Dynamical emergence of instantaneous 3-spaces in a class of models of General Relativity.

Luca Lusanna

Sezione INFN di Firenze
Polo Scientifico
Via Sansone 1
50019 Sesto Fiorentino (FI), Italy
Phone: 0039-055-4572334
FAX: 0039-055-4572364
E-mail: lusanna@fi.infn.it

Massimo Pauri

Dipartimento di Fisica
Universita' di Parma
Parco Area Scienze 7/A
Sezione INFN di Milano Bicocca,
Gruppo Collegato di Parma
43100 Parma, Italy
E-mail pauri@pr.infn.it
(Dated: November 7, 2006, 12.30 am.)

Abstract

The Hamiltonian structure of General Relativity (GR), for both metric and tetrad gravity in a definite continuous family of space-times, is fully exploited in order to show that: i) the *Hole Argument* can be bypassed by means of a specific *physical individuation* of point-events of the space-time manifold M^4 in terms of the *autonomous degrees of freedom* of the vacuum gravitational field (*Dirac observables*), while the *Leibniz equivalence* is reduced to differences in the *non-inertial appearances* (connected to *gauge* variables) of the same phenomena. ii) the chrono-geometric structure of a solution of Einstein equations for given, gauge-fixed, initial data (a *3-geometry* satisfying the relevant constraints on the Cauchy surface), can be interpreted as an *unfolding* in mathematical global time of a sequence of *achronal 3-spaces* characterized by *dynamically determined conventions* about distant simultaneity. This result stands out as an important *conceptual difference* with respect to the standard chrono-geometrical view of Special Relativity (SR) and allows, in a specific sense, for an *endurantist* interpretations of ordinary *physical objects* in GR.

I. INTRODUCTION

The fact that, in the common sense view of ordinary life, phenomena tend to be intuited and described in a non-relativistic 3-dimensional framework independent of any observer, is more or less justified on the basis of the conjunction of the relative smallness of ordinary velocities compared to the velocity of light and of the neurophysiological capabilities of our brain concerning temporal resolution.

Within the physical description of the world furnished by special relativity theory (SR) in terms of the mathematical representation of spatiotemporal phenomena in Minkowski space-time, one is immediately confronted with the problem of defining the 3-dimensional instantaneous space in which ordinary phenomena should be described.

As well known, the only possible resolution of this problem is based on the *relativization* of the description in relation to each observer, as ideally represented by a time-like world-line. This *ideal* observer chooses an arbitrary *convention* for the synchronization of distant clocks, namely an arbitrary foliation of space-time with space-like 3-surfaces: the instantaneous 3-spaces identified by the convention. By exploiting any monotonically increasing function of the world-line proper time and by defining 3-coordinates having their origin on the world-line on each simultaneity 3-surfaces (i.e. a system of radar 4-coordinates), the observer builds either an inertial or a non-inertial frame, instantiating an observer- and frame-dependent notion of 3-space; see Ref.[1] for the contemporary treatment of synchronization and time comparisons in relativistic theories.

In the special case of inertial observers, the simplest way to define distant simultaneity (of a given event with respect to the observer) as well as to coordinatize space-time is to adopt the so-called Einstein convention and exploit two-way light signals with a single clock. Clearly, any two different observers must adopt the same convention in order to describe phenomena in a coherent way (essentially two different origins within the same frame).

The absolute chrono-geometrical structure of SR, together with the existence of the conformal structure of light cones (Lorentz signature) leads to the necessity of looking at Minkowski space-time as a whole 4-dimensional unit.

The way of dealing with such problems within general relativity (GR) has been considered until now as embodying a further level of complication because of the following facts:

i) The universal nature of gravitational interaction;

ii) The fact that the whole inertio-gravitational and chrono-geometrical structure is jointly determined by the metric field tensor g;

iii) The fact that, unlike SR, in GR we have a system of partial differential equation for the dynamical determination of the chrono-geometrical structure of space-time;

iv) The fact that the symmetry group of the theory is no longer a Lie group like in SR but is the infinite group of diffeomorphisms in a pseudo-Riemannian 4-dimensional differentiable manifold M^4. This fact, which expresses the *general covariance* of the theory, concomitantly gives rise to the so-called *Hole phenomenology* (from the famous *Hole Argument*, formulated by Einstein in 1913, see Ref.[2]), which apparently (in Einstein's words, see Ref.[3]) "Takes away from space and time the last remnant of physical objectivity". Furthermore, it renders the Einstein Lagrangian *singular* with the consequence that Einstein's equations do not constitute a hyperbolic system of partial differential equations, a fact that makes the Cauchy initial value problem almost intractable in the configuration space M^4 .

v) The absence of global inertial systems, which is a *global consequence* of the equivalence principle. Consequently, in topologically trivial Einstein's space-times, the only globally

existing frames must be non-inertial.

vi) The necessity of selecting *globally-hyperbolic* space-times in order to get a notion of *global mathematical time* (to be replaced by a physical clock in experimental practice) and avoid the so-called *problem of time* [4].

In this paper we will show that, at least for a definite continuous family of models of GR - analyzed within the Hamiltonian framework - it is possible to accomplish the following program:

i) The metric field is naturally split into two distinct parts:

ia) An *epistemic* part, corresponding to the arbitrary constituents of the metric field (*gauge* variables) that must be completely fixed in order that the Hamilton-Einstein equations became a well-defined hyperbolic system. This complete gauge-fixing defines a *global non-inertial spatiotemporal frame* (called NIF) in which the true dynamics of the gravitational field must be described with all of the *generalized non-inertial effects* made explicit. Once the NIF is fixed, the standard passive 4-diffeomorphisms are subdivided in two classes: those adapted to the NIF, and those which are non-adapted: these latter modify only the 4-coordinates in a way that is not adapted to the NIF.

ib) An *ontic part*, corresponding to the *autonomous degrees of freedom* (2+2) of the vacuum gravitational field (*Dirac observables*, henceforth called DOs) expressed in that NIF.

ii) A *physical individuation* of the point-events in M^4 can be obtained in terms of the DOs in that NIF, an individuation that downgrades the philosophical bearing of the *Hole Argument* (see later). In Ref.[5] we have shown that matter does contribute indirectly to the procedure of physical individuation, and we have suggested how this conceptual individuation could in principle be implemented with a well-defined empirical procedure, as a three-step experimental setup and protocol for positioning and orientation.

iii) A careful reading of the Hamiltonian framework leads to the conclusion that the dynamical nature of the chrono-geometrical structure of every Einstein space-time (or *universe*) of the family considered entails the existence of a *dynamically determined convention* about distant simultaneity. This is tantamount to saying that every space-time of the family considered is *dynamically* generated in terms of a substructure of embedded *instantaneous 3-spaces* that foliates M^4 and defines an associated NIF (modulo gauge transformations, see later). More precisely, once the Cauchy data, i.e. a *3-geometry* satisfying the relevant constraints, are assigned in terms of the DOs on a *initial Cauchy surface, the solution of the Einstein-Hamilton equations embodies the unfolding in mathematical global time of a sequence of instantaneous 3-spaces identified by the Cauchy data chosen.* As a matter of fact, such 3-spaces are obtained, through the solution of an inverse problem, from the extrinsic curvature 3-tensor associated with the 4-metric tensor, solution of the equations in the whole space-time M^4.

Consequently, in a given Einstein space-time, there is *a preferred dynamical convention* for clock synchronization that should be used by every ideal observer (time-like world-line). In practice, since the experimentalist is not aware of what Einstein space-time he lives in, at the beginning the GR observers exploit arbitrary conventions like in SR. Actually, it should be noted that in GR, as in SR, the effective clock synchronization and the setting of

a 3-space grid of coordinates is realized by *purely chrono-geometrical means*[1]. However, the preferred convention can be at least locally identified by making a measurement of g in the 4-coordinate system of the convention chosen and by solving the inverse problem. In this way such GR observers could identify the 3-spaces and re-synchronize the clocks. Under the up-to-now confirmed assumption that the real space-time is an Einstein space, rather than a Weyl space, so that there is no *second clock effect* (see Ref.[6]), all of the clocks should maintain their synchronization on every instantaneous 3-space. Of course, a true physical coordinatization would require a dynamical treatment of realistic matter clocks.

Finally, it must be stressed that, unlike the situation of temporal ordering in SR, the unfolding of the 3-spaces constitutes here a unique *universal B-series ordering* of point-events[2]. Actually this holds true despite the fact that the stratification of M^4 in *achronal 3-spaces* is not gauge-independent. The point is that, *on-shell*, every dynamically admissible gauge transformation is the passive view of an active diffeomorphism within a *definite Einstein's universe*: it changes the NIF, the Hamiltonian (with the *tidal* and *inertial* effects), the world-lines of material objects if present, and the same physical individuation of point-events (see later), in such a way that the *temporal order* of *any pair* of point-events and the *identification* of different material objects as to their relative *order* in space-time are not altered.

Let us conclude with a few philosophical remarks or, rather, specifications. In this paper we only deal with the *theoretical properties* of space-time or space + time, *as mathematically represented* in the three main space-time theories formulated in modern physics, viz. the Newtonian, the special and the general-relativistic. Accordingly, we do not take issue here with such themes as temporal *becoming* (absolute or relational), tensed or de-tensed *existence*, viewed as philosophical questions [7] connected in particular to special or general relativistic theories. Likewise, we exclude from the beginning any statement concerning cosmological issues, either Newtonian or general-relativistic. Above all, we are not concerned with the issue of an alleged *reification* of relativistic space-time as a *real* four-dimensional continuum or a *reification* of the three-dimensional Newtonian space as the *Raum* of our experience or phenomenal space in its various facets. We do believe that such purposes are philosophically misled and grounded on an untutored conflation of autonomous metaphysical issues with a literal interpretation of physical theories.

Certainly in Minkowski space-time there is no absolute fact of the matter as to which an event is *present*. Yet there is no absolute fact of the matter about presentness of events in Newtonian absolute time either, as there is no fact of the matter about presentness in *any* physical *theory* (though of course not in the physicists' practice !). Being valid by assumption at any time[3], a physical theory cannot have the capacity of singling out a particular moment

---

[1] Note that in GR one cannot ascertain in a simple way the usual relativistic effects concerning, e.g., rods contraction and time dilation. In order to show such effects one should have to exploit non-adapted coordinates restricted to a small world-tube in which SR could be considered as a good approximation.

[2] A B-series of temporal determinations concerning events is characterized by purely *relational statements* like "earlier than", "later than" and "simultaneous with". By contrast, an A-series is characterized by *monadic* attributes of single events, like "future", "present" and "past". A-series sentences and their truth-values depend upon the temporal perspective of the utterer, while B-series sentences have truth-values that are time-independent.

[3] Things seem a bit more complicated in cosmology, yet not substantially.

as "the present". Likewise no *physical experiment* can be devised having the capacity of telling whether a particular time signed by the hand of a clock is "the present" or not. This kind of knowledge requires the *conscious awareness* of a living subject, see Ref.[8]. However, no living observer can be forced within the Minkowski space-time (or any general-relativistic space-time model). No living observer can be there to collect in a *factually possible* (specifically *causal*) way the infinite amount of information spread on the space-like Cauchy surface which is necessary to solve the initial value problem according to well-known mathematical theorems and thereby defining the attributes of physical events. There is no living observer who can act to selectively generate a situation in their environment so that this situation, as a cause, will, according to their causal knowledge, give rise later with great probability to the effect which is desirable to them. Finally, there can be no living observer with the freedom to check the very empirical truth of the physical theory itself. Any alleged reification leads to a notion of the world which includes everything, in particular the object of an action and the agent of the same action. *This world, however, is a non-factual world that nobody can "observe", study or control.*

These limitations, which are intrinsic to the nature of the scientific image, can be easily misunderstood. For example, the fact that Minkowski space-time must be considered as a whole 4-dimensional unit gave origin to the *prima facie* appealing but misleading notion of *block* universe, which seemed to entail that in Minkowski space-time there be no *temporal change*, on the grounds that there cannot be any *motion in time*. There is certainly no motion in time in the *ordinary sense of the term*, but *change* is there even if necessarily described in terms of the tenseless language of a (observer-dependent) *B-series*.

The issue of motion in our context can be seriously misinterpreted. The *ordinary sense* of the term - the one used in particular by the experimentalists in their laboratory - refers to the observation that there is some object "moving". Since this "moving" always takes place in "the present" of the experimentalist, as all other phenomena "taking place", one could be easily misled to believe that the *physical essence* of the concept of motion could be captured by the *experience* of an object as "moving", more or less on the same footing in which it is often asserted that there is a "moving now". Of course, there are moments of the motion history of the object that are *now* "past" and characterized as moments of a "remembered present". However, if the notion is no longer taking place "now", we can only describe the former motion as a purely B-series sequence of positions of the object at different times (of a clock), a description which expresses exactly the only *objective physical meaning* of "motion". Accordingly, since there is *no transiency* of the "now" in Minkowski space-time, only *tenseless senses* of the words "becoming", "now" and even of "motion" are admitted in the relativistic idiom. Certainly, the ordinary sense of "motion" is perfectly legitimate in the *practice* of physics and, therefore, in the idiom of the experimentalist. This entails that *locally* and for a *limited length of time*, the experimentalist can project - so to speak - his practical view of the motion into Minkowski space-time as a mental aid for his intuition of the physical process. The experimentalist can consider, e.g., a world-line of a point-mass endowed with a clock and, as time goes by, sign on the world-line certain times indicated by his own synchronized physical clock in the laboratory, as a running chart. As long as the motion takes place, this can be a useful way of reasoning. It should be clear, however, that as soon as the limited allotted time has expired, the intuitive spatiotemporal representation of the motion given by the experimentalist is concluded and *nothing remains* in the Minkowski picture which is different by a section of a standard infinite world-line with

a B-series finite sequence of marked events.

On the other hand, this being said, could we suggest a specific case in connection with the so-called *endurantist/perdurantist* debate ? Briefly, the contrast can be roughly summarized as follows: the *endurantist* takes objects (including people) as lacking temporal extent and persisting by being *wholly present* at each moment of their history, while the *perdurantist* takes objects as persisting by being temporally extended and made up of different *temporal parts* at different times [9]. This opposition, which could appear *prima facie* as a mere matter of terminology at the level of the scientific image, is *formally* vindicated within the orthodox view of relativistic theories. Actually, if material objects were in any relevant sense 3-dimensional and persisted by occupying temporally unextended spatiotemporal regions, how could they fit in with the unavoidable 4-dimensionality of relativistic space-time ?

We believe, in general, that it is remarkably difficult, if not structurally unsound, to devise any conclusive argument from physics to metaphysical issues. It is true that four-dimensionalism, as a philosophical stance, is sometimes used as a shield for perdurantism and three–dimensionalism as a synonym for endurantism. Since, however, we are avowedly averse from any kind of *reification* of the relativistic models of space-time, we should specify the meaning, if any, of notions like *endurantism* or *perdurantism* once restricted to the idiom we deem admissible within the spatiotemporal scientific image. It is clear that, whatever meaning we are ready to allow for the notion of *object* as representable in Minkowski space-time, it cannot be *wholly present* in any sense. Things, however, are apparently different in GR, just in view of our results.

In our description of GR, the 4-dimensional spatiotemporal manifold is *dynamically foliated by gravitation* into global *achronal* 3-spaces at different global times. Therefore the notion of a *wholly present material object* becomes *compatible* with an *endurantist* interpretation of temporal identity. Note that we are avowedly using the restricted adjective *compatible*, because we do not want to be committed to any specific philosophical stance about the issue of the *identity* of objects in general. We shall define a material object (say a *dust* filled sphere) as *wholly present* at a certain global time τ if all of the physical attributes of its constituent events can be obtained by physical information wholly contained in the structure of the 3-space at time τ. Note that such information contains in particular the 3-geometry of the leaf and all the relevant properties of the matter distribution that are also necessary for the formulation of the Cauchy problem of the theory. Having adopted this definition, we will return to the issue of the *endurantism/perdurantism* dispute, restricted to our formulation of GR, only at the end of the paper after expounding all the relevant theoretical features.

Finally, we should add an important remark: since, due to the universal nature of gravitation, SR should be carefully viewed as an approximation of GR rather than an *autonomous* theory, great part of the unending ontological debate about the issues of time, becoming, endurantism, perdurantism etc.. at the special relativistic level, should be reconsidered having in view the results we are going to discuss in the present paper.

In conclusion, here we are only interested in ascertaining whether and to what extent the notion of instantaneous 3-space is physically consistent and univocally definable. With this in view, while sketching the problem of distant simultaneity in the Newtonian and special-relativistic cases for the sake of argument, we will focus on the new and unexpected result concerning the natural, *dynamically ruled*, emergence of a notion of instantaneous 3-space in certain classes of models of GR.

II. NEWTON'S ABSOLUTE DISTANT-SIMULTANEITY.

The absolute space of Newton is, by definition, an instantaneous 3-space at every value of absolute time. Leaving aside foundational problems of Newton and Galilei viewpoints, let us summarize the essential elements of the physicist's viewpoint about non-relativistic mechanics.

The arena of Newton physics is Galilei space-time, in which both time and space have an absolute [4] status (its mathematical structure is the direct product $R^3 \times R$ and can be visualized as a foliation with base manifold the time axis and with Euclidean 3-spaces as fibers[5]). As a consequence, we have the absolute notions of simultaneity, instantaneous Euclidean 3-space and Euclidean spatial distance.

Space is a *container* of material bodies, i.e. objects endowed only with a (inertial) mass. The position of an object, unlike Newton tenets, is a relative frame-dependent notion. Furthermore, since there is an *absolute temporal distance* between events, while Newton's instantaneous 3-space is a *metric* space, Galilei's space-time has a *degenerate metric* structure.

Newton's first law, i.e. *Galilei law of inertia*, states that free objects move on straight lines, eliminating any intrinsic relevance of velocity.

Newton's second law, $\vec{F} = m\,\vec{a}$, identifies acceleration as the basic absolute quantity in the description of motion, where the force is intended to be measured statically.

Galilei relativity principle selects the inertial frames centered on inertial observers as the preferred ones due to the form-invariance of the second law under the Galilei transformations connecting inertial frames.

Gravity is described by an instantaneous action-at-a-distance interaction enjoying the special property of the equality of inertial and gravitational masses (*Galilei equivalence principle*).

In this absolutist point of view, the absolute existence of 3-space allows to develop a well posed kinematics of isolated point-like N-body systems, which can be extended to rigid bodies (and then extended also to deformable ones, see for instance molecular physics). Given the positions $\vec{x}_i(t)$, $i-1,..,N$, of the bodies of mass m_i in a given inertial frame, we can uniquely define their *center of mass* $\vec{x}(t) = \sum_i m_i\,\vec{x}_i(t)/\sum_i m_i$, which describes a *decoupled pseudo-particle in inertial motion*, $\frac{d^2\,\vec{x}(t)}{dt^2} = 0$. All the dynamics is shifted to N-1 relative variables $\vec{r}_a(t)$, $a = 1,..,N-1$. At the Hamiltonian level, where the 3-velocities $\dot{\vec{x}}_i(t) = \frac{d\vec{x}_i(t)}{dt}$ are replaced by the momenta $\vec{p}_i(t) = m_i\,\dot{\vec{x}}_i(t)$, the separation of the center-of-mass conjugate variables $(\vec{x}(t), \vec{p} = \sum_i \vec{p}_i(t) = const.)$ from any set of conjugate relative variables $\vec{r}_a(t)$, $\vec{\pi}_a(t)$, is realized by a canonical transformation, which is a *point transformation* in the coordinates and the momenta separately.

The main property of the non-relativistic notion of center of mass is that it can be determined locally in 3-space in the region occupied by the particles and does not depend on the complementary region of 3-space. Naively, one could say that if we eliminate the

---

[4] In various senses, the most important of which is the statement that time and space are entities independent of the dynamics.

[5] In physics it is always assumed that space, time and space-time can be idealized as suitable mathematical manifolds possibly with additional structure.

decoupled inertial pseudo-particle describing the center of mass, we shift to a *relational* description based on a set of *relative variables* $\vec{r}_a(t)$, $\vec{\pi}_a(t)$. However, as shown in Ref.[10], this is true only if the total barycentric angular momentum of the non-relativistic universe is *zero* [6]. If the total angular momentum is different from zero, Newton relative motion in absolute 3-space satisfies equations of motion which are different from equations of motion having a purely relational structure [10]. This shows why the interpretations of the Newton rotating bucket are so different in the absolute and the relational descriptions.

Let us remark that, since inertial observers are idealizations, all realistic observers are (linearly and/or rotationally) accelerated, so that their spatial trajectories can be taken as the time axis of global (rigid or non-rigid) *non-inertial* frames. It turns out that in Newton's theory this leads only to the appearance of inertial forces proportional to the inertial mass of the accelerated body, rightly called fictitious (or apparent). Let us stress, on the other hand, that all realistic observers do experience such forces and thereby have the problem of disentangling the real dynamical forces from the apparent ones.

---

[6] This is connected with the fact that, while the 3-momentum Noether constants of motion satisfy an Abelian algebra at the Hamiltonian level, the angular momentum Noether constants of motion satisfy a non-Abelian one (only two of them, $\vec{J}^2$ and J_3, have vanishing Poisson brackets). As shown in Ref.[11] this explains why we can decouple the center of mass globally. On the contrary, there is no unique global way to separate 3 rotational degrees of freedom (see molecular dynamics [12]): this gives rise to *dynamical body frames* for deformable bodies (think of the diver and the falling cat). As well known and as shown in Ref.[13], in special relativity there is no notion of center of mass enjoying all the properties of the non-relativistic ones. See Refs.[13, 14] for the types of non-relativistic relative variables admitting a relativistic extension.

III.   SPECIAL RELATIVITY: CONVENTIONAL DISTANT SIMULTANEITY

The arena of special relativity is Minkowski space-time, in which only the (3+1)-dimensional space-time is an absolute notion. Its Lorentz signature and its absolute chrono-geometrical structure allow to distinguish time-like, null and space-like intervals. However, given the world-line of a time-like observer, in each point the observer can only identify the conformal structure of the incoming or outgoing rays of light (the past and future fixed light-cone in that point): for the observer there is no intrinsic notion of simultaneous events, of instantaneous 3-space (to be used as a Cauchy surface for Maxwell equations), of spatial distance, or 1-way velocity of light. The definition of instantaneous 3-space is then completely ruled by the conventions about distant simultaneity.

As well-known, the starting point is constituted by the following basic postulates:
Two *light postulates* - The 2-way (or round-trip) velocity of light (only one clock is involved in its definition) is A) isotropic and B) constant (= c).
The *relativity principle* (replacing the Galilei one) - It selects the relativistic inertial frames, centered on inertial time-like observers, and the Cartesian 4-coordinates x^μ, in which the line element is $ds^2 = \eta_{\mu\nu}\, dx^\mu\, dx^\nu$, $\eta_{\mu\nu} = \epsilon\,(+---)$, $\epsilon = \pm 1$ (according to particle physics or general relativity conventions, respectively).

The *law of inertia* states now that a test body moves along a flat time-like geodesics (a null one for a ray of light). The Poincaré group defines the transformations among the inertial frames. The preferred inertial frames are also selected by *Einstein's convention* for the synchronization of the clock of the inertial observer with any (in general accelerated) distant clock [7], according to which the inertial instantaneous 3-spaces are the Euclidean space-like hyper-planes $x^o = c\,t = const$. Only with this convention the 1-way velocity of light between the inertial observer and any accelerated one coincides with the 2-way velocity c.

The spatial distance between two simultaneous events in an inertial frame is the Euclidean distance along the connecting flat 3-geodesics.

Since inertial frames are still an idealization, we must consider the non-inertial ones centered on accelerated observers. As shown in Refs.[15] (see also Refs.[16, 17]) the standard 1+3 approach, trying to build non-inertial coordinates starting from the world-line of the accelerated observers, meets coordinate singularities preventing their global definition [8]. Let

---

[7] The inertial observer (γ) sends rays of light to another time-like observer γ_1, who reflects them back towards γ. Given the emission (τ_i) and adsorption (τ_f) times on γ, the point P of reflection on γ_1 is assumed to be simultaneous with the point Q on γ where $\tau_Q = \tau_i + \frac{1}{2}(\tau_f - \tau_i) = \frac{1}{2}(\tau_i + \tau_f) \stackrel{def}{=} \tau_P$. With this so-called *Einstein's $\frac{1}{2}$ convention for the synchronization of distant clocks*, the *instantaneous 3-space* is the space-like hyper-plane $x^o = const.$ orthogonal to γ, the point Q is the midpoint between the emission and adsorption points and, since $\tau_P - \tau_i = \tau_f - \tau_P$, the *one-way* velocity of light between γ and every γ_1 is isotropic and equal to the round-trip velocity of light c. Note that in modern metrology clock synchronization is always performed by means of light signals and the velocity of light is assumed as a standard replacing the traditional unit of length.

[8] Fermi coordinates, defined on hyper-planes orthogonal to the observer's 4-velocity become singular where the hyper-planes intersect, i.e. at distances from the world-line of the order of the so-called linear and

9

us remember that the theory of measurements in non-inertial frames is based on the *locality principle* [18]: standard clocks and rods do not feel acceleration and at each instant the detectors of the instantaneously comoving inertial observer give the correct data. Again this procedure fails in presence of electro-magnetic fields when their wavelength is of the order of the acceleration radii [18] (the observer is not static enough during 5-10 cycles of such waves, so that their frequency cannot be measured).

The only known method to overcome these difficulties is to shift to the 3+1 point of view, in which, given the world-line of the observer, one adds as an *independent structure* a 3+1 splitting of Minkowski space-time, which is nothing else than a clock synchronization convention. This allows to define a global non-inertial frame centered on the observer. This splitting foliates Minkowski space-time with space-like hyper-surfaces Σ_τ, which are the instantaneous (Riemannian) 3-spaces [9] associated to the given convention for clock synchronization (in general different from Einstein's). The leaves Σ_τ of the foliation are labeled by any scalar monotonically increasing function of the proper time of the observer. The intersection point of the observer world-line with each Σ_τ is chosen as the origin of scalar curvilinear 3-coordinates. Such observer-dependent 4-coordinates $\sigma^A = (\tau; \sigma^r)$ are called *radar 4-coordinates*. Now the 1-way velocity of light becomes, in general, both anisotropic and point-dependent, while the (Riemannian) spatial distance between two simultaneous points on Σ_τ is defined along the 3-geodesic joining them.

If $x^\mu = z^\mu(\tau, \vec\sigma)$ describes the embedding of the associated simultaneity 3-surfaces Σ_τ into Minkowski space-time, so that the metric induced by the coordinate transformation $x^\mu \mapsto \sigma^A = (\tau, \vec\sigma)$ is $g_{AB}(\tau, \vec\sigma) = \frac{\partial z^\mu(\sigma)}{\partial \sigma^A} \eta_{\mu\nu} \frac{\partial z^\nu(\sigma)}{\partial \sigma^B}$ [10], the basic restrictions on the 3+1 splitting (leading to a nice foliation with space-like leaves) are the Møller conditions [15]

$$\epsilon \, g_{\tau\tau}(\sigma) > 0,$$

$$\epsilon \, g_{rr}(\sigma) < 0, \qquad \begin{vmatrix} g_{rr}(\sigma) & g_{rs}(\sigma) \\ g_{sr}(\sigma) & g_{ss}(\sigma) \end{vmatrix} > 0, \qquad \epsilon \, det\,[g_{rs}(\sigma)] < 0,$$

$$\Rightarrow det\,[g_{AB}(\sigma)] < 0. \tag{3.1}$$

---

rotational *acceleration radii* ($\mathcal{L} = \frac{c^2}{a}$ for an observer with translational acceleration a; $\mathcal{L} = \frac{c}{\Omega}$ for an observer rotating with frequency Ω) [15, 18] (see also Ref.[19]). For rotating coordinates (rotating disk with the associated Sagnac effect) there is a coordinate singularity (the component g_{oo} of the associated 4-metric vanishes) at a distance from the rotation axis, where the tangential velocity becomes equal to c (the so-called horizon problem) [15].

[9] Let us stress that each instantaneous 3-space is a possible Cauchy surface for Maxwell equations. Namely the added structure allows to have a well-posed initial value problem for these equations and to apply to them the theorem on the existence and uniqueness of the solutions of partial differential equations. The price to guarantee *predictability* is the necessity of giving Cauchy data on a non-compact space-like 3-surface inside Minkowski space-time. This is the unavoidable element of *non-factuality* which the 1+3 point of view would like to avoid.

[10] The 4-vectors $z_r^\mu(\tau, \vec\sigma) = \frac{\partial z^\mu(\tau,\vec\sigma)}{\partial \sigma^r}$ are tangent to Σ_τ. If $l^\mu(\tau, \vec\sigma)$ is the unit normal to Σ_τ (proportional to $\epsilon^\mu{}_{\alpha\beta\gamma} [z_1^\alpha z_2^\beta z_3^\gamma](\tau, \vec\sigma)$), we have $z_\tau^\mu(\tau, \vec\sigma) = \frac{\partial z^\mu(\tau,\vec\sigma)}{\partial \tau} = N(\tau, \vec\sigma)\, l^\mu(\tau, \vec\sigma) + N^r(\tau, \vec\sigma)\, z_r^\mu(\tau, \vec\sigma)$, where $N(\tau, \vec\sigma)$ and $N^r(\tau, \vec\sigma)$ are the lapse and shift functions, respectively.

Furthermore, in order to avoid possible asymptotic degeneracies of the foliation, we must make the additional requirement that the simultaneity 3-surfaces Σ_τ must tend to space-like hyper-planes at spatial infinity: $z^\mu(\tau, \vec\sigma) \to_{|\vec\sigma| \to \infty} x_s^\mu(\tau) + \epsilon_r^\mu \sigma^r$ and $g_{AB}(\tau, \vec\sigma) \to_{|\vec\sigma| \to \infty} \eta_{AB}$, with the ϵ_r^μ's being 3 unit space-like 4-vectors tangent to the asymptotic hyper-plane, whose unit normal is ϵ_τ^μ [the ϵ_A^μ form an asymptotic cotetrad, $\epsilon_A^\mu \eta^{AB} \epsilon_B^\nu = \eta^{\mu\nu}$].

As shown in Refs.[15, 16], Eqs.(3.1) forbid rigid rotations: *only differential rotations* are allowed (consistently with the modern description of rotating stars in astrophysics) and the simplest example is given by those 3+1 splittings whose simultaneity 3-surfaces are hyper-planes with rotating 3-coordinates described by the embeddings ($\sigma = |\vec\sigma|$)

$$z^\mu(\tau, \vec\sigma) = x^\mu(\tau) + \epsilon_r^\mu R^r{}_s(\tau, \sigma) \sigma^s,$$

$$R^r{}_s(\tau, \sigma) \to_{\sigma \to \infty} \delta_s^r, \qquad \partial_A R^r{}_s(\tau, \sigma) \to_{\sigma \to \infty} 0,$$

$$R(\tau, \sigma) = \tilde{R}(\beta_a(\tau, \sigma)), \qquad \beta_a(\tau, \sigma) = F(\sigma) \tilde\beta_a(\tau), \quad a = 1, 2, 3,$$

$$\frac{dF(\sigma)}{d\sigma} \neq 0, \qquad 0 < F(\sigma) < \frac{1}{A \sigma}. \tag{3.2}$$

Each $F(\sigma)$ satisfying the restrictions of the last line, coming from Eqs.(3.1), gives rise to a *global differentially rotating non-inertial frame.*

Since physical results in special relativity must not depend on the clock synchronization convention, a description including both standard inertial frames and admissible non-inertial ones is needed. This led to the discovery of *parametrized Minkowski theories.*

As shown in Refs.[20] (see also Refs.[15–17]), given the Lagrangian of every isolated system, one makes the coupling to an external gravitational field and then replaces the external metric with the $g_{AB}(\tau, \vec\sigma)$ associated to a Møller-admissible 3+1 splitting. The resulting action principle $S = \int d\tau d^3\sigma \mathcal{L}(matter, g_{AB}(\tau, \vec\sigma))$ depends upon the system and the embedding $z^\mu(\tau, \vec\sigma)$ and is invariant under frame-preserving diffeomorphisms: $\tau \mapsto \tau'(\tau, \vec\sigma)$, $\sigma^r \mapsto \sigma'^r(\vec\sigma)$. This *special-relativistic general covariance* implies the vanishing of the canonical Hamiltonian and the following 4 first class constraints

$$\mathcal{H}_\mu(\tau, \vec\sigma) = \rho_\mu(\tau, \vec\sigma) - \epsilon \, l_\mu(\tau, \vec\sigma) \mathcal{M}(\tau, \vec\sigma) - \epsilon \, z_{r\mu}(\tau, \vec\sigma) \, h^{rs}(\tau, \vec\sigma) \mathcal{M}_s(\tau, \vec\sigma) \approx 0,$$
$$\{\mathcal{H}_\mu(\tau, \vec\sigma), \mathcal{H}_\nu(\tau, \vec\sigma')\} = 0, \tag{3.3}$$

where $\rho_\mu(\tau, \vec\sigma)$ is the momentum conjugate to $z^\mu(\tau, \vec\sigma)$ and $[\sum_u h^{ru} g_{us}](\tau, \vec\sigma) = \delta_s^r$. $\mathcal{M}(\tau, \vec\sigma) = T_{\tau\tau}(\tau, \vec\sigma)$ and $\mathcal{M}_r(\tau, \vec\sigma) = T_{\tau r}(\tau, \vec\sigma)$ are the energy- and momentum- densities of the isolated system in Σ_τ-adapted coordinates [for N free particles we have $\mathcal{M}(\tau, \vec\sigma) = \sum_{i=1}^N \delta^3(\vec\sigma - \vec\eta_i(\tau)) \sqrt{m_i^2 + h^{rs}(\tau, \vec\sigma) \kappa_{ir}(\tau) \kappa_{is}(\tau)}$, $\mathcal{M}_r(\tau, \vec\sigma) = \sum_{i=1}^N \delta^3(\vec\sigma - \vec\eta_i(\tau)) \kappa_{ir}(\tau)$].

Since the matter variables have only Σ_τ-adapted Lorentz-scalar indices, the 10 constant of the motion corresponding to the generators of the external Poincaré algebra are

$$P^\mu = \int d^3\sigma \, \rho^\mu(\tau, \vec{\sigma}),$$

$$J^{\mu\nu} = \int d^3\sigma \, [z^\mu \rho^\nu - z^\nu \rho^\mu](\tau, \vec{\sigma}). \tag{3.4}$$

The Hamiltonian gauge transformations generated by constraints (3.3) change the form and the coordinatization of the simultaneity 3-surfaces Σ_τ: as a consequence, the embeddings $z^\mu(\tau, \vec{\sigma})$ are *gauge variables*, so that in this framework the choice of the non-inertial frame and in particular of the convention for the synchronization of distant clocks [15, 16] is a *gauge choice*. All the inertial and non-inertial frames compatible with the Møller conditions (3.1) are *gauge equivalent* for the description of the dynamics of isolated systems.

A subclass of the embeddings $z^\mu(\tau, \vec{\sigma})$, in which the simultaneity leaves Σ_τ are equally spaced hyper-planes, describes the standard inertial frames if an inertial observer is chosen as origin of the 3-coordinates. Let us stress that every isolated system intrinsically identifies a special inertial frame, i.e. the rest frame. The use of radar coordinates in the rest frame leads to parametrize the dynamics according to the *Wigner-covariant rest-frame instant form of dynamics* developed in Refs.[20]. This instant form is a special case of *parametrized Minkowski theories* [20] [16] [11], in which the leaves of the 3+1 splitting of Minkowski space-time are inertial hyper-planes (simultaneity 3-surfaces called *Wigner hyper-planes*) orthogonal to the conserved 4-momentum P^μ of the isolated system.

The inertial rest-frame instant form is associated with the special gauge $z^\mu(\tau, \vec{\sigma}) = x_s^\mu(\tau) + \epsilon_r^\mu(u(P)) \sigma^r$, $x_s^\mu(\tau) = Y_s^\mu(\tau) = u^\mu(P)\tau$, selecting the inertial rest frame of the isolated system centered on the Fokker-Pryce 4-center of inertia and having as instantaneous 3-spaces the Wigner hyper-planes.

Another particularly interesting family of 3+1 splittings of Minkowski space-time is defined by the embeddings

$$z^\mu(\tau, \vec{\sigma}) = Y_s^\mu(\tau) + F^\mu(\tau, \vec{\sigma}) = u^\mu(P)\tau + F^\mu(\tau, \vec{\sigma}), \qquad F^\mu(\tau, \vec{0}) = 0,$$
$$\to_{\sigma \to \infty} u^\mu(P)\tau + \epsilon_r^\mu(u(P)) \sigma^r, \tag{3.5}$$

with $F^\mu(\tau, \vec{\sigma})$ satisfying Eqs.(3.1).

In this family the simultaneity 3-surfaces Σ_τ tend to Wigner hyper-planes at spatial infinity, where they are orthogonal to the conserved 4-momentum of the isolated system. Consequently, there are asymptotic inertial observers with world-lines parallel to

---

[11] This approach was developed to give a formulation of the N-body problem on arbitrary simultaneity 3-surfaces. The change of clock synchronization convention may be formulated as a *gauge transformation* not altering the physics and there is no problem in introducing the electro-magnetic field when the particles are charged (see Refs.[20–22]). The rest-frame instant form corresponds to the gauge choice of the 3+1 splitting whose simultaneity 3-surfaces are the intrinsic rest frame of the given configuration of the isolated system. See Ref.[23] for the Hamiltonian treatment of the relativistic center-of-mass problem and for the issue of reconstructing orbits in the 2-body case.

that of the Fokker-Pryce 4-center of inertia, namely there are the *rest-frame conditions* $p_r = \epsilon_r^\mu(u(P)) P_\mu = 0$, so that the embeddings (3.5) define *global Møller-admissible non-inertial rest frames* [12].

Since we are in non-inertial rest frames, the internal energy- and boost- densities contain the inertial potentials source of the relativistic inertial forces (see Ref.[24] for the quantization in non-inertial frames): more precisely, they are contained in the spatial components of the metric $g_{rs}(\tau, \vec{\sigma})$ associated to the embeddings (3.5).

In conclusion *the only notion of instantaneous 3-space* which can be introduced in special relativity is always observer $[X^\mu(\tau)]$- and frame $[\Sigma_\tau]$-dependent. The conceptual difficulties connected with the notion of relativistic center of mass also show that its definition (using the global Poincaré generators of the isolated system) *necessitates a whole instantaneous 3-space* Σ_τ. Therefore, even if we eventually get a decoupled pseudo-particle like in Newton theory [13], we lose the possibility of treating disjoint, non-interacting, sub-systems independently of one another. This is just due to the necessity of choosing a convention for the synchronization of distant clocks.

---

[12] The only ones existing in tetrad gravity, due to the equivalence principle, in globally hyperbolic asymptotically flat space-times without super-translations as we shall see in the next Section.

[13] However, the canonical transformations decoupling it from the relative variables are now non-point; only for free particles they remain point in the momenta, but not in the positions [23].

IV. GENERAL RELATIVITY: DYNAMICALLY DETERMINED DISTANT SI-MULTANEITY

We will show that, contrary to a widespread opinion, general relativity - at least for a particular class of models [14] - contains in itself the capacity for a dynamical definition of instantaneous 3-spaces. Every model of GR in the given class, once completely specified in a precise sense that will be explained presently, entails that space-time *be* essentially the unfolding in time of a dynamically variable form of instantaneous 3-space.

In the years 1913-16 Einstein developed general relativity by relying on the *equivalence principle* (equality of inertial and gravitational masses of non-spinning test bodies in free fall) and on the guiding principle of general covariance. Einstein's original view was that the principle had to express the impossibility of distinguishing a uniform gravitational field from the effects of a constant acceleration by means of local experiments in a sufficiently small region with negligible tidal forces. This led him to the *geometrization* of the gravitational interaction and to the replacement of Minkowski space-time with a pseudo-Riemannian 4-manifold M^4 with non vanishing curvature Riemann tensor.

The *equivalence principle* entails the non existence of global inertial frames (SR relativity holds only in a small neighbourhood of a body in free fall). The principle of *general covariance* (see Ref.[25] for a thorough review), which expresses the tensorial nature of Einstein's equations, has the following two consequences:

i) the invariance of the Hilbert action under *passive* diffeomorphisms (the coordinate transformations in M^4), so that the second Noether theorem implies the existence of first-class constraints at the Hamiltonian level;

ii) the mapping of the solutions of Einstein's equations among themselves under the action of *active* diffeomorphisms of M^4 extended to the tensors over M^4 (*dynamical symmetries* of Einstein's equations).

The basic field of metric gravity is the 4-metric tensor with components $^4g_{\mu\nu}(x)$ in an arbitrary coordinate system of M^4. The peculiarity of gravity is that the 4-metric field, unlike the fields of electromagnetic, weak and strong interactions and the matter fields, has a *double role*:

i) it is the mediator of the gravitational interaction (in analogy to all of the other gauge fields);

ii) it determines *dynamically* the chrono-geometric structure of space-time M^4 through the line element $ds^2 = {}^4g_{\mu\nu}(x)\, dx^\mu\, dx^\nu$.

Consequently, the gravitational field *teaches relativistic causality* to all of the other fields: in particular classical rays of light, photons and gluons, which are the trajectories allowed for massless particles in each point of M^4.

Let us make a comment about the two main existing approaches for quantizing gravity.

1) *Effective quantum field theory and string theory.* This approach contains the standard model of elementary particles and its extensions. However, since the quantization, namely

---

[14] Given the enormous variety of solutions of Einstein's equations, one cannot expect to find general answers to ontological questions.

the definition of the Fock space, requires a background space-time for the definition of creation and annihilation operators, one must use the splitting ${}^4g_{\mu\nu} = {}^4\eta^{(B)}_{\mu\nu} + {}^4h_{\mu\nu}$ and quantize only the perturbation ${}^4h_{\mu\nu}$ of the background 4-metric $\eta^{(B)}_{\mu\nu}$ (usually B is either Minkowski or DeSitter space-time). In this way property ii) is lost (one exploits the fixed non-dynamical chrono-geometrical structure of the background space-time), and gravity is replaced by a field of spin two over the background (and passive diffeomorphisms are replaced by a Lie group of gauge transformations acting in an "inner" space). The only difference between gravitons, photons and gluons lies thereby in their quantum numbers.

2) *Loop quantum gravity*. This approach does not introduce a background space-time but, being inequivalent to a Fock space, has problems in incorporating particle physics. It exploits a fixed 3+1 splitting of the space-time M^4 and quantizes the associated instantaneous 3-spaces Σ_τ (quantum geometry). There is no known way, however, to implement consistent unitary evolution (the problem of the super-Hamiltonian constraint). Furthermore, since the theory is usually formulated in spatially-compact space-times without boundary, it admits no Poincaré symmetry group (and therefore no extra-dimensions as in string theory), and faces a serious problem concerning the definition of time: the so-called *frozen picture* without real evolution.

For outside points of view on loop quantum gravity and string theory see Ref.[26, 27], respectively.

Let us remark that all formulations of the theory of elementary particle and nuclear physics are a chapter of the theory of representations of the Poincaré group in the *inertial frames* of the spatially non-compact Minkowski space-time. As a consequence, if one looks at general relativity from the point of view of particle physics, the main problem to get a unified theory is that of conciliating the Poincaré group and the diffeomorphism group.

Let us now consider the ADM formulation of metric gravity [28] and its extension to tetrad gravity [15] obtained by replacing the ten configurational 4-metric variables ${}^4g_{\mu\nu}(x)$ with the sixteen cotetrad fields ${}^4E^{(\alpha)}_\mu(x)$ by means of the decomposition ${}^4g_{\mu\nu}(x) = {}^4E^{(\alpha)}_\mu(x)\,{}^4\eta_{(\alpha)(\beta)}\,{}^4E^{(\beta)}_\nu(x)$ [(α) are flat indices].

Then, after having restricted the model to globally-hyperbolic, topologically trivial, spatially non-compact space-times (admitting a *global notion of time*), let us introduce a global 3+1 splitting of the space-time M^4 and choose the world-line of a time-like observer. As in special relativity, let us make a coordinate transformation to observer-dependent *radar* 4-coordinates, $x^\mu \mapsto \sigma^A = (\tau, \sigma^r)$, adapted to the 3+1 splitting and using the observer world-line as origin of the 3-coordinates. Again, the inverse transformation, $\sigma^A \mapsto x^\mu = z^\mu(\tau, \sigma^r)$, defines the embedding of the leaves Σ_τ into M^4. These leaves Σ_τ (assumed to be Riemannian 3-manifolds diffeomorphic to R^3, so that they admit global 3-coordinates σ^r and a unique 3-geodesic joining any pair of points in Σ_τ) are both Cauchy surfaces and *simultaneity surfaces* corresponding to a *convention for clock synchronization*. For the induced 4-metric we get

---

[15] This extension is needed to describe the coupling of gravity to fermions; it is a theory of time-like observers each one endowed with a tetrad field, whose time-like axis is the unit 4-velocity of the observer and whose spatial axes are associated with a choice of three gyroscopes.

$$^4g_{AB}(\sigma) = \frac{\partial z^\mu(\sigma)}{\partial \sigma^A} \, ^4g_{\mu\nu}(x) \, \frac{\partial z^\nu(\sigma)}{\partial \sigma^B} = {}^4E_A^{(\alpha)} \, ^4\eta_{(\alpha)(\beta)} \, ^4E_B^{(\beta)} =$$

$$= \epsilon \begin{pmatrix} (N^2 - {}^3g_{rs} \, N^r \, N^s) & -{}^3g_{su} \, N^u \\ -{}^3g_{ru} \, N^u & -{}^3g_{rs} \end{pmatrix} (\sigma).$$

Here $^4E_A^{(\alpha)}(\tau, \sigma^r)$ are adapted cotetrad fields, $N(\tau, \sigma^r)$ and $N^r(\tau, \sigma^r)$ the lapse and shift functions, and $^3g_{rs}(\tau, \sigma^r)$ the 3-metric on Σ_τ with signature $(+ + +)$. We see that, unlike in special relativity, in general relativity the quantities $z_A^\mu = \partial z^\mu / \partial \sigma^A$ are no more cotetrad fields on M^4. Here they are only transition functions between coordinate charts, so that the dynamical fields are now the real cotetrad fields $^4E_A^{(\alpha)}(\tau, \sigma^r)$ and not the embeddings $z^\mu(\tau, \sigma^r)$.

Let us try to identify a class of space-times and an associated family of admissible 3+1 splittings suitable to incorporate particle physics and provide a model for the solar system or our galaxy (and hopefully even allowing an extension to the cosmological context) with the following further requirements [29]:

1) M^4 must be asymptotically flat at spatial infinity and the 4-metric must tend asymptotically to the Minkowski 4-metric there, in every coordinate system (this implies that the 4-diffeomorphisms must tend there to the identity at spatial infinity). In such space-times, therefore, there is an *asymptotic background 4-metric* allowing to avoid the decomposition $^4g_{\mu\nu} = {}^4\eta_{\mu\nu} + {}^4h_{\mu\nu}$ in the bulk.

2) The boundary conditions on the fields on each leaf Σ_τ of the admissible 3+1 splittings must be such to reduce the *Spi* group of asymptotic symmetries (see Ref.[30]) to the ADM Poincaré group. This means that there should not be *super-translations* (direction-dependent quasi Killing vectors, obstruction to the definition of angular momentum in general relativity), namely that all the fields must tend to their asymptotic limits in a direction-independent way (see Refs. [31]). This is possible only if the admissible 3+1 splittings have all the leaves Σ_τ tending to Minkowski space-like hyper-planes orthogonal to the ADM 4-momentum at spatial infinity [29]. In turn this implies that every Σ_τ is *the rest frame of the instantaneous 3-universe* and that there are asymptotic inertial observers to be identified with the *fixed stars*[16]. This requirement implies that the shift functions vanish at spatial infinity $[N^r(\tau, \sigma^r) \to O(1/|\sigma|^\epsilon), \epsilon > 0, \sigma^r = |\sigma| \hat{u}^r]$, where the lapse function tends to 1 $[N(\tau, \sigma^r) \to 1 + O(1/|\sigma|^\epsilon)]$ and the 3-metric tends to the Euclidean 3-metric $[^3g_{rs}(\tau, \sigma^u) \to \delta_{rs} + O(1/|\sigma|)]$.

3) The admissible 3+1 splittings should have the leaves Σ_τ admitting a generalized Fourier transform (namely they should be Lichnerowicz 3-manifolds [32] with involution). This would allow the definition of instantaneous Fock spaces in a future attempt of quantization.

4) All the fields on Σ_τ should belong to suitable weighted Sobolev spaces, so that M^4 has no Killing vectors and Yang-Mills fields on Σ_τ do not present Gribov ambiguities (due to the presence of gauge symmetries and gauge copies) [33].

---

[16] In a final extension to the cosmological context they could be identified with the privileged observers at rest with respect to the background cosmic radiation.

In absence of matter the Christodoulou and Klainermann [34] space-times are good candidates: they are near Minkowski space-time in a norm sense, avoid singularity theorems by relaxing the requirement of conformal completability (so that it is possible to follow solutions of Einstein's equations on long times) and admit gravitational radiation at null infinity.

Since the simultaneity leaves Σ_τ are the rest frame of the instantaneous 3-universe, at the Hamiltonian level it is possible to define *the rest-frame instant form of metric and tetrad gravity* [29, 35]. If matter is present, the limit of this description for vanishing Newton constant will be reduced to the rest-frame instant form description of the same matter in the framework of parametrized Minkowski theories, while the ADM Poincaré generators will tend to the kinematical Poincaré generators of special relativity. In this way we get a model admitting *a deparametrization of general relativity to special relativity*. It is not known whether the rest-frame condition can be relaxed in general relativity without bringing in super-translations, since the answer to this question is connected with the non-trivial problem of boosts in general relativity.

Let us now come back to ADM tetrad gravity. The time-like vector $^4E^A_{(o)}(\tau, \sigma^r)$ of the tetrad field $^4E^A_{(\alpha)}(\tau, \sigma^r)$, dual to the cotetrad field $^4E^{(\alpha)}_A(\tau, \sigma^r)$, may be rotated to become the unit normal to Σ_τ in each point by means of a standard Wigner boost for time-like Poincaré orbits depending on three parameters $\varphi_{(a)}(\tau, \sigma^r)$, $a = 1, 2, 3$: $^4E^A_{(o)}(\tau, \sigma^r) = L^A{}_B(\varphi_{(a)}(\tau, \sigma^r))\,^4\check{E}^B_{(o)}(\tau, \sigma^r)$. This allows to define the following cotetrads adapted to the 3+1 splitting (the so-called *Schwinger time gauge*) $^4\check{E}^{(o)}_A(\tau, \sigma^r) = \Big(N(\tau, \sigma^r); 0\Big)$, $^4\check{E}^{(a)}_A(\tau, \sigma^r) = \Big(N_{(a)}(\tau, \sigma^r); {}^3e_{(a)r}(\tau, \sigma^r)\Big)$, where $^3e_{(a)r}(\tau, \sigma^r)$ are cotriads fields on Σ_τ (tending to $\delta_{(a)r} + O(1/|\sigma|)$ at spatial infinity) and $N_{(a)} = N^r\,{}^3e_{(a)r}$. As a consequence, the sixteen cotetrad fields may be replaced by the fields $\varphi_{(a)}(\tau, \sigma^r)$, $N(\tau, \sigma^r)$, $N_{(a)}(\tau, \sigma^r)$, $^3e_{(a)r}(\tau, \sigma^r)$, whose conjugate canonical momenta will be denoted as $\pi_N(\tau, \sigma^r)$, $\pi_{\vec{N}(a)}(\tau, \sigma^r)$, $\pi_{\vec{\varphi}(a)}(\tau, \sigma^r)$, $^3\pi^r_{(a)}(\tau, \sigma^r)$.

The local invariance of the ADM action entails the existence of 14 first-class constraints (10 primary and 4 secondary):
   i) $\pi_N(\tau, \sigma^r) \approx 0$ implying the secondary super-Hamiltonian constraint $\mathcal{H}(\tau, \sigma^r) \approx 0$;
   ii) $\pi_{\vec{N}(a)}(\tau, \sigma^r) \approx 0$ implying the secondary super-momentum constraints $\mathcal{H}_{(a)}(\tau, \sigma^r) \approx 0$;
   iii) $\pi_{\vec{\varphi}(a)}(\tau, \sigma^r) \approx 0$;
   iv) three constraints $M_{(a)}(\tau, \sigma^r) \approx 0$ generating rotations of the cotriads.

Consequently, there are 14 gauge variables, which, as shown in Refs.[5], describe the *generalized inertial effects* in the non-inertial frame defined by the chosen admissible 3+1 splitting of M^4 centered on an arbitrary time-like observer. The remaining independent "two + two" degrees of freedom are the gauge invariant DOs of the gravitational field describing *generalized tidal effects* (see Refs.[5]). The same degrees of freedom emerge in ADM metric gravity, where the configuration variables N, N^r, $^4g_{rs}$, with conjugate momenta π_N, $\pi_{\vec{N}r}$, $^3\Pi^{rs}$, are restricted by 8 first-class constraints ($\pi_N(\tau, \sigma^r) \approx 0 \rightarrow \mathcal{H}(\tau, \sigma^r) \approx 0$, $\pi_{\vec{N}r}(\tau, \sigma^r) \approx 0 \rightarrow \mathcal{H}^r(\tau, \sigma^r) \approx 0$).

As already said, the first-class constraints are the generators of the Hamiltonian gauge transformations, under which the ADM action is quasi-invariant (second Noether theorem):

i) The gauge transformations generated by the four primary constraints $\pi_N(\tau, \sigma^r) \approx 0$, $\pi_{\vec{N}(a)}(\tau, \sigma^r) \approx 0$, modify the lapse and shift functions, namely how densely the simultaneity surfaces are packed in M^4 and which points have the same 3-coordinates on each Σ_τ.

ii) Those generated by the three super-momentum constraints $\mathcal{H}_{(a)}(\tau, \sigma^r) \approx 0$ change the 3-coordinates on Σ_τ.

iii) Those generated by the super-Hamiltonian constraint $\mathcal{H}(\tau, \sigma^r) \approx 0$ transform an admissible 3+1 splitting into another admissible one by realizing a normal deformation of the simultaneity surfaces Σ_τ [42]. As a consequence, *all the conventions about clock synchronization are gauge equivalent as in special relativity.*

iv) Those generated by $\pi_{\vec{\varphi}(a)}(\tau, \sigma^r) \approx 0$, $M_{(a)}(\tau, \sigma^r) \approx 0$, change the cotetrad fields with local Lorentz transformations.

In the rest-frame instant form of tetrad gravity there are the three extra first-class constraints $P^r_{ADM} \approx 0$ (vanishing of the ADM 3-momentum as *rest-frame conditions*). They generate gauge transformations which change the time-like observer whose world-line is used as origin of the 3-coordinates.

A fundamental technical point, which is of paramount importance for the physical interpretation, is the possibility of performing a separation of the gauge variables from the DOs by means of a Shanmugadhasan canonical transformation [36].

In Ref.[35] a Shanmugadhasan canonical transformation adapted to 13 first class constraints (not to the super-Hamiltonian one, because no one knows how to solve it except in the Post-Newtonian approximation) has been introduced and exploited to clarify the interpretation. There are problems, however, when one introduces matter.

To avoid the above difficulties, a different Shanmugadhasan canonical transformation, adapted only to 10 constraints but allowing the addition of any kind of matter to the rest-frame instant form of tetrad gravity, has been recently found starting from a new parametrization of the 3-metric [37].

The basic idea is that the symmetric 3-metric tensor can be diagonalized with an orthogonal matrix depending on three Euler angles $\theta^i(\tau, \vec{\sigma})$. The three eigenvalues $\lambda_i(\tau, \vec{\sigma})$ are then replaced by the conformal factor $\phi(\tau, \vec{\sigma})$ of the 3-metric and by two *tidal* variables $R_{\bar{a}}(\tau, \vec{\sigma})$, $\bar{a} = 1, 2$. The defining equations and the resulting Shanmugadhasan canonical transformation are:

$$
\begin{aligned}
{}^3g_{rs} &= \sum_{uv} V_{ru}(\theta^n)\, \lambda_u\, \delta_{uv}\, V^T_{vs}(\theta^n) = \\
&= \sum_a \left(V_{ra}(\theta^n)\, \Lambda^a \right) \left(V_{sa}(\theta^n)\, \Lambda^a \right) = \sum_a {}^3\bar{e}_{(a)r}\, {}^3\bar{e}_{(a)s} = \sum_a {}^3e_{(a)r}\, {}^3e_{(a)s},
\end{aligned}
$$

$$
\Lambda^a(\tau, \vec{\sigma}) \overset{def}{=} \sum_u \delta_{au} \sqrt{\lambda_u(\tau, \vec{\sigma})} = \phi^2(\tau, \vec{\sigma})\, e^{\sum_{\bar{a}} \gamma_{\bar{a}b}\, R_{\bar{a}}(\tau, \vec{\sigma})}
$$

$$
\to_{r \to \infty}\ 1 + \frac{M}{4r} + \frac{a^a}{r^{3/2}} + O(r^{-3}),
$$

$$^3e_{(a)r} = R_{(a)(b)}(\alpha_{(c)}) \, ^3\bar{e}_{(b)r},$$

$$^3\bar{e}_{(a)r} = \sum_b {}^3e_{(b)r} \, R_{(b)(a)}(\alpha_{(e)}) = \sum_u \sqrt{\lambda_u} \, \delta_{u(a)} \, V_{ur}^T(\theta^n) = V_{ra}(\theta^n) \, \Lambda^a,$$

$$^3\bar{e}_{(a)}^r = \sum_u \frac{\delta_{u(a)}}{\sqrt{\lambda_u}} \, V_{ru} = \frac{V_{ra}(\theta^n)}{\Lambda^a},$$

$$\phi = (det \, ^3g)^{1/12} = (^3e)^{1/6} = \, ^3\bar{e}^{1/6} = (\lambda_1 \, \lambda_2 \, \lambda_3)^{1/12} = (\Lambda^1 \, \Lambda^2 \, \Lambda^3)^{1/6}.$$

$\varphi_{(a)}$	n	$n_{(a)}$	$^3e_{(a)r}$
≈ 0	≈ 0	≈ 0	$^3\pi_{(a)}^r$

$\longrightarrow$

$\varphi_{(a)}$	$\alpha_{(a)}$	n	$\bar{n}_{(a)}$	$^3\bar{e}_{(a)r}$
≈ 0	≈ 0	≈ 0	≈ 0	$^3\tilde{\tilde{\pi}}_{(a)}^r$

$\longrightarrow$

$\varphi_{(a)}$	$\alpha_{(a)}$	n	$\bar{n}_{(a)}$	θ^r	Λ^r
≈ 0	≈ 0	≈ 0	≈ 0	$\pi_r^{(\theta)}$	$'P_r$

Λ^r
P_r

$\longrightarrow$

ϕ	$R_{\bar{a}}$
π_ϕ	$\Pi_{\bar{a}}$

,

$$\pi_\phi = -\epsilon \, \frac{c^3}{2\pi \, G} \, (^3\bar{e})^{5/6} \, ^3K = 2 \, \frac{\sum_b \Lambda^b \, P_b}{(\Lambda^1 \, \Lambda^2 \, \Lambda^3)^{1/6}}, \tag{4.1}$$

where $\bar{n}_{(a)} = \sum_b n_{(b)} \, R_{(b)(a)}(\alpha_{(e)})$ are the shift functions at $\alpha_{(a)}(\tau, \vec{\sigma}) = 0$, $\alpha_{(a)}(\tau, \sigma^r)$ are three Euler angles and $\theta^r(\tau, \sigma^r)$ are three angles giving a coordinatization of the action of 3-diffeomorphisms on the cotriads $^3e_{(a)r}(\tau, \sigma^r)$. The configuration variable $\phi(\tau, \sigma^r) = \left(det \, ^3g(\tau, \sigma^r) \right)^{1/12}$ is the conformal factor of the 3-metric: it can be shown that it is the unknown quantity in the super-Hamiltonian constraint (also named the Lichnerowicz equation). The gauge variables are n, $\bar{n}_{(a)}$, $\varphi_{(a)}$, $\alpha_{(a)}$, θ^r and π_ϕ, while $R_{\bar{a}}$, $\Pi_{\bar{a}}$, $\bar{a} = 1, 2$, are the DOs of the gravitational field (in general they are not tensorial quantities).

This canonical transformation is the first explicit construction of a York map [38], in which the momentum conjugate to the conformal factor (*the gauge variable controlling the convention for clock synchronization*) is proportional to the trace $^3K(\tau, \vec{\sigma})$ of the extrinsic curvature of the simultaneity surfaces Σ_τ. Both the *tidal* and the *gauge* variables can be expressed in terms of the original variables. Moreover, in a family of completely fixed gauges differing with respect to the convention of clock synchronization, the deterministic Hamilton equations for the *tidal* variables and for *matter* variables contain *relativistic inertial forces* determined by $^3K(\tau, \vec{\sigma})$, which change from attractive to repulsive where the trace changes sign. These inertial forces do not have a non-relativistic counterpart (the Newton 3-space

is absolute) and could perhaps support the proposal of Ref. [39] [17] according to which *dark matter* could be explained as an *inertial effect*. While in the MOND model [41] there is an arbitrary function on the acceleration side of Newton equations in the absolute Euclidean 3-space, here we have the arbitrary gauge function $^3K(\tau, \vec{\sigma})$ on the force side of Hamilton equations.

Finally let us see which Dirac Hamiltonian H_D generates the τ-evolution in ADM canonical gravity. In *spatially compact space-times without boundary* H_D is a linear combination of the primary constraints plus the secondary super-Hamiltonian and super-momentum constraints multiplied by the lapse and shift functions, respectively (a consequence of the Legendre transform). Consequently, $H_D \approx 0$ and, in the reduced phase space, we get a vanishing Hamiltonian. This implies the so-called *frozen picture* and the problem of how to reintroduce a temporal evolution [18]. Usually one considers the normal (time-like) deformation of Σ_τ induced by the super-Hamiltonian constraint as evolution in a local time variable to be identified (the "multi-fingered" time point of view with a local, either extrinsic or intrinsic, time): this is the so-called *Wheeler-DeWitt interpretation* [19].

On the contrary, *in spatially non-compact space-times* the definition of functional derivatives and the existence of a well-posed Hamiltonian action principle (with the possibility of a good control of the surface terms coming from integration by parts) require the addition of the *DeWitt surface term*[44] (living on the surface at spatial infinity) to the Hamiltonian. It can be shown [29] that in the rest-frame instant form this term, together with a surface term coming from the Legendre transformation of the ADM action, leads to the Dirac Hamiltonian

$$H_D = \check{E}_{ADM} + (constraints) = E_{ADM} + (constraints) \approx E_{ADM}. \qquad (4.2)$$

Here $\check{E}_{ADM}$ is the *strong ADM energy*, a surface term analogous to the one defining the electric charge as the flux of the electric field through the surface at spatial infinity in electromagnetism. Since we have $\check{E}_{ADM} = E_{ADM} + (constraints)$, we see that the non-vanishing part of the Dirac Hamiltonian is the *weak ADM energy* $E_{ADM} = \int d^3\sigma \, \mathcal{E}_{ADM}(\tau, \sigma^r)$, namely the integral over Σ_τ of the ADM energy density (in electromagnetism this corresponds to the definition of the electric charge as the volume integral of matter charge density). Therefore there is no frozen picture but a consistent τ-evolution instead.

Note that the ADM energy density $\mathcal{E}_{ADM}(\tau, \sigma^r)$ is a *coordinate-dependent quantity*, because it depends on the gauge variables (namely upon the relativistic inertial effects present in the non-inertial frame): this is nothing else than the old *problem of energy* in general relativity. Let us remark that in most coordinate systems $\mathcal{E}_{ADM}(\tau, \sigma^r)$ does not agree with the pseudo-energy density defined in terms of the Landau-Lifschitz pseudo-tensor.

In order to get a deterministic evolution for the DOs [20] we must fix the gauge completely,

---

[17] The model proposed in Ref.[39] is too naive, as shown by the criticism in Refs.[40].

[18] See Refs.[4] for the problem of time in general relativity.

[19] Kuchar [43] says that the super-Hamiltonian constraint must not be interpreted as a generator of gauge transformations, but as an effective Hamiltonian.

[20] See Refs.[45] for the modern formulation of the Cauchy problem for Einstein equations, which mimics the steps of the Hamiltonian formalism.

that is we must add 14 gauge-fixing constraints satisfying an orbit condition and to pass to Dirac brackets. As already said, the correct way to do so is the following one:

i) Add a gauge-fixing constraint to the secondary super-Hamiltonian constraint [21]. This gauge-fixing fixes the form of Σ_τ, i.e. the convention for the synchronization of clocks. The τ-constancy of this gauge-fixing constraint generates a gauge-fixing constraint to the primary constraint $\pi_N(\tau, \sigma^r) \approx 0$ for the determination of the lapse function.

ii) Add three gauge-fixings to the secondary super-momentum constraints $\mathcal{H}_{(a)}(\tau, \sigma^r) \approx 0$. This fixes the 3-coordinates on each Σ_τ. The τ-constancy of these gauge fixings generates the three gauge fixings to the primary constraints $\pi_{\vec{N}\,(a)}(\tau, \sigma^r) \approx 0$ and leads to the determination of the shift functions (i.e. of the appearances of *gravito-magnetism*).

iii) Add six gauge-fixing constraints to the primary constraints $\pi_{\vec{\varphi}(a)}(\tau, \sigma^r) \approx 0$, $M_{(a)}(\tau, \sigma^r) \approx 0$. This is a fixation of the cotetrad field which includes a convention on the choice and the transport of the three gyroscopes of every time-like observer of the two congruences associated with the chosen 3+1 splitting of M^4 (see Ref.[17, 18].

iv) In the rest-frame instant form we must also add three gauge fixings to the rest-frame conditions $P^r_{ADM} \approx 0$. The natural ones are obtained with the requirement that the three ADM boosts vanish. In this way we select a special time-like observer as origin of the 3-coordinates (like the Fokker-Pryce center of inertia in special relativity [14, 23]).

In this way all the gauge variables are fixed to be either numerical functions or well determined functions of the DOs. This complete gauge fixing is physically equivalent to a definition of the *global non-inertial frame centered on a time-like observer, carrying its pattern of inertial forces* we have called NIF (see Ref.[5]). Note that in a NIF, the ADM energy density $\mathcal{E}_{ADM}(\tau, \sigma^r)$ becomes *a well defined function of the DOs only* and the Hamilton equations for them with E_{ADM} as Hamiltonian are a hyperbolic system of partial differential equations for their determination. For each choice of Cauchy data for the DOs on a Σ_τ, we obtain a solution of Einstein's equations (an Einstein universe) in the radar 4-coordinate system associated with the chosen 3+1 splitting of M^4.

Actually, the Cauchy data are the 3-geometry and matter variables on the Cauchy 3-surfaces of a kinematically possible NIF. Such data are restricted by the super-Hamiltonian and super-momentum constraints (which are four of Einstein's equations).

An Einstein space-time M^4 (a *4-geometry*) *is the equivalence class of all the completely fixed gauges (NIF) with gauge equivalent Cauchy data for the DOs on the associated Cauchy and simultaneity surfaces* Σ_τ.

Once a solution of the hyperbolic Hamilton equations (viz. the Einstein equations after a complete gauge fixing) has been found corresponding to a set of Cauchy data, in each NIF we know the DOs in that gauge (*the tidal effects*) and then the explicit form of the gauge variables (*the inertial effects*). Moreover, *the extrinsic curvature of the simultaneity surfaces* Σ_τ *is determined too.* Since the simultaneity surfaces are asymptotically flat, it is possible to determine their embeddings $z^\mu(\tau, \sigma^r)$ in M^4. As a consequence, *unlike special relativity,* the conventions for clock synchronization and the whole chrono-geometrical structure of M^4 (gravito-magnetism, 3-geodesic spatial distance on Σ_τ, trajectories of light rays in each point of M^4, one-way velocity of light) are *dynamically determined* .

---

[21] The special choice $\pi_\phi(\tau, \sigma^r) \approx 0$ implies that the DOs $R_{\bar{a}}$, $\Pi_{\bar{a}}$, remain canonical even if we do not know how to solve this constraint.

Let us remark that, if we look at Minkowski space-time as a special solution of Einstein's equations with $R_{\bar{a}}(\tau, \sigma^r) = \Pi_{\bar{a}}(\tau, \sigma^r) = 0$ (zero Riemann tensor, no tidal effects, only inertial effects), we find [29] that the dynamically admissible 3+1 splittings (non-inertial frames) must have the simultaneity surfaces Σ_τ *3-conformally flat*, because the conditions $R_{\bar{a}}(\tau, \sigma^r) = \Pi_{\bar{a}}(\tau, \sigma^r) = 0$ imply the vanishing of the Cotton-York tensor of Σ_τ. Instead, in special relativity, considered as an *autonomous theory*, all the non-inertial frames compatible with the Møller conditions are admissible, so that there is much more freedom in the conventions for clock synchronization.

A first application of this formalism [46] has been the determination of post-Minkowskian background-independent gravitational waves in a completely fixed non-harmonic 3-orthogonal gauge with diagonal 3-metric. It can be shown that the requirements $R_{\bar{a}}(\tau, \sigma^r) << 1$, $\Pi_{\bar{a}}(\tau, \sigma^r) << 1$ lead to a weak field approximation based on a Hamiltonian linearization scheme:
i) linearize the Lichnerowicz equation, determine the conformal factor of the 3-metric and then the lapse and shift functions;
ii) find E_{ADM} in this gauge and disregard all the terms more than quadratic in the DOs;
iii) solve the Hamilton equations for the DOs.

In this way we get a solution of linearized Einstein's equations, in which the configurational DOs $R_{\bar{a}}(\tau, \sigma^r)$ play the role of the *two polarizations* of the gravitational wave and we can evaluate the embedding $z^\mu(\tau, \sigma^r)$ of the simultaneity surfaces of this gauge explicitly.

Let us conclude with some remarks about the interpretation of the space-time 4-manifold in general relativity.

In 1914 Einstein, during his researches for developing general relativity, faced the problem arising from the fact that the requirement of general covariance would involve a threat to the physical objectivity of the points of space-time M^4, which in classical field theories are usually assumed to have a *well defined individuality*. This led him to formulate the *Hole Argument*. Assume that M^4 contains a *hole* $\mathcal{H}$, that is an open region where all the non-gravitational fields vanish. It is implicitly assumed that the Cauchy surface for Einstein's equations lies outside $\mathcal{H}$. Let us consider an active diffeomorphism A which re-maps the points inside $\mathcal{H}$, but is the identity outside $\mathcal{H}$. For any point $p \in \mathcal{H}$ we have $p \mapsto D_A p \in \mathcal{H}$. The induced active diffeomorphism on the 4-metric tensor ${}^4g$, solution of Einstein's equations, will map it into another solution $D_A^* {}^4g$ (D_A^* is a dynamical symmetry of Einstein's equations) defined by $D_A^* {}^4g(D_A p) = {}^4g(p) \neq D_A^* {}^4g(p)$. Consequently, we get two solutions of Einstein's equations with the same Cauchy data outside $\mathcal{H}$ and it is not clear how to save the identification of the mathematical points of M^4.

Einstein avoided the problem by means of the pragmatic *point-coincidence argument*: the only real world-occurrences are the (coordinate-independent) space-time coincidences (like the intersection of two world-lines). However, the problem was rebirth by Stachel [47] and then by Earman and Norton [48], and this opened a rich philosophical debate that is still alive today.

We must face the following dilemma:

If we insist on the reality of space-time mathematical points independently of the presence of any physical field (the *substantivalist* point of view of philosophers), we are in trouble with predictability.

If we say that ^4g and $D_A^*\,^4g$ describe the same universe (the so-called *Leibniz equivalence*), we lose any physical objectivity of the space-time points (the *anti-substantivalist* point of view).

Stachel [47] suggested that a physical individuation of the point-events of M^4 could be made only by using *four individuating fields depending on the 4-metric on M^4*, namely that a tensor field on M^4 is needed to identify the points of M^4.

On the other hand, *coordinatization* is the only way to individuate the points *mathematically* since, as stressed by Hermann Weyl [49]: "There is no distinguishing objective property by which one could tell apart one point from all others in a homogeneous space: at this level, fixation of a point is possible only by a *demonstrative act* as indicated by terms like *this* and *there*."

To clarify the situation let us remember that Bergmann and Komar [50] gave *a passive re-interpretation of active diffeomorphisms as metric-dependent coordinate transformations* $x^\mu \mapsto y^\mu(x, {}^4g(x))$ restricted to the solutions of Einstein's equations (i.e. *on-shell*). It can be shown that *on-shell* ordinary passive diffeomorphisms and the *on-shell* Legendre pull-back of Hamiltonian gauge transformations are two (overlapping) dense subsets of this set of *on-shell* metric-dependent coordinate transformations. Since the Cauchy surface for the Hole Argument lies outside the hole (where the active diffeomorphism is the identity), it follows that the passive re-interpretation of the active diffeomorphism D_A^* must be an *on-shell Hamiltonian gauge transformation*, so that the *Leibniz equivalence* reduces to gauge equivalence in the sense of Dirac constraint theory (^4g and $D_A^*\,^4g$ belong to the same gauge orbit). In our language, *Leibniz equivalence* is then reduced to a change of NIF for the same Einstein *universe*.

What remains to be done is to implement Stachel's suggestion according to which the *intrinsic pseudo-coordinates* of Bergmann and Komar [51] should be used as *individuating fields*. These pseudo-coordinates for M^4 (at least when there are no Killing vectors) are four scalar functions $F^A[w_\lambda]$, $A, \lambda = 1, .., 4$, of the four eigenvalues $w_\lambda(^4g, \partial\,^4g)$ of the spatial part of the Weyl tensor. Since these eigenvalues can be shown to be in general functions of the 3-metric, of its conjugate canonical momentum (namely of the extrinsic curvature of Σ_τ) and of the lapse and shift functions, the pseudo-coordinates are well defined in phase space and can be used as a label for the points of M^4.

The final step [5] is to implement the individuation of point-events by considering an arbitrary kinematically admissible 3+1 splitting of M^4 with a given time-like observer and the associated radar 4-coordinates σ^A (a NIF), and imposing the following gauge fixings to the secondary super-Hamiltonian and super-momentum constraints (the only restriction on the functions F^A is the orbit condition):

$$\chi^A(\tau, \sigma^r) = \sigma^A - F^A[w_\lambda] \approx 0. \tag{4.3}$$

In this way we break general covariance completely and we determine the gauge variables θ^r and π_ϕ. Then the τ-constancy of these gauge fixings will produce the gauge fixings determining the lapse and shift functions. After having fixed the Lorentz gauge freedom of the cotetrads, we arrive at a completely fixed gauge in which, after the transition to Dirac brackets, we get $\sigma^A \equiv \tilde{F}^A[r_{\bar{a}}(\sigma), \pi_{\bar{a}}(\sigma)]$, namely the conclusion that the *radar* 4-coordinates

of a point in M^4_{3+1}, the copy of M^4 coordinatized with the chosen non-inertial frame, are determined *off-shell* by the four DOs of that gauge: in other words *the* individuating fields *nothing else than are the genuine* tidal *effects of the gravitational field.* By varying the functions F^A we can make an analogous off-shell identification in every other admissible non-inertial frame. The procedure is consistent, because the DOs are functionals of the metric and the extrinsic curvature on a whole 3-space Σ_τ but in fact know the whole 3+1 splitting M^4_{3+1} of M^4.

Some consequences of this identification of the point-events of M^4 are:

1) *The physical space-time M^4 and the vacuum gravitational field are essentially the same entity.* The presence of matter modifies the solutions of Einstein equations, i.e. M^4, but plays only an indirect role in this identification (see Ref.[5]. On the other hand, matter is fundamental in establishing a (still lacking) dynamical theory of measurement exploiting non-test objects. Consequently, instead of the dichotomy substantivalism/relationism, it seems that this analysis - as a case study limited to the class of space-times dealt with - may offer a new more articulated point of view, which can be named *point structuralism* (see Ref. [52]).

2) The *reduced phase space* of this model of general relativity is the space of *abstract* DOs (pure *tidal* effects without *inertial* effects), which can be thought of as four fields residing on an abstract space-time $\tilde{M}^4$ defined as the equivalence class of all the admissible, non-inertial frames M^4_{3+1} containing the associated inertial effects.

3) Each *radar* 4-coordinate system of an admissible non-inertial frame M^4_{3+1} has an associated *non-commutative structure*, determined by the Dirac brackets of the functions $\tilde{F}^A[r_{\bar{a}}(\sigma), \pi_{\bar{a}}(\sigma)]$ determining the gauge, a fact that could play a role in the quantization of the theory.

As a final remark, let us note that these results on the identification of point-events are *model dependent.* In spatially compact space-times without boundary, the DOs are *constants of the motion* due to the frozen picture. As a consequence, the gauge fixings $\chi^A(\tau, \sigma^r) \approx 0$ (in particular χ^τ) cannot be used to rebuild the temporal dimension: probably only the instantaneous 3-space of a 3+1 splitting can be individuated in this way.

V. CONCLUSIONS.

Our everyday experience of macroscopic objects and processes is scientifically described in terms of Newtonian physics with its separate notions of time (and simultaneity) and (Euclidean instantaneous) 3-space. A huge amount of philosophical literature has been devoted to the analysis of the consequences that follow from the empirical fact that light velocity, as well as that of any causal propagation, has finite magnitude. Our macroscopic experience is dominated by Maxwell equations even for the fact that, from the physical and neurophysiological point of view, all the information that reaches our brain is of electro-magnetic origin. Therefore the consequences of the finite magnitude of the causal propagation of energy and information has a direct bearing on our phenomenological experience. On the other hand, the conventional nature of the definition of distant simultaneity that follows from the analysis of the basic structure of causal influences in SR seems to conflict with every possible notion of *3-dimensional reality* of objects and processes which stands at the basis of our phenomenological experience since it entails that no observer- and frame-independent notions of simultaneity and instantaneous 3-space be possible. Even if - from the technical point of view - the question of the conventionality of simultaneity can be rephrased as a gauge problem, it lasted as source of an unending debate involving old fundamental issues concerning the philosophy of time like that of the nature of now-ness, becoming, reality or unreality of time, past and future, with all possible ramifications and varieties of philosophical distinctions.

It should not be undervalued that relativistic thinking unifies the *physical* notions of space and time in a 4-dimensional structure, whilst space and time maintain a substantial ontological diversity in our phenomenological experience. While time is experienced as "flowing", space is not. Furthermore time, even more than space, plays a fundamental constitutive role for our "being in the world" and for *subjectivity* in general, which manifests itself in living beings with various gradations. There is, therefore, a deep contrast between the formal inter-subjective unification of space and time in the scientific relativistic image, on the one hand, and the ontological diversity of time and space within the subjectivity of experience, on the other. This appears to be the most important and difficult question that physics raises to contemporary philosophy, since it reveals the core of the relation between *reality* of *experience* and *reality-objectivity* of *knowledge*. Dismissing this contrast by a literal adoption of the scientific image is not as much a painless and obvious operation as rather an implicit adoption of a strong physicalist philosophical position that should be argued for itself.

This said, we have faced the question to investigate a possible contribution of the inclusion of gravity (which, as well-known, is a universal interaction that cannot be shielded) to the clarification of the problem of relativistic distant simultaneity. This has been done having in view certainly not a *technical resolution* of the above philosophical contrast, rather as the achievement of a notion of distant simultaneity within the scientific image which be at least *compatible* with our deep experience of what Whitehead called the "cosmic unison".

As a matter of fact, *we have shown that the inclusion of gravity deeply changes the state of affairs about relativistic simultaneity.*

In brief, we have identified a class of curved pseudo-Riemannian space-times in which the following results holds:

i) Outside the solutions of Einstein's equations (i.e. *off-shell*), these space-times admit 3+1 splittings, which can be interpreted as *kinematically possible global non-inertial lab-*

oratories (kinematically possible NIFs) centered on *arbitrary accelerated observers*. The viewpoint following from this concept leads to a frame-dependent notion of *instantaneous 3-space,* which is concomitantly a *clock synchronization convention.* As in SR, all these conventions are gauge equivalent, so that there is *no Wheeler-DeWitt interpretation* of the gauge transformations generated by the super-Hamiltonian constraint.

ii) The *off-shell* Hamiltonian separation of the *tidal* degrees of freedom of the gravitational field from the *gauge variables* implies the interpretation of the latter as *relativistic inertial effects* which are shown in the chosen kinematical NIF. Since in this class of space-times the Hamiltonian is the weak ADM energy plus a combination of the first class constraints, in every completely fixed gauge (a well defined kinematical NIF) it follows deterministic evolution of the *tidal degrees of freedom* in mathematical time (to be replaced by a physical clock when eventually possible) governed by the *tidal forces* and the *inertial forces* of that NIF (note that unlike the Newtonian physics, such forces are in general functions of the tidal degrees of freedom too)[22].

iii) The solution of Hamilton equations in a completely fixed gauge with given Cauchy data for the *tidal* degrees of freedom (and matter if present) determines a solution of Einstein's equations in a well defined 4-coordinated system, which *on-shell* are re-interpretable as coordinates adapted to a *dynamically determined* NIF (one of its leaves is the Cauchy surface on which the Cauchy data have been assigned).

iv) Given any solution of Einstein's equations in a given 4-coordinate system, we can determine the dynamical 3+1 splitting (a *dynamical* NIF) of Einstein's space-time, one of whose simultaneity 3-surfaces is just the Cauchy surface of the solution. Consequently, there is a *dynamical emergence of the instantaneous 3-spaces*, leaves of the dynamical NIF, for each solution of Einstein's equations in a given 4-coordinate system (adapted *on-shell* to the dynamical NIF). Moreover, all the chrono-geometrical structure of Einstein's space-time ($ds^2 = {}^4g_{\mu\nu}(x)\, dx^\mu\, dx^\nu$) is *dynamically determined.*

v) These results and a revisitation of the Hole Argument imply that *space-time and vacuum gravitational field are two faces of the same reality*, and we get a *new kind of structuralism* (with elements of both the *substantivalist* and *relationist* points of view) implying a 4-dimensional holism (see Ref.[5]) resulting from a foliation with 3-dimensional instantaneous 3-spaces [23].

In conclusion what in Newton's theory was an absolute Euclidean instantaneous 3-space reappears in GR as a *dynamicadlly emergent* Riemannian time-varying instantaneous 3-space, which is a simultaneity leaf of a *dynamical* NIF *uniquely associated to a solution of Einstein's equations in 4-coordinates adapted to the NIF itself.* The NIF is centered on a time-like (in general accelerated) observer, whose world-line can be made to coincide with the Fokker-Pryce center of inertia by means of a suitable gauge fixing to the rest-frame conditions. In the post-Newtonian approximation around the Earth we describe the situation in a *quasi-inertial frame* with harmonic 4-coordinates, as those considered in the

---

[22] In these globally hyperbolic space-times there is no *frozen picture of dynamics.*

[23] In spatially compact space-times without boundary, where there is a frozen picture of dynamics and only a local time-evolution according to the Wheeler-DeWitt interpretation, only 3-space, but not the time direction, can be determined from the gravitational field.

IAU conventions for the geocentric celestial reference frame [54]. However, in this case too the IAU frame is not the dynamical NIF associated with the post-Newtonian solution of the Einstein's equations: it has to be determined through the inverse problem starting from the post-Newtonian extrinsic curvature 3-tensor.

Admittedly, all the physical implications of this viewpoint must still be worked out (for instance the determination of non-inertial frames in which the Riemannian distance from the Earth to a galaxy equals the galaxy luminosity distance and the implications for dark matter and dark energy of the dynamical instantaneous 3-spaces).

Let us remark that in SR (and in GR too before identifying the preferred dynamical convention of clock synchronization), an ideal observer has the following freedom in the description of the phenomena around him:

a) the arbitrary choice of the clock synchronization convention, i.e., of the instantaneous 3-spaces;

b) the choice of the 4-coordinate system.

After these choices, the observer has a description of the other world-lines and/or world-tubes simulating the phenomena with Hamiltonian evolution in the chosen time parameter. All these descriptions have been shown to be gauge-equivalent in the previous sections. Every other ideal observer has the same type of freedom in the description of the phenomena.

Each solution of Einstein's equations, i.e. each Einstein *universe*, in our class of models, is an equivalence class of well-defined *dynamical* NIFs (the *epistemic part* of the metric field describing the *generalized relativistic inertial effects*) with their *dynamical clock synchronization conventions*, their *dynamical instantaneous 3-spaces* and their *dynamical individuation of point-events*[24]. The NIFs selected by one solution are *different* from the NIFs selected by a *different solution*. Let us stress, however, that given a solution, the set of the associated NIFs is a *substantially smaller* set than that of the a-priori *kinematically* possible NIFs both in GR and in SR, since the only restrictions at the kinematical level are given by the Møller conditions[25]. Given an Einstein *universe*, all the associated NIFs in the equivalence class are connected by *on-shell Hamiltonian gauge transformations* (containing adapted passive diffeomorphisms) so that they know - as it were - the Cauchy data of the solution. Note moreover that they also contain the freedom of changing the time-like observer's origin of the 3-coordinates on the instantaneous 3-spaces, and the freedom of making an arbitrary (tensorial) passive diffeomorphism leading to non-adapted 4-coordinates.

From the point of view of the 4-dimensional picture with the freedom of passive diffeo-morphisms one could be led to adopt the misleading notion of "block universe" even in GR. However, one should not forget that this "block universe" is the equivalence class of *dynamical* NIFs (stratification or 3+1 splittings with dynamical generated instantaneous 3-spaces). Once the epistemic framework of a NIF is chosen, a *well-defined B-series is established*, since the notion of "earlier than", "later than" and "simultaneous with" is globally defined in terms of the mathematical time of the globally hyperbolic space-time (to be then replaced with a physical clock monotonically increasing in the mathematical time, at least

---

[24] Maybe even realized by means of 4-coordinates not adapted to the NIF; we simplified the exposition by formulating the NIFs with adapted *radar* 4-coordinates

[25] If Minkowski space-time without matter is considered as a special solution of Einstein's equations, its dynamical NIFs have the simultaneity leaves 3-conformally flat [29, 35].

for a finite interval). Unlike in SR, where each observer has their own B-series, we have disclosed the relevant fact that, at least for a specific class of models, GR is characterized by a *universal B-series*. As a matter of fact, a Hamiltonian gauge transformation changes the NIF, the Hamiltonian, the matter distribution, the Cauchy surface and the form in which the Cauchy data are given on it, in such a way that the B-series relations between *any pair of events* is left invariant. The same happens with the freedom of changing the time-like observer that defines the origin of the coordinates on the instantaneous 3-spaces.

Nothing are we willing to say about the relevance of A-series determinations within the scientific image, as already stressed in the Introduction. We maintain that any tensed determination is wholly foreign to any kind of physical description of the world. Clearly, unlike the case of SR, we here have Hamiltonian evolution of the 3-space too, determined by the ADM energy (which depends on the *ontic* tidal effects and on the matter). This evolution, however - as to its temporal characterization - is not substantially different from the Newtonian evolution of matter in absolute time.

It remains to be clarified - as anticipated in the Introduction - the import (if any) of our technical results on the dichotomy *endurantism/perdurantism*. Having already stated our definition of *wholly presentness* of a *physical object* in the Introduction, we must now only ascertain which of the relevant physical features of a general-relativistic space-time with matter we have enumerated so far, tend to support a reasonable notion of *endurantism* or *perdurantism* at the level of the scientific image.

It seems clear that all the attributes which are necessary to define a spatially-extended *physical object* belonging to a dynamically generated 3-space Σ_τ at a certain time τ, can be obtained by the chrono-geometrical features intrinsic to Σ_τ and matter distribution on it. Note - remarkably - that among such attributes there is the 3-geometry of Σ_τ. We can conclude, therefore, that, in this limited sense, our results support an *endurantist* view of physical objects. It is interesting to note, on the other hand, that our analysis does not support a likewise simple *endurantist* view of the space-time structure itself. We have established that the *reality* of the vacuum space-time of GR is ontologically equivalent to the *reality* of the autonomous degrees of freedom of the gravitational field as described by the DOs (viz., the *ontic part* of the metric field). At this point we should look at space-time itself as at something sharing the attributes of a peculiar *physical object*. We could ask accordingly whether and to what extent the mathematical structure of the DOss allows an *endurantist* interpretation of such a peculiar object. The answer is simple: as already said the DOs, though being local fields indexed by the *radar* coordinates σ^A, when considered in relation to the 4-metric field g, are highly non-local functionals of the 3-metric field and of the extrinsic curvature 3-tensor on the whole 3-space Σ_τ[26]. Due to the extrinsic curvature, the structure of the DOs involves therefore an infinitesimal τ-continuum of 3-spaces around Σ_τ. The individuation procedure involves moreover a *temporal gauge* (A=0 in Equation (4.3)). In conclusion, the *physical individuation* of the space-time *point-events*, defined by Equation (4.3), *cannot* be considered as an attribute depending upon information wholly contained in the 3-space considered at time τ. This conclusion, however, should not be viewed as an unexpected and unsatisfactory result, given the double role of the metric field in GR.

---

[26] Let us recall that this 3-tensor describes the *embedding* of the instantaneous 3-space in the Einstein's 4-manifold through its dependence upon the τ-*derivative* of the 3-metric and the *lapse* and *shift* functions.

Let us close by stressing a general fundamental feature of GR. Though Einstein's partial differential equations are defined in a 4-dimensional framework, this framework must be considered as an unfolding of 3-dimensional sub-structures because of the nature of the Cauchy problem. Consequently, the *models* of GR are subdivided into two disjoint classes:

a) the 4-dimensional ones (with spatially-compact space-times), having the *problem of time*, the *frozen picture*, and a lacking *physical individuation* of point-events;

b) the asymptotically-flat space-times, with their 3+1 splitting and *dynamical emergence* of *achronal* 3-spaces, a *non trivial temporal evolution* and a *physical individuation* of point-events.

[1] G.Petit and P.Wolf, *Relativistic Theory for Time Comparisons: a Review*, METROLOGIA, **42**, S138-S144, (2005).

[2] A.Einstein, *Die formale Grundlage der allgemeinen Relativitätstheorie*, in *Preuss. Akad. der Wiss. Sitz.*, pp. 1030–1085;(1914).

[3] A.Einstein, *Die Grundlage der allgemeinen Relativitätstheorie*, *Annalen der Physik* **49**, 769–822, (1916); translation by W. Perrett and G. B. Jeffrey, *The foundation of the general theory of relativity*, in *The principle of relativity*, pp. 117–118, New York: Dover (1952).

[4] C.J.Isham, *Canonical Quantum Gravity and the Problem of Time*, in *Integrable Systems, Quantum Groups and Quantum Field Theories*, eds.L.A.Ibort and M.A.Rodriguez, Salamanca 1993 (Kluwer, London, 1993).
K.Kuchar, *Time and Interpretations of Quantum Gravity*, in Proc.4th Canadian Conf. on *General Relativity and Relativistic Astrophysics*, eds. G.Kunstatter, D.Vincent and J.Williams (World Scientific, Singapore, 1992).

[5] L.Lusanna and M.Pauri, *Explaining Leibniz Equivalence as Difference of Non-Inertial Appearances: Dis-solution of the Hole Argument and Physical Individuation of Point-Events*, talk at the Oxford Conference on Spacetime Theory (2004), to appear in Studies in History and Philosophy of Modern Physics (gr-qc/0604087); *The Physical Role of Gravitational and Gauge Degrees of Freedom in General Relativity. I: Dynamical Synchronization and Generalized Inertial Effects; II: Dirac versus Bergmann Observables and the Objectivity of Space-Time*, Gen.Rel.Grav. **38**, 187 and 229 (2006) (gr-qc/0403081 and 0407007).

[6] V.Perlick, *Characterization of Standard Clocks in General Relativity*, in *Semantical Aspects of Spacetime Theories*, U. Mayer and H.–J. Schmidt (eds.), B. I. Wissenshaftverlag, Mannheim, pp. 169–179, (1994).

[7] *Being and Becoming in Modern Physics*,Stanford Encyclopedia of Philosophy, (http://plato.stanford.edu/entries/spacetime-bebecome/); *Philosophy of Space and Time*, Wikipedia, the free encyclopedia (http://en.wikipedia.org/wiki/$Philosophy_of_space_and_time$).

[8] A.Grünbaum, *The Meaning of Time*, in *Essays in Honor of Carl G. Hempel*, A Tribute in Occasion of his Sixty-Fifth Birthday., D.Reidel Pbublishing Company, Dordrecht, 1972, pp.147–177.

[9] *Temporal Parts*, Stanford Encyclopedia of Philosophy: (http://plato.stanford.edu/entries/temporal-parts/).

[10] D.Lynden-Bell, *A relative Newtonian mechanics*, pp.172-178 of *Mach's principle. From Newton's bucket to quantum gravity*, eds. J.Barbour and H.Pfister (Birkhauser, Boston, 1995).

[11] D.Alba,, L.Lusanna and M.Pauri, *Dynamical Body Frames, Orientation-Shape Variables and Canonical Spin Bases for the Nonrelativistic N-Body Problem*, J.Math.Phys. **43**, 373 (2002) (hep-th/0011014).

[12] R.G.Littlejohn and M.Reinsch, *Gauge Fields in the Separation of Rotations and Internal Motions in the n-Body Problem*, Rev.Mod.Phys. **69**, 213 (1997).

[13] D.Alba, L.Lusanna and M.Pauri, *Centers of Mass and Rotational Kinematics for the Relativistic N-Body Problem in the Rest-Frame Instant Form* J.Math.Phys. **43**, 1677-1727 (2002) (hep-th/0102087).

[14] D.Alba, L.Lusanna and M.Pauri, *New Directions in Non-Relativistic and Relativistic Rotational and Multipole Kinematics for N-Body and Continuous Systems* (2005), in *Atomic and Molecular Clusters: New Research*, ed.Y.L.Ping (Nova Science, New York, 2006) (hep-th/0505005).

[15] D.Alba and L.Lusanna, *Simultaneity, radar 4-Coordinates and the 3+1 Point of View about Accelerated Observers in Special Relativity* (2003) (gr-qc/0311058); *Generalized Radar 4-Coordinates and Equal-Time Cauchy Surfaces for Arbitrary Accelerated Observers* (2005), submitted to Int.J.Mod.Phys. D (gr-qc/0501090).

[16] L.Lusanna, *The Chrono-geometrical Structure of Special and General Relativity: a Re-Visitation of Canonical Geometrodynamics*, Lectures given at the 42nd Karpacz Winter School of Theoretical Physics, "Current Mathematical Topics in Gravitation and Cosmology", Ladek, Poland, 6-11 February 2006 (gr-qc/0604120).

[17] L.Lusanna, *The Chronogeometrical Structure of Special and General Relativity: towards a Background-Independent Description of the Gravitational Field and Elementary Particles* (2004), in *General Relativity Research Trends*, ed. A.Reiner, Horizon in World Physics vol. 249 (Nova Science, New York, 2005) (gr-qc/0404122).
L.Lusanna, *Towards a Unified Description of the Four Interactions in Terms of Dirac-Bergmann Observables*, in *Quantum Field Theory: a 20th Century Profile* of the Indian National Science Academy, ed. A.N.Mitra, foreward F.J.Dyson (Hindustan Book Agency, New Delhi, 2000) (hep-th/9907081).

[18] B.Mashhoon, *The Hypothesis of Locality and its Limitations*, (gr-qc/0303029); *Limitations of Spacetime Measurements*, Phys.Lett. **A143**, 176 (1990). *The Hypothesis of Locality in Relativistic Physics*, Phys.Lett. **A145**, 147 (1990). *Measurement Theory and General Relativity*, in *Black Holes: Theory and Observation*, Lecture Notes in Physics 514, ed. F.W.Hehl, C.Kiefer and R.J.K.Metzler (Springer, Heidelberg, 1998), p.269. *Acceleration-Induced Nonlocality*, in Advances in General Relativity and Cosmology, ed. G.Ferrarese (Pitagora, Bologna, 2003) (gr-qc/0301065).
B.Mashhoon and U.Muench, *Length Measurement in Accelerated Systems*, Ann.Phys. (Leipzig) **11**, 532 (2002).

[19] M.Pauri and M.Vallisneri, *Maerzke-Wheeler Coordinates for Accelerated Observers in Special Relativity*, Foundations Phys. Lett. **13**, 401 (2000).

[20] L.Lusanna, *The N- and 1-Time Classical Descriptions of N-Body Relativistic Kinematics and the Electro-Magnetic Interaction*, Int.J.Mod.Phys. **A12**, 645 (1997).

[21] H.Crater and L.Lusanna, *The Rest-Frame Darwin Potential from the Lienard-Wiechert Solution in the Radiation Gauge*, Ann.Phys. (N.Y.) **289**, 87-177 (2001) (hep-th/0001046).

[22] D.Alba, H.Crater and L.Lusanna, *The Semiclassical Relativistic Darwin Potential for Spinning Particles in the Rest-Frame Instant Form: Two-Body Bound States with Spin 1/2 Constituents*, Int.J.Mod.Phys. **A16**, 3365-3477 (2001) (hep-th/0103109).

[23] D.Alba, H.Crater and L.Lusanna, *Hamiltonian Relativistic Two-Body Problem: Center of Mass and Orbit Reconstruction*, (hep-th/0610200).

[24] D.Alba and L.Lusanna, *Quantum Mechanics in Non-Inertial Frames with a Multi-Temporal Quantization Scheme: I) Relativistic Particles*, Int.J.Mod.Phys. **A21**, 2781 (2006) (hep-th/0502060).
D.Alba, *Quantum Mechanics in Non-Inertial Frames with a Multi-Temporal Quantization Scheme: II) Non-Relativistic Particles*, Int.J.Mod.Phys. **A21**, 3917 (2006) (hep-th/0504060).

[25] J.Norton, *General Covariance and the Foundations of General Relativity: Eight Decades of*

Dispute, Rep.Prog.Phys. **56**, 791-858 (1993).

[26] H.Nicolai, K.Peeters and M.Zamaklar, *Loop Quantum Gravity: an Outside View* (hep-th/0501114).

[27] L.Smolin *How Far are we from Quantum Gravity?* (hep-th/0303185).

[28] R.Arnowitt, S.Deser and C.W.Misner, *The Dynamics of General Relativity*, in L. Witten (ed.), *Gravitation: an Introduction to Current Research*, (pp. 227–265) (Wiley, New York, 1962).

[29] L.Lusanna, *The Rest-Frame Instant Form of Metric Gravity*, Gen.Rel.Grav. **33**, 1579-1696 (2001) (gr-qc/0101048).

[30] R.M.Wald, *General Relativity* (University of Chicago Press, Chicago, 1984).

[31] T.Regge and C.Teitelboim, *Role of Surface Integrals in the Hamiltonian Formulation of General Relativity*, Ann.Phys. (N.Y.) **88**, 286-318 (1974).
R.Beig and N.O'Murchadha, *The Poincaré Group as the Simmetry Group of Canonical General Relativity*, Ann.Phys. (N.Y.) **174**, 463-498 (1987).

[32] A.Lichnerowicz, *Propagateurs, Commutateurs et Anticommutateurs en Relativite Generale*, in Les Houches 1963, *Relativity, Groups and Topology*, eds. C.DeWitt and B.DeWitt (Gordon and Breach, New York, 1964).
C.Moreno, *On the Spaces of Positive and Negative Frequency Solutions of the Klein-Gordon Equation in Curved Space-Times*, Rep.Math.Phys. **17**, 333 (1980).

[33] L.Lusanna, *Classical Yang-Mills Theory with Fermions. I. General Properties of a System with Constraints*, Int.J.Mod.Phys. **A10**, 3531-3579 (1995); *II. Dirac's Observables*, Int.J.Mod.Phys. **A10**, 3675-3757 (1995).

[34] D.Christodoulou and S.Klainerman, *The Global Nonlinear Stability of the Minkowski Space.* (Princeton University Press, Princeton, 1993).

[35] L.Lusanna and S.Russo, *A New Parametrization for Tetrad Gravity*, Gen.Rel.Grav. **34**, 189-242 (2002) (gr-qc/0102074).
R.De Pietri, L.Lusanna, L.Martucci and S.Russo, *Dirac's Observables for the Rest-Frame Instant Form of Tetrad Gravity in a Completely Fixed 3-Orthogonal Gauge*, Gen.Rel.Grav. **34**, 877-1033 (2002) (gr-qc/0105084).

[36] S.Shanmugadhasan, J.Math.Phys. **14**, 677 (1973).
L.Lusanna, *The Shanmugadhasan Canonical Transformation, Function Groups and the Extended Second Noether Theorem*, Int.J.Mod.Phys. **A8**, 4193 (1993).

[37] D.Alba and L.Lusanna, *The York Map as a Shanmugadhasan Canonical Transformation in Tetrad Gravity and the Role of Non-Inertial Frames in the Geometrical View of the Gravitational Field* (gr-qc/0604086).

[38] J.Isemberg and J.E.Marsden, *The York map is a Canonical Transformation* , J.Geom.Phys. **1**,85 (1984).

[39] F.I.Cooperstock and S.Tieu, *General Relativity resolves Galactic Rotation Curves without Exotic Dark Matter* (astro-ph/0507619) and *Perspectives on Galactic Dynamics via General Relativity* (astro-ph/0512048).

[40] M.Korzynski, *Singular Disk of Matter in the Cooperstock-Tieu Galaxy Model*, (astro-ph/0508377).
D.Vogt, *Presence of Exotic Matter in the Cooperstock and Tieu Galaxy Model* (astro-ph/0510750).
D.Garfinkle, *The Need for Dark Matter in Galaxies*, (astro-ph/0511082).
D.J.Cross, *Comments on the Cooperstock-Tieu Galaxy Model*, (astro-ph/0601191).

[41] M.Milgrom, *A Modification of the Newtonian Dynamics as a Possible Alternative to the*

Hidden Mass Hypothesis, Astrophys.J. **270**, 365 (1983); *MOND as Modified Inertia* (astro-ph/0510117).

[42] C.Teitelboim, *The Hamiltonian Structure of Spacetime*, in *General Relativity and Gravitation*, ed. A.Held, vol.I (Plenum, New York, 1980).

[43] K.Kuchar, *Canonical Quantum Gravity*, in G.Kunstatter, D.E.Vincent, and J.G.Williams (eds.), *Cordoba 1992, General Relativity and Gravitation*, (pp.119-150) (gr-qc/9304012).

[44] B.DeWitt, *Quantum Theory of Gravity, I) The Canonical Theory*, Phys.Rev. **160**, 1113-1148 (1967); *II) The Manifestly Covariant Theory*, **162**, 1195-1239 (1967).

[45] A. Rendall, *Local and Global Existence Theorems for the Einstein Equations*, Online Journal Living Reviews in Relativity **1**, n. 4; *ibid.* (2000) **3**, n. 1; (gr-qc/0001008).
H. Friedrich and A. Rendall, *The Cauchy Problem for Einstein Equations*, in B.G.Schmidt (ed.), *Einstein's Field Equations and their Physical Interpretation* (Berlin, Springer, 2000) (gr-qc/0002074).
M.Sa-nchez, *Cauchy Hypersurfaces and Global Lorentzian Geometry*, (math.DG/0604265).

[46] J.Agresti, R.De Pietri, L.Lusanna and L.Martucci, *Hamiltonian Linearization of the Rest-Frame Instant Form of Tetrad Gravity in a Completely Fixed 3-Orthogonal Gauge: a Radiation Gauge for Background-Independent Gravitational Waves in a Post-Minkowskian Einstein Space-Time*, Gen.Rel.Grav. **36**, 1055-1134 (2004) (gr-qc/0302084).

[47] J.Stachel, *Einstein's Search for General Covariance, 1912–1915*. Ninth International Conference on General Relativity and Gravitation, Jena (1980), ed.E.Schmutzer (Cambridge Univ. Press, Cambridge, 1983).

[48] J.Earman and J.Norton, *What Price Spacetime Substantivalism? The Hole Story*, British Journal for the Philosophy of Science **38**, 515-525 (1987).

[49] H.Weyl, *Groups, Klein's Erlangen Program. Quantities*, ch.I, sec.4 of *The Classical Groups, their Invariants and Representations*, 2nd ed., (pp.16) (Princeton University Press, Princeton, 1946).

[50] P.G.Bergmann and A.Komar, *The Coordinate Group Symmetries of General Relativity*, Int.Theor.Phys., **5**, 15-28 (1972).

[51] P.G.Bergmann and A.Komar, *Poisson Brackets between Locally Defined Observables in General Relativity*, Phys.Rev.Lett. **4**, 432-433 (1960).
P.G.Bergmann, *The General Theory of Relativity*, in S.Flugge (ed.), *Handbuch derPhysik*, Vol. IV, *Principles of Electrodynamics and Relativity*, (pp. 247-272) (Springer, Berlin, 1962).

[52] M.Dorato and M.Pauri, *Holism and Structuralism in Classical and Quantum General Relativity*, Pittsburgh-Archive, ID code 1606, forthcoming in (2006). S.French and D.Rickles (eds.), *Structural Foundations of Quantum Gravity* (Oxford University Press, Oxford, 2004).

[53] P.G.Bergmann, *Observables in General Relativity*, Rev.Mod.Phys. **33**, 510-514 (1961).

[54] M.Soffel *et al*, *The IAU 2000 Resolutions for Astrometry, Celestial Mechanics and Metrology in the Relativistic Framework: Explanatory Supplement*, Astron.J., **126**, pp.2687-2706, (2003); (Astroph/0303376).