

**Explaining Leibniz-equivalence as difference of non-inertial  
appearances: dis-solution of the Hole Argument  
and physical individuation of point-events**

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## Abstract

"The last remnant of physical objectivity of space-time" is disclosed in the case of a continuous family of spatially non-compact models of general relativity (GR). The *physical individuation* of point-events is furnished by the autonomous degrees of freedom of the gravitational field, (viz, the *Dirac observables*) which represent - as it were - the *ontic* part of the metric field. The physical role of the *epistemic* part (viz. the *gauge* variables) is likewise clarified as embodying the unavoidable non-inertial aspects of GR. At the end the philosophical import of the *Hole Argument* is substantially weakened and in fact the Argument itself dis-solved, while a specific four-dimensional *holistic and structuralist* view of space-time (called *point-structuralism*) emerges, including elements common to the tradition of both *substantivalism* and *relationism*. The observables of our models undergo real *temporal change*: this gives new evidence to the fact that statements like the *frozen-time* character of evolution, as other ontological claims about GR, are *model dependent*.

### KEYWORDS:

- Hole Argument
- Leibniz equivalence
- Structuralism
- Dirac observables
- Gauge variables
- Non-inertial frames

## I. INTRODUCTION

The fact that the requirement of general covariance might involve a threat to the physical objectivity of the points of space-time as represented by the theory of gravitation was becoming clear to Einstein even before the theory he was trying to construct was completed. It was during the years 1913-1915 that the threat took form with the famous Hole Argument (*Lochbetrachtung*) (Einstein, 1914) <sup>1</sup>. In the literature about classical field theories space-time points are usually taken to play the role of individuals, but it is often implicit that they can be distinguished only by the physical fields they carry. Yet, the Hole Argument apparently forbids precisely this kind of individuation, for it entails that different - so-called diffeomorphic - *models* of general relativity (GR) be taken as *physically equivalent*, under the menace of *indeterminism* for the theory. Since, on the other hand, the Argument is a direct consequence of the *general covariance* of GR, this conflict eventually led Einstein to state (our *emphasis*):

That this *requirement of general covariance*, which *takes away from space and time the last remnant of physical objectivity*, is a natural one, will be seen from the following reflexion...  
(Einstein, 1916, p.117).

Although Einstein quickly bypassed the seeming cogency of the Hole Argument against the implementation of general covariance on the purely instrumentalist grounds of the so-called *Point-Coincidence Argument*<sup>2</sup>, the issue remained in the background of the theory until the Hole Argument received new life in recent years with a seminal paper by John Stachel (1980). This paper, followed seven years later by Earman and Norton's philosophical argument against the so-called space-time *manifold substantivalism* (Earman & Norton, 1987), opened a rich philosophical debate that is still alive today. The Hole Argument was immediately regarded by virtually all participants in the debate (Bartels, 1984; Butterfield, 1984, 1987, 1988, 1989; Maudlin, 1988; Rynasiewicz, 1994, 1996, and many others) as being intimately tied to the deep nature of space and time, at least as they are represented by the mathematical models of GR. From 1987 onward, the debate centered essentially about the ontological position to be taken in interpreting the so-called *Leibniz equivalence*, which is the terminology introduced by Earman and Norton to characterize philosophically the relation between diffeomorphic models of GR satisfying the assumptions of the Hole Argument. It must be acknowledged that until now the debate had a purely philosophical relevance. Indeed, from the physicists' point of view, GR has indeed been immunized against the Hole Argument - leaving aside any underlying philosophical issue - by simply embodying the Argument in the statement that *mathematically different* solutions of Einstein's equations related by *passive* - as well as *active* (see later) - diffeomorphisms are *physically equivalent*. From the technical point of view, the natural reading of the consequences of the Hole Argument was then that the mathematical representation of space-time in GR unavoidably contains *superfluous structure*.

The main scope of this paper is to show that the immunization statement quoted above cannot be regarded as the last word on this matter from both the physical and the philosophical point of view, and that GR contains in itself the remedy for isolating what seems

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<sup>1</sup> For a beautiful historical critique see Norton 1987, 1992, 1993.

<sup>2</sup> The assertion that "*the reality of the world-occurrence (in opposition to that dependent on the choice of reference system) subsists in space-time coincidences.*"

to be superfluous structure of the mathematical representation. In this connection, we wish to remember that, already in 1984, Michael Friedman was very explicit about the unsatisfactory epistemological *status* of the widespread understanding of the relation between diffeomorphic models in terms of Leibniz equivalence, when he wrote (*our emphasis*):

Further, if the above models are indeed equivalent representations of the same situation (as it would seem they must be) then *how do we describe this physical situation intrinsically?* Finding such an *intrinsic* characterization (avoiding quantification over *bare* points) appears to be a non-trivial, *and so far unsolved mathematical problem*. (Note that it will not do simply to replace points with equivalence classes of points: for, in many cases, the equivalence class in question will contain *all* points of the manifold (Friedman, 1984, p.663.)

Friedman's thought was then that the Hole Argument leaves an unsolved problem about the characterization of *intrinsic space-time structure*, rather than an ontological question about the existence of space-time. Now, we claim that we have solved this problem, in this same spirit, to the extent in which a degree of intrinsic-ness can be reached in GR. Clearly, given the enormous mathematical variety of possible solutions of Einstein's equations one should not expect that a clarification of Friedman's question is obtained *in general*. We shall indeed conclude that some of the main questions we discuss can be clarified for the general class of globally hyperbolic space-times, while some others for a definite continuous family of generic solutions corresponding to spatially non-compact, asymptotically-flat space-times (hereafter called by the acronym C-K)<sup>3</sup>, but *not* for the spatially compact ones.

Conceptually, our solution is developed in three parts:

i) unveiling the physical meaning of Leibniz equivalence and thereby, through the disclosure of the alleged superfluous structure, dis-solving the philosophical bearing of the Hole Argument. This can be done for all the globally hyperbolic models of GR;

ii) constructing a *physical individuation* of point-events in terms of the *autonomous degrees of freedom* of the gravitational field (*Dirac observables*, hereafter called DO). This can be achieved for the C-K models of GR, and discloses a *highly non-local and holistic* space-time structure. From the philosophical side, the non-uniformity and dynamical richness of space-time structure so unveiled lends itself to support a *new structuralist view*<sup>4</sup> that we call *point structuralism*. This view appears to be a *tertium quid* that tries in some sense to overcome the crudeness of the historical debate on the nature of space-time by including elements common to the traditions of both *substantivalism* and *relationism*<sup>5</sup>. Note, furthermore, that though conceptually independent of our specific methodology (see later on), the disclosure of

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<sup>3</sup> The Christodoulou-Klainermann space-times (Christodoulou & Klainermann, 1993), which are also privileged from the point of view of the inclusion of elementary particles.

<sup>4</sup> As already advocated by Mauro Dorato some years ago (Dorato, 2000).

<sup>5</sup> The *substantivalist* position is a form of realism about certain spatiotemporal structures, being committed to believing in the existence of those entities that are quantified over by our space-time theories, in particular space-time points. It conceives space-time, more or less, as a substance, that is as something that exists independently of any of the things in it. In particular, the so-called *manifold substantivalism* identifies space-time with the bare mathematical manifold of events glued together by the topological and differential structure. On the other hand, the strong *relationist* position is the view that space-time arises as a mere abstraction from the spatiotemporal properties of other things, so that spatio-temporal relations are derivative and supervenient on physical relations obtaining among events and physical objects. Note

this space-time structure follows from the choice of the individuation procedure and *not* from the Hole Argument in itself. Concerning the Hole Argument, our analysis of it entails only a negative philosophical import. Indeed, any attempt to uphold *manifold substantivalism* or *any other metaphysical doctrine* about space-time points, *in the face of the Hole Argument*, becomes definitely irrelevant. Finally:

iii) as a by-product of our analysis concerning the C-K class, we show a coherent interpretation of the ADM Hamiltonian formulation of metric gravity in which the so-called *weak ADM energy* does generate *real temporal modifications* of the DO. This gives new evidence to the fact that statements like the assertion of the so-called *frozen-time picture of evolution*, as other ontological claims about GR, are *model-dependent*.

The concrete realization of point i) constitutes the conceptual basis for the development of the whole program. We will show that physical equivalence of solutions means much more than mere difference in mathematical description: actually it entails equivalence of the descriptions of gravitational phenomena given in different *global, non-inertial frames*, which are *extended space-time laboratories* (hereafter called NIF), with their (dynamically determined) *chrono-geometrical conventions* and *inertial potentials*<sup>6</sup>. In fact, in developing our program of resolution of Friedman's question, we have been naturally led to rephrase GR in terms of *generalized inertial effects*, viz. those effects which are unavoidably met with by any empirical access to the theory due to a global consequence of the equivalence principle. Incidentally, although abandoned later on, the methodological pre-eminence of non-inertial frames in dealing with GR was evident in Einstein's original attitude towards gravitation. Since then, this attitude has never been recovered in the literature. Note, on the other hand, that today extended laboratories like GPS cannot avoid the issue of inertial effects and that contemporary gravitational experiments in space will tend to get a final clarification of this topic.

Technically, this work is based on a full implementation of Dirac's theory of constraint as applied to metric gravity. The recourse to the Hamiltonian formalism is necessary for our goals for many important reasons.

i) The Hamiltonian approach guarantees that the initial value problem of Einstein's equations is mathematically well-posed, a circumstance that does not occur in a natural way within the configurational Lagrangian framework (or "manifold way") because of the non-hyperbolic nature of Einstein's equations (Friedrich & Rendall, 2000; Rendall, 1998). This is a crucial point which we will come back to in Section VI, with greater technical detail. The Hole Argument, in fact, is inextricably entangled with the *initial-value problem* of GR, even if it has never been explicitly discussed in that context in a systematic way<sup>7</sup>.

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that both a simple anti-substantivalist position and a typical relationist position do not deny the reality of space-time (they are not merely anti-realist), but assert that space-time has no reality independently of the bodies or fields it contains. The crucial question for our notion of spatiotemporal structuralism, is therefore the specification of the nature of fields that are *indispensable* for the very definition of physical space-time (e.g. the gravitational field with its causal structure) as distinguished from *other* physical fields.

<sup>6</sup> A precise definition of NIF is given in Section IV.

<sup>7</sup> Actually, David Hilbert was the first person to discuss the Cauchy problem for Einstein's equations and to realize its connection to the Hole phenomenology (see Hilbert, 1917). He discussed the issue in the context of a general-relativistic generalization of Mie's special-relativistic nonlinear electrodynamics, and

ii) In the context of the Hamiltonian formalism, we can resort to Bergmann and Komar's theory of general coordinate-group symmetries (see Bergmann & Komar, 1972) to clarify the significance of active diffeomorphisms as *dynamical symmetries* of Einstein's equations. This point also is crucial: to fully understand the role played by *active* diffeomorphisms in the original configurational formulation of the Hole Argument, it is necessary to interpret them as the *manifold-way* counterparts of *on shell*<sup>8</sup> Hamiltonian gauge transformations, which are *passive* by definition. It will be seen, again, that a basic misunderstanding on the philosophical bearing of the Hole Argument follows directly from a loose and non-algorithmic account of the Cauchy surface as a purely geometrical entity within the manifold  $M^4$ <sup>9</sup>. On the other hand, we stress that reaching our conclusions within the Lagrangian formulation would be technically quite awkward if not impossible, since the Legendre pull-back of the non-point canonical transformations of the Hamiltonian formulation would require tools like the infinite-jet bundle formalism.

iii) The most important reason in favor of the Hamiltonian approach is that, on the basis of the so-called Shanmugadhasan canonical transformation (Shanmugadhasan, 1973; Lusanna, 1993), this approach provides a neat distinction between *physical observables* (the four DO) connected to the (two) autonomous degrees of freedom of the gravitational field, on one hand, and *gauge variables*, on the other. The latter - which express the typical arbitrariness of the theory and must be fixed (*gauge-fixing*) before solving Einstein's equations for the autonomous degrees of freedom - turn out to play a fundamental role, no less than the DO, in both clarifying the real import of the Hole Argument and, even more, showing the need for skipping the manifestly covariant perspective in order to get a full understanding of the physical basis of GR. As said above, the experimental set up for any kind of measurements made within the GR theoretical framework requires the constitution of a NIF. This is exactly what is done - chrono-geometrically - by a *complete gauge-fixing* which, in turn, amounts to a *complete breaking of general covariance*. It should be stressed once more that breaking general covariance is a theoretical necessity for the procedure of solving Einstein's equation and not a question of free choice, let alone a drawback. The basic role of the gauge variables is, therefore, that of specifying the way in which the generalized inertial effects, typical of any NIF, affect the intrinsic gravitational degrees of freedom described by the four DO.

After the Shanmugadhasan transformation, this mechanism realizes a characteristic functional split of the metric tensor into an *ontic* and an *epistemic* part that can be described as follows: i) the *ontic* part, which is constituted by the four DO and will be found to specify the intrinsic structure of space-time, describes, physically, the *tidal-like* effects<sup>10</sup>; ii) the

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pointed out *the necessity of fixing a special geometrically adapted* ("Gaussian" in his terms, or geodesic normal as known today) *coordinate system*, to assure the causality of the theory (in this connection see Howard & Norton, 1993).

<sup>8</sup> We distinguish *off-shell* considerations, made within the Hamiltonian variational framework before restricting to the dynamical solutions, from *on-shell* considerations, made after such a restriction.

<sup>9</sup> Our stance about the content and the implications of the original Hole Argument contrasts with the manifestly covariant and generalized attitude towards the Hole phenomenology expounded by John Stachel in many papers (see e.g. Stachel & Iftime, 2005, and references therein). We will come back to this point and defend our approach in Sections III and VI.

<sup>10</sup> Note that, unlike the Newtonian case where tidal forces are entirely determined by the variation of the action-at-a-distance generated by the Newton potential of massive bodies on test particles, in GR we can have tidal forces even in absence of matter, since they are due to the autonomous degrees of freedom of

*epistemic* part, which is encoded in the metric at the beginning as arbitrary information and is furnished by the gauge variables, describes, physically, the *generalized inertial* effects and, after a complete gauge-fixing, specifies the way in which the *ontic* component of the metric field manifests itself in a definite NIF. Genuine gravitational effects are always dressed in *inertial-like appearances* which undergo inertial changes when going from a given NIF to another NIF.<sup>11</sup>

iv) Finally, an additional important feature of the solutions of GR dealt with in our Hamiltonian formulation is the following. The ADM formalism (Arnowitt, Deser & Misner, 1962) for *spatially compact* space-times without boundary implies that the Dirac Hamiltonian generates purely harmless gauge transformations, so that, being zero on the reduced phase space (see Section IV), it *cannot engender any real temporal change*. This is the origin of the so-called *frozen evolution* description; in this connection see Earman, 2002, Belot & Earman, 1999, 2001. However, in the case of the C-K family of *spatially non-compact* space-times, *internal mathematical consistency* (requiring the addition of the DeWitt surface term to the Hamiltonian (DeWitt, 1967), see later) entails that the generator of temporal evolution, namely the (now non-weakly vanishing) Dirac Hamiltonian, be instead the so-called *weak ADM energy*. This quantity *does generate real temporal modifications* of the DO.

The *dual role* of the metric field discussed above highlights the fact that, while from the mathematical point of view of the constrained Hamiltonian formalism, GR is a theory like any other (e.g., electromagnetism and Yang-Mills theory), from the *physical* point of view it is *radically different*. Technically, this can be traced to general covariance itself, i.e. to the invariance under a group of diffeomorphisms acting on space-time, instead of invariance under the action of a local *inner* Lie group, like in standard gauge theories. However, physically, matters are much more complex. First of all, in GR we cannot rely from the beginning on empirically validated, gauge-invariant dynamical equations for the *local* fields, as it happens with electro-magnetism, where Maxwell equations can be written in terms of the gauge-invariant electric and magnetic fields. On the contrary, Einstein's general covariance (viz. the basis of the gauge freedom of GR) is such that the introduction of extra (*gauge*) variables does make the *mathematical* description of the geometrical aspects of GR mathematically handy and elegant, but, by ruling out any background structure at the outset, it also makes its physical interpretation more intriguing. In GR the distinction between what is observable and what is not, is unavoidably entangled with the constitution of the very *stage*, space-time, where the play of physics is enacted: a stage, however, which also takes an active part in the play. In other words, the gauge-fixing mechanism also plays the *double role* of making the dynamics *unique* (as in all gauge theories), and of fixing the *appearance* of the spatiotemporal dynamic background. Summarizing, as it will be explained, for both the vacuum case and the case with matter fields, the gauge-fixing (with the correlated break of general covariance) completes the structural properties of the general-relativistic space-time. Such fixing is necessary to solve Einstein's equations, reconstruct the four-dimensional dynamic chrono-geometry emerging from the initial values of the *four Dirac observables*, and allow empirical access to the theory through the definition of a dynamically-selected NIF.

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the gravitational field.

<sup>11</sup> We are perfectly aware that we are here overstating the philosophical import of terms like *ontic* and *epistemic* and their relationships. Nothing, however, hinges on these nuances in what follows.

At the end of our technical developments, it should be clear why people (philosophers, especially) should free themselves from the bewitching fascination of manifest general covariance in order to fully understand the subtle hindrances underlying Friedman's question. For general covariance represents a horizon of *a priori* possibilities for the physical constitution of the space-time, possibilities that must be actualized in non-covariant form within any given solution of the dynamical equations.

The overall perspective emerging from our analysis amounts to a new way of looking at the various aspects of the issue of the *objectivity* of the space-time properties of GR. This will be discussed in detail in Section VI. It will be argued there that the issue of objectivity must be called into question not only for the particular case of point-events, but also for many of the basic spatiotemporal features of the theory, such as causal structure, one-way velocity of light (see Alba & Lusanna, 2005a) and the like. Concerning point events, we shall defend the thesis that, as *relata* within a *structure* they do exist and possess some abstract kind of *intrinsic* properties. However, their *physical* properties are *relational* being *conferred* on them in a *holistic and non-local* way by the whole structure of the metric field and the extrinsic curvature on a simultaneity hyper-surface. In a definite physical sense, point-events are literally *identifiable* with the local values of the autonomous degrees of freedom of the gravitational field (DO). In this way both the metric field and the point-events maintain - to paraphrase Newton - their *own manner of existence* and this justifies our terminology of "*point-structuralism*". Furthermore, this conception does not dissolve physical entities into mathematical structures, so that it supports a moderate *entity-realist* attitude towards both the metric field and its point-events, as well as a *theory-realist* attitude towards Einstein's field equations. However, the degree of *objectivity* that, on the basis of our solution of Friedman's question, should be attributed to the *physical individuation* of point-events as well as to other basic structures of space-time is a matter of discussion. This work should be considered a case study for the defence of a thesis about the physical nature of point-events and other important spatiotemporal elements in certain classes of models of GR. Our conclusion will be that all these structures maintain - in a definite sense - a *weak form of physical objectivity*.

Although great part of the technical developments underlying this work have already been treated elsewhere (Pauri & Vallisneri, 2002; Lusanna & Pauri, 2006-I, 2006-II, hereafter denoted by LPI and LPII, respectively, where additional properties of the Christodoulou-Klainermann family of space-times are also discussed), some technical elements are introduced here for the first time. For a more general philosophical presentation, see Dorato & Pauri, 2004.

Section II is devoted to a synopsis of Noether and dynamical symmetries of GR within the configurational description in a mathematical manifold  $M^4$ , together with a brief presentation of the  $Q$  group of Bergmann & Komar (1972). This is the largest group of *passive dynamical symmetries* of Einstein's equations and is instrumental to our understanding of the Hole Argument. The latter is expounded in detail in Section III. The basic ingredients of the ADM formulation as applied to the Christodoulou-Klainermann family of space-times and its canonical reduction, the chrono-geometric meaning of the gauge-fixings, and the constitution of the NIF are discussed in Section IV, together with the issue of temporal change. In Section V we show how the *ontic* part of the metric (the autonomous degrees of freedom of



the gravitational field) may confer a *physical individuation* of space-time points<sup>12</sup>. In Section VI we take up the results obtained in the previous Sections and re-discuss them with respect to the issue of the objectivity of space-time structures in general. The concluding remarks are devoted to a philosophical assessment of our results in view of the traditional debate between *substantivalism* and *relationism*, as well as in view of some more recent discussions about *structural realism*.

## II. NOETHER AND DYNAMICAL SYMMETRIES

Standard general covariance, which essentially amounts to the statement that Einstein's equations for the metric field  ${}^4g(x)$  have a tensor character, implies first of all that the basic equations are form invariant under general coordinate transformations (*passive* diffeomorphisms), so that the Lagrangian density in the Einstein-Hilbert action is singular. Namely, passive diffeomorphisms are *local Noether symmetries* of the action, so that Dirac constraints appear correspondingly in the Hamiltonian formulation. The singular nature of the variational principle of the action entails in turn that four of Einstein's equations be in fact *Lagrangian constraints*, namely restrictions on the Cauchy data, while four combinations of Einstein's equations and their gradients vanish identically (*contracted Bianchi identities*). Thus, the ten components of the solution  ${}^4g_{\mu\nu}(x)$  are in fact functionals of only two "deterministic" dynamical degrees of freedom and eight further degrees of freedom which are left *completely undetermined* by Einstein's equations *even once the Lagrangian constraints are satisfied*. This state of affairs makes the treatment of both the Cauchy problem of the non-hyperbolic system of Einstein's equations and the definition of observables within the Lagrangian context (Friedrich & Rendall, 2000; Rendall, 1998) extremely complicated.

For the above reasons, standard general covariance is then interpreted, in modern terminology, as the statement that a *physical solution of Einstein's equations* properly corresponds to a *4-geometry*, namely the equivalence class of all the 4-metric tensors, solutions of the equations, written in all possible 4-coordinate systems. This equivalence class is usually represented by the quotient  ${}^4Geom = {}^4Riem / {}_P Diff M^4$ , where  ${}^4Riem$  denotes the space of metric tensor solutions of Einstein's equations and  ${}_P Diff$  is the infinite group of *passive* diffeomorphisms (general coordinate transformations). On the other hand, any two *inequivalent* Einstein space-times are different 4-geometries or "universes".

Consider now the abstract differential-geometric concept of *active* diffeomorphism  $D_A$  and its consequent action on the tensor fields defined on the differentiable manifold  $M^4$  [see, for example, (Wald, 1984, pp.438-439)]. An *active* diffeomorphism  $D_A$  maps points of  $M^4$  to points of  $M^4$ :  $D_A : p \rightarrow p' = D_A \cdot p$ . Its tangent map  $D_A^*$  maps tensor fields  $T \rightarrow D_A^* \cdot T$  in such a way that  $[T](p) \rightarrow [D_A^* \cdot T](p) \equiv [T'](p)$ . Then  $[D_A^* \cdot T](p) = [T](D_A^{-1} \cdot p)$ . It is seen that the transformed tensor field  $D_A^* \cdot T$  is a *new* tensor field whose components in general will have at  $p$  values that are *different* from those of the components of  $T$ . On the other hand, the components of  $D_A^* \cdot T$  have at  $p'$  - by construction - the same values that the components of the original tensor field  $T$  have at  $p$ :  $T'(D_A \cdot p) = T(p)$  or  $T'(p) = T(D_A^{-1} \cdot p)$ . The new

<sup>12</sup> There is an unfortunate ambiguity in the usage of the term *space-time points* in the literature: sometimes it refers to elements of the mathematical structure that is the first layer of the space-time model, other times to the points interpreted as *physical events*. We will adopt the term *point-event* in the latter sense and simply *point* in the former.

tensor field  $D_A^* \cdot T$  is called the *drag-along* (or *push-forward*) of  $T$ . There is another, non-geometrical - so-called *dual* - way of looking at the active diffeomorphisms (Norton, 1987). This *duality* is based on the circumstance that in each region of  $M^4$  covered by two or more charts there is a one-to-one correspondence between an *active* diffeomorphism and a specific coordinate transformation. The coordinate transformation  $\mathcal{T}_{D_A} : x(p) \rightarrow x'(p) = [\mathcal{T}_{D_A}x](p)$  which is *dual* to the active diffeomorphism  $D_A$  is defined so that  $[\mathcal{T}_{D_A}x](D_A \cdot p) = x(p)$ . Essentially, this *duality* transfers the functional dependence of the new tensor field in the new coordinate system to the old system of coordinates. By analogy, the coordinates of the new system  $[x']$  are said to have been *dragged-along* with the *active* diffeomorphism  $D_A$ . It is important to note here, however, that the above *dual view* of active diffeomorphisms, as particular *coordinate*-transformations, is only *implicitly* defined for the moment .

In the abstract coordinate-independent language of differential geometry, Einstein's equations for the vacuum

$${}^4G_{\mu\nu}(x) \stackrel{def}{=} {}^4R_{\mu\nu}(x) - \frac{1}{2} {}^4R(x) {}^4g_{\mu\nu}(x) = 0. \quad (2.1)$$

can be written as  $G = 0$ , where  $G$  is the Einstein 2-tensor ( $G = G_{\mu\nu}(x) dx^\mu \otimes dx^\nu$  in the coordinate chart  $x^\mu$ ). Under an *active diffeomorphism*  $D_A : M^4 \mapsto M^4$ ,  $D_A \in {}_A\text{Diff } M^4$ , we have  $G = 0 \mapsto D_A^* G = 0$ , which shows that active diffeomorphisms are *dynamical symmetries* of Einstein's tensor equations, i.e. *they map solutions into solutions*.

We have clarified elsewhere (LPI) the *explicit* relationships<sup>13</sup> existing between passive and active diffeomorphisms on the basis of an important paper by Bergmann and Komar (1972), in which it is shown that *the largest group of passive dynamical symmetries of Einstein's equations* is not  ${}_P\text{Diff } M^4 [x'^\mu = f^\mu(x^\nu)]$ , but rather a larger group of transformations of the form

$$Q : \quad x'^\mu = f^\mu(x^\nu, {}^4g_{\alpha\beta}(x)),$$

$${}^4g'_{\mu\nu}(x'(x)) = \frac{\partial h^\alpha(x', {}^4g'(x'))}{\partial x'^\mu} \frac{\partial h^\beta(x', {}^4g'(x'))}{\partial x'^\nu} {}^4g_{\alpha\beta}(x). \quad (2.2)$$

In the case of completely Liouville-integrable systems, dynamical symmetries can be re-interpreted as maps of the space of Cauchy data onto itself. Although we do not have a general proof of the integrability of Einstein's equations, we know that if the initial value problem is well-posed and defined<sup>14</sup>, as it is in the ADM Hamiltonian description, *the space of Cauchy data is partitioned in gauge-equivalent classes of data*: all of the Cauchy

<sup>13</sup> At least for the infinitesimal active transformations.

<sup>14</sup> It is important to stress that in looking for global solutions of Einstein's equations as a system of partial differential equations, a number of preliminary specifications must be given. Among other things: a) the topology of space-time; b) whether the space-time is spatially compact or asymptotically flat at spatial infinity; c) whether or not in the spatially compact case there is a spatial boundary; d) the nature of the function space and the class of boundary conditions, either at spatial infinity or on the spatial boundary, for the 4-metric and its derivatives (only in the spatially compact case without boundary there is no

data in a given class identify a single 4-geometry or "universe". Therefore, under the given hypothesis, *the dynamical symmetries of Einstein's equations fall into two classes only*: a) those mapping different "universes" among themselves, and b) those acting within a single Einstein "universe", mapping gauge-equivalent Cauchy data among themselves. It is remarkable that, at least for the subset  $Q' \subset Q$  (passive counterpart of a subset  ${}_A\text{Diff}' M^4 \subset {}_A\text{Diff} M^4$ ) which corresponds to *mappings among gauge-equivalent Cauchy data*, the transformed metrics do indeed belong to the *same 4-geometry*, i.e. the same equivalence class generated by applying all *passive diffeomorphisms* to the original 4-metrics:  ${}^4\text{Geom} = {}^4\text{Riem}/{}_P\text{Diff} M^4 = {}^4\text{Riem}/Q'^{15}$ .

Note finally that: a) an *explicit* passive representation of the infinite group of  ${}_A\text{Diff} M^4$  is necessary anyway for our Hamiltonian treatment of the Hole Argument, as well as for any comparison of the various viewpoints existing in the literature concerning the *solutions* of Einstein's equations; b) the group  $Q'$  describes the dynamical symmetries of Einstein's equations which are *also* local Noether symmetries of the Einstein-Hilbert action. The 4-metrics reached by using passive diffeomorphisms are, so to speak, only a dense subset of the metrics obtainable by means of the group  $Q$ .

In conclusion, what is known as a *4-geometry*, is also an equivalence class of solutions of Einstein's equations *modulo* the subset of dynamical symmetry transformations  ${}_A\text{Diff}' M^4$ , whose passive counterpart is  $Q'$ . Therefore, following Bergmann & Komar (1972), Wald (1984), we can state <sup>16</sup>

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need of boundary conditions, replaced by periodicity conditions, so that these models of GR show the well-known Machian aspects which influenced Einstein and Wheeler). After these specifications have been made, a *model* of GR is identified. What remains to be worked out is the characterization of a well-posed initial value problem. Modulo technicalities, this requires choosing a 4-coordinate system and finding which combinations of the equations are of *elliptic* type (restrictions on the Cauchy data) and which are of *hyperbolic* type (evolution equations), namely the only ones requiring an initial value problem. At the Hamiltonian level, the elliptic equations are the first-class constraints identifying the constraint sub-manifold of phase space (see Section IV), while the hyperbolic equations are the Hamilton equations in a fixed gauge (a completely fixed Hamiltonian gauge corresponds *on-shell* to a 4-coordinate system, see Section IV). When the gauge variables can be separated from the Dirac observables, only the latter need an initial value problem (the gauge variables are *arbitrary*, modulo restrictions upon their range coming from the structure of the gauge orbits inside the constraint sub-manifold). Finally, given a space-like Cauchy surface in a 4-coordinate system (or in a fixed Hamiltonian gauge), *each admissible set of Cauchy data gives rise to a different "universe" with the given boundary conditions*. Clearly, each universe is defined modulo passive diffeomorphisms changing both the 4-coordinate system and the Cauchy surface (or modulo the Hamiltonian gauge transformations changing the gauge and the Cauchy surface) and also modulo the (*on-shell*) active diffeomorphisms.

<sup>15</sup> Note, incidentally, that this circumstance is mathematically possible only because  ${}_P\text{Diff} M^4$  is a *non-normal dense* sub-group of  $Q'$ .

<sup>16</sup> Eqs.(2.3) are usually taken for granted in mathematical physics, at least at the heuristic level. Since, however, the control in large of the group manifold of infinite-dimensional groups like  ${}_P\text{Diff} M^4$  and  $Q'$  is, as yet, an open mathematical issue, one cannot be more rigorous on this point: see also the end of Section III. For more details about these issues, the interested reader should see LPI and LPII, where a new subset  $Q_{can}$  of  $Q$  is introduced, namely the Legendre pullback of the *on-shell* Hamiltonian canonical

$${}^4\text{Geom} = {}^4\text{Riem}/{}_P\text{Diff } M^4 = {}^4\text{Riem}/Q' = {}^4\text{Riem}/{}_A\text{Diff}' M^4. \quad (2.3)$$

### III. THE HOLE ARGUMENT AND ITS DIS-SOLUTION: A FIRST LOOK

Although the issue could not be completely clear to Einstein in 1916, as shown by Norton (1987, 1992, 1993), it is precisely the nature of dynamical symmetry of the *active diffeomorphisms* that has been considered as expressing the *physically relevant* content of *general covariance*, as we shall presently see.

Remember, first of all, that a *mathematical model* of GR is specified by a four-dimensional mathematical manifold  $M^4$  and by a metrical tensor field  $g$ , where the latter represents *both* the chrono-geometrical structure of space-time *and* the potential for the inertial-gravitational field. Non-gravitational physical fields, when they are present, are also described by dynamical tensor fields, which appear to be sources of Einstein's equations. Assume now that  $M^4$  contains a *hole*  $\mathcal{H}$ : that is, an open region where all the non-gravitational fields vanish so that *the metric obeys the homogeneous Einstein equations*. On  $M^4$  we can define an *active* diffeomorphism  $D_A^*$  that re-maps the points inside  $\mathcal{H}$ , but blends smoothly into the identity map outside  $\mathcal{H}$  and on the boundary. By construction, for any point  $x \in \mathcal{H}$  we have (in the abstract tensor notation)  $g'(D_A x) = g(x)$ , but of course  $g'(x) \neq g(x)$  (in the same notation). The crucial fact is that from the general covariance of Einstein's equations it follows that if  $g$  is one of their solutions, so is the *drag-along* field  $g' \equiv D_A^* g$ .

What is the correct interpretation of the new field  $g'$ ? Clearly, the transformation involves an *active redistribution of the metric over the points of the manifold in  $\mathcal{H}$* , so the critical question is whether and how the points of the manifold are primarily *individuated*. Now, *if* we think of the points of  $\mathcal{H}$  as *intrinsically individuated physical events*, where *intrinsic* means that their identity is autonomous and independent of any physical field, the metric in particular - a claim that is associated with any kind of *manifold substantivalism* - then  $g$  and  $g'$  must be regarded as *physically distinct* solutions of Einstein's equations (after all,  $g'(x) \neq g(x)$  at the *same* point  $x$ ). This appeared as a devastating conclusion for the causality of the theory, because it implied that, even after we specify a physical solution for the gravitational and non-gravitational fields outside the hole - in particular, *on a Cauchy surface for the initial value problem* - we are still unable to predict a unique physical solution within the hole.

According to Earman and Norton (1987), the way out of the Hole Argument lies in abandoning *manifold substantivalism*: they claim that if diffeomorphically related metric fields were to represent different physically possible "universes", then GR would turn into an *indeterministic* theory. And since the issue of whether determinism holds or not at the *physical* level cannot be decided by opting for a *metaphysical* doctrine like *manifold substantivalism*, they conclude that one should refute any kind of such *substantivalism*. Since, however, relationism does not amount to the mere negation of substantivalism, and since the literature contains so many conflicting meanings of the term "relationism", they do

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transformations. In LPI and LPII it is shown that it also holds  ${}^4\text{Geom} = {}^4\text{Riem}/Q_{can}$ , since, modulo technicalities, we have  $Q_{can} = Q'$ . Note that  ${}_P\text{Diff } M^4 \cap Q_{can}$  are the *passive diffeomorphisms which are re-interpretable as Hamiltonian gauge transformations*.

not simply conclude that space-time is relational. They state the more general assumption (which - they claim - is applicable to all space-time theories) that "diffeomorphic models in a space-time theory represent the same physical situation", i.e. must be interpreted as describing the same "universe" (*Leibniz equivalence*).

The fact that the Leibniz equivalence seems here no more than a sophisticated re-phrasing of what physicists consider a foregone conclusion for general relativity, should not be taken at face value, for the real question for an opposing "weak substantivalist" is whether or not space-time should be simply identified with the bare manifold deprived of any physical field, and of *the metric field* in particular, as Earman and Norton do, instead of with a set of points each endowed with its own *metrical fingerprint*<sup>17</sup>. Actually, this "weak substantivalist" could sustain the conviction - as we ourselves do - that the metric field, because of its basic *causal structure*, has ontological priority (Pauri, 1996) over all other fields and, therefore, it is *not* like any other field, as Earman and Norton would have it.

We do believe that the bare manifold of points, deprived of the infinitesimal Pythagorean structure defining the basic distinction between temporal and spatial directions, let alone the causal structure which teaches all the other fields how to move, can hardly be seen as space-time. We insist, therefore, that in order to be able to speak of space-time the definition of a metric field is a necessary condition. Consequently, and in agreement with Stachel (1993), we believe that asserting that  $g$  and  $D_A^*g$  represent *one and the same gravitational field* implies that *the mathematical individuation* of the points of the differentiable manifold by their coordinates *has no physical content unless a metric tensor is specified*<sup>18</sup>. Stachel stresses that if  $g$  and  $D_A^*g$  must represent the same gravitational field, they cannot be physically distinguished in any way. Accordingly, when we act on  $g$  with  $D_A^*$  to create the *drag-along* field  $D_A^*g$ , no element of physical significance can be left behind: in particular, *nothing* that could identify a point  $x$  of the manifold itself as the *same point* of space-time for both  $g$  and  $D_A^*g$ . Instead, when  $x$  is mapped onto  $x' = D_A^*x$ , it *carries over its identity*, as specified by  $g'(x') = g(x)$ . This means, for one thing, that "the last remnant of physical objectivity" of space-time points, if any, should be sought for in the physical content of the metric field itself. Note, however, that while this point of view about the Hole Argument casts new light upon the fact that  $g$  and  $g'$  must represent the same physical situation, it still does not explain the source of the superfluous structure of the mathematical representation and, therefore, not even the origin of the *prima facie difference* between  $g$  and  $g'$ .

Anyway, these remarks led Stachel to the important conclusion that *vis á vis* the physical point-events, the metric actually plays the role of *individuating field*. Precisely, Stachel suggested that this individuating role could be implemented by four invariant functionals of the metric, which Komar and Bergmann (Komar 1958; Bergmann & Komar, 1960) had already considered. Stachel, however, did not follow up this proposal by providing a concrete realization in terms of solutions of Einstein's equations, something that we instead will presently do. At the same time we will show in Section VI that Stachel's suggestion, as it stands, remains at a too abstract level and fails to exploit the crucial distinction between

<sup>17</sup> See, for example, Bartels, 1994 and Maudlin, 1988.

<sup>18</sup> *Coordinatization* is the only way to individuate the points *mathematically* since, as stressed by Hermann Weyl: "There is no distinguishing objective property by which one could tell apart one point from all others in a homogeneous space: at this level, fixation of a point is possible only by a *demonstrative act* as indicated by terms like *this* and *there*." (Weyl, 1946, p. 13). See also Schlick, 1917, quoted in M.Friedman, 2003, p.165.

*ontic* and arbitrary *epistemic* content of the Bergmann-Komar invariant functionals of the metric. This content, in fact, must be specified in order to calculate the functionals on the solutions.

We conclude this Section by summarizing the implications of our analysis of the meaning and philosophical import of the Hole Argument. The force of the *indeterminacy argument* apparently rests on the following basic facts: (i) a solution of Einstein's equations must be *preliminarily* individuated outside (and, of course, inside) the Hole, otherwise there would be no 'meat' for the Argument itself. Although the original formulation of the Hole Argument, as well as many subsequent expositions of it, are silent on this point, we will see that the Hole Argument is unavoidably entangled with the initial value problem; (ii) the *active* diffeomorphism  $D_A^*$ , which is purportedly chosen to be the *identity* outside the *hole*  $\mathcal{H}$ , is a *dynamical symmetry* of Einstein's equations, so that it maps solutions onto solutions, which in general will be equivalent (as 4-geometries or Einstein "universes") or not ; (iii) since  $D_A^*$  is, by hypothesis, the identity on the Cauchy hyper-surface, it cannot map a solution defining a given Einstein "universe" onto a different "universe", *which would necessarily correspond to inequivalent Cauchy data*; but (iv) nevertheless, we are told by the Hole Argument that  $D_A^*$  engenders a "different" solution inside the Hole.

As we shall later see with greater evidence, in spite of the *prima facie* geometric obviousness of the identity condition required for  $D_A^*$  outside the Hole, it is quite illusory to try to explain all the facets of the relations of the Argument with the initial value problem in the purely abstract way of differential geometry. The point is that the configurational four-dimensional formulation cannot exploit the advantage that the Hamiltonian formulation possesses of working *off-shell*. This is crucial since in the present context the four-dimensional active diffeomorphisms - *qua* dynamical symmetries of Einstein's equations - must be directly applied to *solutions* of GR. These solutions, however, cannot be exhaustively managed in the four-dimensional configurational approach in terms of initial data because of the non-hyperbolicity of Einstein's equations<sup>19</sup>. The Hole Argument needs the Cauchy problem to be formulated outside the Hole explicitly and *in advance*, a fact that requires abandoning the Lagrangian way in favor of Hamiltonian methods. At this point, the results of the previous Section (the passive counterpart of  $D_A^*$  must belong to  $Q'$ , or belong to  $Q$  but not to  $Q'$ ) leave us with the sole option that, *once rephrased in the passive Hamiltonian language*, the *active* diffeomorphism  $D_A^*$  exploited by the Argument must lie in the subset  $Q' ({}_A\text{Diff}' M^4)$ . Then, however, *it must necessarily map Cauchy data onto gauge-equivalent Cauchy data*, precisely those gauge-equivalent data that generate the allegedly "different" solution within the Hole. In the end, the "difference" will turn out to correspond to a mere *different choice of the gauge* for the *same solution*. Thus *Leibniz equivalence* boils down to mere *gauge equivalence* in its strict sense<sup>20</sup>, an effect that - let us stress it again - cannot be transparently displayed in the configurational geometric descrip-

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<sup>19</sup> The reader should avoid the impression that our criticisms about the weakness of the configurational approach be dictated by personal preferences for Hamiltonian methods or be simple questions of taste. The issue is in fact very important. In LP2 we gave a detailed analysis of the drawbacks one can encounter by adopting trusting confidence in a mixing of Lagrangian and geometric considerations involving a non-algorithmic attitude towards the Cauchy surface. A brief reconsideration of this analysis is given in Section VI.

<sup>20</sup> The physical meaning of this equivalence will be clarified in Section IV.

tion. On the other hand, were the *active* diffeomorphism  $D_A^*$ , once passively rephrased, to belong to the group  $Q$  but not to the subset  $Q'$  (i.e. were it originally lying in  ${}_A\text{Diff } M^4$ , but not in  ${}_A\text{Diff}' M^4$ ), then it would not correspond to a mere gauge equivalence and it would necessarily modify the Cauchy data outside the Hole. Therefore it would lead to a really different Einstein "universe" but it would violate the assumption of the Hole Argument that  $D_A^*$  be the identity on the Cauchy hyper-surface. In any case, it is seen that the disappearance of the "indeterminacy" rests upon the necessity of formulating the Cauchy problem *before* talking about the relevant properties of the solutions.

We conclude that - to the extent that the Cauchy problem is well-posed, i.e. *in every globally hyperbolic space-time* and not necessarily in the C-K class only - exploiting the original Hole Argument to the effect of asking ontological questions about the general relativistic space-time is an enterprise devoid of real philosophical impact, in particular concerning the menace of indeterminism. There is clearly no room left for upholding "manifold substantivalism", "different worlds", "metric essentialism" or any other metaphysical doctrine about space-time points *in the face of the Hole Argument*. Of course, such metaphysical doctrines could still be defended, yet independently of the Hole Story.

#### IV. CHRISTODOULOU-KLAINERMANN SPACE-TIMES, 3+1 SPLITTING, AND ADM CANONICAL REDUCTION

The Christodoulou-Klainermann space-times are a continuous family of space-times that are non-compact, globally hyperbolic, asymptotically flat at spatial infinity (asymptotic Minkowski metric, with asymptotic Poincaré symmetry group) and topologically trivial ( $M^4 \equiv R^3 \times R$ ), supporting global 4-coordinate systems.

The ADM Hamiltonian approach starts with a 3+1 splitting of the 4-dimensional manifold  $M^4$  into constant-time hyper-surfaces  $\Sigma_\tau \equiv R^3$ , indexed by the *parameter time*  $\tau$ , each equipped with coordinates  $\sigma^a$  ( $a = 1,2,3$ ) and a three-metric  ${}^3g$  (in components  ${}^3g_{ab}$ ). The *parameter time*  $\tau$  and the coordinates  $\sigma^a$  ( $a = 1,2,3$ ) are in fact *Lorentz-scalar, radar coordinates adapted* to the 3+1 splitting (Alba & Lusanna, 2003, 2005a). They are defined with respect to an arbitrary, in general accelerated, observer, a centroid  $X^\mu(\tau)$ , chosen as origin of the coordinates, whose proper time may be used as the parameter  $\tau$  labelling the hyper-surfaces. On each hyper-surface *all the clocks are conventionally synchronized to the value*  $\tau$ . The simultaneity (and Cauchy) hyper-surfaces  $\Sigma_\tau$  are described by the embedding functions  $x^\mu = z^\mu(\tau, \sigma^a) = X^\mu(\tau) + F^\mu(\tau, \sigma^a)$ ,  $F^\mu(\tau, 0^a) = 0$ .

All this machinery builds up, at the chrono-geometric level, a *global, extended frame of reference*, realizing a *non-rigid, non-inertial, laboratory* (the only one existing in GR due to the equivalence principle)<sup>21</sup>. This global laboratory will be called a NIF. As we shall presently show, any *chrono-geometrically possible* NIF is the result of a *complete gauge-fixing*, a procedure that determines the *appearance of gravitational phenomena* by uniquely specifying *the form* of the inertial forces (Coriolis, Jacobi, centrifugal,...) in each point of a NIF. A crucial difference of this structure in GR with respect to the Newtonian case (see also footnote 10), besides the fact that inertial effects are unavoidable (they cannot be traced here to "apparent forces" in that they cannot be removed by a choice of reference frame),

<sup>21</sup> All the details of this structure can be found in LPI, where it is shown in particular how two congruences of time-like observers are naturally associated to a NIF .

is that the inertial potentials may also depend upon *generalized tidal effects* in addition to the coordinates of the non-inertial frame<sup>22</sup>.

An important point to be kept in mind is that the *explicit functional form* of the embedding functions and - consequently - of the chrono-geometry of the 3 + 1 splitting of  $M^4$ , thought to be implicitly given at the outset, remains undefined until the solution of Einstein's equations is worked out in a fixed gauge. Likewise, it is only after the solution emerges from given initial data of the four DO in that gauge that a subset of the *chrono-geometrically possible* NIF becomes *dynamically selected* NIF together with their dynamically determined "conventions": see later on.

Now, start at a point on  $\Sigma_\tau$ , and displace it infinitesimally in a direction that is normal to  $\Sigma_\tau$ . The resulting change in  $\tau$  can be written as  $\Delta\tau = Nd\tau$ , where  $N$  is the so-called *lapse function*. Moreover, the time displacement  $d\tau$  will also shift the spatial coordinates:  $\sigma^a(\tau + d\tau) = \sigma^a(\tau) + N^a d\tau$ , where  $N^a$  is the *shift vector*. Then the interval between  $(\tau, \sigma^a)$  and  $(\tau + d\tau, \sigma^a + d\sigma^a)$  is:  $ds^2 = N^2 d\tau^2 - {}^3g_{ab}(d\sigma^a + N^a d\tau)(d\sigma^b + N^b d\tau)$ . The *configurational* variables  $N, N^a, {}^3g_{ab}$  (replacing the 4-metric  $g$ ) together with their 10 conjugate momenta, index a 20-dimensional phase space<sup>23</sup>. Expressed (*modulo* surface terms) in terms of the ADM variables, the Einstein-Hilbert action is a function of  $N, N^a, {}^3g_{ab}$  and their first time-derivatives, or equivalently of  $N, N^a, {}^3g_{ab}$  and the extrinsic curvature  ${}^3K_{ab}$  of the hyper-surface  $\Sigma_\tau$ , considered as an embedded manifold.

Since Einstein's original equations are not hyperbolic, it turns out that the canonical momenta are not all functionally independent, but satisfy four conditions known as *primary* constraints (they are given by the vanishing of the *lapse* and *shift* canonical momenta). Another four *secondary* constraints arise when we require that the primary constraints be preserved through evolution (the secondary constraints are called the *super-hamiltonian*  $\mathcal{H}_0 \approx 0$ , and the *super-momentum*  $\mathcal{H}_a \approx 0$ , ( $a = 1, 2, 3$ ) constraints, respectively). The eight constraints are given as functions of the canonical variables that vanish on the *constraint surface*. The existence of such constraints implies that not all the points of the 20-dimensional phase space physically represent meaningful states: rather, we are restricted to the *constraint surface* where all the constraints are satisfied, i.e. to a 12-dimensional (20 - 8) *surface* which, however, does not possess the geometrical structure of a true phase space. When used as generators of canonical transformations, the eight constraints map points on the constraint surface to points on the same surface; these transformations are known as *gauge transformations*.

In order to obtain the correct dynamics for the constrained system, we must consider the Dirac Hamiltonian, which is the sum of the DeWitt surface term (DeWitt, 1967)<sup>24</sup>

<sup>22</sup> Let us stress that the *scalar radar coordinates* are *intrinsically frame-dependent* since they parametrize a NIF centered on the arbitrary observer. Furthermore, they are not ordinary coordinates  $x^\mu$  in a chart of the Atlas  $\mathcal{A}$  of  $M^4$ . They should be properly called *pseudo-coordinates* in a chart of the Atlas  $\tilde{\mathcal{A}}$  defined by adding to  $M^4$  the extra-structure of all its admissible 3+1 splittings: actually the new coordinates are *adapted* to this extra-structure. If the embedding of the constant-time hyper-surfaces  $\Sigma_\tau$  of a 3+1 splitting into  $M^4$  is described by the functions  $z^\mu(\tau, \sigma^a)$ , then the transition functions from the *adapted radar-coordinates*  $\sigma^A = (\tau; \sigma^a)$  to the ordinary coordinates are  $\frac{\partial z^\mu(\tau, \sigma^a)}{\partial \sigma^A}$ .

<sup>23</sup> Of course, all these *variables* are in fact *fields*.

<sup>24</sup> The DeWitt surface term is *uniquely* determined as the sum of two parts: a) the surface integral to be extracted from the Einstein-Hilbert action to get the ADM action; b) a surface integral due to an



[present only in spatially non-compact space-times and becoming the ADM energy after suitable manipulations (Lusanna, 2001; DePietri & Lusanna & Martucci & Russo, 2002)], of the *secondary* constraints multiplied by the lapse and shift functions, and of the *primary* constraints multiplied by arbitrary functions (the so-called *Dirac multipliers*). If, following Dirac, we make the reasonable demand that the evolution of all *physical variables* be unique - otherwise we would have real physical variables that are indeterminate and therefore neither *observable* nor *measurable* - then the points of the constraint surface lying on the same *gauge orbit*, i.e. linked by gauge transformations, must describe the *same physical state*. Conversely, only the functions in phase space that are *invariant with respect to gauge transformations* can describe physical quantities.

In order to eliminate this ambiguity and create a one-to-one mapping between points in the phase space and physical states, we must impose further constraints, known as *gauge conditions* or *gauge-fixings*. The gauge-fixings can be implemented by arbitrary functions of the canonical variables, except that they must intersect each gauge orbit exactly once (*orbit conditions*) in order to allow a well-posed definition of the *reduced phase space*. The number of independent fixings must be equal to the number of independent gauge variables (i.e., 8 in our case). The canonical reduction follows a cascade procedure. Precisely, the gauge-fixings to the *super-hamiltonian* and *super-momentum* constraints come first (call it  $\Gamma_4$ ): they determine the 3-coordinate system and the *off-shell* shape of  $\Sigma_\tau$  (i.e., the *off-shell* convention for clock synchronization); the requirement of their time constancy then determines the gauge-fixings to the *primary* constraints: they determine the *lapse* and *shift* functions. Finally, the requirement of time constancy for these latter gauge-fixings determines the Dirac multipliers. Therefore, the first level of gauge-fixing gives rise to a *complete* gauge-fixing, say  $\Gamma_8$ , and is sufficient to remove all the gauge arbitrariness.

The  $\Gamma_8$  procedure reduces the original 20-dimensional phase space to a *copy*  $\Omega_4$  of the *abstract reduced phase-space*  $\tilde{\Omega}_4$  having 4 degrees of freedom per point (12 - 8 gauge-fixings). Abstractly, the reduced phase-space  $\Omega_4$  with its symplectic structure is defined by the quotient of the constraint surface with respect to the 8-dimensional group of gauge transformations and represents *the space of the abstract gauge-invariant observables of GR: two configurational and two momentum variables*. These observables carry the physical content of the theory in that they represent the *autonomous degrees of freedom of the gravitational field* (remember that at this stage we are dealing with a pure gravitational field without matter).

A  $\Gamma_8$ -dependent *copy*  $\Omega_4$  of the *abstract*  $\tilde{\Omega}_4$  is realized in terms of the symplectic structure (Dirac brackets) defined by the given gauge-fixings and coordinatized by four DO [call such field observables  $q^r, p_s$  ( $r,s = 1,2$ )]. The functional form of these DO (*concrete realization of the gauge-invariant abstract observables in the given complete gauge*  $\Gamma_8$ ) in terms of the original canonical variables depends upon the chosen gauge, so that such observables - a

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integration by parts required by the Legendre transformation from the ADM action to phase space [see (Lusanna, 2001) after Eq.(5.5) and (Hawking & Horowitz, 1996)]. By adding a surface term different from the ADM one, we would get another action with the same equations of motion but an a-priori different canonical formulation. Still another option is to consider the metric and the Christoffel connection as independent configuration variables: this is the first-order Palatini formalism, which has much larger gauge freedom, also including second class constraints. All these canonical formulations must lead anyway to the same number of physical degrees of freedom.

priori - are neither tensors nor invariant under  ${}_P Diff$ . In each gauge  $\Gamma_8$ , the original 8 gauge variables are now uniquely determined functions of the DO. Yet, *off shell*, barring sophisticated mathematical complications, *any two copies of  $\Omega_4$  are diffeomorphic images of one another*.

It is very important to understand qualitatively the geometric meaning of the eight infinitesimal *off-shell* Hamiltonian gauge transformations and thereby the geometric significance of the related gauge-fixings. i) The transformations generated by the four *primary* constraints modify the *lapse* and *shift* functions which, in turn, determine both how densely the space-like hyper-surfaces  $\Sigma_\tau$  are distributed in space-time and the appearance of *gravito-magnetism* on them; ii) the transformations generated by the three *super-momentum* constraints induce a transition on  $\Sigma_\tau$  from one given 3-coordinate system to another; iii) the transformation generated by the *super-hamiltonian* constraint induces a transition from one *a-priori* given "form" of the 3+1 splitting of  $M^4$  to another (namely, from a given notion of distant simultaneity to another), by operating deformations of the space-like hyper-surfaces in the normal direction.

It should be stressed that the *manifest effect* of the gauge-fixings related to the above transformations emerges only *at the end* of the canonical reduction and *after* the solution of the Einstein-Hamilton equations has been worked out (i.e. *on shell*). This happens because the role of the gauge-fixings is essentially that of choosing the *functional form* in which all the gauge variables depend upon the DO, i.e. - physically - of fixing the *form* of the *inertial potentials* of the associated *chrono-geometrically possible* NIF. It must also be emphasized that this important physical aspect is completely lost within the *abstract reduced phase space*  $\tilde{\Omega}_4$ , which could play, nevertheless, another important role (see Sections V and VI). It is only after the initial conditions for the DO have been arbitrarily selected on a Cauchy surface that the whole four-dimensional chrono-geometry of the resulting *Einstein "universe"* is *dynamically* determined, including the embedding functions  $x^\mu = z^\mu(\tau, \vec{\sigma})$  (i.e. the *on-shell* shape of  $\Sigma_\tau$ ), and therefore even the *dynamically admissible* NIF within the set of *chrono-geometrically possible* NIF. In particular, since the transformations generated by the super-Hamiltonian modify the rules for the synchronization of distant clocks, *all* the relativistic *conventions* (including those for gravito-magnetism) associated to all of the NIF in a given *Einstein "universe"*, turn out to be *dynamically-determined, gauge-related options*<sup>25</sup>.

Two important points must be emphasized.

i) In order to carry out the canonical reduction *explicitly*, before implementing the gauge-fixings we must perform a basic canonical transformation at the *off-shell* level, the so-called Shanmugadhasan transformation (Shanmugadhasan, 1973; Lusanna, 1993), moving from the original canonical variables to a new basis including the DO as a canonical subset<sup>26</sup>. It should be stressed here that it is not known whether the Shanmugadhasan canonical transformation,

<sup>25</sup> Unlike the special relativistic case where the various possible conventions are non-dynamical options.

<sup>26</sup> In practice, this transformation is adapted to seven of the eight constraints (Lusanna, 2001; DePietri, Lusanna, Martucci & Russo, 2002): they are replaced by seven of the new momenta whose conjugate configuration variables are the gauge variables describing the *lapse* and *shift* functions and the choice of the spatial coordinates on the simultaneity surfaces. The new basis contains the conformal factor (or the determinant) of the 3-metric, which is determined by the super-Hamiltonian constraint (though as yet no solution has been found for this equation, also called the Lichnerowicz equation), and its conjugate

and therefore the GR observables, can be defined *globally* in C-K space-times. In most of the spatially compact space-times this cannot be done for topological reasons. A further problem is that in field theory in general the status of the canonical transformations is still heuristic. Therefore the only tool (viz. the Shanmugadhasan transformation) we have for a systematic search of GR observables in every type of space-time still lacks a rigorous definition. In conclusion, *the mathematical basis of our analysis regarding the objectivity of space-time structures is admittedly heuristic, yet our arguments are certainly no more heuristic than the overwhelming majority of the theoretical and/or philosophical claims concerning every model of GR.*

The Shanmugadhasan transformation is highly *non-local* in the metric and curvature variables: although, at the end, for any  $\tau$ , the DO are *fields* indexed by the coordinate point  $\sigma^a$ , they are in fact *highly non-local functionals of the metric and the extrinsic curvature over the whole off-shell surface  $\Sigma_\tau$* . We can write, *symbolically* ( ${}^3\pi^{cd}$  are the momenta conjugate to  ${}^3g_{ab}$ ):

$$\begin{aligned} q^r(\tau, \vec{\sigma}) &= \mathcal{F}_{[\Sigma_\tau]^r}[(\tau, \vec{\sigma}) | {}^3g_{ab}, {}^3\pi^{cd}] \\ p_s(\tau, \vec{\sigma}) &= \mathcal{G}_{[\Sigma_\tau]^s}[(\tau, \vec{\sigma}) | {}^3g_{ab}, {}^3\pi^{cd}], \quad r, s = 1, 2. \end{aligned} \quad (4.1)$$

ii) Since, as already mentioned, in *spatially compact* space-times the original canonical Hamiltonian in terms of the ADM variables is zero, the Dirac Hamiltonian happens to be written solely in terms of the eight constraints and Lagrangian multipliers. This means, however, that this Hamiltonian generates purely harmless gauge transformations, so that *it cannot engender any real temporal change*. Therefore, in spatially compact space-times, in a completely fixed Hamiltonian gauge we have a vanishing Hamiltonian, and the canonical DO are constants of the motion, i.e.  $\tau$ -independent.

In such models of GR with *spatially compact* space-times without boundary (nothing is known if there is a boundary), one must re-introduce the *appearance of evolution* in a frozen picture. Without deeply entering this debated topic (see the viewpoints of Earman, 2002, 2003, Maudlin, 2002, Rovelli, 1991, 2002, as well as the criticisms of Kuchar, 1992, 1993, and Unruh, 1991), we only add a remark on the *problem of time*. In all of the globally hyperbolic space-times (the only ones admitting a canonical formulation), the mathematical time  $\tau$ , labeling the simultaneity (and Cauchy) surfaces, must be related to some empirical notion of time (astronomical ephemerides time, laboratory clock,...). In a GR model with frozen picture there is no physical Hamiltonian governing the evolution in  $\tau$ <sup>27</sup> and consequently there exists the problem of defining a local evolution in terms of a clock built with GR observables (with a time monotonically increasing with  $\tau$ ), as well as the problem of parametrizing other GR observables in terms of this clock<sup>28</sup>.

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momentum (the last gauge variable whose variation describes the normal deformations of the simultaneity surfaces, namely the changes in the clocks' synchronization convention).

<sup>27</sup> Unless, following Kuchar (1993), one states that the super-Hamiltonian constraint is not a generator of gauge transformations but an effective Hamiltonian instead.

<sup>28</sup> See the concept of evolving constants of motion, and the partial and complete observables of Rovelli (1991, 2002), as well as a lot of other different point of views.

Our advantage point, however, is that, in the case of *spatially non-compact* space-times of the C-K class, the generator of  $\tau$ -temporal evolution is the *weak ADM energy*<sup>29</sup>. Indeed, this quantity *does generate real  $\tau$ -temporal changes of the canonical variables*, changes which can subsequently be rephrased in terms of some empirical clock monotonically increasing in  $\tau$ . It is important to stress that since the density  $\mathcal{E}_{ADM}(\tau, \vec{\sigma})$  of the weak

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<sup>29</sup> The ADM energy is a Noether constant of motion representing the *total mass* of the instantaneous "3-universe", just one among the ten asymptotic ADM Poincarè *charges* that, due to the absence of super-translations, are the only asymptotic symmetries existing in C-K space-times. Consequently, the Cauchy surfaces  $\Sigma_\tau$  must tend to space-like hyper-planes normal to the ADM momentum at spatial infinity. This means that: (i) such  $\Sigma_\tau$ 's are the *rest frame* of the instantaneous "3-universe"; (ii) asymptotic inertial observers exist and should be identified with the *fixed stars*, and (iii) an asymptotic Minkowski metric is naturally defined. This *asymptotic background* allows to avoid a split of the metric into a background metric plus a perturbation in the weak field approximation. Due to point ii) the C-K space-times provide a model of both the *solar system* and our *galaxy* but, as yet, not a well-defined model for cosmology. If gravity is switched off, the C-K space-times collapse to Minkowski space-time and the ADM Poincarè charges become the Poincarè special relativistic generators. *These space-times provide, therefore, the natural model of GR for incorporating particle physics which, in every formulation, is a chapter of the theory of representations of the Poincarè group on Minkowski space-time in inertial frames, the elementary particles being identified by the mass and spin invariants.* If we change the boundary conditions, allowing the existence of super-translations, the asymptotic ADM Poincarè group is enlarged to the infinite-dimensional asymptotic SPI group (Wald, 1984) and we lose the possibility of defining the spin invariant. Note that in spatially-compact space-times *with boundary* it could be possible to define a boundary Poincarè group (lacking in the absence of boundary), but we know of no result about this case. The mathematical background of these results can be found in Lusanna, 2001; Lusanna & Russo, 2002; DePietri, Lusanna, Martucci & Russo, 2002; Agresti, DePietri, Lusanna & Martucci, 2004, and references therein.

Let us add some further comments:

- A) The fact that particle physics is defined in the spatially non-compact Minkowski space-time implies that speaking of, e.g., nucleosynthesis in spatially compact cosmologies entails a *huge extrapolation*.
- B) Classical string theories and super-gravity theories include particles, but their quantization requires the introduction of a background space-time for defining the particle Fock space. The only well-developed form of background-independent quantum gravity (loop quantum gravity), obtained by quantizing either the connection or the loop representation of GR, leads to a quantum formulation inequivalent to Fock space, so that it is as yet not known how to incorporate particle physics. We hope that our viewpoint, taking into account the non-inertial aspects of GR, can be developed to the extent of being able to reopen the program of canonical quantization of gravity in a background independent way by *quantizing the DO only*. See Alba & Lusanna, (2005b), for a preliminary attempt to define relativistic and non-relativistic quantum mechanics in non-inertial frames in Galilei and Minkowski space-times, respectively, in such a way that the gauge variables describing the inertial effects (the *appearances*) remain c-numbers.
- C) Finally, quantum field theory in background curved space-times does not admit a useful particle interpretation of its states due to the absence of the notion of Fourier transform (no way of defining the sign of energy and the usual Fock space). As a consequence, the particle notion is replaced by the notion of detector and in this approach it is not clear how to recover the results of particle physics needed for astrophysics.

ADM energy  $\int d^3\sigma \mathcal{E}_{ADM}(\tau, \vec{\sigma})$  contains the potentials of the inertial forces explicitly, it is a *gauge-dependent quantity*. This is nothing else than another aspect of the gauge-dependence problem of the energy density in GR.

Thus, the final Einstein-Dirac-Hamilton equations for the DO, in a complete gauge, are

$$\dot{q}^r = \{q^r, H_{ADM}\}^*, \quad \dot{p}_s = \{p_s, H_{ADM}\}^*, \quad r, s = 1, 2, \quad (4.2)$$

where  $H_{ADM}$  is intended to be the restriction of the *weak ADM energy* to  $\Omega_4$  and where the  $\{\cdot, \cdot\}^*$  are the Dirac brackets.

In conclusion, within the Hamiltonian formulation, we found a class of solutions in which - unlike what has been correctly argued by Earman (Earman, 2002; Belot & Earman, 1999, 2001) for spatially-compact space-times - there is a *real, NIF-dependent, temporal change*. But this of course also means that the *frozen-time* picture, being model dependent, is not a *typical* feature of GR.

On the other hand, it is not clear whether the formulation of a cosmological model for GR is necessarily limited to spatially compact space-times without boundary. As already said, our model is suited for the solar system and the galaxy. It cannot be excluded, however, that our asymptotic inertial observers (up to now identifiable with the fixed stars) might be identified with the preferred frame of the cosmic background radiation with our 4-metric including some pre-asymptotic cosmological term.

## V. THE INTRINSIC GAUGE AND THE DYNAMICAL INDIVIDUATION OF POINT-EVENTS

We know that only two of the ten components of the metric are physically essential: it seems plausible then to suppose that only this subset can act as an individuating field, and that the remaining components play a different role.

Consider the following *four scalar invariant functionals* (the eigenvalues of the Weyl tensor), written here in Petrov's compressed notation (see, e.g., Kramer, Stephani, MacCallum & Herlit, 1980):

$$\begin{aligned} w_1 &= \text{Tr}(gWgW), \\ w_2 &= \text{Tr}(gW\epsilon W), \\ w_3 &= \text{Tr}(gWgWgW), \\ w_4 &= \text{Tr}(gWgW\epsilon W), \end{aligned} \quad (5.1)$$

where  $g$  is the 4-metric,  $W$  is the Weyl tensor, and  $\epsilon$  is the Levi-Civita totally antisymmetric tensor.

Bergmann and Komar (Komar, 1958; Bergmann & Komar, 1960; Bergmann, 1961, 1962) proposed a set of invariant *intrinsic pseudo-coordinates* as four suitable functions of the  $w_T$ <sup>30</sup>,

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<sup>30</sup> Modulo the equations of motion, the eigenvalues  $w_T$  are functionals of the 4-metric and its first derivatives.

$$\hat{I}^{[A]} = \hat{I}^{[A]}[w_T[g(x), \partial g(x)]], \quad A = 0, 1, 2, 3. \quad (5.2)$$

Indeed, under the hypothesis of no space-time symmetries, the  $\hat{I}^{[A]}$  can be used to label the point-events of space-time, at least locally. Since they are *scalars*, the  $\hat{I}^{[A]}$  are invariant under passive diffeomorphisms (therefore they do not define a coordinate chart in the usual sense, precisely as it happens with *radar-pseudo-coordinates*).

Clearly, our attempt to use intrinsic pseudo-coordinates to provide a physical individuation of point-events would *prima facie* fail in the presence of symmetries, when the  $\hat{I}^{[A]}$  become degenerate. This objection was originally raised by Norton (see Norton, 1988, p.60) as a critique to manifold-plus-further-structure (MPFS) substantivalism (see for instance Maudlin, 1988, 1990). Several responses are possible. Firstly, although to this day all the *known* exact solutions of Einstein's equations admit one or more symmetries, these mathematical models are very idealized and simplified; in a realistic situation (for instance, even with two masses alone) space-time would be filled with the excitations of the gravitational degrees of freedom, and would admit no symmetries at all. Secondly, the parameters of the symmetry transformations can be used as supplementary individuating fields, since, as noticed by Stachel (1993), they also depend on the metric field, through its isometries. Thirdly, and most importantly, in our analysis of the physical individuation of points we are arguing a question of principle, and therefore we must consider *generic* solutions of Einstein's equations rather than the null-measure set of solutions with symmetries.

It turns out that the four Weyl scalar invariants can be re-expressed in terms of the ADM variables, namely the lapse  $N$  and shift  $N^a$  functions, the 3-metric  ${}^3g_{ab}$  and its conjugate canonical momentum (the extrinsic curvature  ${}^3K_{a,b}$ )<sup>31</sup>. Consequently the  $\hat{I}^{[A]}$  can be exploited to implement four gauge-fixing constraints *involving a hyper-surface  $\Sigma_\tau$  and its embedding in  $M^4$* . On the other hand, in a completely fixed gauge  $\Gamma_8$ , the  $\hat{I}^{[A]}$  become *gauge dependent* functions of the DO of that gauge.

Writing

$$\hat{I}^{[A]}[w_T(g, \partial g)] \equiv \hat{Z}^{[A]}[\hat{w}_T({}^3g, {}^3\pi, N, N^a)], \quad A = 0, 1, 2, 3; \quad (5.3)$$

and selecting a *completely arbitrary, radar-pseudo-coordinate system*  $\sigma^A \equiv [\tau, \sigma^a]$  adapted to the  $\Sigma_\tau$  surfaces, we apply the *intrinsic gauge-fixing* defined by

$$\chi^A \equiv \sigma^A - \hat{Z}^{[A]}[\hat{w}_T[{}^3g(\sigma^B), {}^3\pi(\sigma^D), N(\sigma^E), N^a(\sigma^F)]] \approx 0, \quad A, B, D, E, F = 0, 1, 2, 3; \quad (5.4)$$

to the *super-hamiltonian* ( $A = 0$ ) and the *super-momentum* ( $A = 1, 2, 3$ ) constraints. This is a good gauge-fixing provided that the functions  $\hat{Z}^{[A]}$  are chosen to satisfy the fundamental *orbit conditions*  $\{\hat{Z}^{[A]}, \mathcal{H}_B\} \neq 0$ , ( $A, B = 0, 1, 2, 3$ ), which ensure the independence of

<sup>31</sup> Bergmann and Komar have shown that the four eigenvalues of the *spatial part* of the Weyl tensor depend only upon the 3-metric and its conjugate momentum.

the  $\chi^A$  and *carry information about the Lorentz signature*. Then the complete  $\Gamma_8$  *intrinsic gauge-fixing* procedure leads to the final result

$$\sigma^A \equiv \tilde{Z}^{[A]}[q^a(\sigma^B), p_b(\sigma^D)|\Gamma], \quad A, B, D = 0, 1, 2, 3; \quad a, b = 1, 2; \quad (5.5)$$

where the notation  $|\Gamma$ ) means the functional form assumed in the chosen gauge  $\Gamma_8$ .

*On-shell* the last equation amounts to a *definition* of the *radar-pseudo-coordinates*  $\sigma^A$  as four *scalars* providing a *physical individuation of any point–event, in terms of the gravitational degrees of freedom  $q^a$  and  $p_b$* . Therefore the scalars  $\tilde{Z}^{[A]}$ , *strongly identical* to the radar pseudo-coordinates, define the enlarged Atlas  $\mathcal{A}$  of  $M^4$  referred to in footnote 22. In this way, each of the point–events of space-time is endowed with its own *metrical fingerprint* extracted from the tensor field, i.e. the value of the four scalar functionals of the DO (exactly four!)<sup>32</sup>. The price that we have paid for this achievement is that *we have necessarily broken general covariance!* As already repeatedly stressed, every choice of 4-coordinates for a point (every gauge-fixing, in the Hamiltonian language), in any procedure whatsoever for solving Einstein’s equations, amounts to a breaking of general covariance, by definition. On the other hand, the whole extent of general covariance can be recovered by exploiting the gauge freedom. Our construction does *not* depend on the selection of a set of physically preferred intrinsic pseudo-coordinates, because by modifying the functions  $I^{[A]}$  we have the possibility of implementing *any* (adapted) radar-coordinate system. Passive diffeomorphism-invariance reappears in a different suit: we find exactly the same functional freedom of  ${}_P\text{Diff } M^4$  in the functional freedom of the choice of the *pseudo-coordinates*  $Z^{[A]}$  (i.e. of the gauge-fixing). What matters here is that *any* adapted radar-coordinatization of the manifold can be seen as embodying the physical individuation of points, because it can be implemented as the Komar–Bergmann *intrinsic pseudo-coordinates* after we choose the correct  $Z^{[A]}$  and select the proper gauge.

In conclusion, as soon as the Einstein-Dirac-Hamilton equations are solved *in the chosen gauge*  $\Gamma_8$ , starting from given initial values of the DO on a Cauchy hyper-surface  $\Sigma_{\tau_0}$ , the evolution in  $\tau$  throughout  $M^4$  of the DO themselves, whose dependence on space (and on parameter time) is indexed by the chosen coordinates  $\sigma^A$ , yields the following *dynamically-determined* effects: i) *reproduces* the  $\sigma^A$  as the Bergmann-Komar *intrinsic pseudo-coordinates*; ii) reconstructs space-time as a (*on-shell*) foliation of  $M^4$ ; iii) defines the associated *dynamically-admissible* NIF; iv) determines a *simultaneity* and *gravito-magnetism convention*.

Now, what happens if matter is present? Matter changes the Weyl tensor through Einstein’s equations and, in the new basis constructed by the Shanmugadhasan transformation, contributes to the separation of gauge variables from DO through the presence of its own DO. In this case we have DO for both the gravitational field and the matter fields, which satisfy *coupled Einstein-Dirac-Hamilton equations*. Since the gravitational DO will still provide the individuating fields for point-events according to our procedure, *matter will come to influence the evolution of the gravitational DO and thereby the physical individuation of point-events and the dynamically-admissible NIF*. Of course, a basic role of matter is the

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<sup>32</sup> The fact that there are just *four* independent invariants for the vacuum gravitational field should not be regarded as a coincidence. On the contrary, it is crucial for the purpose of point individuation and for the gauge-fixing procedure we are proposing.

possibility of building apparatuses for the measurement of the gravitational field, i.e. for an empirical localization of point-events. As shown elsewhere (Pauri & Vallisneri, 2002; LPI and LPII), as a dynamical theory of measurement is lacking, the epistemic circuit of GR can be approximately closed via an *experimental three-step procedure* that, starting from concrete radar measurements and using test-objects, ends up in a complete and empirically coherent *intrinsic individuating gauge fixing*, i.e. in an empirical construction of a net of radar coordinates and in a measurement of the metric in such coordinates.

Finally, let us emphasize that, even in the case with matter, time evolution is still ruled by the *weak ADM energy*. Therefore, the temporal variation corresponds to a *real change* and not merely to a harmless gauge transformation as in other models of GR. The latter include, as already stressed in Section IV, the spatially compact space-time without boundary. Note furthermore that, since the DO of every completely fixed gauge in these spatially compact models are  $\tau$ -independent, the gauge fixing with  $A = 0$  in (5.5) is inconsistent: it is therefore impossible to realize the *time-direction* in terms of DO, and *the individuation of point-events breaks down*. This is compatible with the Wheeler-DeWitt interpretation according to which in such models we have only a *local time evolution* (in the direction normal to  $\Sigma_\tau$ ) generated by the super-hamiltonian constraint (see for instance Kuchar, 1993). It is seen that *our individuation procedure fails in spatially compact models of GR on the same grounds that prevent a real time evolution for them*. More precisely, in such models it is possible to get at best a physical individuation of the point-events belonging to the 3-space on a fixed time slice, but not to space-time on the whole <sup>33</sup>.

## VI. "THE LAST REMNANT OF PHYSICAL OBJECTIVITY OF SPACE-TIME": A FINAL LOOK

The main results we have so far obtained are: i) a *NIF-dependent temporal evolution* of the physical observables; ii) the dis-solution of the Hole Argument; iii) a *NIF-dependent physical individuation* of point-events in terms of the autonomous degrees of freedom of the gravitational field (the *metrical fingerprint* we were looking for); while results i) and iii) are valid for the C-K class only, result ii) is valid for every globally-hyperbolic space-time.

We want to scrutinize such results from the point of view of *the issue of objectivity* of general relativistic space-time structures.

Concerning the first result, we can only stress that the NIF-dependence of the generator of temporal evolution is nothing else than another manifestation of the endless problem of energy of GR.

Concerning the searched for *explanation* of Leibniz equivalence, our analysis of the correspondence between symmetries of the Lagrangian configurational approach and those of the Hamiltonian formulation has shown the following. Solutions of Einstein's equations that, in the configurational approach, differ within the Hole by elements of the subset  ${}_A \text{Diff}' M^4$ ,

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<sup>33</sup> This, by the way, is just what happens in loop quantum gravity: one starts with a fixed classical time slice (a Cauchy surface) as a given 3-space and makes the quantization. This 3-space appears explicitly in all the construction (see Nicolai, Peeters and Zamaklar, 2005). Up to now there has been no accepted solution for the issue of temporal evolution in spatially compact space-times (the problem of the super-hamiltonian constraint) and, therefore, of the spatiotemporal interpretation of the quantization.



which correspond to mappings among gauge-equivalent Cauchy data, belong to the *same 4-geometry*, i.e. the same equivalence class generated by applying all *passive diffeomorphisms* to any of the original 4-metrics:  ${}^4\text{Geom} = {}^4\text{Riem}/{}_P\text{Diff } M^4 = {}^4\text{Riem}/Q'$ . In this case, as seen at the Hamiltonian level, they are simply solutions differing by a *harmless Hamiltonian gauge transformation on shell* and describing, therefore, the same Einstein "universe". Furthermore, it is possible to engender these allegedly different *models* of GR within the hole, by appropriate choices of the initial gauge fixing (the functions  $\hat{Z}^{[A]}$ ). Since we know that the physical role of the gauge-fixings is essentially that of choosing the functional form of the inertial potentials in the NIF defined by the complete gauge (the *epistemic* part of the game), the "differences" among the solutions generated within the Hole by the allowed active diffeomorphisms amount to the different *inertial appearances* of the autonomous gravitational phenomena (the *ontic* part of the game) in different NIFs.

*In the end, this is what, physically, Leibniz equivalence reduces to. This conclusion, together with the physical individuation of point-events achieved by exploiting the intrinsic gauge, make up our answer to Friedman's question. The extent of intrinsic-ness of such an answer will be specified presently.*

As already anticipated, our analysis contrasts with Stachel's attitude towards the Hole Argument. Leaving aside Stachel's broad perspective on the significance and the possibility of generalizations of the Hole story (see Stachel & Iftime, 2005), let us confine ourselves to a few comments about Stachel's original proposal for the physical individuation of points of  $M^4$  by means of a fully covariant exploitation of the Bergmann-Komar invariants  $\hat{I}^{[A]}[w_T[g(x), \partial g(x)]]$ ,  $A = 0, 1, 2, 3$ . First of all, remember again that the effect of the Hole Argument reveals itself on *solutions* of Einstein's equations and that the active diffeomorphisms that purportedly maintain the physical identity of the points are, therefore, dynamical symmetries. Now, how are we guaranteed that the functional dependence of the covariant quantities  $\hat{I}^{[A]}[w_T[g(x), \partial g(x)]]$  be *concretely characterized* as relating to actual solutions of Einstein's equations? Since in the actual case we know that these quantities depend upon 4 DO and 8 gauge variables, we have, hidden under general covariance, a gauge arbitrariness that unavoidably transfers itself on the individuation procedure and *leaves it undefined*. Indeed, speaking of general covariance in an abstract way hides the necessity of getting rid of the above arbitrariness by a gauge-fixing that, in turn, necessarily breaks general covariance. In other words, a definite individuation entails a concrete characterization of the *epistemic* part of the game, which is precisely what we have done. The result is, in particular, exactly what Stachel's suggestion was intended for, for our *intrinsic gauge* shows that *active diffeomorphisms* of the first kind (i.e. those belonging to  $Q'$  in their passive interpretation) do map individuations of point-events into *physically* equivalent individuations. Indeed, since the *on-shell* Hamiltonian gauge transformation connecting two different gauges is the passive counterpart in  $Q'$  of an *active* diffeomorphisms  $D_A \in {}_A\text{Diff}' M^4$ , it determines the *drag-along coordinate transformation*  $\mathcal{T}_{D_A}$  of Section II connecting the 4-radar-coordinates of the two gauges, i.e. the *dual view* of the active diffeomorphism. While the active diffeomorphism *carries along* the identity of points by assumption, its passive view *attributes different physically-individuated radar-coordinates to the same (mathematical) point*. It is seen, therefore, that for any point-event a given individuation by means of DO is mapped into a physically-equivalent, NIF-dependent, individuation.

It is worth stressing again that the main reason why we succeeded in carrying out a concrete realization of Stachel's original suggestion to its natural end lies in the possibility that the Hamiltonian method offers of working *off-shell*. In fact, the 4-D active diffeomorphisms,

*qua* dynamical symmetries of Einstein's equations, must act on solutions at every stage of the procedure and fail to display the arbitrary *epistemic* part of the scalar invariants. On the other hand, the Hamiltonian separation of the gauge variables (characterizing the NIF and ruling the *generalized inertial effects*) from the DO (characterizing generalized tidal effects) *is an off-shell procedure* that brings in the wanted *metrical fingerprint* by working independently of the initial value problem. Once again, this mechanism is a typical consequence of the special role played by gauge variables in GR<sup>34</sup>.

Consider now the results achieved by exploiting the *intrinsic gauge*. First of all, let us state that results iii) are derived on the following assumptions only: a) the recourse to Hamiltonian methods, which are necessary to keep the initial value problem under control; b) the analysis of the  $Q$  group of Bergmann & Komar that provides the unique way for connecting the Bergmann-Komar *intrinsic pseudo-coordinates* to our *radar coordinates*. It should, therefore, be stressed that the uniqueness of the mathematical basis (the way in which the four scalar eigenvalues of the Weyl tensor can be equated to four *scalar radar pseudo-coordinates* by means of the *intrinsic gauge*) shows that this methodology constitutes the only possible way of disclosing the proper point-events ontology of the class of space-times we are referring to. For given initial data of the DO (identifying an Einstein's "universe"), any other kind of gauge-fixing procedure would lead to gauge-equivalent solutions in which the underlying point-events ontology simply *would not be manifestly shown*. Therefore, there are no different formulations or methodologies to compare that could affect the conclusions (philosophical or not) to be drawn from the theory.

As to the physical individuation, our results are tantamount to claiming that the *physical role of the gravitational field without matter* is exactly that of individuating *physically* the points of  $M^4$  as point-events, by means of the four independent phase-space degrees of freedom. As pointed out above, the mathematical structure of the canonical transformation that separates the DO from the gauge variables is such that the DO are *highly non-local functionals of the metric and the extrinsic curvature over the whole (off-shell) hyper-surface*

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<sup>34</sup> According to a *main conjecture* we have advanced elsewhere (see LPI & LPII), a canonical basis of *scalars* (coordinate-independent quantities), or at least a Poisson algebra of them, should exist, making the above distinction between DO and gauge variables *fully invariant*. An evaluation of the degrees of freedom in connection with the Newman-Penrose formalism for tetrad gravity (Stewart, 1993) tends to corroborate the conjecture. In the Newman-Penrose formalism we can define ten coordinate-independent quantities, namely the ten Weyl scalars. If we add ten further scalars built using the extrinsic curvature, we have a total of twenty scalars from which one should extract a canonical basis replacing the 4-metric and its conjugate momenta. Consequently, it should be possible to find *scalar* DO (the *Bergmann observables*, see LPII) and some *scalar* gauge variables (for instance a scalar version of the shift functions, allowing a coordinate-independent description of gravito-magnetism). In any case, the three gauge variables connected to the choice of the 3-coordinates (or at least certain combinations of them) cannot be made scalar, since they appear in those terms inside the ADM energy density which describe the potentials of the intrinsically coordinate-dependent inertial effects. The individuating functions of (5.3) would depend on scalars only and the distinction between DO and gauge observables would become *fully invariant*. Yet, even in the case that the main conjecture might be proved, the gauge-fixing procedure would always break general covariance and one should not forget, furthermore, that the concept of *radar-coordinates* contains a built-in *frame-dependence* (see Section IV). Finally, and above all, the energy density  $\mathcal{E}_{ADM}(\tau, \vec{\sigma})$  would remain a NIF-dependent quantity anyway.

$\Sigma_\tau$ . The same is clearly true for the *intrinsic pseudo-coordinates* [see Eq.(5.3)].

This said, we can even state that the existence of physical *point-events* in our models of general relativity appears to be synonymous with the existence of the DO for the gravitational field. We advance accordingly the *ontological* claim that - physically - *Einstein's vacuum space-time in our models is literally identifiable with the autonomous degrees of freedom of such a structural field*, while the specific (NIF-dependent) functional form of the *intrinsic pseudo-coordinates* associates such coordinates to the points of  $M^4$ . The autonomous gravitational degrees of freedom are - so to speak - *fully absorbed in the individuation of point-events*. On the other hand, when matter is present, the individuation methodology maintains its validity and shows how matter comes to influence the physical individuation of point-events.

At this point, looking back at our results on their whole but with a special attention to iii), we should make a clear assessment of the *degree of physical objectivity* of our individuation procedure *vis a' vis* the radical statement made by Einstein in the passage quoted in the Introduction. What matters, of course, is the NIF-dependence of the *physical identity* of point-events we have achieved for a given Einstein's "universe". Clearly, a *really different* physical individuation is obtained starting with different initial conditions for the *Dirac observables* (i.e. for a different "universe").

Now, can the freedom of choice among the dynamically-possible NIF be equated with "taking away from space and time the last remnant of physical objectivity" as Einstein claimed? We do believe that the answer is certainly "no". On the other hand, however, we should acknowledge that what we have gained does not seem to be a kind of objectivity in the usual sense, since the values of the DO which individuate the point-events are NIF-dependent. It is only in the *abstract reduced phase space*  $\tilde{\Omega}_4$ , defined in Section IV by taking the quotient with respect to all the gauge inertial effects, that *abstract* DO live: they are the "*strictly intrinsic*" *qualifiers* of the generalized tidal effects (i.e. of the proper degrees of freedom of the gravitational field), which are then concretely realized by the ordinary Dirac observables in each NIF. For any given Einstein's "universe" with its topology, the abstract DO in  $\tilde{\Omega}_4$  are locally functions of the points  $x$  of an abstract mathematical manifold  $\tilde{M}_4$  that is the equivalence class of all our *concrete realizations* of space-time, each one equipped with its gauge-dependent individuation of points, NIF and inertial forces. The point over which such fields reside could be called *intrinsic* to the extent that they are no longer NIF-dependent, and synthesize the essential properties of all the appearances shown by the gauges. Admittedly, the global existence of  $\tilde{\Omega}_4$  over  $\tilde{M}_4$  is subjected to a huge set of mathematical hypotheses which we will not take into account here. Locally, however, the Dirac fields certainly exist and we could introduce a coordinate system defined by their values as intrinsic individuating system for the given "universe". Given the abstract nature of the NIF-independent DO, these considerations possess a purely mathematical value. At any rate, we take them as meaningful enough to justify the affix "point" to our notion of "point-structuralism"<sup>35</sup>.

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<sup>35</sup> Recall the following passage by Bergmann and Komar: "[...] in general relativity the identity of a world point is not preserved under the theory's widest invariance group. This assertion forms the basis for the conjecture that some physical theory of the future may teach us how to dispense with world points as the ultimate constituents of space-time altogether." (Bergmann & Komar, 1972, p.27). Now the *abstract reduced phase space*  $\tilde{\Omega}_4$  would be just the germ of such a theory. The theory would be an *abstract and highly*

The important question, however, is another one, namely, what kind of would-be fully objective spatiotemporal structures of our models of GR should the discovered NIF-dependence of the *point-events individuation* be compared to ? Here, again, the fascination of general covariance must be put to the test, and again we will find that the problem has to do with an excessively self-confident utilization of the configurational (and fully general covariant) geometric interpretation of a complex mathematical notion like that of Cauchy-surface. This point was dealt with in great detail in LP11, Section 3. Here we shall limit ourselves to exploiting the notion of *Bergmann observable* (BO) as an 'acid test' for any off-hand attribution of "true objectivity", or NIF-independence, or "unique predictability", to spatiotemporal structures in GR. Briefly, a BO is a configurational quantity defined in  $M^4$  which is both coordinate-independent (i.e. it is a scalar field or an invariant under  $PDiff$ ) and also "*uniquely predictable from initial data*". The essential point is that - under the current general covariance wisdom - quantities which are "invariant" under the passive diffeomorphisms are often confidently, but wrongly, taken to be also "uniquely predictable from the initial data". In LP11,3 we considered, in particular, the example of the four-dimensional scalar curvature  $R(p)$ , calculated at any point  $p$  of  $M^4$  lying in the "future" of a Cauchy-surface. This quantity, taken to be a BO, has been considered by Earman (see Earman, 2002) to the effect of showing that the observables in GR cannot undergo *any kind of change at all*, let alone *temporal change*. Now, the problem is exactly the same raised above for the Bergmann-Komar scalars in Stachel's perspective: how could we be sure by definition that a scalar field - *qua* fully covariant entity - when rewritten in terms of ADM variables, does not contain arbitrary gauge elements that can jeopardize the conclusion to be drawn from its seemingly symmetric simplicity ? For this is exactly what happens in this case: as a matter of fact,  $R(p)$  turns out to be a *gauge-dependent* (or NIF-dependent) quantity and therefore *not predictable*; in conclusion *it is not a BO!* This lack of predictability, however, cannot be perceived within the configurational and fully covariant approach of  $M^4$ : the shallow geometric interpretation of the Cauchy surface *fails the 'acid test'*. The same NIF-dependence characterizes, for instance, quantities like the following *off-shell scalars* with respect to  $PDiff M^4$ : the bilinears  ${}^4R_{\mu\nu\rho\sigma} {}^4R^{\mu\nu\rho\sigma}$ ,  ${}^4R_{\mu\nu\rho\sigma} \epsilon^{\mu\nu\alpha\beta} {}^4R_{\alpha\beta}{}^{\rho\sigma}$  and - as already said - the *four eigenvalues of the Weyl tensor* exploited in Section V. What is more important is that the same *does hold*, in particular, for the *one-way velocity of light*, for the *line element*  $ds^2$  and, therefore, for the very *causal structure of space-time*.

This technical de-tour highlights the important fact that, as soon as one leaves the rarefied atmosphere of full general covariance and soils his hands with the dirty facts of the empirical front of GR, i.e. - theoretically - with the *epistemic component* of the inertio-gravitational field, one realizes that *all* of the fundamental features of space-time structure are - in our language - NIF-dependent. While the local equivalence principle dissolves the absolute structures of the special theory of relativity, the *global consequences* of the (local)

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*non-local theory* of classical gravitation that, transparency aside, would be stripped of all the *epistemic machinery* (the gauge freedom) which is indispensable for both an empirical access to the theory and the reconstruction of the *local field*  $g_{\mu\nu}(x)$ . In other words - inversely seen - the *gauge* structure contributes to the *re-construction* of the spatiotemporal local and continuum representation. We see that even in the context of classical gravitational theory, the spatiotemporal *continuum* plays the role of an *epistemic precondition* of our sensible experience of macroscopic objects, playing a role which is not too dissimilar from that enacted by Minkowski *micro-space-time* in the local relativistic quantum field theory (see Pauri, 2000).

equivalence principle are precisely the pervasive non-inertial factors that show themselves as NIF-dependence. It seems therefore reasonable that we do not expect for *the physical individuation of point-events*, in our models, a degree of objectivity greater than that of all the other physically relevant structures in GR. To conclude, we will state that all of these structures are - unavoidably - *weakly, or NIF-objective*.

We would like to surmise that the disclosure of the physical meaning of Leibniz equivalence renders even more glaring the ontological diversity of the gravitational field with respect to all other fields, even beyond its prominent causal role. It seems substantially difficult to reconcile the nature of the gravitational field with the standard approach of theories based on a *background space-time* (to wit, string theory and perturbative quantum gravity in general). Any attempt at linearizing such theories unavoidably leads to looking at gravity from the perspective of a spin-2 theory in which the graviton stands at the same ontological level as other quanta. In the standard approach of background-dependent theories of gravity, photons, gluons and gravitons all live on the stage on an equal footing. From the point of view set forth in this paper, however, *non-linear gravitons* are at the same time both the stage and the actors within the causal play of photons, gluons, and other *material characters* such as electrons and quarks.

Note finally that the individuating relation (5.5) is a numerical identity that has an *in-built non-commutative structure*, deriving from the Dirac–Poisson structure hidden in its right-hand side. The individuation procedure transfers, as it were, the non-commutative Poisson-Dirac structure of the DO onto the individuated point-events, even though the coordinates on the l.h.s. of the identity are c-number quantities. One could guess that such a feature might deserve some attention in view of quantization, for instance by maintaining that the identity, interpreted as a relation connecting mean values, could still play some role at the quantum level.

## VII. CONCLUDING REMARKS: AN INSTANTIATION OF STRUCTURAL REALISM AS "POINT-STRUCTURALISM"

We conclude by spending a few words about the implications of our results for some issues surrounding the recent debate on scientific structural realism, as well as for the traditional debate on the absolutist/relationist dichotomy.

It is well-known that the term *scientific realism* has been interpreted in a number of different ways within the literature on philosophy of science, in connection with the progressive sophistication of our understanding of scientific knowledge. Such ways concern, e.g., realism about *observable or unobservable entities*, and realism about *theories*. A further ramification of meanings has been introduced more recently by the so-called *structural realism* (the only attainable reality are relations between (unobservable) objects), which originated a division between the so-called *epistemic* structural realists (entity realism is unwarranted) and the *ontic* structural realists (the relations exhaust what exists) (see Simon, 2003).

From the logical point of view, we can assume that the concept of *structure* refers to a (stable or not) set of *relations* among a set of some kind of *constituents* that are put in relations (the *relata*). The specification expressed by the notion of *structural realism* introduces *some kind* of ontological distinction between the role of the relations and that of the constituents. At least two main exemplary possibilities present themselves as obvious: (i)

there are relations in which the constituents are (ontologically) primary and the relation secondary; (ii) there are relations in which the relation is (ontologically) primary while the constituents are secondary, and this even without any prejudice against the ultimate ontological consistency of the constituents. In the case of *physical* entities, following Stachel (see Stachel, 2005) one could cautiously recover in this connection the traditional distinction between *essential* and *non-essential* properties (*accidents*) in order to characterize the *degree of (ontological) primacy* of the relations versus the *relata* and vice versa (and this independently of any metaphysical flavor possibly connected to the above distinctions). For example, one could say that in the extreme case (i) only *accidental* properties of the constituents can depend upon the relational structure, while in the extreme case (ii) at least one *essential* property of the constituents depends upon the relational structure (saying that *all* the essential properties of the relata depend upon the relation would be tantamount to claiming that there exist only relations without constituents, as the *ontic structural realist* has it).

A further complication is connected to the nature of the structure we are considering. For while at the logical level (leaving aside the deep philosophical issue concerning the relationships between mathematical structures and substances) the concept of *mathematical structure* (e.g. a system of differential equations, or even the bare mathematical manifold of point which provides the first layer of our *representations* of the real space-time ) can be taken to be sufficiently clear for our purposes, the definition of *physical structure* immediately raises existential philosophical problems. For example, we believe that it is very difficult to define a *physical structure* without bringing in its *constituents*, and thereby granting them *some kind of existence* and defending some sort of *entity realism*. Analogously, we believe that it is very difficult to defend *structural realism* without also endorsing a *theory realism* of some sort. However, both theses are not universally shared.

Having said this, let us come back to the results we obtained in the previous sections. The analysis based on our *intrinsic gauge* has disclosed a remarkable and rich local structure of the general-relativistic space-time for the considered models of GR. In correspondence to every *intrinsic gauge* (5.5) we achieved a gauge-related *physical individuation of point-events* in terms of the DO of that gauge, i.e. in terms of the *ontic* part of the gravitational field, as represented in the clothes furnished by the NIF. Such individuation is characterized by a highly *non-local* functional dependence of the DO upon the values of the metric and the extrinsic curvature over the whole (*off-shell*) space-like hyper-surface  $\Sigma_\tau$  of distant simultaneity. Since the extrinsic curvature has to do with the embedding of the simultaneity hyper-surface in  $M^4$ , the DO do *involve geometrical elements external to the hyper-surface* itself. In fact, the temporal gauge (fixed by the scalar  $Z^{[0]}$ ) in the identity (5.5) refers to a continuous interval of hyper-surfaces, and the gauge-fixing identity itself is *intrinsically four-dimensional*. We have, therefore, an instantiation of *metrical holism* which, though local in the temporal dimension and characterized by a *dynamic stratification in 3-hypersurfaces*, is *four-dimensional*. Admittedly, the distinction between *ontic* and *epistemic* parts, as well as the *form* of the space-like surfaces of distant simultaneity, are NIF-dependent.

Thus we have discovered that ontologically the identity of point-events is *conferred* upon them by a complex relational structure in which they are holistically enmeshed. This relational structure includes all the elements of the *complete gauge fixing*  $\Gamma_8$  summarized by a NIF, and supported by a definite solution of Einstein's equations throughout  $M^4$ , corresponding to given initial values for the DO in that gauge (a definite Einstein "universe"). We propose to define such physical identity of point-events as *weakly-objective* or *NIF-objective*,

with the important notice, however, that this weak degree of objectivity is the maximum that can be attained for *all* of the relevant spatiotemporal structures in a formulation in which the initial value problem for the C-K models of GR is well-posed. Although the *holistic structure* appears to be *ontologically prior* to its *constituents* (the point-events) as to their physical identity, we cannot agree with Cao's assertion (see Cao, 2003, p.111) that the constituents, as mere *place-holders*, derive their meaning or *even their existence* from their function and place in the structure. Indeed, at any level of GR, the empirical level above all, one cannot avoid quantifying over points, and we have just attributed a physical meaning<sup>36</sup> to the radar-coordinate indexing of such point-events. Such an indexing makes the latter as ontologically equivalent to the existence of the gravitational field in vacuum as an extended entity, since the autonomous degrees of freedom of the gravitational field are - so to speak - *fully absorbed in the individuation of point-events*. From this point of view, our dressed point-events are not under-determined by empirical evidence. Quite in general, we cannot see how a place-holder can have any ontological function in an evolving network of relationships without possessing at least *some kind* of properties. For our results do in fact confer a sort of causal power on the gravitationally-dressed points.

Furthermore, as already said, we could even dare a little more concerning the identity of points, since some kind of *abstract intrinsic individuality* survives beneath the variety of descriptions displayed by all the gauge-related NIF and common to all these *appearances*. This kind of *intrinsic identity* - in the vein suggested by Bergmann and Komar - is just furnished by the *abstract Dirac fields* residing within the phase-space  $\tilde{\Omega}_4$ , which is nothing else but a quotient with respect to all of the concrete realizations and appearances of the NIF. Accepting this suggestion for the sake of argument we would be led to a peculiar space-time *structure* in which the relation/relata correspondence does not fit with any of the extreme cases listed above, for one could assert that while the abstract *essential* properties belong to the constituents as seen in  $\tilde{\Omega}_4$  (so that abstract point-events in  $\tilde{M}^4$  would be like *natural kinds*), the totality of the physically concrete *accidents* are displayed by means of the holistic relational structure. This is the reason why we propose calling this peculiar kind of space-time structuralism *point-structuralism*. Admittedly, it is important not to be misled into thinking that this abstract intrinsic-ness has a direct physical meaning.

Summarizing, this view holds that space-time point-events (the *relata*) do exist and we continue to quantify over them; however, their properties can be viewed both as *extrinsic* and *relational*, being conferred on them in a holistic way by the whole structure of the metric field and the extrinsic curvature on a simultaneity hyper-surface, and, at the same time - at an abstract level only - as *intrinsic*, being coincident with the autonomous degrees of freedom of the gravitational field represented by the abstract NIF-independent Dirac fields in  $\tilde{\Omega}_4$ . In this way - although point-events cannot be viewed as *genuine individuals* - both the metric field and point-events maintain their *own manner of existence*, so that the structural texture of space-time in our models does not force us to abandon an *entity realist* stance about both the metric field and its points. We must, therefore, deny the thesis according to which metrical relations can exist without their constituents (the point-events).

Concerning the traditional debate on the dichotomy substantialism/relationism, we believe that our analysis may indeed offer a *tertium-quid* solution to the debate.

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<sup>36</sup> Even operationally, in principle (see Pauri & Vallisneri, 2002; LPI and LPII, already quoted in Section V).

First of all, let us recall that, in remarkable diversity with respect to the traditional historical presentation of Newton's absolutism *vis á vis* Leibniz's relationism, Newton had a much deeper understanding of the nature of space and time. In two well-known passages of *De Gravitatione*, Newton expounds what could be defined an original *proto-structuralist view* of space-time (see also Torretti, 1987, and DiSalle, 1994). He writes (our *emphasis*):

Perhaps now it is maybe expected that I should define extension as substance or accident or else nothing at all. But by no means, for it has *its own manner of existence* which fits neither substance nor accidents [ . . . ] The parts of space derive their character from their positions, so that if any two could change their positions, they would change their character at the same time and each would be converted numerically into the other *qua* individuals. The parts of duration and space are only understood to be the same as they really are because of their mutual order and positions (*propter solum ordinem et positiones inter se*); nor do they have any other *principle of individuation* besides this order and position which consequently cannot be altered. (Hall & Hall, 1962, p.99, p.103.)

On the other hand, in his relationist arguments, Leibniz could exploit the conjunction of the Principle of Sufficient Reason and the Principle of the Identity of Indiscernibles because Newtonian space was *uniform*, as the following passage lucidly explains (our *emphasis*):

Space being *uniform*, there can be neither any *external* nor *internal reason*, by which to distinguish its parts, and to make any choice between them. For, any external reason to discern between them, can only be grounded upon some internal one. Otherwise we should discern what is indiscernible, or choose without discerning. (Alexander, 1956, p.39).

Clearly, if the parts of space were real, Leibniz Principles would be violated. Therefore, for Leibniz, space is not real. The upshot, however, is that space (space-time) in general relativity, far from being *uniform* may possess, as we have seen, a *rich structure*. This is just the reason why - in our sense - it is *real*, and why Leibniz equivalence called upon for general relativity happens to hide the very nature of space-time, instead of disclosing it.

We claim that our results lead to a new kind of *structuralist* conception of space-time. Such structuralism is not only richer than that of Newton, as one could expect because of the dynamical structure of Einstein space-time, but richer in an even deeper sense. Not only the independent degrees of freedom of the metric field are able to characterize the "mutual order and positions" of points *dynamically*, since each point-event "is" - so to speak - the "values" of the autonomous degrees of freedom of the gravitational field; their capacity is even stronger, since *such mutual order is altered by the presence of matter*.

This *new structuralist* conception turns out to include elements common to the tradition of both *substantialism* and *relationism*. Although the metric field does *not* embody the traditional notion of *substance*, it is taken to represent a genuine and primitive element of physical reality and its definition is a necessary condition in order to be able even to *speak* of space-time. In this sense *exists* and plays a role for the individuation of point-events by means of its *structure*. On the other hand, our *point-structuralism* does not support even the standard relationist view. In fact, the holistic relationism we defend does not reduce the whole of spatiotemporal relations to physical relations (i.e. it is not eliminativist), nor does it entails that space-time does not exist as such, being reducible to physical relations. Our dressed point-events are "individuals" in a peculiar sense: they exist as autonomous constituents, but one cannot claim that their properties do not depend on the properties of



others. Not only relations, but also their carriers do exist, even if they do bring intrinsic properties in a very special sense.

Let us finally consider what John Earman wrote in his 1989 book:

The absolute-relational contrast is far from being a dichotomy. A possible, third alternative, which I shall call the property-view of space-time, would take something from both camps: it would agree with the relationist in rejecting a substantial substratum for events while joining with the absolutist in recognizing monadic properties of spatiotemporal locations. (Earman, 1989, p.14).

Clearly, *stricto sensu*, we cannot say that our view of space-time is a *property-view* in Earman's sense. All quantities definable with respect to our gravitationally-dressed point-events are not irreducible monadic spatiotemporal properties. For they are reducible on the grounds that the *physical* identity of our point-events is NIF-relational. Still, due to the underlying abstract structure of autonomous gravitational degrees of freedom represented by the abstract NIF-independent Dirac fields in  $\hat{\Omega}_4$ , we feel allowed to talk of a *weak-property view* of space-time.

We acknowledge that the validity of our results is restricted to the class of models of GR we worked with. Yet, we were interested in exemplifying a question of principle, so that we can claim that there is a class of models of GR embodying both a *real notion of NIF-dependent temporal change*, a *NIF-dependent physical individuation of points* and a new *structuralist and holistic view of space-time*.

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