal{H} = (\mathcal{H}_p) \otimes \left( \bigotimes_{i=1}^N \mathcal{H}_i \right) \quad (7)$$

In the particle  $P$ , the two eigenstates of the spin operator  $S_{P,\vec{v}}$  in direction  $\vec{v}$  are  $|\uparrow\rangle$  and  $|\downarrow\rangle$ . In each particle  $P_i$ , the two eigenstates of the spin operator  $S_{i,\vec{v}}$  in direction  $\vec{v}$  are  $|\uparrow_i\rangle$  and  $|\downarrow_i\rangle$ . Therefore, a pure initial state of  $U$  reads

$$|\Psi_0\rangle = (a|\uparrow\rangle + b|\downarrow\rangle) \otimes \left( \bigotimes_{i=1}^N (\alpha_i|\uparrow_i\rangle + \beta_i|\downarrow_i\rangle) \right) \quad (8)$$

where  $|a|^2 + |b|^2 = |\alpha_i|^2 + |\beta_i|^2 = 1$ . If the self-Hamiltonians  $H_P$  of  $P$  and  $H_i$  of  $P_i$  are taken to be zero, and there is no interaction among the  $P_i$ , then the total Hamiltonian  $H$  of the composite system  $U$  is given by the interaction between the particle  $P$  and each particle  $P_i$ . For instance (see Zurek 1982),

$$H = \frac{1}{2} (|\uparrow\rangle\langle\uparrow| - |\downarrow\rangle\langle\downarrow|) \otimes \sum_{i=1}^N \left( g_i (|\uparrow_i\rangle\langle\uparrow_i| - |\downarrow_i\rangle\langle\downarrow_i|) \otimes \left( \bigotimes_{\substack{j=1 \\ j \neq i}}^N I_j \right) \right) \quad (9)$$

where  $I_j = |\uparrow_j\rangle\langle\uparrow_j| + |\downarrow_j\rangle\langle\downarrow_j|$  is the identity operator on the subspace  $\mathcal{H}_j$  and the  $g_i$  are the coupling constants.

### 3.1. Decomposition I

In the typical situation studied by the EID approach, the open system  $S$  is the particle  $P$  and the remaining particles  $P_i$  play the role of the environment  $E$ :  $S = P$  and  $E = \bigcup_{i=1}^N P_i$ . Then, the TPS for this case is

$$\mathcal{H} = \mathcal{H}_S \otimes \mathcal{H}_E = (\mathcal{H}_P) \otimes \left( \bigotimes_{i=1}^N \mathcal{H}_i \right) \quad (10)$$

and the relevant observables  $O_R$  of  $U$  are those corresponding to the particle  $P$ :

$$O_R = O_S \otimes I_E = (s_{\uparrow\uparrow}|\uparrow\rangle\langle\uparrow| + s_{\uparrow\downarrow}|\uparrow\rangle\langle\downarrow| + s_{\downarrow\uparrow}|\downarrow\rangle\langle\uparrow| + s_{\downarrow\downarrow}|\downarrow\rangle\langle\downarrow|) \otimes \left( \bigotimes_{i=1}^N I_i \right) \quad (11)$$

The expectation value of these observables in the state  $|\psi(t)\rangle = |\Psi_0\rangle e^{-iHt}$  is given by (Castagnino *et al.* 2010a)

$$\langle O_R \rangle_{\psi(t)} = |a|^2 s_{\uparrow\uparrow} + |b|^2 s_{\downarrow\downarrow} + 2 \operatorname{Re}(ab^* s_{\downarrow\uparrow} r(t)) = \Sigma^d + \Sigma^{nd}(t) \quad (12)$$

where

$$r(t) = \langle \varepsilon_{\downarrow}(t) | \varepsilon_{\uparrow}(t) \rangle = \prod_{i=1}^N (|\alpha_i|^2 e^{-ig_i t} + |\beta_i|^2 e^{ig_i t}) \quad (13)$$

By means of numerical simulations it is shown that, for  $N \gg 1$ , in general  $|r(t)|^2 \rightarrow 0$  and, therefore,  $\Sigma^{nd}(t) \rightarrow 0$ : the particle  $P$  decoheres in interaction with a large environment  $E$  composed by  $N$  particles  $P_i$  (see Schlosshauer 2007; for larger values of  $N$  and realistic values of the  $g_i$  in typical models of spin interaction, see Castagnino *et al.* 2010a).

### 3.2. Decomposition 2

Although in the usual presentations of the model the system of interest is  $P$ , there are different ways of splitting the whole closed system  $U$ . For instance, we can decide to observe a particular particle  $P_j$  of what was previously considered the environment, and to consider the remaining particles as the new environment:  $S = P_j$  and  $E = P \cup \left( \bigcup_{i=1, i \neq j}^N P_i \right)$ . The total Hilbert space of the closed composite system  $U$  is still given by eq.(7), but now the TPS is

$$\mathcal{H} = \mathcal{H}_S \otimes \mathcal{H}_E = \left( \mathcal{H}_j \right) \otimes \left( \mathcal{H}_P \otimes \left( \bigotimes_{\substack{i=1 \\ i \neq j}}^N \mathcal{H}_i \right) \right) \quad (14)$$

and the relevant observables  $O_R$  are those corresponding to the particle  $P_j$ :

$$O_R = O_S \otimes I_E = \left( \zeta_{\uparrow\uparrow}^j |\uparrow^j\rangle\langle\uparrow^j| + \zeta_{\uparrow\downarrow}^j |\uparrow^j\rangle\langle\downarrow^j| + \zeta_{\downarrow\uparrow}^j |\downarrow^j\rangle\langle\uparrow^j| + \zeta_{\downarrow\downarrow}^j |\downarrow^j\rangle\langle\downarrow^j| \right) \otimes \left( I_P \otimes \left( \bigotimes_{\substack{i=1 \\ i \neq j}}^N I_i \right) \right) \quad (15)$$

The expectation value of these observables in the state  $|\psi(t)\rangle$  is given by (Castagnino *et al.* 2010a)

$$\langle O_R \rangle_{\psi(t)} = \left( |\alpha_j|^2 \zeta_{\uparrow\uparrow}^j + |\beta_j|^2 \zeta_{\downarrow\downarrow}^j \right) + 2 \operatorname{Re} \left( \alpha_j \beta_j^* \zeta_{\downarrow\uparrow}^j e^{ig_j t} \right) = \Sigma^d + \Sigma^{nd}(t) \quad (16)$$

In this case, numerical simulations are not necessary to see that the time-depending term of eq.(16) is an oscillating function which, therefore, has no limit for  $t \rightarrow \infty$ . This result is not surprising, but completely reasonable from a physical point of view. In fact, with the exception of the particle  $P$ , the remaining particles of the environment  $E$  are uncoupled to each other: each  $P_i$  evolves as a free system and, as a consequence,  $E$  is unable to reach a final stable state.

## 4. A generalized spin-bath model

Let us consider a closed system  $U = A \cup B$  where:

- (i) The subsystem  $A$  is composed of  $M$  spin-1/2 particles  $A_i$ , with  $i = 1, 2, \dots, M$ , each one represented in its Hilbert space  $\mathcal{H}_{A_i}$ : in each  $A_i$ , the two eigenstates of the spin operator  $S_{A_i, \vec{v}}$  in direction  $\vec{v}$  are  $|\uparrow_i\rangle$  and  $|\downarrow_i\rangle$ .
- (ii) The subsystem  $B$  is composed of  $N$  spin-1/2 particles  $B_k$ , with  $k = 1, 2, \dots, N$ , each one represented in its Hilbert space  $\mathcal{H}_{B_k}$ : in each  $B_k$ , the two eigenstates of the spin operator  $S_{B_k, \vec{v}}$  in direction  $\vec{v}$  are  $|\uparrow_k\rangle$  and  $|\downarrow_k\rangle$ .

The Hilbert space of the composite system  $U = A \cup B$  is, then,

$$\mathcal{H} = \mathcal{H}_A \otimes \mathcal{H}_B = \left( \bigotimes_{i=1}^M \mathcal{H}_{A_i} \right) \otimes \left( \bigotimes_{k=1}^N \mathcal{H}_{B_k} \right) \quad (17)$$

and a pure initial state of  $U$  reads

$$|\Psi_0\rangle = |\Psi_A\rangle \otimes |\Psi_B\rangle = \left( \bigotimes_{i=1}^M (a_i |\uparrow_i\rangle + b_i |\downarrow_i\rangle) \right) \otimes \left( \bigotimes_{k=1}^N (\alpha_k |\uparrow_k\rangle + \beta_k |\downarrow_k\rangle) \right) \quad (18)$$

with  $|a_i|^2 + |b_i|^2 = |\alpha_k|^2 + |\beta_k|^2 = 1$ . As in the original spin-bath model, the self-Hamiltonians  $H_{A_i}$  and  $H_{B_k}$  are taken to be zero, and there is no interaction among the particles  $A_i$  nor among the particles  $B_k$ . As a consequence, the total Hamiltonian  $H = H_A \otimes H_B$  of the composite system  $U$  is given by

$$H = \sum_{i=1}^M \left( \frac{1}{2} (|\uparrow_i\rangle\langle\uparrow_i| - |\downarrow_i\rangle\langle\downarrow_i|) \otimes \left( \bigotimes_{\substack{j=1 \\ j \neq i}}^M I_{A_j} \right) \right) \otimes \left( \sum_{k=1}^N \left( g_k (|\uparrow_k\rangle\langle\uparrow_k| - |\uparrow_k\rangle\langle\downarrow_k|) \otimes \left( \bigotimes_{\substack{l=1 \\ l \neq k}}^N I_{B_l} \right) \right) \right) \quad (19)$$

where  $I_{A_j} = |\uparrow_j\rangle\langle\uparrow_j| + |\downarrow_j\rangle\langle\downarrow_j|$  and  $I_{B_l} = |\uparrow_l\rangle\langle\uparrow_l| + |\downarrow_l\rangle\langle\downarrow_l|$  are the identity operators on the subspaces  $\mathcal{H}_{A_j}$  and  $\mathcal{H}_{B_l}$ , respectively. Let us notice that the eq.(9) of the original model is the particular case of eq.(19) for  $M = 1$ .

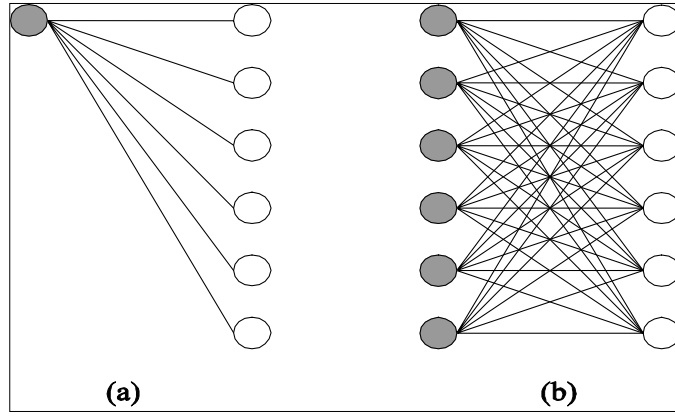


Figure 1. Schema of the interactions among the particles of the open system  $A$  (grey circles) and of the open system  $B$  (white circles): (a) original spin-bath model ( $M = 1$ ), and (b) generalized spin-bath model ( $M \neq 1$ )

#### 4.1. Decomposition 1

We can consider the decomposition where  $A$  is the open system  $S$  and  $B$  is the environment  $E$ . This is a generalization of Decomposition 1 in the traditional spin-bath model: the only difference is that here  $S$  is composed of  $M \geq 1$  particles instead of only one. Then, the TPS is

$$\mathcal{H} = \mathcal{H}_S \otimes \mathcal{H}_E = \left( \bigotimes_{i=1}^M \mathcal{H}_{A_i} \right) \otimes \left( \bigotimes_{k=1}^N \mathcal{H}_{B_k} \right) \quad (20)$$

and the relevant observables  $O_R$  are those corresponding to  $A$ :

$$O_R = O_S \otimes I_E = O_A \otimes \left( \bigotimes_{i=1}^N I_i \right) \quad (21)$$

When the expectation value  $\langle O_R \rangle_{\psi(t)} = \Sigma^d + \Sigma^{nd}(t)$  of the observables  $O_R$  in the state  $|\psi(t)\rangle$  is computed, two cases can be distinguished:

➤ **Case (a):**  $M \ll N$

Numerical simulations show that  $\Sigma^{nd}(t) \rightarrow 0$  very fast for increasing time (see Figure 2 of Castagnino *et al.* 2010a). This means that, as expected, a small open system  $S = A$  of  $M$  particles decoheres in interaction with a large environment  $E = B$  of  $N \gg M$  particles.

➤ **Case (b):**  $M \gg N$  or  $M \simeq N$

Numerical simulations show that  $\Sigma^{nd}(t)$  exhibits an oscillating behavior and, then, it does not approach zero for increasing time (see Figures 3 and 4 of Castagnino *et al.* 2010a). This means that, when the environment  $E = B$  of  $N$  particles is not large enough when compared with the open system  $S = A$  of  $M$  particles,  $S$  does not decohere.

#### 4.2. Decomposition 2

In this case we decide to observe only one particle of  $A$ . This amounts to splitting the closed system  $U$  into two new subsystems: the open system  $S$  is, say, the particle  $A_M$  and the environment is  $E = \left(\bigcup_{i=1}^{M-1} A_i\right) \cup B = \left(\bigcup_{i=1}^{M-1} A_i\right) \cup \left(\bigcup_{k=1}^N B_k\right)$ . Let us notice that the Decomposition 2 of the traditional spin-bath model is a particular case of this one, for  $N=1$  (where  $N$  plays the role of the  $M$  of this case). The TPS here is

$$\mathcal{H} = \mathcal{H}_S \otimes \mathcal{H}_E = \left(\mathcal{H}_{A_M}\right) \otimes \left( \left( \bigotimes_{i=1}^{M-1} \mathcal{H}_{A_i} \right) \otimes \left( \bigotimes_{k=1}^N \mathcal{H}_{B_k} \right) \right) \quad (22)$$

and the relevant observables  $O_R$  are those corresponding to  $A_M$ :

$$O_R = O_S \otimes I_E = O_{A_M} \otimes \left( \left( \bigotimes_{i=1}^{M-1} I_i \right) \otimes \left( \bigotimes_{k=1}^N I_k \right) \right) \quad (23)$$

When the expectation value  $\langle O_R \rangle_{\psi(t)} = \Sigma^d + \Sigma^{nd}(t)$  is computed, numerical simulations show that, if  $N \gg 1$ ,  $\Sigma^{nd}(t) \rightarrow 0$  very fast for increasing time (see Figures 5, 6 and 7 of Castagnino *et al.* 2010a). This means that the particle  $A_M$  decoheres when  $N \gg 1$ , independently of the value of  $M$ . But since the particle  $A_M$  was arbitrarily selected, the same argument holds for any particle  $A_i$  of  $A$ . Then, when  $N \gg 1$  and independently of the value of  $M$ , any particle  $A_i$  decoheres in interaction with its environment  $E$  of  $N + M - 1$  particles. On the other hand, the symmetry of the whole system  $U$  allows us to draw analogous conclusions when the system  $S$  is one of the particles of  $B$ : when  $M \gg 1$  and independently of the value of  $N$ , any particle  $B_k$  decoheres in interaction with its environment  $E$  of  $N + M - 1$  particles.



## 5. Decoherence as a relative phenomenon

### 5.1. Analyzing results

Let us consider the generalized spin-bath model when  $M \approx N \gg 1$ . In this case, the subsystem  $A = \bigcup_{i=1}^M A_i$  does not decohere (Decomposition 1), but the particles  $A_i$ , considered independently, do decohere (Decomposition 2). In other words, in spite of the fact that certain particles decohere and may behave classically, the subsystem composed by all of them retains its quantum nature. We have also seen that, since  $M \gg 1$ , all the particles  $B_k$ , considered independently, decohere. Then, in this case not only all the  $A_i$ , but also all the  $B_k$  decohere. This means that all the particles of the closed system  $U = \left(\bigcup_{i=1}^M A_i\right) \cup \left(\bigcup_{k=1}^N B_k\right)$  may become classical when considered independently, although the whole system  $U$  certainly does not decohere and, therefore, retains its quantum character.

The fact that certain particles may be classical or quantum depending on how they are considered sounds paradoxical in the context of an approach that explains decoherence as the result of an interaction between open systems. This difficulty can also be seen as a manifestation of the “looming big” problem of defining the open systems involved in decoherence. The irony of this story is that such a problem is the consequence of what has been considered to be the main advantage of the decoherence program, its open-system perspective, according to which particles interacting with other particles are well-defined open systems, and the collections of those particles are open systems too. So, the problem is to decide which one of all these open systems is the system  $S$  that decoheres or, in other words, where to place the cut between the system  $S$  and its environment  $E$ .

The open-system approach not only leads to the “looming big” problem, but in a certain sense also disregards the well-known holism of quantum mechanics: a quantum system is not the mere collection of its parts and the interactions among them. In order to retain its holistic nature, a quantum system has to be considered as a whole: the open “subsystems” are only partial descriptions of the whole closed system. On the basis of this closed-system perspective, we can develop a different conceptual viewpoint for understanding decoherence.

### 5.2. A closed-system perspective

As we have seen, a TPS expresses the decomposition of the closed system  $U$  into two open systems  $S_A$  and  $S_B$ , which amounts to the split of the whole space  $\mathcal{O} = \mathcal{H} \otimes \mathcal{H}$  of the observables of  $U$  into the subspaces  $\mathcal{O}_A = \mathcal{H}_A \otimes \mathcal{H}_A$  and  $\mathcal{O}_B = \mathcal{H}_B \otimes \mathcal{H}_B$  such that  $\mathcal{O} = \mathcal{O}_A \oplus \mathcal{O}_B$ . In particular, the total Hamiltonian of  $U$ ,  $H \in \mathcal{O}$ , can be expressed as  $H = H_A \otimes I_B + I_A \otimes H_B + H_{AB}$ , where  $H_A \in \mathcal{O}_A$

is the Hamiltonian of  $S_A$ ,  $H_B \in \mathcal{O}_B$  is the Hamiltonian of  $S_B$ , and  $H_{AB} \in \mathcal{O}$  is the interaction Hamiltonian, representing the interaction between the open systems  $S_A$  and  $S_B$ .

In general, a quantum system  $U$  admits a variety of TPSs, that is, of decompositions into  $S_A$  and  $S_B$ , each one defined by the space of observables  $\mathcal{O}_A$  of  $S_A$  and  $\mathcal{O}_B$  of  $S_B$  (Harshman and Wickramasekara 2007). Among all these possible decompositions, there may be a particular TPS that remains *dynamically invariant*. This is the case when there is no interaction between  $S_A$  and  $S_B$ ,  $H_{AB} = 0$ , and, then,

$$[H_A \otimes I_B, I_A \otimes H_B] = 0 \quad \Rightarrow \quad \exp[-iHt] = \exp[-iH_A t] \exp[-iH_B t] \quad (24)$$

Therefore,

$$\rho_A(t) = Tr_{(B)}\rho(t) = Tr_{(B)} \left[ e^{-iHt} \rho_0 e^{iHt} \right] = e^{iH_A t} \left[ Tr_{(B)}\rho_0 \right] e^{-iH_A t} = e^{iH_A t} \rho_{0A} e^{-iH_A t} \quad (25)$$

$$\rho_B(t) = Tr_{(A)}\rho(t) = Tr_{(A)} \left[ e^{-iHt} \rho_0 e^{iHt} \right] = e^{iH_B t} \left[ Tr_{(A)}\rho_0 \right] e^{-iH_B t} = e^{iH_B t} \rho_{0B} e^{-iH_B t} \quad (26)$$

This means that, even if the initial state  $\rho_0$  of  $U$  is an entangled state with respect to the TPS  $\mathcal{H} = \mathcal{H}_A \otimes \mathcal{H}_B$ ,  $S_A$  and  $S_B$  are *dynamically independent*: each one evolves unitarily under the action of its own Hamiltonian. As a consequence, the subsystems  $S_A$  and  $S_B$  resulting from this particular TPS do not decohere.

Once we have excluded the dynamically invariant TPS, all the remaining TPSs of  $U$  define subsystems  $S_A$  and  $S_B$  such that  $H_{AB} \neq 0$ . As a result of the interaction,  $S_A$  and  $S_B$  evolve non-unitarily; then, depending on the particular  $H_{AB}$ , they may decohere. But the point to stress here is that there is no privileged non-dynamically invariant decomposition of  $U$ : each partition of the closed system into  $S_A$  and  $S_B$  is just a way of selecting the spaces of observables  $\mathcal{O}_A$  and  $\mathcal{O}_B$ .

When we adopt this closed-system perspective, it turns out to be clear that there is no essential criterion for identifying the “open system” and its “environment”. Given the closed system  $U$ , that identification requires two steps: (i) to select a TPS  $\mathcal{H} = \mathcal{H}_A \otimes \mathcal{H}_B$  such that  $U = S_A \cup S_B$ , and (ii) to decide that one of the systems resulting from the decomposition, say  $S_A$ , is the open system  $S$ , and the other,  $S_B$ , is the environment  $E$ . Since the TPS is defined by the spaces of observables  $\mathcal{O}_A$  and  $\mathcal{O}_B$ , the decomposition of  $U$  is just the adoption of a descriptive perspective: the identification of  $S$  and  $E$  amounts to the selection of the relevant observables in each situation. But since the split can be performed in many ways, with no privileged decomposition, there is no need of an unequivocal criterion for deciding where to place the cut between “the” system and “the” environment. Decoherence is not a yes-or-not process, but a phenomenon relative to the chosen decomposition of the whole closed quantum system. When viewed from this closed-system perspective, Zurek’s

“looming big problem” is not a real threat to the decoherence program: the supposed challenge dissolves once the relative nature of decoherence is taken into account.

From this perspective, the perplexities derived from the generalized spin-bath model vanish. In fact, when we consider the whole closed system  $U$ , there is no difficulty in saying that from the viewpoint of the space of observables, say,  $\mathcal{O}_{A_1}$  (corresponding to the particle  $A_1$ ) there is decoherence, but from the viewpoint of the space of observables  $\mathcal{O}_A$  (corresponding to the open subsystem  $A = \bigcup_{i=1}^M A_i$ ) there is no decoherence. Moreover, even if there is decoherence from the viewpoint of all the  $\mathcal{O}_{A_i}$ , this does not imply decoherence from the viewpoint of  $\mathcal{O}_A$  since, as it is well-known,  $\mathcal{O}_A$  is not the mere union of the  $\mathcal{O}_{A_i} \otimes \left( \bigotimes_{j=1, j \neq i}^M I_j \right)$ . In other words, in agreement with quantum holism, the open subsystem  $A$  is not the mere collection of the particles  $A_i$ ; then, it is reasonable to expect that the behavior of  $A$  cannot be inferred from the behavior of all the  $A_i$ . In the same sense, it is not surprising that there is no decoherence from the viewpoint of the total space of observables  $\mathcal{O}$  of  $U$ , in spite of the fact that there is decoherence from the viewpoint of anyone of the  $\mathcal{O}_{A_i}$  and  $\mathcal{O}_{B_k}$ , corresponding to the particles  $A_i$  and  $B_k$  respectively. And since the privileged viewpoint does not exist, the conclusions about decoherence have to be relativized to the particular observational perspective selected in each case.

### 5.3. Decoherence and dissipation

As pointed out in the Introduction, certain presentations of the EID approach suggest the existence of a certain relationship between decoherence and dissipation, as if decoherence were a physical consequence of energy dissipation. Some particular models studied in the literature on the subject tend to reinforce this idea by describing the behavior of a small open system –typically, a particle– immersed in a large environmental bath. On this basis, the EID approach has been considered a “dissipative” approach, by contrast to “non-dissipative” accounts of decoherence that constitute the “heterodoxy” in the field (see Bonifacio *et al.* 2000, Ford and O’Connell 2001, Frasca 2003, Sicardi Shifino *et al.* 2003, Gambini *et al.* 2006).

The fact that energy dissipation is not a condition for decoherence has been clearly stressed by Schlosshauer (2007), who says that “*decoherence may, but does not have to, be accompanied by dissipation, whereas the presence of dissipation also implies the occurrence of decoherence*” (p.93). This fact is explained by stressing that the loss of energy from the system is a classical effect, leading to thermal equilibrium in the *relaxation time*, whereas decoherence is a pure quantum effect that takes place in the *decoherence time*, many orders of magnitude shorter than the relaxation time: “*If*

*dissipation and decoherence are both present, they are usually quite easily distinguished because of their very different timescales*” (Schlosshauer 2007, p.93). According to the author, it is this crucial difference between relaxation and decoherence timescales what explains why we observe macroscopic objects to follow Newtonian trajectories –effectively “created” through the action of decoherence– with no manifestation of energy dissipation, such as a slowing-down of the object. Schlosshauer recalls an example used by Joos (Joos *et al.* 1996): the planet Jupiter has been revolving around the sun on a Newtonian trajectory for billions of years, while its motional state has remained virtually unaffected by any dissipative loss.

This explanation, although correctly stressing the difference between decoherence and dissipation, seems to present both phenomena on the same footing: an open system would first become classical through decoherence, and would then relax due to energy dissipation. According to this picture, whereas dissipation involves the loss of energy from the system to the environment, decoherence amounts to a sort of “dissipation” of coherence which leads the open system, in a very short time, to the classical regime: the environment plays the role of a “sink” that carries away the information about the system (Schlosshauer 2007, p.85). The results obtained in the generalized spin-bath model show that the coherence-dissipation or information-dissipation picture has to be considered with great caution, as a mere metaphor. In fact, to the extent that decoherence is a relative phenomenon, no flow of a non-relative quantity from the open system to the environment can account for decoherence. In particular, although energy dissipation and decoherence are in general easily distinguished because of their different timescales, the very reason for their difference is that energy dissipation is not a relative phenomenon, whereas decoherence is relative to the observational partition of the whole closed system selected in each situation. On the other hand, decoherence can be explained in terms of the flow of information from the open system to the environment if information is also conceived as a relative magnitude (Lombardi 2004, 2005).

## *6. Conclusions*

The aim of this paper has been to argue that environment-induced decoherence can be viewed from a closed-system perspective, which improves the understanding of the phenomenon. For this purpose, we have analyzed the results obtained in the traditional spin-bath model and in a generalization of that model. By considering different partitions of the whole closed system in both cases, we have shown how decoherence depends on the way in which the relevant observables are selected. On this basis, the following conclusions can be drawn:

- (i) Decoherence is a phenomenon relative to which degrees of freedom of the whole closed system are considered relevant and which are disregarded in each situation.
- (ii) Since there is no privileged or essential decomposition of the closed system, there is no need of an unequivocal criterion for identifying the systems involved in decoherence. Therefore, the “looming big problem” –which, according to Zurek, poses a serious threat to the whole decoherence program– dissolves in the light of the relativity of decoherence.
- (iii) Due to its relative nature, decoherence cannot be accounted for in terms of dissipation of energy or of any other non-relative magnitude.

Once the phenomenon of decoherence is “de-substantialized” in this way, one might ask in what sense it can be still understood as the result of the action of an environment that destroys the coherence between the states of a quantum system by its incessant “monitoring” of the observables associated with the preferred states (Paz and Zurek 2002, Zurek 2003). One might consider whether it is not time to leave aside the picture according to which it is the environment what “distills” the classical essence from quantum systems (Castagnino *et al.* 2010b).

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