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A Model-Based Method of Extension and Intension

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Abstract: The semantic method of extension and intension presented by Rudolf Carnap may have some problems from the current perspective in cognitive science. For example, that method is strongly linked to logic, and it is hard to determine where its state-descriptions come from, how many they are, and what the relation of accessibility between them is. This paper tries to solve difficulties such as those by updating the method resorting to the theses and results of a contemporary reasoning theory: the theory of mental models. Basically, the update consists of replacing the state-descriptions of the original method with the possibilities of the theory of mental models. That allows reformulating several relevant concepts in Carnap's method in order to better adjust them to the real characteristics of human language.

Keywords: *fact, mental model, necessity, possibility, state-description*

Introduction

In 1947, Rudolf Carnap presented his semantic method of extension and intension (Carnap, 1947), which continues to be a method with a great potential. For example, it has allowed the analysis of different linguistic, logical, and philosophical problems (e.g., López-Astorga, 2018; 2019). However, that method has several difficulties.

On the one hand, it is possible that it is away from the actual way people understand language. Some researches in the area of cognitive science have

shown that the human intellectual activity is not always coherent with what is provided by logic (e.g., Johnson-Laird *et al.*, 2015). Thus, given that the method of extension and intension is based upon logic, one might think that it does not really describe the human language.

On the other hand, one of the general problems of modal logic, and hence of Carnap's (1947) method in particular, which is linked to modal logic, is that, when used to analyze reasoning or human mental activities, some inconveniences emerge. Some of them can be that the number of worlds which can be taken as possible and which people can access from their actual world can be excessively high (e.g., Partee, 1979), and that it is required to explain where the possible worlds individuals consider come from, or, if preferred, in terms of modal logic, what the relation of accessibility from one world to another exactly is (see also, e.g., Johnson-Laird & Ragni, 2019; Khemlani *et al.*, 2017).

This being the case, the present paper proposes an adaptation of the semantic method of extension and intension, following the results and developments of the theory of mental models (e.g., Bucciarelli & Johnson-Laird, 2019; Byrne & Johnson-Laird, 2020; Khemlani & Johnson-Laird, 2019; Quelhas *et al.*, 2019). That will be done because it may allow removing difficulties such as those pointed out above for Carnap's method. Nevertheless, this paper will not present a complete readjustment of the method of extension and intension. It will only update some of its most significant concepts that can be used in the analysis of sentences and sentential connectives such as the conditional, conjunction, and disjunction. This will be so for at least two reasons. First, although the method provided by Carnap (1947) can have many other applications, it seems that it can be utilized more easily in those kinds of analyses (see, e.g., López-Astorga, 2019). Second, from what will be described below for sentences and sentential connectives, the necessary settings to deal with other linguistic elements taken into account by Carnap (1947) in his method (e.g., classes, proprieties, designators, etc.) can be obvious and trivial to address.

Thus, to do all this, first, the concepts of the semantic method of extension and intension that will be reviewed in order to establish a new proposal will be indicated. Then, an explanation of the general framework of the theory of mental models will be given. Finally, a redefinition from this very theory of the concepts offered by Carnap (1947) will be presented.

Carnap's method and sentences

As Carnap (1947) acknowledges, Frege was the first writer to distinguish concepts such as those of extension and intension. Following that differentiation, according to the former, as other linguistic elements, sentences have an extension and an intension. Their truth value should be understood as the extension, and the proposition referred to in each case should be deemed as the intension.

Carnap's argument to link extension to truth value is based upon the idea that, when the same degree is shared by two predicators, and that degree is equal to or higher than 1, the extension of those predicators is also the same when there is logical equivalence between them. For instance, if P and Q are predicators with degree 3, then their extension is identical if and only if

(1) $\forall xyz\,(Pxyz \leftrightarrow Qxyz)$

Where x, y, and z are variables for predicators, $\forall$ is the universal quantifier, and $\leftrightarrow$ stands for biconditionality or logical equivalence.

This, following Carnap (1947), authorizes to assume that, if the degree is 0, then their extensions match when their truth values match, that is, when:

(2) $P \leftrightarrow Q$

And this is because, in (2), P and Q are logically equivalent because, precisely, their truth values are not different.

As far as the intension is concerned, the proposition described means that sentences in different languages can refer to the same intension if they transmit exactly the same proposition. Perhaps this is beyond Carnap's (1947) interest, since his limits seem to be English and logical language. Nonetheless, it appears to imply that, for instance, the intension that can be attributed to (3), which is an English sentence, (4), which is the translation of (3) into Spanish, and (5), which is the translation of (3) into Portuguese, is the same.

(3) She goes to work by car

(4) Ella va al trabajo en auto

(5) Ela vai ao trabalho de carro

Indeed, one might think that the intensions that can be assigned to (3), (4), and (5) are not distinct because they represent the same fact: she uses a car to go to work.

However, another important point of the semantic method of extension and intension is that, if two sentences share their extension when they are equivalent, they also share their intension when they are L-equivalent. So, it is necessary to account for what this last relation is, too.

For Carnap (1947), 'L-concepts' are concepts considering all the possible worlds. So, something is L-true if and only if its value is true in every conceivable, accessible, and imaginable possible world, that is, if and only if it is analytically true or necessary (the influence that Carnap's work had on modal logic approaches, such as those of Kripke, 1963, 1965, is well known). Nevertheless, he does not use the expression 'possible world', but 'state-description.' In this way, it can be stated that a sentence is L-true if and only if there is no state-description in which it is false. Likewise, two sentences are L-equivalent if and only if, when one of them is true in a particular state-description, the other one is true in that very state-description as well. Evidently, this last definition leads to assume that those two sentences that are L-equivalent are also two sentences related to an identical intension.

Clearly, from all of this, other definitions can be derived too. One example can be the one of F-truth or factual truth. According to Carnap (1947), something is F-true if and only if it is that by virtue of empirical criteria. The advantage of definitions such as this one is that they, in turn, allow better understanding of concepts such as the one of L-truth, as they lead to conclude that something is L-true if and only if it is that by virtue of the language or a priori (in the Kantian sense of this last expression).

Furthermore, as pointed out, L-truth and necessity are the same in Carnap's (1947) method, and possibility, as usual in modal logic, refers to the fact that something happens in, at a minimum, one state-description. Therefore, what is possible is not true by virtue of the language and is more linked to factual truth.

Obviously, the semantic method of extension and intension addresses much more linguistic and logical elements, and develops many more aspects. Nevertheless, for the aims of this paper, which focuses on sentences and sentential connectives, those described can be enough.

The theory of mental models and modality

The theory of mental models is also an approach with important developments and dealing with very different aspects related to reasoning and cognition. For this reason, it is necessary to make a selection of its theses here too and consider only those that are relevant for the goals of this article. In this way, maybe it is important to mention two points.

On the one hand, it is a dual-process theory (e.g., Johnson-Laird & Ragni, 2019). Hence, following it, people have two mental systems—System 1 and System 2. If they only use System 1, which is related to intuition, they can be aware of less possibilities associated to a sentence. Only if they use System 2, which is related to reflection and analysis, they can discover all of the possibilities the theory of mental models assigns to a particular sentence. Nonetheless, this paper will only take into account perfect situations in which the individual resorts to System 2 and identify all the possibilities.

On the other hand, for exposition purposes, some of its concepts will be simplified as well. The theory of mental models not only speaks about possibilities, but also, for example, about presuppositions (e.g., Johnson-Laird & Ragni, 2019). However, given that the main aim here is a reformulation of Carnap's (1947) method, basically only possibilities will be considered. Thus, the trend will be to ignore the other concepts of the theory more linked to the manner individuals reason.

Having said that, it can be stated that, in general, for the theory of mental models, sentential connectives allow building 'compound assertions', and that compound assertions relate sets of possibilities by means of conjunctions (e.g., Khemlani *et al.*, 2017; Johnson-Laird & Ragni, 2019). In this way, the theory addresses, even since its inception (see, e.g., Oakhill & Garnham, 1996) and in other previous versions of it (see, e.g., Johnson-Laird, 2012), the traditional logical connectives assigning a set of possibilities to each of them.

In the last version of the theory, as said, those sets of possibilities are 'conjunctions of possibilities.' However, precisely conjunction, as a sentential connective, can be an exception in this sense. It only expresses one possible situation, and is really a statement of a fact, and not of a possibility (e.g., Johnson-Laird & Ragni, 2019). So, given a conjunction such as

(6) P and Q

only one situation can be thought, the one in which both P and Q are true, that is,

(7) P & Q

Actually, (7) is a model that iconically reproduces reality in the individual's mind. Of course, the philosophical and metaphysical interest an exploration of the real nature of iconic models such as (7) can have is indisputable. Nevertheless, that discussion is beyond the purposes of this paper and, perhaps, what is interesting now is that the case of the disjunction is different.

Indeed, disjunction refers to several models and, therefore, several possibilities. For instance, if it is inclusive and expressed in a way akin to this one:

(8) Either P or Q, or both of them

Following the literature (e.g., Khemlani *et al.*, 2014; Johnson-Laird & Ragni, 2019), three models or possibilities can be derived from it. As indicated and shown in (9), the set is really a conjunction of possibilities.

(9) Possible (P & Q) & Possible (P & not-Q) & Possible (not-P & Q)

Obviously, each conjunct in (9) is a model iconically describing a possible reality. Nonetheless, according to the same literature, the set can be modified if the particular disjunction is exclusive and is such as (10).

(10) Either P or Q, but not both of them

The possibilities of (10) cannot be the same as those of (8). Now, the first conjunct in (9) has to disappear, the resulting conjunction being as follows:

(11) Possible (P & not-Q) & Possible (not-P & Q)

Regarding the conditional, that is, sentences such as

(12) If P, then Q

Papers supporting the theory of mental models similar to those cited above reveal that the conjunction of possibilities would be:

(13) Possible (P & Q) & Possible (not-P & Q) & Possible (not-P & not-Q)

Lastly, the literature of the theory of mental models also informs of the case of the biconditional. In this way, if the sentence is of this kind:

(14) P if and only if Q

Its conjunction would have this structure:

(15) Possible (P & Q) & Possible (not-P & not-Q)

However, all this is, as said, in ideal circumstances, that is, when individuals use System 2 and consider all of the possibilities. The theory also explains what occurs when only System 1 is used and shows which the possibilities identified in each of the cases accounted for—(6), (8), (10), (12), and (14)—are in those situations (see, e.g., Table 2 in Johnson-Laird & Ragni, 2019, where the possibilities corresponding to the sentential connectives reviewed and other expressions are indicated, whether System 1 or System 2 is utilized). But maybe the most interesting point of the theory for this section is that it proposes, as an essential part of it, the action of modulation. Modulation can act by virtue of semantics and pragmatics, and its main result is that it can alter the conjunctions of possibilities (see, e.g., Johnson-Laird *et al.*, 2015). One example can be the case of (16).

(16) "Pat visited Milan or she visited Italy"
(Johnson-Laird *et al.*, 2015, p. 204).

In principle, one might think that (16) is a disjunction. In fact, it could be inclusive, as Pat can be in Milan and Italy at once. However, that is not the case because its conjunction of possibilities is not (9), but (17).

(17) Possible (P & Q) & Possible (not-P & Q)

The missing conjunct is the second one in (9), and the reason for its absence is clear: if Pat goes to Milan, she necessarily goes to Italy too.

According to the theory of mental models, phenomena such as the one of (16) and (17) allow explaining experimental results that are very hard to account for based on an exclusively formal logical framework. For example, this deduction is correct in classical logic:

(18) Q $\therefore$ P $\vee$ Q

Where $\therefore$ is the symbol for logical deduction and $\vee$ denotes inclusive disjunction.

Nevertheless, people tend to assess (18) as a wrong inference (e.g., Orenes & Johnson-Laird, 2012). But, following the theory, the reason for the rejection is obvious: it cannot be accepted that (8) can be inferred from Q because (8) refers

to conjunction of possibilities (9), and in (9) there is a possibility (the second one) in which Q is false. So, to accept an inference as (18) would be to accept an inference in which the conclusion admits as a possibility that the premise can be false (Orenes & Johnson-Laird, 2012).

And the theory of mental models can go even further. One of its predictions is that, in the cases in which the conclusion is a sentence such as (16), that is, a sentence whose conjunction of possibilities is such as (17), and the premise is the statement of the second disjunct, that is, of the disjunct that appears in the two possibilities of (17), a derivation in natural language with the structure of (18) should be accepted. The reason is evident here as well—the premise would be negated by no possibility in the conclusion. This prediction, as well as other similar ones of the theory, has been experimentally confirmed (e.g., Orenes & Johnson-Laird, 2012).

There are much more cognitive problems or phenomena reported by the scientific literature which logic cannot solve and for which, however, the theory of mental models does offer an explanation. Nonetheless, this last one implying (8), (9), (16), (17), and (18) can be enlightening enough to understand how this theory works and why it is so relevant in cognitive science today.

The method of extension and intension and the theory of mental models

Links between Carnap's (1947) semantic method and the theory of mental models have already been provided (e.g., López-Astorga, 2020). In addition, in the same way as the method of extension and intension, the theory of mental models has also been used to review some linguistic, logical, or philosophical problems (see, e.g., the references in this direction in López-Astorga, 2019). However, the aim in this paper is not just to find relations between the two frameworks. It is about updating Carnap's (1947) method from the theses of the theory of mental models.

At the beginning of this paper, it was said that the method of extension and intension by Carnap (1947) has problems because of its links to logic. Therefore, to relate the method to the theory of mental models can remove those problems. The reason for that is simple: The theory of mental models is not logic. The possibilities in the conjunctions of possibilities do not correspond to rows in

truth tables. First, they are iconic models of reality, and not formulae representing propositions (e.g., Johnson-Laird, 2012). Second, while the rows in truth tables cannot be accepted at once, the sets of possibilities are conjunctions. This means that, unlike the rows in truth tables, the possibilities must be true at the same time, since conjunction requires that (e.g., Johnson-Laird & Ragni, 2019). In addition, as shown in most of the works supporting the theory of mental models cited above, in contrast to the possible worlds in modal logic, the possibilities the theory assigns to sentences are very few (usually, at most three). That makes the possibilities easy to deal with. Furthermore, the theory clearly explains how the possibilities are generated (for comparisons of the theory of mental models with modal logic, see, e.g., Johnson-Laird & Ragni, 2019; Khemlani *et al.*, 2017).

Hence, to update Carnap's (1947) method by means of the theory of mental models is not a reinterpretation of that method from an alternative (e.g., non-classical) logic. It is only a reinterpretation from a cognitive framework trying to account for the way people make inferences. Thus, the goal is not to offer new cognitive data or a new interpretation of empirical data in cognitive science. The present paper is only intended to adapt the semantic method of extension and intension to the manner, following the theory of mental models, people reason. Beyond the fact that that manner does not match with the requirements logic provides, if the theory is right, what the arguments below will present is an adaptation of Carnap's method to the real way individuals think.

Besides, an update of this kind implies that the concepts of extension and intension are not semantic concepts (in the logical sense of the adjective 'semantic') anymore. They need to be linked to possibilities such as those the theory of mental models proposes, and not to truth values and state-descriptions (or possible worlds). Then, the way all of this could be done is described.

To start a dialogue between the two approaches, it seems to be necessary to determine the manner the central elements in the semantic method of extension and intension could be understood from the theory of mental models. Thus, it could be claimed that the possibilities of the latter correspond to the state-descriptions of the former. Nevertheless, the possibilities of the theory of mental models do not have the same characteristics as state-descriptions. For this reason, the acceptance of the theory of mental models can require certain changes and nuances in Carnap's method. The first step in this sense could be, for example, to replace the expression 'state-descriptions' with the word 'possibilities' and acknowledge their iconic nature.

In this way, another important point is that the possibilities of the theory of mental models are not, as said, values in a row in a truth table. They express contradictory possible situations and, accordingly, if they were values of that kind, they could not be related by means of conjunctions (e.g., Johnson-Laird & Ragni, 2019). Therefore, it could not be stated that, under the theory of mental models, the extension of a sentence continues to be a truth value. What should be said is that, under that theory, the extension refers to the fact that the sentence is or is not true in a particular possibility. So, from this perspective, it could be claimed that two sentences have identical extension, and hence are equivalent, if and only if both of them happen or both of them do not happen in the particular possibility being considered.

With regard to intension, at this point, the adaptation would be easy. It would be enough to assume that two sentences have identical intension, and hence are L-equivalent, if and only if they occur in exactly the same possibilities. As far as this point is concerned, it is clear that, for instance, (3), (4), and (5) denote the same situation. So, what is indicated by them has to happen in the same possibilities.

Furthermore, all of this would be coherent with an adaptation to the definitions that the proponents of the theory of mental models generally attribute to concepts such as 'possible', 'necessary', and 'fact.' According to the theory, given a conjunction of possibilities related to a sentence, if something appears in, at a minimum, one of those possibilities, it is possible; if something happens in all of the possibilities of the conjunction, it is necessary; and, if there is no conjunction of possibilities, but just one, and something occurs in that only possibility, it is a fact (Khemlani *et al.*, 2017).

So, it would be still plausible to keep using the concepts of L-truth and F-truth, which, evidently, could also be linked, from this new perspective, to those of possibility and necessity such as they are interpreted by the theory of mental models. A sentence would be F-true if and only if it is true in, at a minimum, one possibility of a conjunction of possibilities, and L-true if and only if it is true in all of the possibilities of a conjunction of possibilities.

This adaptation of Carnap's (1947) semantic method of extension and intension would continue to have the same potential to account for logical, linguistic, and philosophical problems. Maybe this is obvious and almost trivial to show. However, for the purpose of illustration only, just an example is described below. It refers to the analysis that can be made of an axiom that has already been

considered from the original version of the method provided by Carnap (López-Astorga, 2018). As it can be checked then, it is not hard to also address it from this model-based update of the method.

The axiom is exactly one of those presented by Prior (2012) in order to prove Hintikka's theorem, which, as is well known, links the fact of something being impossible to the fact of that being forbidden. The axiom is as follows:

(19) $N(\mathrm{P} \rightarrow \mathrm{Q}) \rightarrow (O\mathrm{P} \rightarrow OQ)$

Where N stands for the modal operator of necessity, $\rightarrow$ is the conditional, and O represents the deontic operator of obligation.

Axiom (19) is, with different symbols, axiom [I] in López-Astorga (2018), or axiom (3) in Øhrstrøm, Zeller, and Sandborg-Petersen (2012), and, as pointed out, was revised from the method of extension and intension. The result of that review was that (19) should be assumed and that, under Carnap's (1947) method, is L-true (López-Astorga, 2018). Nevertheless, the new version of that method in the present paper can lead to the same conclusions, and in a very easy way.

The first point to consider is that (19) establishes a conditional relation between its antecedent and its consequent, that is, between $N(\mathrm{P} \rightarrow \mathrm{Q})$ and $O\mathrm{P} \rightarrow OQ$. As shown above, if the antecedent of a conditional is false, the value of the consequent does not matter. The two last conjuncts in (13) reveal that, in those cases, both if the consequent is true and if it is false, the complete compound assertion is correct. So, it is only required to examine the cases in which the antecedent is true.

Given that the antecedent in (19) claims that the conditional relation between P and Q is necessary, it states that, in its conjunction of possibilities, there cannot be a conjunct in which P is true and Q is not, that is, it states that

(20) Not-possible (P & not-Q)

Indeed, (20) represents the only impossibility for a necessary conditional such as the antecedent of the axiom (for an explanation of how, according to the theory of mental models, people note impossible situations, see, e.g., Johnson-Laird & Ragni, 2019). Hence, (13) refers to the only possible conjuncts for (19). Of course, this is in perfect situations and without modulation (for a presentation of the different interpretations that a conditional can have in daily life following previous versions of the theory of mental models, see, e.g., Johnson-Laird &

Byrne, 2002). But, beyond that, what is more important now is that the first conjunct in (13) describes the only situation in which P can be true, and, in that very situation, Q is also true. This means that, in a world, reality, or situation in which P were obligatory, Q would have to occur as well. And this last fact reveals that if P is obligatory, Q is that too ($OP \rightarrow OQ$), and, in turn, that (19) is correct and needs to be accepted. Besides, for evident reasons, the hypothetical case in which P were not obligatory can be ignored. It would have an influence in no way on the argumentation presented, and it would raise no doubts regarding the acceptance of (19).

Conclusions

An old debate in philosophy and logic, which seems to come from the account of the conditional given by Chrysippus of Soli, is that related to the kinds of impossibility, since one might think that it can be logical or empirical (see, e.g., Gould, 1970). It is clear that this type of discussion can disappear by resorting to a model-based method of extension and intension such as the one proposed here. Nevertheless, in spite of that, one might keep asking whether or not an update such as the one provided in this paper is necessary.

There are facts that seem to show that it is. As mentioned in the introduction, the original version by Carnap (1947) has some problems that this reformulation removes, or at least helps to remove. One of those problems is linked to the relations between logic, reasoning, and human language. That relation is not evident and that fact can be a challenge for Carnap's (1947) method (it can also be a challenge even for modal logic systems such as those presented by Kripke, 1963, 1965; but the detailed analysis of this point is beyond the goal of the present paper). Nonetheless, if the cognitive science literature is reviewed, it can be said that the theory of mental models seems to be able to explain most intellectual behaviors people have in reasoning tasks. Those behaviors usually appear to be inconsistent with logic. However, the theory of mental models does not have important difficulties to even predict them. One example has been described above by means of (8), (9), (16), (17), and (18), but, as also indicated, there are much more cognitive phenomena coherent with the theses of the theory of mental models. Some of the most important works showing that are recorded in Table 3 in Johnson-Laird and Ragni (2019), and they are Bell and Johnson-Laird (1998), Bucciarelli and Johnson-Laird (2005), Evans, Handley,

Harper and Johnson-Laird (1999), Hinterecker, Knauff and Johnson-Laird (2016), Khemlani, Barbey and Johnson-Laird (2014), Khemlani and Johnson-Laird (2009), Orenes and Johnson-Laird (2012), and a paper in preparation titled 'Reasoning about possibilities' by Ragni and Johnson-Laird.

Nevertheless, this reformulation appears to eliminate other disadvantages as well. The theory of mental models only assigns a limited number of possibilities (linked by conjunctions) to sentences. Thus, one can clearly know which they are, where they come from, and how they are. This allows even developing software packages emulating human reasoning and based upon the theory (e.g., Khemlani *et al.*, 2017). Given that there is no single way to define relations such as the one of accessibility between possible worlds (and hence between state-descriptions), the framework described in the works supporting the theory of mental models makes it clear that those developments seem hardly to do with methods such as the one of extension and intension. Nonetheless, because the semantics of the theory of mental models is clearer, its case can be simpler (for all of these points, see also, e.g., Johnson-Laird & Ragni, 2019).

So, it only remains to work in two directions. As indicated, Carnap's (1947) method of extension and intension has a much broader scope. Therefore, it would be very interesting to check whether or not this adaptation continues to be acceptable when its most particular details are addressed from the theory of mental models. On the other hand, the review of other formulae, axioms, and linguistic problems that have already been dealt with from Carnap's method would also be advisable. This is because it would allow checking whether the model-based approach proposed above can offer an analysis of such formulae, axioms, and problems without difficulties, too. In principle, it can be thought that the result of the latter task would be obvious. However, maybe the verification in practice of that result could also be an important point. Undoubtedly, it would offer more arguments for keeping using the theory of mental models in the future as a method of analysis of language more applicable than the one of Rudolf Carnap.

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