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Research Article

Multiparty Evolutionary Game Model in Coal Mine Safety Management and Its Application

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Coal mine safety management involves many interested parties and there are complex relationships between them. According to game theory, a multiparty evolutionary game model is established to analyze the selection of strategies. Then, a simplified three-party model is taken as an example to carry out detailed analysis and solution. Based on stability theory of dynamics system and phase diagram analysis, this article studies replicator dynamics of the evolutionary model to make an optimization analysis of the behaviors of those interested parties and the adjustment mechanism of safety management policies and decisions. The results show how the charge of supervision of government department and inspection of coal mine enterprise impact the efficiency of safety management and the effect of constraint measures and incentive and other measures in safety management.

1. Introduction

On October 31, 2016, a particularly serious gas explosion occurred in Jinshangou Coal Industry Co., Ltd., Yongchuan District of China's Chongqing, resulting in 33 deaths and 1 injury and direct economic losses of more than 36 million yuan. Shortly after that, on December 3, another particularly serious gas explosion occurred in Baoma Mining Co., Ltd., Chifeng City of the Inner Mongolia Autonomous Region, resulting in 32 deaths and 20 injuries, and the direct economic loss was more than 43 million yuan. The investigation team established by the State Council identified that both accidents were liability accidents of production safety. Both the Jinshangou Coal Mine and the Baoma Coal Mine had been involved in illegal cross-border mining for a long time. They adopted the laneway type coal mining technology expressly prohibited by the state and refused to implement the supervision orders issued by the regulatory authorities. They used false drawings and false information and other illegal means to avoid safety supervision. The investigation team also deemed that relevant departments of local governments failed to perform their duties conscientiously. Their investigation and supervision of the cross-border mining of coal mines were ineffective and they violated the legal procedure of the annual inspection of mining permits [1].

Cases of such accidents still occur from time to time in today's China. Management chaos, illegal operations, poor policy implementation, and regulatory dislocation have become the prominent factors affecting the safety of coal mine production. The emergence of these problems is the result of fierce gambling on the interests of production and safety and benefits by the relevant interested parties in the coal mine production. There are multiple levels of safety inspection and supervision involved in the process of coal mine safety management, such as mutual supervision among colleagues, inspection groups of coal mine enterprises, and supervision departments of local governments and social groups (news media, volunteers, etc.). These parties have different interests and aspirations, thus forming a complex contradictory body that is both interdependent and relatively independent, mutually unifying, and mutually antagonistic. Differences in inputs and incomes lead to interested parties playing the game of safety regulatory [2, 3]. The process of safety management is essentially the process of redistribution of regulatory interests. How to adjust the redistribution of regulatory interests is the key to improve the efficiency of safety regulations, and it is also the main reason for this study. With the extensive application of game theory in social economics and safety management, many scholars have realized that the study of game behavior can more effectively find out the essential law of safety management [4–11].

The rest of this paper is organized as follows. In Section 2, we build a fundamental multiparty game model of safety regulation in coal mine production to discuss the interests relationship between the incomes of each interested group and the costs, the fine maybe received, and the detection rate of mistake behavior in the process of safety management. In Section 3, taking the three-party game model as an example, we carry out the further theoretical analysis and model solving and the qualitative analysis of evolutionary of tripartite game system. In Section 4, based on the theoretical analysis of evolutionary stability strategy, we will investigate the incentive measures and restraint measures in the coal mine safety management by considering incentives and constrain measures. We finally conclude our paper in Section 5.

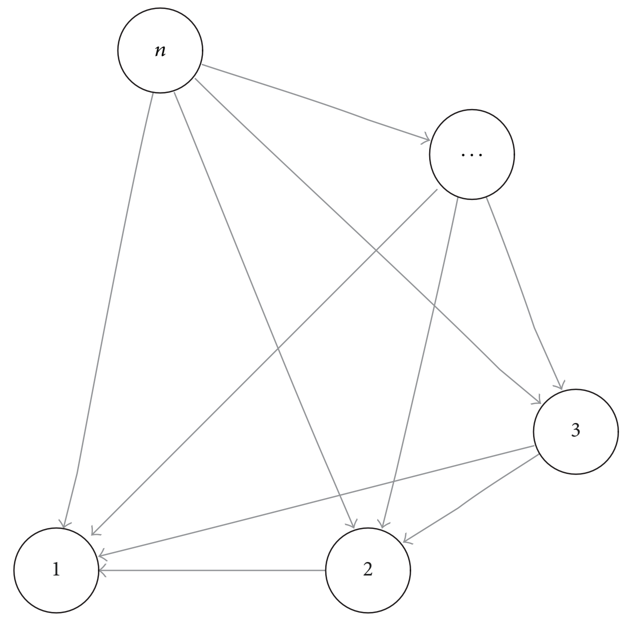

Figure 1: The hierarchy network diagram of a coal mine safety regulatory game.

2. Multiparty Game Model of Safety Regulation in Coal Mine Production

2.1. Basic Assumptions and Symbolic Conventions. It is assumed that there are n interested groups involved in the game process in the coal mine safety management system. The first participant (player I) is coal miners on the front line and the rest are the coal mine safety regulators at all levels within and outside the enterprise. In this way, the relationship between regulators and regulated objects will form a hierarchy of network structure (see Figure 1). We also assume that regulators at all levels are directly involved in the regulation of player I (coal miners), while they indirectly carry out supervision over their subordinate safety supervision departments. For example, they directly perform inspections on the miners' daily attendance, technical standards and work efficiency, and so on. As long as regulators found some unsafe behaviors of the miners, the subordinate safety supervision departments would also be punished because the subordinate departments were considered not performing the duty of safety supervision correctly.

Taking into account the relationship between coal mine safety interested groups, we also make the following assumptions.

(1) The game strategies of player I (coal miners) are safe operation and unsafe operation, respectively. The safety investment cost (manpower, material resources, time, etc.) that produces the normal benefit of 1 unit is c_1 if the strategy of safe operation is chosen. If player I chooses the strategy of unsafe operation, the corresponding safety investment cost is zero. However, they would receive a fine of f_1 when the unsafe operation is checked out.

(2) The game strategies of other participants (safety regulation departments at all levels) are safety regulation and no regulation (or safety supervision and no supervision), respectively. The regulation cost is c_i, $i = 2, 3, \ldots, n$, respectively, if the strategy of safety regulation is selected. When the superior supervision department finds out the unsafe behavior of player I, it also will punish the lower level supervision departments; the fine will be f_i, $i = 2, 3, \ldots, n - 1$.

(3) The inspection and supervision work of the coal mine safety management at all levels is independent, while results of regulation are shared with each other. When an inspector checks out player I's unsafe behavior, he will immediately stop the unsafe behavior and make a punishment, so that the supervision at the upper levels is not necessary to carry out repeatedly.

(4) The detection rate of player I's unsafe behaviors of supervision department at all levels is noted as θ_i ($i = 1, 2, \ldots, n$), which, respectively, represents the probability of corresponding regulators finding player I's unsafe behavior.

According to the actual situation in the process of coal mine production and coal mine safety regulation, we assume that the parameters described above satisfy the following conditions:

(i) $c_2 \le c_3 \le c_n \le c_1 \le 1$; (ii) $f_i \ge 1$, $i = 1, 2, \ldots, n$; (iii) $0 \le \theta_n \le \cdots \le \theta_2 \le \theta_1 \le 1$.

2.2. Income Functions of Interested Parties. The game process of the parties in the coal mine safety management is mainly related to the distribution of the benefits and safety responsibilities. The probability (or the ratio) of player I choosing the strategy of safe operation is noted as x_1, and the probability (or the ratio) of regulators at all levels choosing the strategy of inspection or supervision is noted as $x_i \in [0, 1]$ ($i = 2, 3, \ldots, n$), respectively. Because the wages of coal miners, the subsidies of safety inspection/supervision departments, and other benefits are fixed values in a certain period of time, we will ignore them for convenience.

The income function of player I (coal miners) is

$$\begin{aligned}\pi_1 &= -x_1c_1 - (1-x_1)(1-\theta_1) \\ &\quad\cdot\left[x_2\theta_2 + (1-x_2)x_3\theta_3 + \cdots + x_n\theta_n\prod_{i=2}^{n-1}(1-x_i)\right]f_1 \\ &= -x_1c_1 - (1-x_1)(1-\theta_1) \\ &\quad\cdot\left[x_2\theta_2 + \sum_{j=3}^{n}\left(x_j\theta_j\prod_{i=2}^{j}(1-x_i)\right)\right]f_1.\end{aligned} \tag{1}$$

The income functions of other players (regulators at all levels), respectively, are

$$\begin{aligned}\pi_2 &= -x_2c_2 + (1-x_1)x_2\theta_2f_1 - (1-x_1)(1-x_2) \\ &\quad\cdot\left[x_3\theta_3 + \sum_{j=4}^{n}\left(x_j\theta_j\prod_{i=3}^{j}(1-x_i)\right)\right]f_2; \\ \pi_3 &= -x_3c_3 + \prod_{i=1}^{2}(1-x_i)x_3\theta_3(f_1+f_2) - \prod_{i=1}^{3}(1-x_i) \\ &\quad\cdot\left[x_4\theta_4 + \sum_{j=5}^{n}\left(x_j\theta_j\prod_{i=4}^{j}(1-x_i)\right)\right]f_3; \\ &\vdots \\ \pi_n &= -x_nc_n + x_n\theta_n(f_1+f_2+\cdots+f_{n-1})\prod_{i=1}^{n-1}(1-x_i),\end{aligned} \tag{2}$$

where π_i $(i = 2, 3, \ldots, n)$ is, respectively, the income of safety regulators at various levels from low to high in coal mine safety management system. For example, π_n represents the income of regulatory agency at the highest level. It is difficult to quantify the income function of social organizations such as the news media and the surrounding masses, as well as the loss of reputation of enterprises, so we do not specifically consider them in this paper.

2.3. Analysis of the Multiparty Game Model. We assume that the interested parties involved in the game are all rational and all aim to maximize their own interests. Due to the asymmetric information, although the selection of game behavior is in sequence (usually the coal miners do their job firstly; then the regulators check), it is difficult for one to know the other players' game behavior selection and the probability of the selection. At this point, the game model can be seen as a static game under incomplete information. Simple theoretical analysis of the game model (see formulae (1) and (2)) can draw some significant conclusions.

(1) The total income of each party in the game is $\sum_{i=1}^{n}\pi_i = -\sum_{i=1}^{n}c_ix_i \le 0$. This shows the following:

(i) Safety regulation is costly, and the higher the number of levels of safety inspection, the greater the total cost to pay.

(ii) In the past, the thought of *emphasizing economic efficiency and ignoring safety management* prevailed within the coal enterprises and within some local government departments. In order to reduce economic losses, interested parties compromise each other and reduce safety inspection efforts $(x_1, x_2, \ldots, x_n)$, even reduce the level of safety supervision (supervision of dislocation), and seek to achieve a win-win economic benefit. This is exactly the intrinsic cause of the inefficiency of China's coal production safety management and the low level of safety management over the years.

(iii) In order to reduce the complexity and compromise of interested parties and improve the efficiency of safety management, coal mine safety regulation should not remain inside the enterprises. It also requires the external supervision power of government departments and social organizations to enhance the safety awareness of coal mine enterprises.

(2) The derivatives of income function π_i to variable c_i are

$$\frac{\partial\pi_i}{\partial c_i} = -x_i \le 0, \quad i = 1, 2, \ldots, n. \tag{3}$$

This shows that the income of each participant is negatively related to the cost of safety operation (or safety regulation), and the income function will increase when the cost is reduced.

(i) For player I, to reduce cost (c_1) requires coal miners to improve their productivity level, including coal mine technology level, safety operation skills and safety knowledge, and so on.

(ii) For the supervision agencies at all levels, to reduce the costs of safety regulation (c_i, $i = 2, 3, \ldots, n$) means they should have strong ability of safety supervision and law enforcement. Meanwhile, it also requires the coal mine enterprises to actively cooperate with the supervision and inspection work of the safety regulation departments in process of coal mine production with no confrontation and no concealment.

(3) The derivatives of income function π_i to variable f_i are

$$\begin{aligned}\frac{\partial\pi_1}{\partial f_1} &= -(1-x_1)(1-\theta_1) \\ &\quad\cdot\left[x_2\theta_2 + \sum_{j=3}^{n}\left(x_j\theta_j\prod_{i=2}^{j}(1-x_i)\right)\right] < 0, \\ \frac{\partial\pi_i}{\partial f_i} &= -\prod_{j=1}^{i}(1-x_j)\end{aligned}$$

$$\cdot \left[x_{i+1}\theta_{i+1} + \sum_{k=i+2}^{n} \left(x_k\theta_k \prod_{l=i+1}^{k-1} (1-x_l) \right) \right] < 0,$$

$$i = 2, 3, \ldots, n-1. \tag{4}$$

The incomes of each participant π_i are negatively related to the fines (f_i) they may receive. Punitive measures are very important in the process of coal mine safety management. However, high-intensity punitive measures will result in damage to the interests of players, which may lead to participants (especially coal miners) losing their work enthusiasm and even may affect the production effect and safety management.

(4) The derivatives of income function π_i $(i = 2, 3, \ldots, n)$ to variable f_j $(j < i)$ are

$$\frac{\partial \pi_i}{\partial f_j} = x_i\theta_i \prod_{k=1}^{i-1} (1-x_k) \geq 0, \quad 1 \leq j < i \leq n. \tag{5}$$

This shows that the fines that supervisory agencies imposed on regulated objects (including their subordinate regulatory agencies and coal miners) are the driving force of their work of regulations. This also shows that the supervisory agencies not only supervise the production of coal miners (player I) but also promote the work of the subordinate departments in charge of safety regulation.

(5) Through calculating the derivatives of income function π_i to variable θ_i, we can obtain $\partial \pi_i / \partial \theta_i \geq 0$ $(i = 1, 2, \ldots, n)$.

This indicates that the income of every participant is positively related to its detection rate of player I's unsafe behaviors. That means all interested parties involved in coal mine production and safety management will make their own income increase if they have more advanced production technology, safety operation skills, and safety awareness. So it is necessary for coal mine enterprises to improve their professional qualifications and technical capabilities through vocational and technical training and introduction of highly qualified workers. The regulatory intensity of the government departments should be kept at a certain level in order to achieve the expected regulatory effect.

2.4. First-Order Optimization of Game Model. The derivatives of income function π_i to variable x_i $(i = 1, 2, \ldots, n)$ are

$$\begin{aligned}
\frac{\partial \pi_1}{\partial x_1} &= -c_1 \\
&\quad + (1-\theta_1) \left[x_2\theta_2 + \sum_{j=3}^{n} \left(x_j\theta_j \prod_{i=2}^{j} (1-x_i) \right) \right] f_1; \\
\frac{\partial \pi_2}{\partial x_2} &= -c_2 + (1-x_1)\theta_2 f_1 \\
&\quad + (1-x_1) \left[x_3\theta_3 + \sum_{j=4}^{n} \left(x_j\theta_j \prod_{i=3}^{j} (1-x_i) \right) \right] f_2; \\
&\;\;\vdots \\
\frac{\partial \pi_n}{\partial x_n} &= -c_n + \theta_n (f_1 + f_2 + \cdots + f_{n-1}) \prod_{i=1}^{n-1} (1-x_i).
\end{aligned} \tag{6}$$

As can be easily concluded from the above formulae, the model parameters f_i, θ_i (fines and detection rates) are positively related to $\partial \pi_i / \partial x_i$, while the safety inspection cost c_i and $\partial \pi_i / \partial x_i$ are negatively related, respectively $(i = 1, 2, \ldots, n)$.

Let $\partial \pi_1 / \partial x_1 = \partial \pi_2 / \partial x_2 = \cdots = \partial \pi_n / \partial x_n = 0$; we get the first-order optimization conditions of the game model:

$$\begin{aligned}
&(1-\theta_1) \left[x_2\theta_2 + \sum_{j=3}^{n} \left(x_j\theta_j \prod_{i=2}^{j} (1-x_i) \right) \right] f_1 = c_1; \\
&(1-x_1)\theta_2 f_1 \\
&\quad + (1-x_1) \left[x_3\theta_3 + \sum_{j=4}^{n} \left(x_j\theta_j \prod_{i=3}^{j} (1-x_i) \right) \right] f_2 \\
&\quad = c_2; \\
&\;\;\vdots \\
&\theta_n (f_1 + f_2 + \cdots + f_{n-1}) \prod_{i=1}^{n-1} (1-x_i) = c_n.
\end{aligned} \tag{7}$$

Since multiparty game model contains too many model variables and parameters, its solution and analysis become very difficult. Therefore, for a clearer and more accurate description, the following part of this article mainly takes the three-party game (tripartite game) model as an example to solve and analyze.

3. Solution and Analysis of Tripartite Game in Coal Mine Safety Management

3.1. Tripartite Game in Coal Mine Safety Management. Even though we neglect the voluntary supervision of social groups on the externality of coal mine production, coal mine safety management generally involves at least three participants: coal miners (denoted as player I), safety inspection groups inside the coal mine (denoted as player II), and safety regulatory departments of local government (denoted as player III). We also assume the following:

(i) There is a mutual supervision within the same group of workers in a timely and effective manner which has been completed before induction.

(ii) The supervision and inspections of various regulatory agencies are scientific and rigorous. Once the supervision and inspection are conducted, they will certainly find the unsafe behaviors of player I.

In this case, $\theta_1 = 0$ and $\theta_i = 1, i = 2, 3, \ldots, n$. The tripartite game model can be simplified as follows:

$$\begin{aligned} \pi_1 &= -x_1 c_1 - (1 - x_1)[x_2 + (1 - x_2) x_3] f_1 \\ \pi_2 &= -x_2 c_2 + (1 - x_1)[x_2 f_1 - (1 - x_2) x_3 f_2] \\ \pi_3 &= -x_3 c_3 + (1 - x_1)(1 - x_2) x_3 (f_1 + f_2). \end{aligned} \quad (8)$$

The corresponding first-order optimal conditions of the tripartite game model are

$$\begin{aligned} [x_2 + (1 - x_2) x_3] f_1 &= c_1; \\ (1 - x_1)(f_1 + x_3 f_2) &= c_2; \\ (1 - x_1)(1 - x_2)(f_1 + f_2) &= c_3. \end{aligned} \quad (9)$$

Calculating the first-order optimal conditions (see formulae (9)), we obtain the solution of the tripartite game model; that is,

$$\begin{aligned} x_1 &= 1 - \frac{2c_2}{(f_1 + f_2)(1 \pm \sqrt{\gamma})} \\ &= 1 - \frac{c_3 f_1}{2 f_2 (f_1 - c_1)} (1 \mp \sqrt{\gamma}); \\ x_2 &= 1 - \frac{c_3}{2c_2} (1 \pm \sqrt{\gamma}); \\ x_3 &= 1 - \frac{f_1 + f_2}{2 f_2} (1 \mp \sqrt{\gamma}) = 1 - \frac{2c_2 (f_1 - c_1)}{c_3 (1 \pm \sqrt{\gamma})}, \end{aligned} \quad (10)$$

where $\gamma = 1 - 4c_2 f_2 (f_1 - c_1)/c_3 f_1 (f_1 + f_2)$.

Lemma 1. *According to the parameter values of the model assumption, it is obvious that $\gamma < 1$ (see formula (10)).*

Lemma 2. *The necessary condition of existence of model solutions (see formulae (10)) is that $\gamma \geq 0$; that is, $4c_2 f_2 (f_1 - c_1) \leq c_3 f_1 (f_1 + f_2)$.*

The Nash equilibrium (NE) is a traditional solution concept in game theory. A property of Nash equilibrium is that all participants can predict the emergence of a specific Nash equilibrium, and no one has motivation to adopt different choice of behavior with equilibrium. It is assumed that players are aware of the structure of the game and consciously try to predict the behaviors of their opponents and to maximize their own payoffs. In addition, it is presumed that all the players know this. These assumptions are then used to explain why players choose Nash equilibrium strategies [12, 13].

For example, if the parameter values are given as $(c_1, c_2, c_3, f_1, f_2) = (0.2, 0.05, 0.1, 0.5, 1.5)$, the Nash equilibrium point of mixed strategy of the tripartite game model can be obtained as $(0.927, 0.316, 0.123)$. The mixed strategy Nash equilibrium is a stochastic stable state. When people involved in strategy selection deviate from mixed strategy equilibrium, the probability that the opponent chooses to produce better results will increase. At this point, the game is dynamic. Through the simulation of the game model under different parameter values, it is found that the Nash equilibrium of nondegenerate mixed strategy (pure strategy Nash equilibrium is regarded as a degenerate mixed strategy equilibrium) may exist or may not.

Because of the long-term process of games, chances for people to make mistakes are always possible. Therefore, the most likely outcome of the game is in fact determined by the degree of completeness of the information obtained by the participants, such as how much experience the participants have for the game and the participants' expectations for others' game behaviors [14–17].

Thus, in addition to strict implementation of the supervision and inspection work in accordance with the requirements of laws and regulations, coal mine enterprises must give full play to safety training, safety awareness education, and other auxiliary means to improve the safety awareness and self-protection ability of workers in the process of daily management. Besides, the enterprises should take full use of radio, television, newspaper, pictures, and other forms to propagate safety knowledge and carry out extensive activities of safety education.

3.2. Game Strategy Selection and Evolutionary Stabilization. In the traditional game theory, it is assumed that the participants are completely rational and under the condition of complete information, which are very difficult to achieve in their really economic life [13, 18]. The sources of incomplete information and the bounded rationality are the differences between the participants and the complexity of economic environment and the game itself. Once a participant changes his selection strategy, other participants will change constantly. Therefore, we apply evolutionary game theory to study the stability of strategic choices of all parties in the safety regulation game. The results of evolutionary game model can be a theoretical basis for establishing adjustment mechanism of rules and measures in coal mine safety management system.

The evolutionarily stable strategy (ESS) is a refinement of the Nash equilibrium, which cannot be invaded by any alternative strategy that is initially rare [12, 13, 19]. After the 1980s, with the inherent defects of neoclassical economics and game theory being gradually recognized, the concept of bounded rationality has been generally accepted and widely used in behavioral ecology and economics and other scientific fields [20–22]. In evolutionary game theory, each participant dynamically adjusts his decision-making through learning, imitation, and other behaviors. Not only is the result of the equilibrium dependent on the initial state of the game, but also the change of the external environment will affect the evolutionary path of the game. Based on the bounded rationality of the participants, evolutionary game theory combines some research results of classical game theory and ecological theory and uses the dynamic analysis method to analyze the various factors that affect the behavior of participants and to examine the evolutionary trend of group behavior [13]. Traditional game theory tells people that there are many possible Nash equilibriums in one game. Evolutionary game theory further states which one is the real equilibrium in reality and it is not necessarily Pareto optimal equilibrium [23–25].

Replicator dynamics (RDs) is an important concept to describe the evolutionary advantage of population in evolutionary game theory [12, 19]. The evolutionary advantage of a population is reflected in the proportion of participants; that is, the growth rate $\dot{x}/x$ of each player equals the difference between its adaptability and average adaptability $\pi_i|_{x_i=1} - \pi_i$ $(i = 1, 2, 3)$. So we obtain

$$\dot{x_1} = x_1(1-x_1)\left[(x_2 + (1-x_2)x_3)f_1 - c_1\right];$$
$$\dot{x_2} = x_2(1-x_2)\left[(1-x_1)(f_1 + x_3 f_2) - c_2\right];$$
$$\dot{x_3} = x_3(1-x_3)\left[(1-x_1)(1-x_2)(f_1+f_2) - c_3\right]. \quad (11)$$

Obviously,

$$\dot{x_i} = x_i(1-x_i)\frac{\partial \pi_i}{\partial x_i}, \quad i = 1, 2, 3. \quad (12)$$

3.3. Stability Analysis of Evolutionary Game Model Dynamical System. The dynamical system (see formulae (11)) has eight pure strategy equilibrium points: $(0,0,0), (1,0,0), (0,1,0), (0,0,1), (1,1,0), (1,0,1), (0,1,1), (1,1,1)$. The system may also have a mixed strategy equilibrium point as we get in Section 3.1.

Constructing the Jacobian matrix of dynamical system of game model,

$$J = \frac{\partial(\dot{x_1}, \dot{x_2}, \dot{x_3})}{\partial(x_1, x_2, x_3)} = \begin{bmatrix} (1-2x_1)\left[f_1(x_2+x_3-x_2x_3)-c_1\right] & x_1(1-x_1)(1-x_3)f_1 & x_1(1-x_1)(1-x_2)f_1 \\ x_2(1-x_2)(f_1+f_2x_3) & (1-2x_2)\left[(1-x_1)(f_1+x_3f_2)-c_2\right] & x_2(1-x_1)(1-x_2)f_2 \\ -x_3(1-x_3)(1-x_2)(f_1+f_2) & -x_3(1-x_3)(1-x_1)(f_1+f_2) & (1-2x_3)\left[(1-x_1)(1-x_2)(f_1+f_2)-c_3\right] \end{bmatrix}. \quad (13)$$

Eigenvalue method can be used to determine the stability of equilibrium points in dynamic system [26–29]. The results of calculation show that all the equilibrium points of the dynamic system are saddle points and global stability cannot be achieved under the current parameter assumption. The dynamic system of the game does not satisfy the self-control.

4. Adjustment Mechanism of Government Policies and Enterprise Decisions in Coal Mine Safety Management System

In the past, the formulation of China's safety management policies was mostly temporary measures according to the specific situation of certain accident. The policies always were adjusted along with the accidents, and the relevant regulatory measures also tended to have the characteristics of the accident type. Lack of a comprehensive analysis of the coal mine safety system is one of the important causes of frequent occurrence of safety accidents. Based on the conclusion of multiparty game model, this paper attempts to study the optimization of behavior strategy of relevant players in coal mine safety management and analyzes the stability of strategy selection so as to reveal the inherent mechanism and operating rules of coal mine safety system. Considering the long-term and repetitive nature of the game of coal mine safety and the limited rationality of game participants, it seems reasonable to formulate restraint and incentive measures based on the conclusion of evolutionary game. A comparative analysis of current safety policies and measures of safety management systems provides a theoretical basis for government departments to formulate and improve the policies and measures.

4.1. Incentives and Stability of Game Strategy Selection. In the range of current parameter values of the model, the equilibrium points of the dynamical system (see formulae (11)) have no evolutionary stability. Considering that the purpose of this paper is to improve the safety management of coal mine production, we choose to change the value of the parameters in the model and then use the Jacobian matrix (see Table 1) to analyze the evolutionary stability (ES) of the equilibrium points of safety operation strategy (i.e., $x_1 = 1$).

$c_2 < 0$ and $c_3 < 0$, respectively, represent that the cost of player II's inspection and the cost of player III's supervision are negative, which may happen when the inspection/supervision costs are borne and subsidized by the coal mine enterprises or the government. As can be seen from Table 1, when $c_2 < 0$, equilibrium point $(1,1,0)$ is stable in the dynamical system of the tripartite game; when $c_3 < 0$, equilibrium point $(1,0,1)$ is stable; and $(1,1,1)$ is stable when $c_2 < 0$ and $c_3 < 0$ are met at the same time. Under these conditions, player II (inspection groups) or player III (supervision departments of the government) selects the safety inspection/supervision strategy with probability 1, and player I (coal miners) would choose safety operation strategy with probability 1; then the process of coal mining can be guaranteed stably. Theoretically speaking, equilibrium point $(1,0,0)$ is also stable in the three-party game if $c_1 < 0$, but this situation that productivity cost is negative is unreasonable and not consistent with the actual situation.

Table 1: Stability analysis of equilibrium points under incentive measures.

Equilibrium points	(1, 0, 0)	(1, 1, 0)	(1, 0, 1)	(1, 1, 1)
J	$\mathrm{diag}(c_1, -c_2, -c_3)$	$\mathrm{diag}(c_1 - f_1, c_2, -c_3)$	$\mathrm{diag}(c_1 - f_1, -c_2, c_3)$	$\mathrm{diag}(c_1 - f_1, c_2, c_3)$
Change of parameter value	No	$c_2 < 0$	$c_3 < 0$	$c_2 < 0, c_3 < 0$
Stability	Saddle point	ES	ES	ES

Through the above analysis, we find that appropriate incentives like subsidies and allowances reduce the regulation costs and may effectively encourage coal miners to increase input in production safety and enhance safety awareness.

4.2. Selection of Evolutionary Stable Strategy under Constraint Measures. We mainly consider two kinds of constraint measures on the impact of the evolution of game strategies: one is that the coal mine safety related government departments can use constraints, such as the specified number of times of supervision behavior within a certain period; the other is that the choice for the internal inspection groups of coal mine enterprises can also be constrained by provisions of safety inspection within a certain period not less frequently than a lower limit.

4.2.1. Constraint Measures on the Safety Supervision Strategy of Player III. In this part, we consider a particular situation that it is mandatory to stipulate the times of player III's safety supervision; that is, the value of the model parameter x_3 is fixed as a constant. At this point, there are only two participants in the game of safety supervision: coal miners (player I) and safety inspection groups inside coal mine enterprise (player II).

The income functions of player I and player II, respectively, are

$$\pi_1 = -x_1 c_1 - (1 - x_1)[x_2 + (1 - x_2) x_3] f_1$$
$$\pi_2 = -x_2 c_2 + (1 - x_1)[x_2 f_1 - (1 - x_2) x_3 f_2]. \quad (14)$$

The corresponding first-order conditions of the two-player game model are

$$[x_2 + (1 - x_2) x_3] f_1 = c_1;$$
$$(1 - x_1)(f_1 + x_3 f_2) = c_2. \quad (15)$$

The solution of the game model is

$$x_1^* = 1 - \frac{c_2}{f_1 + x_3 f_2}; \quad (16)$$

$$x_2^* = \frac{c_1 - x_3 f_1}{(1 - x_3) f_1}. \quad (17)$$

Obviously, formula (16) shows that x_1^* is positively related to x_3, f_1, and f_2. We also can find that x_2^* is negatively related to x_3 and f_1 when the derivatives of x_2^* on the variables x_3, f_1, respectively, are obtained; that is, $\partial x_2^*/\partial x_3 = (c_1 - f_1)/f_1(1 - x_3)^2 < 0$ and $\partial x_2^*/\partial f_1 = -c_1/f_1^2(1 - x_3) < 0$.

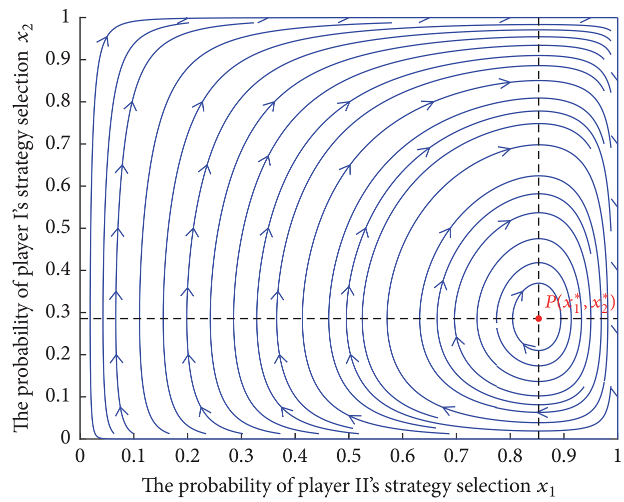

Figure 2: Phase diagram of the dynamical system as x_3 is fixed ($\mathbf{x_3 = 0.3}$; $c_1 = 0.5$; $c_2 = 0.2$; $f_1 = 1$; $f_2 = 1.2$).

The replicator dynamic equations (Replicator Dynamics) corresponding to the game model are

$$\dot{x}_1 = x_1 (1 - x_1)[(x_2 + (1 - x_2) x_3) f_1 - c_1];$$
$$\dot{x}_2 = x_2 (1 - x_2)[(1 - x_1)(f_1 + x_3 f_2) - c_2]. \quad (18)$$

Phase space of a dynamical system is a space in which all possible states of a system are represented with each possible state corresponding to one unique point [30–33]. As can be seen from the phase diagram of dynamical system (see Figure 2), the equilibrium point $P(x^*, y^*)$ is an unstable center of the dynamical system of the two-player game model.

Let $\dot{x}_1 = 0$; we get two stable states: $x_1 = 0$ and $x_1 = 1$. The evolutionary stable strategy requires $d\dot{x}_1/dx_1 < 0$. So $x_1 = 0$ is an ESS if $x_2 < x_2^* = (c_1 - x_3 f_1)/(1 - x_3) f_1$ and $x_1 = 1$ is an ESS when $x_2 > x_2^*$. This shows that when the probability of supervision strategy is greater than x_2^*, coal miners tend to choose the safe production strategy. On the other hand, when the intensity of supervision is relatively small, coal miners may have no desire to increase safety investment.

Similarly, let $\dot{x}_2 = 0$; we get two stable states of $x_2 = 0$ and $x_2 = 1$. When $x_1 < x_1^* = (c_1 - x_3 f_1)/(1 - x_3) f_1$, $x_2 = 1$ is an ESS; and $x_2 = 0$ is an ESS when $x_1 > x_1^*$. This means that the competent departments will continue to strengthen the safety supervision until the proportion of coal miners choosing safety operation exceeds a certain level, and supervisors will tend to choose no supervision strategy while miners will choose that the probability of safe production strategy exceeds x_1^*.

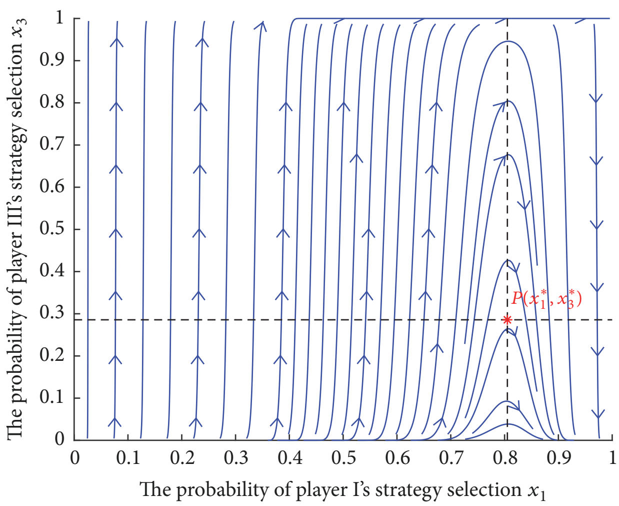

FIGURE 3: Phase diagram of the dynamical system as x_2 is fixed as a smaller value ($\mathbf{x_2 = 0.3}$; $c_1 = 0.5$; $c_2 = 0.2$; $c_3 = 0.3$; $f_1 = 1$; $f_2 = 1.2$).

4.2.2. Constraint Measures on the Safety Inspection Strategy of Player II. In this part, it is mandatory to stipulate the times of player II's safety inspection; that is, x_2 is fixed as a constant. There are also only two participants in the game of safety supervision: coal miners (player I) and safety supervision department of the government (denoted as player III as well).

The income functions of player I and player III, respectively, are

$$\pi_1 = -x_1 c_1 - (1 - x_1)[x_2 + (1 - x_2) x_3] f_1;$$

$$\pi_3 = -x_3 c_3 + (1 - x_1)(1 - x_2) x_3 (f_1 + f_2). \quad (19)$$

The corresponding first-order conditions of the two-player game model are

$$\begin{aligned} [x_2 + (1 - x_2) x_3] f_1 &= c_1; \\ (1 - x_1)(1 - x_2)(f_1 + f_2) &= c_3. \end{aligned} \quad (20)$$

The solution of the game model is

$$\begin{aligned} x_1^* &= 1 - \frac{c_3}{(1 - x_2)(f_1 + f_2)}; \\ x_3^* &= \frac{c_1 - x_2 f_1}{(1 - x_2) f_1}. \end{aligned} \quad (21)$$

Obviously, formula (22) shows that x_1^* is positively related to x_2, f_1, and f_2. However, x_3^* is negatively related to x_2 and f_1, because $\partial x_3^* / \partial x_2 = (c_1 - f_1)/f_1(1-x_2)^2 < 0$ and $\partial x_3^* / \partial f_1 = -c_1/f_1^2(1 - x_2) < 0$.

The replicator dynamic equations (RDs) corresponding to the game model are

$$\dot{x}_1 = x_1 (1 - x_1)[(x_2 + (1 - x_2) x_3) f_1 - c_1]; \quad (22)$$

$$\dot{x}_3 = x_3 (1 - x_3)[(1 - x_1)(1 - x_2)(f_1 + f_2) - c_3]. \quad (23)$$

The Jacobian matrix of the dynamic system of game model is

$$J_{13} = \frac{\partial(\dot{x}_1, \dot{x}_3)}{\partial(x_1, x_3)} = \begin{bmatrix} (1 - 2x_1)[f_1(x_2 + x_3 - x_2 x_3) - c_1] & x_1(1 - x_1)(1 - x_2) f_1 \\ -x_3(1 - x_3)(1 - x_2)(f_1 + f_2) & (1 - 2x_3)[(1 - x_1)(1 - x_2)(f_1 + f_2) - c_3] \end{bmatrix}, \quad (24)$$

where $a_1 = (1 - x_2)(f_1 + f_2) - c_3; a_2 = x_1^*(1 - x_1^*)(1 - x_2) f_1; a_3 = x_3^*(1 - x_3^*)(1 - x_2)(f_1 + f_2)$. Obviously, $a_2 > 0, a_3 > 0$. $a_1 > 0$ is also obtained because $(f_1, f_2) \gg c_3$ and x_2 is not approximated to 1. As shown in Table 2, all of the equilibrium points $(0, 0), (1, 0), (0, 1), (1, 1)$ and (x_1^*, x_3^*) are unstable unless x_2 is larger than f_1/c_1. So the analysis followed is concretely discussed in two cases: $x_2 < f_1/c_1$ and $x_2 > f_1/c_1$.

Case 1 ($x_2 < f_1/c_1$). The value of x_2 is relatively small ($c_1 > x_2 f_1$), and the equilibrium point $P(x_1^*, x_3^*)$ is the internal center of the dynamical system, but not an ESS (see Figure 3 and Table 2). At this time, the labor cost of coal miners is relatively large, while the strength of inspection (x_2) and the punishment (f_1) is relatively low.

Let $\dot{x}_1 = 0$; we can get two stable states: $x_1 = 0$ and $x_1 = 1$. The evolutionary stable strategy requires $d\dot{x}_1/dx_1 < 0$. So $x_1 = 0$ is an ESS if $x_3 < x_3^* = (c_1 - x_2 f_1)/(1 - x_2) f_1$ and $x_1 = 1$ is an ESS when $x_3 > x_3^*$. This shows that only when the regulatory intensity exceeds a certain level ($x_3 > x_3^*$) will coal miners (player I) have a desire to increase their safety investment and tend to select the strategy of safety operation.

Similarly, let $\dot{x}_3 = 0$; we get two stable states of $x_3 = 0$ and $x_3 = 1$. When $x_1 < x_1^* = (c_1 - x_3 f_1)/(1 - x_3) f_1$, $x_3 = 1$ is an ESS; and $x_3 = 0$ is an ESS when $x_1 > x_1^*$. This shows that the supervision of the government department (player III) will continue to be strengthened until the proportion of safety investment of coal miners (player I) exceeds a certain level.

Case 2 ($x_2 > f_1/c_1$). In this case, the value of x_2 is relatively large (i.e., $x_2 > f_1/c_1$); there is no internal equilibrium in the dynamic system of game model (see Figure 4, $P(x_1^*, x_3^*)\nexists$). Let $\dot{x}_1 = 0$; we can get two stable states: $x_1 = 0$ and $x_1 = 1$. The result is that $x_1 = 1$ is an ESS because $(d\dot{x}_1/dx_1)|_{x_1=1} = -[f_1(x_2 + x_3 - x_2 x_3) - c_1] < 0, \forall x_3 \in [0, 1]$. This result also can be seen in Table 2; the pure strategy $(1, 0)$ is the stable node of the dynamic system and is also the evolutionary stabilization

Table 2: Stability analysis of equilibrium points of RD system (x_2 = Cons.).

Equilibrium points	$(0, 0)$	$(1, 0)$	$(0, 1)$	$(1, 1)$	(x_1^*, x_3^*)
J_{13}	$\mathrm{diag}(x_2 f_1 - c_1, a_1)$	$\mathrm{diag}\left(c_1 - x_2 f_1, -c_3\right)$	$\mathrm{diag}(f_1 - c_1, -a_1)$	$\mathrm{diag}(c_1 - f_1, c_3)$	$\begin{pmatrix} 0 & a_2 \\ -a_3 & 0 \end{pmatrix}$
Stability	Saddle	Stable node (if $c_1 < x_2 f_1$) Saddle point (if $c_1 > x_2 f_1$)	Saddle	Saddle	Center (unstable)

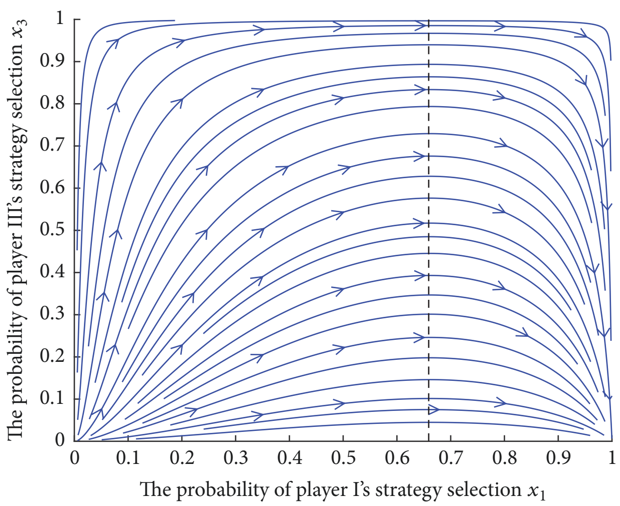

Figure 4: Phase diagram of the dynamical system as x_2 is fixed relatively larger ($\mathbf{x_2 = 0.6}$; $c_1 = 0.5$; $c_2 = 0.2$; $c_3 = 0.3$; $f_1 = 1$; $f_2 = 1.2$).

strategy of the game model. This means that coal miners (player I) prefer to choose the strategy of safety operation no matter the probability of strategy selection of the supervisors. The reason is that the supervision intensity of player II (x_2) and the punishment maybe received during selection of safety operation strategy (f_1) are relatively large (to be precise, the product of the two is relatively large compared to the producer's cost input; i.e., $x_2 f_1 > c_1$). So the coal miners (player I) are no longer willing to choose the unsafe operation strategy.

5. Conclusions

Considering the present situation of coal mine safety management in China and the complex relationship between interested groups, a multiparty evolutionary game model was first established to analyze the selection of strategies. Analysis of the multiparty game model shows essential reasons for the deficiencies and problems of safety management now and in the past. Taking the tripartite game model as an example, this paper studies its further theoretical analysis and model solving and studies the stability of the evolutionary game model. The result of numerical simulation shows that the equilibrium points of the evolutionary game model are all saddle points under current model hypothesis and the dynamic system of the game model is not self-controlled. Therefore, based on theoretical analysis of evolutionary stability strategy, some incentive measures and restraint measures of coal mine safety management are put forward. It is found that game strategy selection is controllable under these corresponding measures. So appropriate incentive and constraint measures can fully mobilize the initiative of inspection and supervision agencies and urge coal miners to actively increase input of production safety. Then the effect and related process of coal production safety management can be guaranteed.

Some details in the evolutionary equilibrium also have an important influence on the selection of game equilibrium. Based on the results of stability analysis of dynamical system, the following conclusions are confirmed:

(1) A good safety supervision system of the government will encourage coal mine enterprises to take measures to improve their professional quality.

(2) If the supervision strength is not high, the coal mine enterprise will appear to reduce investment in safety situation; and with the supervision strength increasing, enterprises will have to increase safety investment of coal mine. When the strength of regulation is too low, the stability of safety management is damaged, which may cause the chaos of safety management.

(3) In the long run, the coal mine enterprise's safety training for employees is a long-term and strategic investment, which is an important measure for the enterprise to improve the level of safety management.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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