Empirically Adequate but Observably False Theories

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1 Introduction

According to van Fraassen (1980, 8, emphasis removed), scientifc realism is the view that

> [s]cience aims to give us, in its theories, a literally true story of what the world is like; and acceptance of a scientific theory involves the belief that it is true.

Van Fraassen's constructive empiricism does not differ from scientific realism in its interpretation of scientific theories. They are interpreted literally and as either true or false (van Fraassen 1980, 10). Rather, constructive empiricism claims a different aim of science and a different condition of theory acceptance (van Fraassen 1980, 12, emphasis removed):

> Science aims to give us theories which are empirically adequate; and acceptance of a theory involves as belief only that it is empirically adequate.

The motivation for this position is that the truth of a theory is elusive. For not only is the empirical adequacy of a theory but "indirect or partial evidence for its truth" (van Fraassen 1980, 82; cf. 1989, 191), it is also the only evidence possible. Explanation, for example, cannot provide "evidence for the truth of a theory that goes beyond any evidence we have for its providing an adequate description of the phenomena", that is, beyond any evidence we have for its empirical adequacy (van Fraassen 1980, 156–157, 154). There is, then, nothing beyond empirical adequacy that could speak for a theory's truth.

In an informal overview, van Fraassen (1980, 12) states that "a theory is empirically adequate exactly if what it says about the observable things and events

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in this world, is true", although he stresses that the notion of empirical adequacy "will have to be spelt out very carefully if it is not to bite the dust among hackneyed objections" (van Fraassen 1980, 19). I will argue that van Fraassen has not spelled out the notion of empirical adequacy carefully enough, for a theory may be empirically adequate according to his definition even though it can be observationally determined that the theory is false.

2 Truth and empirical adequacy

Towards a precise definition of empirical adequacy, van Fraassen (1980, 64) suggests describing theories in terms of model theory:

> To present a theory is to specify a family of structures, its *models*; and secondly, to specify certain parts of those models (the *empirical substructures*) as candidates for the direct representation of observable phenomena.

Furthermore the models of the theory "are describable only up to structural isomorphism" (van Fraassen 2008, 238; cf. 2002, 22). A formal paraphrase of van Fraassen's notion is given in definition 1. Realism now claims that science strives for the truth of a theory, where a "theory is true if the real world itself is (or is isomorphic to) one of [the theory's] models" (van Fraassen 1989, 226).[1]

For the definition of empirical adequacy, van Fraassen (2008, 283) strictly distinguishes between the phenomena and their appearances: "*Phenomena* are observable entities (objects, events, processes,...) of any sort, *appearances* are the contents of measurement outcomes". And he suggests describing appearances by structures: "The structures which can be described in experimental and measurement reports we can call *appearances*" (van Fraassen 1980, 64; cf. 2008, 286). This is paraphrased in definition 2. Van Fraassen (2008, 284) is careful to note that each

> appearance is determined jointly by the measurement set-up (involving both apparatus and the system to which it is applied), the experimental practice, and the theoretical conceptual framework in which the target and measurement procedure are classified, characterized, and understood.

Thus even though the phenomena are objective, the appearances are in part determined by the theory, specifically by its conceptual framework. Van Fraassen assumes that the appearances are given not by a single structure, but a set of them, which becomes explicit in his central definition of empirical adequacy: *A "theory is empirically adequate if it has some model such that all appearances are isomorphic*

[1]As is clear from the context, van Fraassen here substitutes 'if' for 'if and only if'. The parenthetical remark is justified because the family of theories is closed under isomorphism.

to empirical substructures of that model" (van Fraassen 1980, 64, my emphasis; cf. 1991, 12).[2] To be empirically adequate, a theory thus has to have one specific model with empirical substructures (plural) such that all appearances (plural) are isomorphic to some of those substructures (see definition 3).

Since the phenomena are part of the world, the appearances have to be substructures of the world's structure. Thus even though the world is objective, the *structure* of the world is determined also by the theory's conceptual framework and, as far as the appearances are concerned, by the features of the experiments. In a sense the structure of the world mediates between the world and a theory in the same way that the appearances mediate between the phenomena and a theory's empirical claims: The truth of a theory's claims about the world is determined by the relation of the theory's models to the structure of the world, and the truth of a theory's claims about the phenomena is determined by the relation of the theory's empirical substructures to the appearances.

The definition of the truth of a theory involves a subtlety that is easily overlooked: It is not enough for the truth of a theory that there is an isomorphism from the structure of the world to one of the theory's models. Additionally, the isomorphism has to respect the appearances, that is, its restriction to the domain of an appearance has to be an isomorphism between that appearance and an empirical substructure of the model. Otherwise, there can be true theories that are not empirically adequate (claim 1). Definition 4 of truth includes a paraphrase of this condition.

3 To save constructive empiricism

Since the appearances are substructures of the structure of the world, a theory can only be true if one of its models is isomorphic to an extension of all appearances,[3] and there's the rub: The empirical adequacy of a theory does not ensure that there is such an extension of the appearances. An even stronger claim can be made: There can be theories that are empirically adequate *and* only make claims about phenomena, but for which there is still no such extension (claim 2). This is because for empirical adequacy it suffices that there is some model of the theory so that for each appearance, there is some, any isomorphism to an empirical substructure of that model. These isomorphisms can be chosen independently from each other, and specifically, two isomorphisms may map two different objects of their respective domains to the same object in the domain of the model. In such a case it can happen that there is no isomorphism between the structure of the world and the model of the theory simply because there is no appropriate bijection between their domains. In short, the problem is that van Fraassen's definition of empirical adequacy only demands that the relation of a theory to each appearance is evaluated individually, even though considering all appearances to-

[2] Van Fraassen again substitutes 'if' for 'if and only if'.

[3] A structure $\mathfrak{B}$ is and extension of a structure $\mathfrak{A}$ if and only if $\mathfrak{A}$ is a substructure of $\mathfrak{B}$.

gether can reveal that a theory is false. This result does not even require that for the truth of a theory, the isomorphism between the structure of the world and the model of the theory must respect the appearances.

Constructive empiricism therefore is in trouble: According to van Fraassen himself, the informal description of empirical adequacy is insufficient for avoiding "hackneyed objections", but his precise notion of empirical adequacy is not in general "partial evidence" for a theory's truth. It can on the contrary happen that a theory that is empirically adequate is certainly false given the appearances. Conversely, that a theory is not certainly false given the appearances can be established by comparing the extensions of the appearances and the models of the theory, and hence there can be evidence for a theory's truth that goes beyond evidence for its empirical adequacy. Science, then, should aim for more than empirical adequacy.

It seems that to save constructive empiricism, van Fraassen's definition of empirical adequacy must be modified. A straightforward modification that ensures that the isomorphisms between appearances and empirical substructures are not independent anymore uses the notion of coordinated isomorphisms. The appearances are *coordinately isomorphic* to some empirical substructures if and only if there is a single bijection that, when restricted to the domain of an appearance, is an isomorphism to an empirical substructure (cf. definition 5). With this notion, it is easy to define a stricter notion of empirical adequacy: *A theory is strictly empirically adequate if and only if it has some model such that all appearances are coordinately isomorphic to empirical substructures of that model* (definition 6). Strict empirical adequacy has a neat connection to truth, because a theory is true if and only if it has some model such that all appearances *and* the world are coordinately isomorphic to empirical substructures of that model and *and* the model itself (claim 3). And unlike in the case of empirical adequacy, a theory is strictly empirically adequate if and only if the theory is true in an extension of all appearances (claim 4). It therefore holds generally that a model of a strictly empirically adequate theory could be true, for all we can observe. Strict empirical adequacy is hence indeed partial evidence for a theory's truth and, in van Fraassen's account, is all the evidence one can have.

4 Conclusion

Since its conception, constructive empiricism has relied on the wrong relation between theory and appearances. For van Fraassen's definition of empirical adequacy is incompatible with the central motivation of constructive empiricism: that any theory whose claims about the phenomena are true could be true itself, and that there is nothing beyond the truth of these claims that can provide evidence for the theory's truth. But as with previous criticisms, constructive empiricism is resilient enough to be saved by an amendment (cf. Muller and van Fraassen 2008). Constructive empiricism should be the claim that *science aims to* *give us theories which are strictly empirically adequate; and acceptance of a theory involves as belief only that it is strictly empirically adequate.* Since this amendment is designed to recover the central motivation of constructive empiricism, most discussions of this motivation can probably remain unchanged. But it remains to be seen which discussions of empirical adequacy must be reevaluated.

Appendix: Definitions, claims, and proofs

Definition 1. A *theory* $\langle \{\mathfrak{T}_n\}_{n\in N}, \{\mathbf{E}_n\}_{n\in N}\rangle$ contains a family of structures (the *models of the theory*) and for each structure $T_n, n \in N$, a set $\mathbf{E}_n$ of *empirical substructures*, such that for each $\mathfrak{E} \in \mathbf{E}_n$, $\mathfrak{E} \subseteq \mathfrak{T}_n$. With each model, a theory also contains every isomorphic structure and its corresponding[4] empirical substructures.

In the following, $\langle \mathfrak{T}_n, \mathbf{E}_n\rangle := \langle \{\mathfrak{T}_n\}_{n\in N}, \{\mathbf{E}_n\}_{n\in N}\rangle$ and $T_n := |\mathfrak{T}_n|$.

Definition 2. The *appearances* are given by a set $\mathbf{A}$ of structures. A structure $\mathfrak{A} \in \mathbf{A}$ is an appearance.

Definition 3. A theory $\langle \mathfrak{T}_n, \mathbf{E}_n\rangle$ is *empirically adequate* for the set $\mathbf{A}$ of appearances if and only if there is some $n \in N$ such that the structures in $\mathbf{A}$ are isomorphic to some structures in $\mathbf{E}_n$.

Claim 1. *There are a theory $\langle \mathfrak{T}_n, \mathbf{E}_n\rangle$ and a structure $\mathfrak{B}$ of the world with appearances $\mathbf{A}$ such that $\langle \mathfrak{T}_n, \mathbf{E}_n\rangle$ is not empirically adequate given $\mathbf{A}$, but $\mathfrak{B}$ is isomorphic to some $\mathfrak{T}_n, n \in N$.*

Proof. Let $\mathfrak{B} = \langle \{a,b,c\}, \{a,c\}\rangle$ and $\mathbf{A} = \{\langle \{a,b\}, \{a\}\rangle\}$,[5] where a, b, and c are distinct objects. Let the theory be given by the family with the member $\mathfrak{T}_1 = \langle \{1,2,3\}, \{1,2\}\rangle$ and the set of empirical substructures $\mathbf{E}_1 = \{\langle \{1,2\}, \{1,2\}\rangle\}$. Let all other models of the theory be isomorphic to $\mathfrak{T}_1$ and have the corresponding empirical substructures. Since $\langle \{a,b\}, \{a\}\rangle$ is not isomorphic to $\langle \{1,2\}, \{1,2\}\rangle$, the theory is not empirically adequate. But $\mathfrak{B}$ is isomorphic to $\mathfrak{T}_1$. $\square$

Definition 4. A theory $\langle \mathfrak{T}_n, \mathbf{E}_n\rangle$ is *true* in structure $\mathfrak{B}$ with appearances $\mathbf{A}$ if and only if there are an $n \in N$ and an isomorphism f from $\mathfrak{B}$ to $\mathfrak{T}_n$ such that for any $\mathfrak{A} \in \mathbf{A}$, $f|_A : A \longrightarrow E$ is an isomorphism from $\mathfrak{A}$ to some $\mathfrak{E} \in \mathbf{E}_n$.

Claim 2. *There are a theory $\langle \mathfrak{T}_n, \mathbf{E}_n\rangle$ with $T_n = \bigcup\{E : \mathfrak{E} \in \mathbf{E}_n\}$ and appearances $\mathbf{A}$ such that $\langle \mathfrak{T}_n, \mathbf{E}_n\rangle$ is empirically adequate given $\mathbf{A}$, but there is no extension of all $\mathfrak{A} \in \mathbf{A}$ that is isomorphic to some $\mathfrak{T}_n, n \in N$.*

[4] To be precise: If $f : T_m \longrightarrow T_n$ is an isomorphism from $\mathfrak{T}_m$ to $\mathfrak{T}_n$, then the set $\mathbf{E}_n$ of empirical substructures that corresponds to $\mathbf{E}_m$ contains all and only those structures $\mathfrak{E}'$ for which there is an $\mathfrak{E} \in \mathbf{E}_m$ such that $f|_E$ is an isomorphism from $\mathfrak{E}$ to $\mathfrak{E}'$.

[5] As is customary, the first element of the tuple is the domain, the second a relation.

Proof. Let the appearances be given by the set of the two structures $\mathbf{A} = \{\langle\{a,b\},\{a,b\}\rangle, \langle\{c,d\},\{c\}\rangle\}$, where a, b, c, and d are distinct objects. Let the theory be given by the family with the member $\mathfrak{T}_1 = \langle\{1,2,3,4\},\{1,2\}\rangle$ and the set of empirical substructures $\mathbf{E}_1 = \{\langle\{1,2\},\{1,2\}\rangle, \langle\{2,3\},\{2\}\rangle\}$. Let all other models of the theory be isomorphic to $\mathfrak{T}_1$ and have the corresponding empirical substructures. Then the theory is empirically adequate. An isomorphism from $\mathfrak{T}_1$ to a structure with the two appearances as substructures would have to be a bijection from $\{1,2,3,4\}$ to $\{a,b,c,d\}$, and thus the structure has to be $\langle\{a,b,c,d\},\{a,b,c\}\rangle$. But this structure is not isomorphic to $\mathfrak{T}_1$. □

Definition 5. The elements of a set $\mathbf{A}$ of structures are *coordinately isomorphic* to the elements of a set $\mathbf{E}$ of structures if and only if there is a bijection $i : \bigcup\{A : \mathfrak{A} \in \mathbf{A}\} \longrightarrow \bigcup\{E : \mathfrak{E} \in \mathbf{E}\}$ such that for each $\mathfrak{A} \in \mathbf{A}$, $i|_A : A \longrightarrow E$ is an isomorphism from $\mathfrak{A}$ to some $\mathfrak{E} \in \mathbf{E}$ and for each for each $\mathfrak{E} \in \mathbf{E}$, $i^{-1}|_E : E \longrightarrow A$ is an isomorphism from $\mathfrak{E}$ to some $\mathfrak{A} \in \mathbf{A}$. i is called a *coordinating bijection* from the elements of $\mathbf{A}$ to the elements of $\mathbf{E}$, and its restrictions $i|_A$ are called *coordinated isomorphisms*.[6]

Definition 6. A theory $\langle\mathfrak{T}_n, \mathbf{E}_n\rangle$ is *strictly empirically adequate* for the set $\mathbf{A}$ of all appearances if and only if there is some $n \in N$ such that the structures in $\mathbf{A}$ are coordinately isomorphic to some structures in $\mathbf{E}_n$.

Claim 3. *A theory $\langle\mathfrak{T}_n, \mathbf{E}_n\rangle$ is true in the structure $\mathfrak{B}$ with appearances $\mathbf{A}$ if and only if there is some $n \in N$ such that the structures in $\{\mathfrak{B}\} \cup \mathbf{A}$ are coordinately isomorphic to $\mathfrak{T}_n$ and some structures in $\mathbf{E}_n$.*

Proof. Every isomorphism from $\mathfrak{B}$ to $\mathfrak{T}_n$ that establishes a theory's truth is also a coordinating bijection from $\{\mathfrak{B}\} \cup \mathbf{A}$ to $\{\mathfrak{T}_n\} \cup \mathbf{E}^*_n$ for some $\mathbf{E}^*_n \subseteq \mathbf{E}_n$, and *vice versa*. □

Claim 4. *A theory $\langle\mathfrak{T}_n, \mathbf{E}_n\rangle$ is strictly empirically adequate for the set $\mathbf{A}$ of appearances if and only if there is an extension $\mathfrak{B}$ of all $\mathfrak{A} \in \mathbf{A}$ such that $\langle\mathfrak{T}_n, \mathbf{E}_n\rangle$ is true in $\mathfrak{B}$ with appearances $\mathbf{A}$.*

Proof. For the inference from right to left, assume that the isomorphism f from $\mathfrak{B}$ to $\mathfrak{T}_n$ establishes the theory's truth. Let $\mathbf{E}^*_n$ be the minimal set such that $f|_A : A \longrightarrow E$ for some $\mathfrak{E} \in \mathbf{E}^*_n$. Then it follows straightforwardly from the definition that f is a coordinating bijection from the elements of $\mathbf{A}$ to those of $\mathbf{E}^*_n$.

For the inference from left to right, assume that there is a coordinating bijection i from all appearances to the empirical substructures in $\mathbf{E}^*_n \subseteq \mathbf{E}_n$ of $\mathfrak{T}_n$. For any two $\mathfrak{A}, \mathfrak{A}' \in \mathbf{A}$, then, the restrictions of their respective relations,

[6] Incidentally, it is straightforward to show that the empirical substructures in $\mathbf{E}_n$ correspond to those in $\mathbf{E}_m$ if and only if the isomorphism from $\mathfrak{T}_m$ to $\mathfrak{T}_n$ is a coordinating bijection from the elements of $\mathbf{E}_m$ to those of $\mathbf{E}_n$ (see note 4).

functions, and constants to $\mathfrak{A} \cap \mathfrak{A}'$ are identical. Thus there is a unique $\tilde{\mathfrak{B}}$ with $\tilde{B} = \bigcup\{A : \mathfrak{A} \in \mathbf{A}\}$ and $\mathfrak{A} \subseteq \tilde{\mathfrak{B}}$ for all $\mathfrak{A} \in \mathbf{A}$.

Now define $\mathfrak{T}'_n$ as an expansion of $\mathfrak{T}_n$ by the unary predicates $P_{\mathfrak{E}}$ interpreted by E for each $\mathfrak{E} \in \mathbf{E}^*_n$, and define $\tilde{\mathfrak{B}}'$ analogously as an expansion of $\tilde{\mathfrak{B}}$ by the unary predicates $P_{\mathfrak{A}}$ interpreted by A for each $\mathfrak{A} \in \mathbf{A}$. It follows from the definition of coordinating bijection that i is an embedding of $\tilde{\mathfrak{B}}'$ in $\mathfrak{T}'_n$. Therefore there is an extension $\mathfrak{B}'$ of $\tilde{\mathfrak{B}}'$ and an isomorphism f from $\mathfrak{B}'$ to $\mathfrak{T}'_n$ (Hodges 1993, ex. 1.2.4b). By the definition of $\tilde{\mathfrak{B}}'$, the reduct $\mathfrak{B}$ of $\mathfrak{B}'$ to the non-logical constants of $\tilde{\mathfrak{B}}$ is an extension of $\tilde{\mathfrak{B}}$ and, since the substructure relation is transitive, also an extension of all $\mathfrak{A} \in \mathbf{A}$. Furthermore, by the definitions of $\tilde{\mathfrak{B}}'$ and $\mathfrak{T}'_n$, f is an isomorphism from $\mathfrak{B}$ to $\mathfrak{T}_n$ such that for every $\mathfrak{A} \in \mathbf{A}$, $f|_A : A \longrightarrow E$ is an isomorphism for some $\mathfrak{E} \in \mathbf{E}_n$ (namely those in $\mathbf{E}^*_n$). $\square$

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