Generalizing Empirical Adequacy II: Partial Structures

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Abstract

The companion piece to this article captures and generalizes empirical adequacy in terms of vagueness sets. In this article, I show that previous attempts to capture and generalize empirical adequacy in terms of partial structures fail. Indeed, the motivations for the partial structures approach are better met by vagueness sets, which can be used to generalize the partial structure approach.

Keywords: empirical adequacy; partial structures; vagueness sets; approximation; partial isomorphism; constructive empiricism

Contents

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1 Introduction

Constructive empiricism, with its central notion of empirical adequacy, and the partial structures approach, with its central notions of quasi-truth and partial isomorphism, are two major lines of research and influence in the semantic view, according to which scientific theories are best represented by model- or set theoretic structures. Combining the two approaches, da Costa and French (1990), Bueno (1997), and da Costa et al. (1998) have discussed a means to describe empirical adequacy as quasi-truth, and Bueno (1997) has used partial isomorphisms and quasi-truths to generalize empirical adequacy.

The initial motivation for Bueno's generalizations was Suárez's criticism of constructive empiricism. I briefly show that both the criticism (§B) and Bueno's reply (§C) fail. What is more, I argue that the attempts to describe empirical adequacy and to generalize empirical adequacy using partial structures fail as well (§§3.1, 4).

In the companion piece to this article (Lutz 2011), I have shown that empirical adequacy is under certain conditions equivalent to generalized approximate truth, and have suggested a generalization of empirical adequacy in terms of vagueness sets that can deal with lack of knowledge and approximations. In this article, I will show that the central concepts of the partial structure approach, quasi-truth (§2.2) and partial isomorphisms (§4.1) can be captured with the help of vagueness sets, too, and that the generalizations of empirical adequacy in terms of approximate truth achieves the goals of the generalizations in terms of partial structures (§3.2).

2 Two formalisms

I will rely on the standard notations used in model theory.[1] A *structure* $\mathfrak{A}$ is a pair $\langle A, \mathscr{I} \rangle$ consisting of a domain A and a function $\mathscr{I}$ from a set of n_i-place relation symbols R_i, n_j-place function symbols F_j, and constant symbols c_k to, respectively, n_i-ary relations, n_j-ary functions, and constants on A. Unless otherwise noted, all structures and partial structures will be for the symbols $\{R_i, F_j, c_k\}_{i \in I, j \in J, k \in K}$ with their respective arity. In the following, I will sometimes refer to symbols as 'terms' when this does not lead to ambiguity. Sometimes, I

[1] I defend this choice of notation and the definitions given in §2.1 in this article's companion piece.

use indexed structures $\mathfrak{M}_i$ instead of $\mathfrak{A}$, $\mathfrak{B}$, etc. A will always be the domain $|\mathfrak{A}|$ of $\mathfrak{A}$, $B = |\mathfrak{B}|$ etc. If $\mathfrak{A} = \langle A, \mathscr{I} \rangle$, I write $R_i^{\mathfrak{A}}$ instead of $\mathscr{I}(R_i)$, and analogously for functions and constants. $R_i^{\mathfrak{B}}$ is the relation in $\mathfrak{B}$ that *corresponds* to relation $R_i^{\mathfrak{A}}$ in $\mathfrak{A}$, and analogous for functions and constants. In displayed form, I write a structure $\mathfrak{A}$ as $\langle A, R_1^{\mathfrak{A}}, \ldots, R_s^{\mathfrak{A}}, F_1^{\mathfrak{A}}, \ldots, F_t^{\mathfrak{A}}, c_1^{\mathfrak{A}}, \ldots, c_u^{\mathfrak{A}} \rangle$ or, for possibly infinite index sets, $\langle A, R_i^{\mathfrak{A}}, F_j^{\mathfrak{A}}, c_k^{\mathfrak{A}} \rangle_{i \in I, j \in J, k \in K}$.

2.1 Empirical adequacy

Within constructive empiricism, van Fraassen (1980, 64) gives the following definition (cf. van Fraassen 2008, 238):

Definition 1. A *theory* is a family $\{\mathfrak{T}_n\}_{n \in N}$ of structures (the *models of the theory*) such that each of its members $\mathfrak{T}_n = \langle T_n, R_i^{\mathfrak{T}_n}, F_j^{\mathfrak{T}_n}, c_k^{\mathfrak{T}_n} \rangle_{i \in I_n, j \in J_n, k \in K_n}$ has a set $\mathbf{E}_n$ of *empirical substructures*, such that for each $\mathfrak{E} \in \mathbf{E}_n$, $\mathfrak{E} \subseteq \mathfrak{T}_n$. With each model, a theory also contains every isomorphic structure and its corresponding[2] empirical substructures.

With O being the set of observable objects, van Fraassen (1980, 64) gives

Definition 2. *Appearances* are given by a set $\mathbf{P}$ of structures such that the domain of each $\mathfrak{P} \in \mathbf{P}$ is a subset of O. A structure $\mathfrak{P} \in \mathbf{P}$ is an appearance.

Definition 3. A theory $\{\mathfrak{T}_n\}_{n \in N}$ is *empirically adequate* for appearances $\mathbf{P}$ if and only if there is some $n \in N$ such that for every $\mathfrak{P} \in \mathbf{P}$, there is an $\mathfrak{E} \in \mathbf{E}_n$ with $\mathfrak{E} \cong \mathfrak{P}$.

In this terminology, van Fraassen (1980, 12, emphasis removed) defines constructive empiricism as the claim that science "aims to give us theories which are empirically adequate; and acceptance of a theory involves as belief only that it is empirically adequate".

2.2 Quasi-truth

The partial structures approach is motivated by a simple epistemological point: Most of the time, scientists do not have enough information about a domain to determine its structure with arbitrary precision. For most relations, it is at best known of *some* tuples of objects that they fall under the relation and known of *some* objects that they do not fall under it. For many if not most tuples this is unknown. Similarly, the value of a function is not know for all of its possible arguments. Partial structures are defined to take this lack of knowledge into account.

[2] If $f : T_m \longrightarrow T_n$ is an isomorphism between $\mathfrak{T}_m$ and $\mathfrak{T}_n$, then the corresponding empirical substructures $\mathbf{E}_n$ are those structures for which f is an isomorphism to an element of $\mathbf{E}_m$.

While most works on partial structures in the philosophy of science (e. g., da Costa and French 1990, Bueno 1997, da Costa and French 2000) do not consider functions, and the foundational paper by Mikenberg et al. (1986) does not consider constants, the respective definitions can be easily combined to give

Definition 4. $\tilde{\mathfrak{A}}$ is a *partial structure* for the symbols $\{R_i, F_j, c_k\}_{i\in I, j\in J, k\in K}$ if and only if

$$\tilde{\mathfrak{A}} = \langle A, R_i^{\tilde{\mathfrak{A}}}, F_j^{\tilde{\mathfrak{A}}}, c_k^{\tilde{\mathfrak{A}}} \rangle_{i\in I, j\in J, k\in K}, \tag{1}$$

where $A \neq \varnothing$, $R_i^{\tilde{\mathfrak{A}}} = \langle R_i^{\tilde{\mathfrak{A}},+}, R_i^{\tilde{\mathfrak{A}},-}, R_i^{\tilde{\mathfrak{A}},\circ} \rangle$ is a tripartition of A^{m_i} for each $i \in I$, $F_j^{\tilde{\mathfrak{A}}} : C_{\tilde{\mathfrak{A}},j} \longrightarrow A$ is a function with domain $C_{\tilde{\mathfrak{A}},j} \subseteq A^{n_j}$ for each $j \in J$, and $c_k^{\tilde{\mathfrak{A}}} \in A$ for each $k \in K$.

The definition of partial structures by Mikenberg et al. (1986, def. 1) is recovered for $K = \varnothing$, the definition by da Costa and French (1990, 255f) and da Costa et al. (1998, 605) for $J = \varnothing$.[3] Lack of knowledge is represented by non-empty sets $R_i^{\tilde{\mathfrak{A}},\circ}$ and sets $C_{\tilde{\mathfrak{A}},j} \subset A^{n_j}$.

Taking into account background knowledge, expressed by a set $\tilde{\Pi}$ of sentences, the *primary statements*, Mikenberg et al. (1986, def. 2.ii) and da Costa and French (1990, 256) give

Definition 5. Structure $\mathfrak{B}$ is $\tilde{\mathfrak{A}}$-normal for primary statements $\tilde{\Pi}$ if and only if $B = A$, $R_i^{\tilde{\mathfrak{A}},+} \subseteq R_i^{\mathfrak{B}} \subseteq A^{m_i} - R_i^{\mathfrak{A},-}$ for each $i \in I$, $F_j^{\mathfrak{B}}|C_j = F_j^{\tilde{\mathfrak{A}}}$ for each $j \in J$, $c_k^{\mathfrak{B}} = c_k^{\tilde{\mathfrak{A}}}$ for each $k \in K$, and $\mathfrak{B} \vDash \tilde{\Pi}$.

This allows to define quasi-truth, also called 'pragmatic truth' or 'partial truth':

Definition 6. Sentence φ is quasi-true in partial structure $\tilde{\mathfrak{A}}$ relative to primary statements $\tilde{\Pi}$ if and only if there is a structure $\mathfrak{B}$ that is $\tilde{\mathfrak{A}}$-normal for $\tilde{\Pi}$ and $\mathfrak{B} \vDash \varphi$.

3 Capturing empirical adequacy

3.1 Empirical adequacy as quasi-truth

Since a theory may be given by a family $\{\mathfrak{T}_n\}_{n\in N}$ rather than a set of sentences, da Costa and French (1990, 256f) give[4]

[3]While da Costa and French (1990, 255) and da Costa et al. (1998, 605) define partial structures *only* for relations, their further definition of $\tilde{\mathfrak{A}}$-normal structures presumes that partial structures can contain constants as well.

[4]Da Costa and French (1990) actually assume that a theory is given by a set **T**, rather than a family of structures. In such cases, I will always silently assume the family $\{\mathfrak{T}_{\mathfrak{T}} \,|\, \mathfrak{T} \in \mathbf{T}\}$ associated with the set.

Definition 7. If $\{\mathfrak{T}_n\}_{n\in N}$ is a theory, then $\{\mathfrak{T}_n\}_{n\in N}$ is *quasi-true* in partial structure $\tilde{\mathfrak{A}}$ relative to $\tilde{\Pi}$ if and only if for some $n \in N$, $\mathfrak{T}_n$ is $\tilde{\mathfrak{A}}$-normal for $\tilde{\Pi}$.

If thus $\{\mathfrak{T}_n\}_{n\in N}$ is the set of models of some set H of sentences, $\mathbf{T}$ is quasi-true in $\tilde{\mathfrak{A}}$ relative to $\tilde{\Pi}$ if and only if there is some $\tilde{\mathfrak{A}}$-normal structure for $\tilde{\Pi}$ in which all elements of H are true.

Restricting their discussion to relational structures (which contain only relation symbols), da Costa and French (1990, 255) suggests to formalize our knowledge of a domain by first giving a partial structure $\tilde{\mathfrak{O}} = \langle O, R_i^{\tilde{\mathfrak{O}}}\rangle_{i\in I}$ for the set of relation symbols $\{R_i\}_{i\in I}$, where O is the set of observable objects. Then, they suggest to extend the domain O by the domain $U, O \cap U = \varnothing$ of unobservable objects and introduce further relations $\{R_i\}_{i\in I'}, I' \cap I = \varnothing$ over $O \cup U$ to arrive at a partial structure $\tilde{\mathfrak{A}} = \langle O \cup U, R_i^{\tilde{\mathfrak{A}}}\rangle_{i\in I\cup I'}$ (da Costa and French 1990, 256).[5] While they do not discuss explicitly what happens to the partial relations $\{R_i^{\tilde{\mathfrak{O}}}\}_{i\in I}$ when the domain is extended, I take it that $R_i^{\tilde{\mathfrak{A}},+} = R_i^{\tilde{\mathfrak{O}},+}$ and $R_i^{\tilde{\mathfrak{A}},-} = R_i^{\tilde{\mathfrak{O}},-}$ for each $i \in I$. In other words, the relation symbols $\{R_i\}_{i\in I}$ do not serve to describe the unobservable objects.

Da Costa and French (1990, 257) then suggest that $\tilde{\mathfrak{A}}$ and the primary statements "can be taken to represent [a theory's] 'empirical substructures', if [the primary statements are] restricted to observation statements only." And this then, they claim, "leads us to understand the 'empirical adequacy' of a theory as its pragmatic truth". But it is at least not obvious how we would do this: For one, it is not clear in what sense $\tilde{\mathfrak{A}}$ "represents" the empirical substructures. As Bueno (1997, 595) furthermore points out, if it *was* clear, and empirical adequacy was then taken to be quasi-truth, it would follow that a theory is empirically adequate if and only if it is quasi-true according to its own empirical substructure. Thus the appearances would play no role whatsoever in the empirical adequacy of a theory.

I will now show that under specific assumptions, there *is* a way of capturing empirical adequacy in terms of partial structures, because empirical adequacy was inspired by a semantics for vague terms, and this semantics allows to capture partial structures as well.

3.2 Quasi-truth as approximate truth

One of van Fraassen's inspiration for the use of empirical substructures was given by Przełęcki (1969; 1976), who suggested to capture the interpretation of vague terms with the help of a multiplicity of structures, rather than one single structure. The denotation of an m_i-place relation symbol R_i that is vague over some domain A tripartitions the product domain A^{m_i} into R_i's positive extension R_i^+ of definite instances, R_i's negative extension R_i^- of definite non-instances, and R_i's neutral extension R° of borderline cases. The denotation of an n_j-place function symbol

[5]Incidentally, the introduction of the nonobservational terms $\{R_i\}_{i\in I'}$ amounts to a bipartition of the set of symbols akin to that in the received view on scientific theories (cf. Lutz 2011, §3).

F_j that is vague over A does not assign a single element $b \in A$ to an n_j-tuple $(a_1,\ldots,a_{n_j}) \in A^{n_j}$, but rather a set $F_j^{+\circ}(a_1,\ldots,a_{n_j}) = B \subseteq A$. I will refer to the set $\{(a_1,\ldots,a_{n_j},b) \mid a_1,\ldots,a_{n_j} \in A, b \in F_j^{+\circ}(a_1,\ldots,a_{n_j})\}$ as the non-negative extension $F^{+\circ}$ of F_j. If $F_j^{+\circ}(a_1,\ldots,a_{n_j})$ is a singleton set, I will say that F_j has a positive extension for $(a_1,\ldots,a_{n_j})$. The denotation of a constant symbol c_k that is vague over A is a set $c_k^{+\circ} \subseteq A$. If it is a singleton set, I will say that c_k has a positive extension.

Vague terms lead to sets of structures (cf. Lutz 2011, def. 7):

Definition 8. Let the terms $\{R_i, F_j, c_k\}_{i\in I, j\in J, k\in K}$ be vague over domain A with positive, negative, and non-negative extensions $\{R_i^+, R_i^-, F_j^{+\circ}, c_k^{+\circ}\}_{i\in I, j\in J, k\in K}$ and penumbral connections $W(R_i, F_j, c_k)_{i\in I, j\in J, k\in K}$. Then $\mathfrak{M}$ is in the terms' *vagueness set* **M** for A if an only if it fulfills the penumbral connections and

$$M = A, \tag{2}$$

$$R_i^+ \subseteq R_i^{\mathfrak{M}} \subseteq A^{m_i} - R_i^- \text{ for all } i \in I, \tag{3}$$

$$F_j^{\mathfrak{M}} \subseteq F_j^{+\circ} \text{ for all } j \in J, \text{ and} \tag{4}$$

$$c_k^{\mathfrak{M}} \in c_k^{+\circ} \text{ for all } k \in K\ . \tag{5}$$

The penumbral connections exclude structures from the vagueness set that cannot be excluded by the positive, negative, and non-negative denotations of the terms.

Every partial structure can be expressed with the help of a vagueness set:

Definition 9. Let $\tilde{\mathfrak{A}} = \langle A, R_i^{\tilde{\mathfrak{A}}}, F_j^{\tilde{\mathfrak{A}}}, c_k^{\tilde{\mathfrak{A}}}\rangle_{i\in I, j\in J, k\in K}$ be a partial structure, $\tilde{\Pi}$ a set of primary statements, and let $W(R_i, F_j, c_k)_{i\in I, j\in J, k\in K}$ express $\tilde{\Pi}$. Then the vagueness set for $\{R_i^+, R_i^-, F_j^{+\circ}, c_k^{+\circ}\}_{i\in I, j\in J, k\in K}$ over A with penumbral connections $W(R_i, F_j, c_k)_{i\in I, j\in J, k\in K}$ and $R_i^+ = R_i^{\tilde{\mathfrak{A}},+}$, $R_i^- = R_i^{\tilde{\mathfrak{A}},-}$, $F_j^{+\circ} = F_j^{\tilde{\mathfrak{A}}} \cup (A^{n_j} - C_{\tilde{\mathfrak{A}},j}) \otimes A$, and $c_k^{+\circ} = \{c_k^{\tilde{\mathfrak{A}}}\}$ for $i \in I, j \in J, k \in K$ *corresponds* to $\tilde{\mathfrak{A}}$ and $\tilde{\Pi}$.

Claim 1. *For any partial structure $\tilde{\mathfrak{A}}$, the set of $\tilde{\mathfrak{A}}$-normal structures relative to $\tilde{\Pi}$ is the corresponding vagueness set.*

Proof. '$\Rightarrow$': Let $\mathfrak{B}$ be $\tilde{\mathfrak{A}}$-normal for $\tilde{\Pi}$. Then $B = A$ and $\mathfrak{B}$ fulfills $W(R_i, F_j, c_k)_{i\in I, j\in J, k\in K}$. Furthermore, $R_i^+ = R_i^{\tilde{\mathfrak{A}},+} \subseteq R_i^{\mathfrak{B}} \subseteq A^{m_i} - R_i^{\tilde{\mathfrak{A}},-} = A^{m_i} - R_i^-$ for each $i \in I$, $F_j^{\mathfrak{B}} | C_{\tilde{\mathfrak{A}},j} = F_j^{\tilde{\mathfrak{A}}}$ so that $F_j^{\mathfrak{B}} \subseteq F_j^{\tilde{\mathfrak{A}}} \cup (A^{n_j} - C_{\tilde{\mathfrak{A}},j}) \otimes A = F_j^{+\circ}$ for each $j \in J$, and $c_k^{\mathfrak{B}} = c_k^{\tilde{\mathfrak{A}}} \in \{c_k^{\tilde{\mathfrak{A}}}\} = c_k^{+\circ}$ for each $k \in K$. Thus $\mathfrak{B}$ is in the vagueness set corresponding to $\tilde{\mathfrak{A}}$ and $\tilde{\Pi}$.

'$\Leftarrow$': Let $\mathfrak{B}$ be in the vagueness set for $\{R_i^+, R_i^-, F_j^{+\circ}, c_k^{+\circ}\}_{i\in I, j\in J, k\in K}$ over A with penumbral connections $W(R_i, F_j, c_k)_{i\in I, j\in J, k\in K}$. Then $B = A$ and $\mathfrak{B} \vDash \tilde{\Pi}$.

Furthermore, $R_i^{\tilde{\mathfrak{A}},+} = R_i^+ \subseteq R_i^{\mathfrak{B}} \subseteq A^{m_i} - R_i^+ = A^{m_i} - R_i^{\mathfrak{A},-}$ for each $i \in I$, $F_j^{\mathfrak{B}} \subseteq F_j^{+\circ} = F_j^{\tilde{\mathfrak{A}}} \cup (A^{n_j} - C_{\tilde{\mathfrak{A}},j}) \otimes A$ so that $F_j{}^{\mathfrak{B}} | C_{\tilde{\mathfrak{A}},j} = F_j^{\tilde{\mathfrak{A}}}$, and $c_k^{\mathfrak{B}} \in c_k^{+\circ} = \{c_k^{\tilde{\mathfrak{A}}}\}$ so that $c_k^{\mathfrak{B}} = c_k^{\tilde{\mathfrak{A}}}$. Therefore $\mathfrak{B}$ is $\tilde{\mathfrak{A}}$-normal for $\tilde{\Pi}$. □

If the vagueness set corresponding to $\tilde{\mathfrak{A}}$ and $\tilde{\Pi}$ is a singleton set $\{\mathfrak{B}\}$, then I will say that $\mathfrak{B}$ corresponds to $\tilde{\mathfrak{A}}$ and $\tilde{\mathfrak{A}}$ corresponds to $\mathfrak{B}$.

Since not every vagueness set corresponds to a partial structure, the notion of a vagueness set is a proper generalization of the notion of a partial structure. Incidentally, this generalization solves two problems stand in the way of many applications of partial structures in the analysis of scientific theories: While the interpretations of relation symbols in partial structures can capture fairly general cases of lack of knowledge, there can be no lack of knowledge whatsoever when it comes to constant symbols, because they are interpreted uniquely. And the interpretation of function symbols, practically important because many mathematized theories are teeming with functions, captures only a kind of lack of knowledge encountered very seldomly in the sciences. For in a partial structure $\tilde{\mathfrak{A}}$, the values of a function $F_j^{\tilde{\mathfrak{A}}}$ are known with arbitrary precision over $C_{\tilde{\mathfrak{A}},j}$, but not at all over $A^{n_j} - C_{\tilde{\mathfrak{A}},j}$. In contradistinction, the measurement of, say, the time averaged intensity $\overline{\psi}$ of a light wave over some spatial interval $[x_1, x_2]$ will typically have a finite precision, giving a range of possible intensity values $[y_1, y_2] \subset \mathbb{R}^{\geq 0}$ for each point $x \in [x_1, x_2]$. While this can be neatly captured by vagueness sets (Lutz 2011, §4.3), a partial structure can only capture measurements that, at any point x, give either a precise value $\overline{\psi}(x) = y_3 \in \mathbb{R}^{\geq 0}$, or no value at all.

Przełęcki suggests to use vagueness sets to define approximate truth and call a set H of sentences approximately true in a vagueness set $\mathbf{M}$ if and only if there is at least one element of $\mathbf{M}$ in which all elements of H are true. In other words, H is approximately true if and only if one of its models is in $\mathbf{M}$. This suggest the following definition of approximate truth for theories:

Definition 10. If $\mathbf{M}$ is a vagueness set and $\{\mathfrak{T}_n\}_{n\in N}$ a theory, then $\{\mathfrak{T}_n\}_{n\in N}$ is *approximately true* in $\mathbf{M}$ if and only if for some $n \in N$, $\mathfrak{T}_n \in \mathbf{M}$.

Thus if $\{\mathfrak{T}_n\}_{n\in N}$ is the family of models of a set of sentences H, $\{\mathfrak{T}_n\}_{n\in N}$ is approximately true if and only if H is approximately true.

The relation between quasi-truth and approximate truth is given by

Claim 2. *$\{\mathfrak{T}_n\}_{n\in N}$ is quasi-true in partial structure $\tilde{\mathfrak{A}}$ for $\tilde{\Pi}$ if and only if $\{\mathfrak{T}_n\}_{n\in N}$ is approximately true in the corresponding vagueness set.*

Proof. Immediately from claim 1. □

Therefore approximate truth is a generalization of quasi-truth. Note that if a partial structure corresponds to a structure, quasi-truth in the partial structure is equivalent to truth in the corresponding structure.

A simple relation between quasi-truth and empirical adequacy is now given by

Claim 3. *Let* $\tilde{\Pi} = \varnothing$, $\tilde{\mathfrak{O}}$ *correspond to the appearances* $\{\mathfrak{P}\}$ *and* $\tilde{\mathbf{A}}$ *be the set of partial structures* $\tilde{\mathfrak{A}} = \langle O \cup U, R_i^{\tilde{\mathfrak{A}}} \rangle_{i \in I}$ *with* $R_i^{\tilde{\mathfrak{A}},+} = R_i^{\tilde{\mathfrak{O}},+}$ *and* $R_i^{\tilde{\mathfrak{A}},-} = R_i^{\tilde{\mathfrak{O}},-}$ *for each* $i \in I$. *Let* $\{\mathfrak{T}_n\}_{n \in N}$ *be such that* $\mathfrak{T}_n | O \in \mathbf{E}_n$ *whenever* $O \subseteq T_n$. *Then* $\{\mathfrak{T}_n\}_{n \in N}$ *is quasi-true in some* $\tilde{\mathfrak{A}} \in \tilde{\mathbf{A}}$ *if and only if* $\{\mathfrak{T}_n\}_{n \in N}$ *is empirically adequate for* $\{\mathfrak{P}\}$.

Proof. By claim 2, $\{\mathfrak{T}_n\}_{n \in N}$ is quasi-true in some $\tilde{\mathfrak{A}} \in \tilde{\mathbf{A}}$ if and only if $\{\mathfrak{T}_n\}_{n \in N}$ is approximately true in the corresponding vagueness set. By claim 1 of the companion piece, the union of the corresponding vagueness sets is a generalized vagueness set, so that $\{\mathfrak{T}_n\}_{n \in N}$ is quasi-true in one of the vagueness sets if and only if $\{\mathfrak{T}_n\}_{n \in N}$ is generalized approximately true. Since $|\{\mathfrak{P}\}| = 1$, by claim 2 of the companion piece $\{\mathfrak{T}_n\}_{n \in N}$ is generalized approximately true if and only if $\{\mathfrak{T}_n\}_{n \in N}$ is empirically adequate. □

Claim 3 shows that the attempt by da Costa and French (1990) of capturing empirical adequacy as quasi-truth is, under some additional assumptions, successful when modified in two crucial ways: First, $\{\mathfrak{T}_n\}_{n \in N}$ does not have to be quasi-true in one specific partial structure, but may be quasi-true in any partial structure with the positive and negative extensions as described. Second, these partial structures must not be taken to represent the empirical substructures of a theory, but must be taken to represent the appearances. Bueno's objection that the appearances play no role in da Costa and French's attempt is thereby avoided.

However, claim 3 relies on the additional assumption that the appearances have to be given by a single structure. This is an assumption that van Fraassen decidedly does not make[6], and thus even in this reconceptualization, da Costa and French's attempt to capture empirical adequacy in terms of quasi-truth is not wholly successful.

4 Generalizing empirical adequacy

Bueno (1997) suggests to use the partial structures approach not to capture, but rather to generalize the notion of empirical adequacy. While his two generalizations are meant to allow the formalization of common scientific practices within constructive empiricism and have been the basis of further development and applications of the partial structures approach, they have initially been suggested as a response to a criticism by Suárez (1995; 2005) of empirical adequacy. I argue in appendix B that this initial motivation is not relevant because Suárez's criticism fails.

Both of Bueno's generalizations rely on the basic idea that the appearances are seldom known well enough to be described in a set $\mathbf{P}$ of structures, and that usually a set of partial structures must suffice, leading to the *partial appearances* $\tilde{\mathbf{P}}$. This is more than just a formal modification of the notion of empirical adequacy: Van Fraassen is arguably justified in assuming that the appearances are represented

[6]Personal communication.

by structures, because a theory has to be empirically adequate with respect to *all* the appearances, known in all precision (cf. Monton and Mohler 2008, §1.5). While van Fraassen's notion of empirical adequacy thus relies exclusively on what the appearances are like, Bueno's notion includes our lack of knowledge about them. Bueno thus gives a generalization of empirical adequacy that takes epistemic states into account, and should therefore be equivalent to Van Fraassen's notion in the special case that there is no epistemic uncertainty about the phenomena.

I will argue that both of Bueno's generalizations of empirical adequacy are inadequate, and that his intend is better captured by generalizations suggested in this article's companion piece.

4.1 Partial empirical adequacy

Bueno's first suggested generalization can be motivated by the simple observation that if the appearances are given by partial structures, there can be no isomorphisms between the appearances and the empirical substructures of any theory, because substructures and isomorphisms are defined only for structures. Bueno (1997, 596) therefore defines the partial substructures of a relational partial structure, which can easily be generalized to partial structures as follows:

Definition 11. $\tilde{\mathfrak{A}}$ is a *partial substructure* of partial structure $\tilde{\mathfrak{B}}$, $\tilde{\mathfrak{A}} \subseteq \tilde{\mathfrak{B}}$, if and only if $A \subseteq B$, $R_i^{\tilde{\mathfrak{A}},+} = R_i^{\tilde{\mathfrak{B}},+} \cap A^{m_i}$, $R_i^{\tilde{\mathfrak{A}},-} = R_i^{\tilde{\mathfrak{B}},-} \cap A^{m_i}$, $R_i^{\tilde{\mathfrak{A}},\circ} = R_i^{\tilde{\mathfrak{B}},\circ} \cap A^{m_i}$ for all $i \in I$, $F_j^{\tilde{\mathfrak{A}}} = F_j^{\tilde{\mathfrak{B}}}|A^{n_j}$ for all $j \in J$, and $c_k^{\tilde{\mathfrak{A}}} = c_k^{\tilde{\mathfrak{B}}}$ for all $k \in K$.

It is easy to see that if $\mathfrak{A}$ corresponds to $\tilde{\mathfrak{A}}$ and $\mathfrak{B}$ corresponds to $\tilde{\mathfrak{B}}$, then $\tilde{\mathfrak{A}} \subseteq \tilde{\mathfrak{B}}$ if and only if $\mathfrak{A} \subseteq \mathfrak{B}$.

Further, Bueno defines the concept of partial isomorphy between two relational partial structures, which can also easily be generalized to partial structures:

Definition 12. $f : A \longrightarrow B$ is a *partial isomorphism* between partial structures $\mathfrak{A}$ and $\mathfrak{B}$ if and only if f is a bijection and it holds that $f(R_i^{\tilde{\mathfrak{A}},+}) = R_i^{\tilde{\mathfrak{B}},+}$, and $f(R_i^{\tilde{\mathfrak{A}},-}) = R_i^{\tilde{\mathfrak{B}},-}$ for each $x_1,\ldots,x_{m_i} \in A$, $i \in I$, $f(C_{\tilde{\mathfrak{A}},j}) = C_{\tilde{\mathfrak{B}},j}$ and $f(F_j^{\tilde{\mathfrak{A}}} x_1 \ldots x_{n_j}) = F_j^{\tilde{\mathfrak{B}}} f(x_1) \ldots f(x_{m_j})$ for each $x_1,\ldots,x_{n_j} \in C_{\tilde{\mathfrak{A}},j}$, $j \in J$, and $f(c_k^{\tilde{\mathfrak{A}}}) = c_k^{\tilde{\mathfrak{B}}}$ for each $k \in K$.[7]

$\tilde{\mathfrak{A}}$ and $\tilde{\mathfrak{B}}$ are called *partially isomorphic* ($\tilde{\mathfrak{A}} \cong \tilde{\mathfrak{B}}$) if and only if there is a partial isomorphism between $\tilde{\mathfrak{A}}$ and $\tilde{\mathfrak{B}}$.[8] It is easy to see that if $\mathfrak{A}$ corresponds to $\tilde{\mathfrak{A}}$ and $\mathfrak{B}$ corresponds to $\tilde{\mathfrak{B}}$, then $\tilde{\mathfrak{A}} \cong \tilde{\mathfrak{B}}$ if and only if $\mathfrak{A} \cong \mathfrak{B}$.

[7] For a relation R over A^n, $f(R_i)$ is the *image* of R under f, that is, $f(R)f(x_1)\ldots f(x_n)$ if and only if $Rx_1 \ldots x_n$.

[8] The concept given by definition 12 must not be confused with the concept that French and Ladyman (1997, 371, n. 16) name 'partial isomorphism', the isomorphism between substructures (cf. Ebbinghaus et al. 1984, §XI.1).

Bueno (1997, 596) then assumes that theories are given as families $\{\tilde{\mathfrak{T}}_n\}_{n\in N}$ of partial structures, with the set of partial empirical substructures $\tilde{\mathbf{E}}_n$ for each $n \in N$.[9] But this assumptions is problematic: In constructive empiricism, theories and their empirical substructures are assumed to be given by normal structures, not partial structures.[10] And it is not obvious how a theory $\{\mathfrak{T}_n\}_{n\in N}$ given by normal structures can be described as a set of partial structures. Of course, every set of normal structures corresponds to a set of partial structures with $R^\circ = \varnothing$, but in this identification quasi-truth, partial substructures, and partial isomorphisms amount to truth, substructures, and isomorphisms, respectively, so nothing is gained. There may be a solution to this problem: Given $\{\mathfrak{T}_n\}_{n\in N}$ with $\{\mathbf{E}_n\}_{n\in N}$, one can try to find a set $M \subseteq \wp N$ of subsets of N such that for each $m \in M$, $\{\mathfrak{T}_n \mid n \in m\}$ is a vagueness set and $\bigcup_{n\in m} \mathbf{E}_n$ is a union of vagueness sets. Then for each M, $\{\mathfrak{T}_n \mid n \in m\}$ corresponds to a partial structure $\tilde{\mathfrak{T}}_m$, and $\bigcup_{n\in m} \mathbf{E}_n$ is the union of vagueness sets that correspond to elements of a set $\tilde{\mathbf{E}}_m$ of partial structures. This provides a partial theory $\{\tilde{\mathfrak{T}}_m\}_{m\in M}$ with partial empirical substructures $\{\tilde{\mathbf{E}}_m\}_{m\in M}$.[11] In this way, definition 13 may become non-trivially applicable, although it is not clear how many scientific theories have non-trivial vagueness sets and sets of empirical substructures.

With these concepts in place, Bueno (1997, 596) gives

Definition 13. A partial theory $\{\tilde{\mathfrak{T}}_n\}_{n\in N}$ is *partially empirically adequate* for the partial appearances $\tilde{\mathbf{P}}$ if and only if there is some $n \in N$ such that for every $\tilde{\mathfrak{P}} \in \tilde{\mathbf{P}}$, there is an $\tilde{\mathfrak{E}} \in \tilde{\mathbf{E}}_n$ with $\tilde{\mathfrak{E}} \cong \tilde{\mathfrak{P}}$.

As noted, capturing the appearances as a set of partial structures rather than a set of structures introduces an epistemic element into empirical adequacy. If there is no lack of knowledge about the appearances, $R_i^{\tilde{\mathfrak{A}},\circ} = \varnothing$ and $C_{\tilde{\mathfrak{A}},j} = A^{n_j}$ for all $i \in I, j \in J$ and partial structures $\tilde{\mathfrak{A}}$ that describe the knowledge about the appearances, so that there are structures corresponding to the partial structures. Since furthermore partial substructures and -isomorphisms generalize substructures and isomorphisms, definition 13 generalizes definition 3 of empirical adequacy to partial theories and partial appearances. Therefore, for full information, partial empirical adequacy is equivalent to empirical adequacy, as demanded.

Since Suárez's criticism fails, it is not important that partial empirical adequacy cannot cope with Suárez's criticism (appendix C.1). A real problem is that almost no theory is ever partially empirically adequate, and if so, only for a very short time (with some notable exceptions). This is because the partial appearances are meant to represent our lack of knowledge of the appearances. And to be partially empirically adequate, the partial theory must have a partial empirical substructure

[9] Bueno (2000, §3.2) relies on this assumption as well.

[10] Note that da Costa and French (1990, 256f) take this into account in their discussion of empirical adequacy.

[11] In the following, I will rename the index set M back to N.

that is partially isomorphic to the partial appearances, and therefore describe our state of knowledge about the appearances. Formally, for each relation $R_i^{\tilde{\mathfrak{P}}}$ of a partial appearance, there has to be a relation $R_i^{\tilde{\mathfrak{E}}}$ of a partial empirical substructure of the partial theory such that a partial isomorphism maps all and only elements in $R_i^{\tilde{\mathfrak{P}},\circ}$ to elements in $R_i^{\tilde{\mathfrak{E}},\circ}$. Therefore, if a theory asserts more about any appearance than what is currently known, it is not partially empirically adequate. This point becomes even clearer when considering Bueno's claim that partial isomorphisms to partial substructures amount to "approximate embeddings" (Bueno 1997, 597): If the partial appearances are meant to describe the approximation with which we know the appearances, then a partial theory is partially empirically adequate only if for some $n \in N$, the partial empirical substructures in $\tilde{\mathbf{E}}_n$ contain the *exact* approximations up to which we know the appearances. Therefore, if a partial theory describes the appearances up to some approximation, the theory is not partially empirically adequate until we have determined the appearances with exactly that approximation. Of course, if later the we know a little bit more, the theory ceases to be partially empirically adequate.[12]

The concept of approximate empirical adequacy defined in the companion piece (Lutz 2011, 16) avoids these problems by using, instead of a set $\tilde{\mathbf{P}}$ of partial structures, a set $\mathbf{P}$ of vagueness sets for the appearances, which generalizes the kind of approximations for functions and constants that can be captured in partial appearances.

Definition 14. Given the approximate appearances $\mathbf{P}$, a theory $\{\mathfrak{T}_n\}_{n\in N}$ is *approximately empirically adequate* for $\mathbf{P}$ if and only if there is some $n \in N$ such that for every $\mathbf{Q} \in \mathbf{P}$, there are a $\mathfrak{P} \in \mathbf{Q}$ and an $\mathfrak{E} \in \mathbf{E}_n$ with $\mathfrak{E} \cong \mathfrak{P}$.

That definition 14 should be acceptable to Bueno follows from his reason for dismissing a different suggestion for generalizing empirical adequacy. Instead of demanding empirical adequacy, one could demand that there be some n such that for every $\tilde{\mathfrak{P}} \in \tilde{\mathbf{P}}$, there are a $\tilde{\mathfrak{P}}$-normal structure $\mathfrak{P}$ with $\mathfrak{P} \in \mathbf{E}_n$. However, because of the definition of '$\tilde{\mathfrak{P}}$-normal structure', this would require that the theory has a set of empirical substructures with the same domains as the partial appearances. The problem with this requirement, so Bueno (1997, 594), is that "[i]n practice [...] we would hardly, if ever, see such a constraint satisfied". Definition 14 generalizes a demand that avoids exactly the constraint that Bueno criticizes:

Claim 4. *If each approximate appearance in the approximate appearances* $\mathbf{P}$ *correspond to a partial appearance in the partial appearances* $\tilde{\mathbf{P}}$*, a theory* $\{\mathfrak{T}_n\}_{n\in N}$ *is approximately empirically adequate for* $\mathbf{P}$ *if and only if there is some* $n \in N$ *such that for every* $\tilde{\mathfrak{P}} \in \tilde{\mathbf{P}}$*, there are a* $\tilde{\mathfrak{P}}$*-normal structure* $\mathfrak{P}$ *and an* $\mathfrak{E} \in \mathbf{E}_n$ *with* $\mathfrak{E} \cong \mathfrak{P}$.

[12]Similar criticisms apply to the applications of the notion of partial isomorphism by Bueno (1999, 67), French (2000, 110f), Bueno (2000, §3.2), da Costa and French (2003, 73, 102, 150, 193), French (2003, §§4f), and Bueno and Colyvan (2011, §5.3).

Proof. Immediately from claim 1. □

The demand that Bueno dismisses is recovered by substituting '=' for '≅' in definition 14 and claim 4. Given that the identity of empirical substructures and appearances was Bueno's *sole* reason for rejecting the generalization, definition 14 should be acceptable. It furthermore avoids the need to introduce partial theories.

Definition 14 does not rely on the notion of a partial isomorphism, but since that notion has been the basis of much further development in the partial structure approach (cf. Bueno 1999, 65f; French 2000, 106; Bueno 2000, 278ff; Bueno et al. 2002, 499; da Costa and French 2003, 48ff; French 2003, 1480; Bueno and Colyvan 2011, 14) I show in §A that the concept can be captured and generalized with vagueness sets.

4.2 Quasi empirical adequacy

Bueno's second generalization (Bueno 1997, 599ff) starts from the ideas that the structures of data are often changed in multiple steps into structures of theories (Suppes 1962), and that theories are not compared directly to data, but more refined structures (Bogen and Woodward 1988). To capture both points, Bueno (1997, 600f) gives an informal description of a hierarchy of partial structures (cf. French and Ladyman 1999, 113). Since he makes a distinction between partial structures with finite and with infinite domains, this can be captured in two more formal definitions.

Definition 15. A *hierarchy of finite partial structures* is a finite sequence $\langle\tilde{\mathfrak{A}}_l\rangle_{1\leq l\leq m}$ of relational partial structures with finite domains for the n_i-place relation symbols $\{R_i\}_{i\in I}$,[13] such that (i) $\tilde{\mathfrak{A}}_m$ corresponds to a structure, and (ii) for all $1 \leq l < m$ and $i \in I$, $\mathrm{card}(R_i^{\tilde{\mathfrak{A}}_l,\circ}) > \mathrm{card}(R_i^{\tilde{\mathfrak{A}}_{l+1},\circ})$, and $\mathrm{card}(A_l) \geq \mathrm{card}(A_{l+1})$.[14]

Definition 16. A *hierarchy of infinite partial structures* is a finite sequence $\langle\tilde{\mathfrak{A}}_l\rangle_{1\leq l\leq m}$ of relational partial structures with finite domains for the n_i-place relation symbols $\{R_i\}_{i\in I}$, such that (i) $\tilde{\mathfrak{A}}_m$ corresponds to a structure and (ii) for all $1 \leq l < m$ and $i \in I$, $R_i^{\tilde{\mathfrak{A}}_l,\circ} \supset R_i^{\tilde{\mathfrak{A}}_{l+1},\circ}$.

A *hierarchy of partial structures* is either a hierarchy of finite or a hierarchy of infinite partial structures.

Bueno (1997, 601) further defines a *reduced partial structure* $\tilde{\mathfrak{A}}$ of a partial structure $\tilde{\mathfrak{B}}$ to be a partial substructure of $\tilde{\mathfrak{B}}$ such that every element of $\tilde{\mathfrak{A}}$'s domain has been measured at some point with a specific measurement result.

[13]That all structures are for the same relation symbols is only implicitly suggested by (my paraphrase of) Bueno's informal description of the partial models of the phenomena below.

[14]The cardinality condition on the domains is implicit in Bueno's claim that from $\mathrm{card}(R_i^{\tilde{\mathfrak{A}}_l,\circ}) > \mathrm{card}(R_i^{\tilde{\mathfrak{A}}_{l+1},\circ})$, it follows that $\mathrm{card}(R_i^{\tilde{\mathfrak{A}}_l,+}) < \mathrm{card}(R_i^{\tilde{\mathfrak{A}}_{l+1},+})$ or $\mathrm{card}(R_i^{\tilde{\mathfrak{A}}_l,-}) < \mathrm{card}(R_i^{\tilde{\mathfrak{A}}_{l+1},-})$ given that all structures in the domain are for the same relation symbols.

Finally, Bueno (1997, 601, 607) seems to define the *partial models of the phenomena* to be a hierarchy of partial structures (a) whose lowest partial structure represents the data, (b) whose highest partial structure represents the appearances, and (c) where there is a cardinal number w such that the partial structure at each level has a reduced partial structure with a domain of cardinality w.[15] By definition 11, there is for every partial structure $\tilde{\mathfrak{A}}$ and set $B \subseteq A$ a partial substructure $\tilde{\mathfrak{C}} \subseteq \tilde{\mathfrak{A}}$ with $C = B$. Therefore, since partial structures have non-empty domains, condition (iii) simply demands that for each $1 \leq l \leq m$, at least one element of domain A_l has been measured.

Bueno (1997, 601, my notation) claims that partial models of the phenomena formalize the idea that

> at each level, partial relations R, which were not yet defined at a lower level (whose elements thus belong to R°) come to be defined: either taking their elements as belonging to the domain of R (i. e. R^+) or belonging to the complement of R (that is, R^-). The partial relations are extended until one obtains normal (total) structures. These are then to be compared to scientific theories in testing them.

This description is rather misleading, for one because it suggests that each new level in the hierarchy introduces new relations, which is not true. What Bueno probably means is that at each level $\tilde{\mathfrak{A}}_l$ and for each relation symbol R_i, tuples of members of the domain that were borderline cases at the lower level $\tilde{\mathfrak{A}}_{l-1}$ (and thus belong to $R_i^{\tilde{\mathfrak{A}}_{l-1},\circ}$) come to be included either in the positive extension of R_i (i. e., $R_i^{\tilde{\mathfrak{A}}_l,+}$) or in the negative extension of R_i (i. e., $R_i^{\tilde{\mathfrak{A}}_l,-}$).[16]

The definition of a hierarchy of partial structures has a number of quirks that become problems in the definition of the partial models of the phenomena. One is that a partial structure $\tilde{\mathfrak{A}}_l$ with $R_i^{\tilde{\mathfrak{A}}_l,\circ} = \varnothing$ for some, but not all $i \in I$ cannot occur in a hierarchy. For condition (ii) demands that $\mathrm{card}\big(R_i^{\tilde{\mathfrak{A}}_l,\circ}\big) > \mathrm{card}\big(R_i^{\tilde{\mathfrak{A}}_{l+1},\circ}\big)$ or $R_i^{\tilde{\mathfrak{A}}_l,\circ} \supset R_i^{\tilde{\mathfrak{A}}_{l+1},\circ}$ for *all* $i \in I$. If thus $R_i^{\tilde{\mathfrak{A}}_l,\circ} = \varnothing$ for some $i \in I$, condition (ii) cannot be fulfilled by $R_i^{\tilde{\mathfrak{A}}_{l+1},\circ}$, so that $\mathfrak{A}_l$ must be the highest level of the hierarchy. But this is impossible because the highest level must correspond to a structure, so that $R_i^{\tilde{\mathfrak{A}}_l,\circ} = \varnothing$ for *all* $i \in I$. This problem can easily be solved by slightly modifying condition (ii), demanding only $\mathrm{card}\big(R_i^{\tilde{\mathfrak{A}}_l,\circ}\big) \geq \mathrm{card}\big(R_i^{\tilde{\mathfrak{A}}_{l+1},\circ}\big)$ in definition 15 and $R_i^{\tilde{\mathfrak{A}}_l,\circ} \supseteq R_i^{\tilde{\mathfrak{A}}_{l+1},\circ}$ in definition 16 for all $i \in I$. To have an asymmetric relation, one could further demand that the respective converse relation does not hold for all $i \in I$.

[15] Conditions (a) and (b) are inferred from Bueno's informal discussion.

[16] Bueno (personal communication) has confirmed that this is an accurate paraphrase.

A bigger problem is that the demands on the relations are radically different for hierarchies of finite and infinite domains. Infinite structures form a hierarchy if from one level to the next, some borderline cases of each relation symbol become members of its positive or negative extension, but not vice versa. The relations of finite structures, on the other hand, only have to have a decreasing number of borderline cases, no matter which ones they are. This means that the concept of a hierarchy of infinite partial structures does not generalize the concept of a hierarchy of finite partial structures: If $\tilde{\mathfrak{A}}_l = \langle A, R_1^{\tilde{\mathfrak{A}}_l} \rangle$ with $R_1^{\tilde{\mathfrak{A}}_l,\circ} := \{a, b\}$, and $\tilde{\mathfrak{A}}_{l+1}$ is such that $R_1^{\tilde{\mathfrak{A}}_{j+1},\circ} := \{c\}$, then $\mathrm{card}(R_i^{\tilde{\mathfrak{A}}_l,\circ}) > \mathrm{card}(R_i^{\tilde{\mathfrak{A}}_{j+1},\circ})$, but $R_i^{\tilde{\mathfrak{A}}_l,\circ} \not\supseteq R_i^{\tilde{\mathfrak{A}}_{j+1},\circ}$. Note also that for infinite domains, the elements in $R_i^{\tilde{\mathfrak{A}}_l,\circ}$ for any $i \in I$, $1 \leq l \leq m$ have to be in every $R_i^{\tilde{\mathfrak{A}}_l,\circ}$, $k \leq l$, so that, since $R_i^{\tilde{\mathfrak{A}}_l,\circ} \neq 0$ for all $i \in I$, $1 \leq l < m$, partial structures at all levels except the last one have to have some elements in common. In contradistinction, the domains of the finite structures can change completely from one level to the next.

This has a number of odd implications. First and foremost, there is no obvious reason for the change in condition (ii). If the subset-relation is justified for infinite domains, its lack of justification for finite domains first has to be shown. Second, it is not clear how to define a hierarchy of partial structures some of which have finite, and some of which have infinite domains. Third, any concept relying on the concept of a hierarchy of partial structures will also have very different properties for finite and infinite domains. This, though, renders typical scientific methods unusable, for example the use of limiting cases, where a very big but finite domain is used as approximation of an infinite domain. Results for finite domains thus will typically have little bearing on results for infinite domains, and vice versa.

The biggest problem of the concept of a hierarchy of partials structures, however, is starkly illustrated by Bueno's own informal description (quoted above) of each level in the hierarchy as "extending" the relations of the partial structure from the previous level. In the rest of his article, Bueno (e. g., 1997, 592–593) uses the concept of one relation "extending" another only to state that the relations $\{R_i^{\mathfrak{A}}\}_{i\in I}$ in an $\tilde{\mathfrak{A}}_l$-normal structure $\mathfrak{A}$ extend the relations $\{R_i^{\tilde{\mathfrak{A}}_l}\}_{i\in I}$. This entails that to $\mathfrak{A}$ corresponds a partial structure $\tilde{\mathfrak{A}}$ so that for all $i \in I$, $R_i^{\tilde{\mathfrak{A}}_l,+} \subseteq R_i^{\tilde{\mathfrak{A}},+}$ and $R_i^{\tilde{\mathfrak{A}}_l,-} \subseteq R_i^{\tilde{\mathfrak{A}},-}$. The natural generalization of this notion is that the same subset condition also holds between all relations of any two partial structures where the relations of the one extend the relations of the other. But understood in this way, Bueno's informal description goes against his definition of hierarchies of infinite and of finite partial structures. For infinite domains, a member of the positive extension $R_i^{\tilde{\mathfrak{A}}_l,+}$ may be in the negative extension $R_i^{\tilde{\mathfrak{A}}_{l+1},-}$ and vice versa. For finite domains, any instance, non-instance, or borderline case can become an instance, non-instance, or borderline case on the next level of the hierarchy, as long as the number of borderline cases overall decreases. It is thus a stretch to say that the hierarchy is "built in such a way that, at each level, there is a gain of information regarding the phenomena being modeled" (Bueno 1997, 603), since at each level, any information one has about the domain can be negated on the next level. For then one would convey information by saying: "They are certainly scoundrels, though they may not be".[17]

This problem is so severe that the relation of the highest level of a pragmatic empirical hierarchy to the rest of the hierarchy is almost trivial:

Claim 5. *Let $\tilde{\mathfrak{B}}_m$ be any partial structure for $\{R_i\}_{i\in I}$ that corresponds to a structure. If $\langle A_l\rangle_{1\le l\le m}$ is any hierarchy of infinite partial structures for $\{R_i\}_{i\in I}$, then $\langle\tilde{\mathfrak{A}}_l,\tilde{\mathfrak{B}}_m\rangle_{1\le l<m}$ is also a hierarchy of infinite partial structures. If $\langle\tilde{\mathfrak{A}}_l\rangle_{1\le l\le m}$ is any hierarchy of finite partial structures for $\{R_i\}_{i\in I}$ with $\mathrm{card}(A_{m-1})\ge\mathrm{card}(B_m)$, then $\langle\tilde{\mathfrak{A}}_l,\tilde{\mathfrak{B}}_m\rangle_{1\le l<m}$ is also a hierarchy of finite partial structures. (This holds whether the definitions use '$\supset$', '$\supseteq$', '$>$', or '$\ge$' in condition (ii).)*

Proof. If condition (ii) is formulated with '$\supset$', then $R_i^{\tilde{\mathfrak{A}}_{m-1},\circ}\neq\emptyset$ for all $i\in I$. Since $R_i^{\tilde{\mathfrak{B}}_m,\circ}=\emptyset$ for all $i\in I$ and the empty set is a proper subset of every non-empty set, condition (ii) is fulfilled. If condition (ii) is formulated with '$>$', then $\mathrm{card}\big(R_i^{\tilde{\mathfrak{A}}_{m-1},\circ}\big)>0=\mathrm{card}\big(R_i^{\tilde{\mathfrak{B}}_m,\circ}\big)$ for all $i\in I$. If condition (ii) is formulated with '$\supseteq$' or '$\ge$', the proof is immediate. □

Therefore any level of a hierarchy of partial structures except the highest one only restricts the relation symbols of the partial structure at the highest level (and in the finite case also the maximal size of the domain).[18]

This almost trivial restriction of the highest level of the hierarchy of partial structures by the lower levels is not strengthened in Bueno's definition of the partial models of the phenomena, for the only additional restriction on the structure at the highest level of the hierarchy is that at least one object in its domain must have been measured.

Bueno (1997, 602, my notation) goes on to define that a theory

> is empirically adequate if it is pragmatically true in the (partial) empirical substructure $\tilde{\mathfrak{E}}$ according to a structure $\mathfrak{A}$, where $\mathfrak{A}$ is the last level of the hierarchy of models of phenomena (being thus a total structure).

A sentence is "pragmatically true in $\tilde{\mathfrak{E}}$ according to $\mathfrak{A}$" if and only if it is true in $\mathfrak{A}$ and $\mathfrak{A}$ is $\tilde{\mathfrak{E}}$-normal (Bueno 1997, 592, my notation). Note that this definition of what I will call 'pragmatic empirical adequacy' assumes that the theory is

[17] In some contexts, of course, this sentence can convey information by way of conversational implicature (for an example, see Hammett 1928, 115–120).

[18] These criticisms also apply to the applications of the hierarchies by Bueno (1999, §3.1), French and Ladyman (1999, 112–114) Bueno et al. (2002, §2), da Costa and French (2003, 68–74), and Bueno and Colyvan (2011, §5.3, n. 25).

given by a sentence or possibly a set thereof. A theory is thus pragmatically empirically adequate if and only if it is true in a structure that is $\tilde{\mathfrak{E}}$-normal for a partial empirical substructure $\tilde{\mathfrak{E}}$ of the theory and is the highest level of the partial models of the phenomena.

This definition has a number of peculiar features that cast its adequacy into doubt. For one, it defines pragmatic empirical adequacy relative to a single hierarchy of partial structures, with a single structure on its highest level that represents the appearances. Hence the definition presumes, contrary to van Fraassen, that there is only one appearance. Second, a theory is pragmatically empirically adequate if and only if it is true in the highest level of the partial models of the phenomena, that is, in the single appearance. Thus empirical adequacy is the same as truth in the structure of the appearance, which trivializes van Fraassen's idea of substituting empirical adequacy for truth as the goal of science. Third, since $\mathfrak{A}$ is $\tilde{\mathfrak{E}}$-normal, $A = E$, so that all the objects that occur in the theory also have to occur in its partial empirical substructure, which thereby is not a proper substructure. Fourth, there is only the almost trivial relation between the appearances at the highest level and the data at the lowest level of the models of the phenomena. Claim 5 shows that the last level is almost independent from the lower levels, so that, if the data are meant to determine the appearances, the notion of pragmatic empirical adequacy is almost trivial. If, on the other hand, the appearances are determined on their own, then pragmatic empirical adequacy simply presumes that the appearance is given by a structure, not a partial structure, and thus does not differ in this respect from empirical adequacy. A corollary of these four points is that pragmatic empirical adequacy clearly does not properly generalize empirical adequacy. The effects of these modifications of empirical adequacy on Suárez's criticism are discussed in §C.2.

The fifth peculiarity of the definition is that it contradicts the presupposition of Bueno's claim that "there is no problem with the fact that $\mathfrak{E}$ and $\mathfrak{A}$ have distinct domains" (Bueno 1997, 603, my notation). There is indeed no problem, but only because $\tilde{\mathfrak{E}}$ and $\mathfrak{A}$ have the *same* domain, since $\mathfrak{A}$ is $\mathfrak{E}$-normal. Oddly, Bueno (1997, 603, n. 16) seems to acknowledge this in a footnote, in which he states that "one of the conditions for a total structure to be employed as one that extends the relations of a partial structure, is that both of them have the same domain. Such a condition obviously could not hold here."[19] This identity of the domain of the partial appearance and the partial empirical substructure should be fatal for the definition according to Bueno's own standards, since he has dismissed another suggestion for generalizing empirical adequacy on exactly these grounds.

Finally, the definition is in tension with Bueno's informal description. Bueno

[19]He adds that "the condition can be easily changed for a weaker one, still preserving the main features of the concept of pragmatic truth. I have pursued this line of argument elsewhere [. . .]." The reference (Bueno and de Souza 1996), however, gives no indication on how to avoid the identity of the domains of $\tilde{\mathfrak{E}}$ and $\mathfrak{A}$, as it does not discuss possible relations between partial structures with different domains.

(1997, 603) states:

> The basic point of [the definition] consists in the fact that, intuitively, a theory is empirically adequate if that part of it which is concerned with the observable phenomena (its empirical substructures) can be extended to a total structure that represents the information provided by the observational side of 'experience' (the last level in the hierarchy of partial models of phenomena).

If partial empirical substructures $\tilde{\mathfrak{E}}$ are given through vagueness sets, the $\tilde{\mathfrak{E}}$-normal structures are just empirical substructures of $\{\mathfrak{T}_n\}_{n\in N}$. The highest level of the models of the phenomena also corresponds to a structure $\mathfrak{P}$. If a structure is represented by another through an isomorphism between the two, a theory is then pragmatically empirically adequate if and only if one of its empirical substructures is isomorphic to the appearance $\mathfrak{P}$. Thus pragmatic empirical adequacy differs from empirical adequacy only in that there can be only one appearance, and there must also be an almost unrelated data structure.

This article companion piece contains a generalization of empirical adequacy that avoids all of the above criticisms. First, it provides a substitute for Bueno's notion of the models of the phenomena:

Definition 17. A *restricted hierarchy of approximate appearances* is a sequence $\langle \mathbf{P}_l \rangle_{l\in L}$ of approximate appearances such that for any $l \leq m$ with $l, m \in L$, there is a bijection $b : \mathbf{P}_l \longrightarrow \mathbf{P}_m$ such that for all $\mathbf{Q} \in \mathbf{P}_l$, $b(\mathbf{Q}) \subseteq \mathbf{Q}$.

This definition leads to a real increase of information from one point of the hierarchy to the next, allows more than one appearance, and is not restricted to hierarchies of finitely many levels.

Second, one can define which appearances are possible given the knowledge ones has about them at some point:

Definition 18. Given approximate appearances $\mathbf{P}$, $\mathbf{P}'$ are *approximately possible appearances* if and only if $\mathbf{P}' = \{e(\mathbf{Q}) \,|\, \mathbf{Q} \in \mathbf{P}\}$, where e is any function from $\mathbf{P}_m$ to $\bigcup \mathbf{P}$ with $e(\mathbf{Q}) \in \mathbf{Q}$.

Thus for every vagueness set in the approximate appearances, any of its members can be a member of the appearances.

These definitions lead to the following result:

Claim 6. *$\{\mathfrak{T}_n\}_{n\in N}$ is approximately empirically adequate at all points of all restricted hierarchies of approximate appearances with the initial sequence $\langle \mathbf{P}_l \rangle_{l\leq m}$ if and only if $\{\mathfrak{T}_n\}_{n\in N}$ is empirically adequate for all appearances that are epistemically possible given $\mathbf{P}_l$.*

Proof. (Lutz 2011, proof of claim 7) □

Thus the growth of information can be represented by the restricted hierarchy of approximate appearances, and at each point of the hierarchy, it is clear which theories are approximately empirically adequate.

5 Conclusion

The notion of a vagueness set generalizes the notion of a partial structure to allow imprecise functions and constants, and allows to capture the notion of a partial isomorphism without the use of partial functions. These results are helpful in that they show that vagueness sets can be used to formalize all theories, models, and appearances that can be formalized in partial structures, and can in fact formalize more. That vagueness sets do not rely on any new technical notions allows a simple transfer of known results from model theory. Furthermore, vagueness sets are the basis of the notions of approximate empirical adequacy and restricted hierarchy of approximate empirical appearances, which seem capture the basic intuition behind Bueno's notions of partial and pragmatic empirical adequacy without succumbing to their problems.

Together with the results of this article's companion piece, these results show the adequacy and flexibility of vagueness sets, approximate empirical adequacy, and hierarchies of approximate empirical appearances.

A Partial isomorphisms in terms of vagueness sets

Claim 7. *Let* $\mathbf{A}$ *and* $\mathbf{B}$ *be the vagueness sets corresponding to partial structures* $\tilde{\mathfrak{A}}$ *and* $\tilde{\mathfrak{B}}$ *with* $\tilde{\Pi} = \varnothing$*. Then* f *is a partial isomorphism between* $\tilde{\mathfrak{A}}$ *and* $\tilde{\mathfrak{B}}$ *if and only if for all* $\mathfrak{A} \in \mathbf{A}$ *there is a* $\mathfrak{B} \in \mathbf{B}$ *and for all* $\mathfrak{B} \in \mathbf{B}$ *there is an* $\mathfrak{A} \in \mathbf{A}$ *such that* f *is an isomorphism between* $\mathfrak{A}$ *and* $\mathfrak{B}$.

Proof. '$\Rightarrow$': Let $f : A \longrightarrow B$ be a partial isomorphism between $\tilde{\mathfrak{A}}$ and $\tilde{\mathfrak{B}}$ and let $\mathfrak{A}$ be in $\mathbf{A}$. By claim 1, $\mathfrak{A}$ is $\tilde{\mathfrak{A}}$-normal for $\varnothing$. Since f is a bijection, it is an isomorphism from $\mathfrak{A}$ to $\mathfrak{B}$ with $R_i^{\mathfrak{B}} = f\left(R_i^{\mathfrak{A}}\right)$ for each $i \in I$, $F_j^{\mathfrak{B}} x_1 \ldots x_{n_j} = f\left(F_j^{\mathfrak{A}} f^{-1}(x_1) \ldots f^{-1}(x_{m_j})\right)$ for each $x_1, \ldots, x_{n_j} \in B^{n_j}$, $j \in J$, and $c_k^{\mathfrak{B}} = f\left(c_k^{\mathfrak{A}}\right)$ for each $k \in K$. Furthermore, $\mathfrak{B}$ is $\tilde{\mathfrak{B}}$-normal, because first, $R_i^{\tilde{\mathfrak{B}},+} = f\left(R_i^{\tilde{\mathfrak{A}},+}\right) \subseteq f\left(R_i^{\mathfrak{A}}\right) = R_i^{\mathfrak{B}} \subseteq f\left(R_i^{\tilde{\mathfrak{A}},-}\right) = R_i^{\tilde{\mathfrak{B}},-}$. Second, $C_{\tilde{\mathfrak{B}},j} = f\left(C_{\tilde{\mathfrak{A}},j}\right)$ and hence $F_j^{\mathfrak{B}} x_1 \ldots x_{n_j} = f\left(F_j^{\mathfrak{A}} f^{-1}(x_1) \ldots f^{-1}(x_{n_j})\right) = f\left(F_j^{\tilde{\mathfrak{A}}} f^{-1}(x_1) \ldots f^{-1}(x_{n_j})\right) = F_j^{\tilde{\mathfrak{B}}} x_1 \ldots x_{n_j}$ for all $x_1, \ldots, x_{n_j} \in C_{\tilde{\mathfrak{B}}}$. Third, $c_k^{\mathfrak{B}} = f\left(c_k^{\mathfrak{A}}\right) = f\left(c_k^{\tilde{\mathfrak{A}}}\right) = c_k^{\tilde{\mathfrak{B}}}$. By claim 1, $\mathfrak{B} \in \mathbf{B}$. By the same reasoning, if $\mathfrak{B} \in \mathbf{B}$, there is an $\mathfrak{A} \in \mathbf{A}$ such that f^{-1} and thus f is an isomorphism between $\mathfrak{A}$ and $\mathfrak{B}$.[20]

'$\Leftarrow$': Assume that $f : A \longrightarrow B$ is a bijection but not a partial isomorphism between $\tilde{\mathfrak{A}}$ and $\tilde{\mathfrak{B}}$. Then there are an $i \in I$ and some $x_1, \ldots, x_{m_i} \in A$ such that (i) $R_i^{\mathfrak{A},+} x_1 \ldots x_{m_i}$ and not $R_i^{\mathfrak{B},+} f(x_1) \ldots f(x_{m_i})$ or (ii) not $R_i^{\mathfrak{A},+} x_1 \ldots x_{m_i}$ and $R_i^{\mathfrak{B},+} f(x_1) \ldots f(x_{m_i})$ or (iii) $R_i^{\mathfrak{A},-} x_1 \ldots x_{m_i}$ and not $R_i^{\mathfrak{B},-} f(x_1) \ldots f(x_{m_i})$ or (iv)

[20]This half of the proof generalizes the rough proof given by Bueno (2000, 279f).

not $R_i^{\mathfrak{A},-} x_1 \dots x_{m_i}$ and $R_i^{\mathfrak{B},-} f(x_1) \dots f(x_{m_i})$, or there is a $j \in J$ such that for some $x_1, \dots, x_{m_j} \in A$, (v) $C_{\tilde{\mathfrak{A}},j} x_1 \dots x_{m_j}$ and not $C_{\tilde{\mathfrak{B}},j} f(x_1) \dots f(x_{m_j})$ or (vi) not $C_{\tilde{\mathfrak{A}},j} x_1 \dots x_{m_j}$ and $C_{\tilde{\mathfrak{B}},j} f(x_1) \dots f(x_{m_j})$ or (vii) for some $(x_1, \dots, x_{m_j}) \in C_{\tilde{\mathfrak{A}},j}$, $f\left(F_j^{\tilde{\mathfrak{A}}} x_1 \dots x_{n_j}\right) \neq F_j^{\tilde{\mathfrak{B}}} f(x_1) \dots f x_{n_j}$, or (viii) there is a $k \in K$ such that $f c_k^{\tilde{\mathfrak{A}}} \neq c_k^{\tilde{\mathfrak{B}}}$. It is to be shown that (∗) there is an $\mathfrak{A} \in \mathbf{A}$ for which there is no $\mathfrak{B} \in \mathbf{B}$ or there is an $\mathfrak{B} \in \mathbf{A}$ for which there is no $\mathfrak{A} \in \mathbf{A}$ such that f is an isomorphism between $\mathfrak{A}$ and $\mathfrak{B}$.

If (i) holds for some $i \in I$ and $x_1, \dots, x_{m_i}$, then choose an $\tilde{\mathfrak{A}}$-normal structure with $R_i^{\mathfrak{A}} x_1 \dots x_{m_i}$, if (ii) holds for some $i \in I$ and $x_1, \dots, x_{m_i}$, then choose an $\tilde{\mathfrak{B}}$-normal structure with $R_i^{\mathfrak{B}} f(x_1) \dots f(x_{m_i})$, and analogously for (iii) and (iv). Then, if f is an isomorphism between $\mathfrak{A}$ and $\mathfrak{B}$, in case (i) $\mathfrak{B}$ is not $\tilde{\mathfrak{B}}$-normal because $R_i^{\mathfrak{B},+} f(x_1) \dots f(x_{m_i})$ does not hold, in case (ii) $\mathfrak{A}$ is not $\tilde{\mathfrak{A}}$-normal because $R_i^{\mathfrak{A},+} x_1 \dots x_{m_i}$ does not hold, and analogously for (iii) and (iv). If (v) holds for some $j \in J$ and $x_1, \dots, x_{n_j} \in A$, choose $F_j^{\mathfrak{B}} f(x_1), \dots, f(x_{n_j}) \neq f\left(F_j^{\tilde{\mathfrak{A}}} x_1 \dots x_{n_j}\right)$, and analogously for (vi). Then, if f is an isomorphism between $\mathfrak{A}$ and $\mathfrak{B}$, in case (v) $\mathfrak{A}$ is not $\tilde{\mathfrak{A}}$-normal because $F_j^{\mathfrak{A}} x_1 \dots x_{n_j} = f^{-1}\left(F_j^{\mathfrak{B}} f(x_1) \dots f(x_{n_j})\right) \neq F_j^{\tilde{\mathfrak{A}}} x_1 \dots x_{n_j}$, and analogously for (vii). For any $\tilde{\mathfrak{A}}$-normal structure $\mathfrak{A}$ and any $\tilde{\mathfrak{B}}$-normal structure $\mathfrak{B}$, if (vii) holds, $F_j^{\mathfrak{A}} x_1, \dots, x_{n_j} = F_j^{\tilde{\mathfrak{A}}} x_1 \dots x_{n_j} \neq F_j^{\tilde{\mathfrak{B}}} x_1, \dots, x_{n_j} = F_j^{\mathfrak{B}} x_1, \dots, x_{n_j}$ and if (viii) holds, $c_k^{\mathfrak{A}} = c_k^{\tilde{\mathfrak{A}}} \neq c_k^{\tilde{\mathfrak{B}}} = c_k^{\mathfrak{B}}$. (∗) follows by claim 1. □

B Suárez's criticism of empirical adequacy

According to a criticism by Suárez (1995; 2005), van Fraassen's notion of empirical adequacy does not, *pace* van Fraassen (1980, 12), capture what it means that a theory "saves the phenomena". The structure of Suárez's argument is simple: In the 1930s, electromagnetic theory was considered to save the phenomenon of superconductivity, but electromagnetic theory has no empirical substructure[21] that is isomorphic to this phenomenon and is thus not empirically adequate. Therefore, the two notions cannot be equivalent.

In slightly more detail, Suárez (2005, §6.1) considers the classic electromagnetic theory to include Maxwell's equations in matter, which give the relations between the electric field $\vec{E}$, magnetic field $\vec{B}$, the electric displacement field $\vec{D}$, the magnetizing field $\vec{H}$, the charge density ρ, the current density $\vec{j}$, the time t, and the speed of light c. It also includes the acceleration equation, which gives an additional

[21]With a reference to Suppes (2002, 62), Suárez (2005, 38) defines a substructure of $\mathfrak{T}$ to have possibly fewer relations than $\mathfrak{T}$. This is rather the definition of a relativized reduct (Lutz 2011, §4.1), and misconstrues van Fraassen's notion of empirical adequacy and Suppes's definition. Suárez (2005, 39) weakens empirical adequacy even further (cf. Lutz 2011, §2.1), but since he argues that empirical adequacy is *too strong*, these changes do not threaten his conclusion.

relation between $\vec{H}$, the magnetic field $\vec{H}_0$ at time $t = 0$, and the penetration depth Λ, and which was meant to deal with the behavior of the magnetic flux in superconductors. The acceleration equation therefore describes the empirical substructure of the classic electromagnetic theory as far as superconductivity is concerned. But the magnetic field $\vec{H}$ in superconductors in fact is, with slight idealizations, correctly described by the Londons' equation for Λ, c, and $\vec{H}$, which therefore describes the structure of the appearances.

There is, as is demanded for empirical adequacy, a bijection between the domains of the empirical substructure and the structure of the appearances, since, according to Suárez (2005, 54)

> the two domains are isomorphic: for every physical entity (i. e. $\vec{j}$, $\vec{H}$, etc.) in the domain over which the 'acceleration equation' model is defined, there is a corresponding entity in the domain of the London model.

But the Londons' equation is incompatible with the acceleration equation (i. e., their conjunction is inconsistent), so that the structures cannot be isomorphic. There can also be no other empirical substructure of the classic electromagnetic theory that could be isomorphic to the Londons' equation, since then the theory would yield incompatible predictions for the same domain and would thus be inconsistent.

One could also take electromagnetic theory to consist only of Maxwell's equations, without the acceleration equation. This theory is compatible with the Londons' equation, but, says Suárez (2005, 55), it

> can provide neither an accurate nor an inaccurate representation of superconductivity phenomena—it can provide no representation at all! The phenomenon of superconductivity simply lies outside the theory's domain of empirical adequacy, and the question of embedding [...] of [the phenomena] simply does not arise.

There are a number of problems with Suárez's argument. The main one is its overly impressionistic nature. Suárez does not define a family of structures for the classic electromagnetic theory, nor does he define its empirical substructures. Instead, he mentions the equations that hold for the functions that occur in the respective structures. For this reason, his description of the domain of the theory may seem more plausible than it is: According to Suárez, the domain of the empirical substructure contains the physical quantities $\vec{j}$ and $\vec{H}$, so that, apparently, the domain of the classical electromagnetic theory contains the physical quantities that occur in Maxwell's equations and the acceleration equation: $\vec{E}, \vec{B}, \vec{D}, \vec{H}, \vec{H}_0, \rho, \vec{j}, c$, and Λ. (I am unsure about the way in which time development would be represented in this formalization.) The relations that hold between these objects are thus, presumably, the equations themselves. Suárez (2005, 54) furthermore seems to consider any equation that follows from these equations to be part of the structure, since he claims on the ground that one equation follows from both the Londons' and the acceleration equation that "at least one relation over the domain is isomorphic".

The empirical substructures of the classic electromagnetic theory therefore are structures that contain some, but not necessarily all, of the quantities that occur in the classic electromagnetic theory itself. The first part of Suárez's argument then is that the relations that are given by Maxwell's equation and the acceleration equation cannot be fulfilled by the same objects that fulfill the relation given by the Londons' equation, since the equations are incompatible. The second, rather loosely formulated part of his argument could be reconstructed as follows: Maxwell's equations alone do not entail the Londons' equation, and therefore there is no relation in any structure of the electromagnetic theory (as described by Maxwell's equations) that corresponds to the Londons' equation. Since any substructure of a structure has the same relations as the structure itself, no empirical substructure of the electromagnetic theory can be isomorphic to a structure that contains the Londons' equation.

Suárez's formalization of the electromagnetic theory is very close to the received view: The physical quantities in the empirical substructures can be seen as the observation terms, and the relations that occur in all of these substructures are the implications of the theory in the observational vocabulary. The relations that occur in none are incompatible with the theory. On the other hand, a formalization in van Fraassen's spirit would rather describe the phase space of the systems with the respective physical quantities (van Fraassen 1980, 67). In the phase space description of a system of finitely many objects, each quantity of each object is assigned a dimension of the phase space, so that at each time, the values of all quantities of all objects are given by a single point in phase space. For a field, each point in physical space can have a different value, so that each quantity of each point in physical space would have to be assigned a dimension in phase space. This would result in a non-denumerable dimensional phase space, which would be, at best, very unwieldy.

Not as close to van Fraassen's preferred formalization but still typical in the semantic view would be to give a structure with physical spacetime in its domain, the physical quantities $\vec{E}$, $\vec{B}$, $\vec{D}$, $\vec{H}$, and $\vec{J}$ as functions from spacetime points to triples of real numbers, the physical quantity ρ as a function from spacetime points to real numbers, $\vec{H}_0$ as a constant in $\mathbb{R}^3$, and c and Λ as real constants. Maxwell's, the Londons', and the acceleration equation would then be demanded to hold between these functions and constants of the structures. Since the Londons' equation describes superconductors, the empirical substructures of electromagnetic theory that have to be isomorphic to the appearances are those whose domains (subsets of spacetime) contain only superconducting material. In a formalization that follows this sketch, Suárez's argument against the empirical adequacy of classic electromagnetic theory is valid, since a the functions and constants cannot simultaneously fulfill two incompatible equations. However, Suárez's argument against the empirical adequacy of the electromagnetic theory as described by Maxwell's equations alone fails: A structure with the points in physical space and the real numbers as its domain and the physical quantities given above as its relations is a structure of the electromagnetic theory if and only if the physical quantities fulfill Maxwell's equations. Since Maxwell's equations are compatible with the Londons' equations for restricted domains, there are empirical substructures of the electromagnetic theory that are isomorphic to the structure of the phenomenon of superconductivity as described by the Londons' equation. Therefore, the electromagnetic theory as described by Maxwell's equations is empirically adequate for the phenomenon of superconductivity.

Therefore, Suárez's criticism is not fatal for the concept of empirical adequacy, because in a plausible formalization, only the classic electromagnetic theory fails to be empirically adequate. And this is as it should be, since the acceleration equation describes the appearances incorrectly, and the conjunction of Maxwell's equation and the acceleration equation can therefore not be considered to save the appearances.

C Bueno's reply to Suárez

C.1 Partial empirical adequacy

According to Bueno (1997, §4.4, my notation), classic electromagnetic theory as described by Maxwell's equation and the acceleration equation is partially empirically adequate, so that his generalization of empirical adequacy to partial empirical adequacy avoids Suárez first criticism:

> Indeed, the kind of new relation discovered at the phenomenological level might be represented by one of those partial relations that initially were not defined for some elements of the domain under investigation (whose elements thus belong to its R°-component), and with the introduction of further bits of information, came to be defined—hence, the relevant elements belong then either to its domain (R^{+}-component), or to its complement (R^{-}-component).

This reply to Suárez is rather vague, but it seems that the Londons' equation is the "new relation discovered" between objects of the domain, or better: It is the discovery that some relation holds between objects of the domain (Bueno is thus relying on Suárez's formalization).

A major problem with this reply is that it is not clear in what way the notion of partial empirical adequacy is time dependent, and thus it is not clear how the discovery that some elements of the domain have some relation can be represented as partial empirical adequacy. Another problem is that it is utterly artificial to use partial relations to formalize a situation in which an equation is discovered to hold between physical quantities: Before the Londons' equation was discovered to hold for Λ, c, and $\vec{H}$, no one was of the opinion that it held for, say c, c, and $\vec{D}$ and was wondering whether it also held for other physical quantities of the above list. Staying in Suárez's formalization, it was rather that the Londons discovered a *new* relation—just as Bueno puts it informally in his reply quoted above. Finally, Suárez (2005, §6.2) points out that, since the acceleration equation is incompatible with the Londons' equation, Λ, c, and $\vec{H}$ are, according to the acceleration equation, certainly not in the relation described by the Londons' equation. That is, in any partial appearance $\tilde{\mathfrak{P}}$, $R_i^{\tilde{\mathfrak{P}},+}\Lambda c\vec{H}$ holds,[22] where R_i is given by the Londons' equation. The partial isomorphism f to the empirical substructure then gives $R_i^{\tilde{\mathfrak{E}},+}f(\Lambda)f(c)f(\vec{H})$ for the analogue of the Londons' equation in the theory. But this would lead to inconsistency, since according to the classic electromagnetic theory, $R_i^{\tilde{\mathfrak{E}},-}f(\Lambda)f(c)f(\vec{H})$.

In the formalization that I have sketched, the classic electromagnetic theory is not partially empirically adequate either: The domain of the Londons' equation consists of the space-time volumes that contain superconductors, and its terms are Λ, c, and $\vec{H}$. In each appearance of superconductivity, these terms' interpretations fulfill the Londons' equation. Since this is known, every partial structure $\tilde{\mathfrak{P}}$ of the appearances must be such that no $\tilde{\mathfrak{P}}$-normal structure fulfills the Londons' equation. The classic electromagnetic theory, on the other hand, must be such that for no partial empirical substructure $\tilde{\mathfrak{E}}$, there is an $\tilde{\mathfrak{E}}$-normal structure that fulfills the Londons' equation, since it is part of the theory that the Londons' equation is not fulfilled by superconductors. Since no $\tilde{\mathfrak{P}}$-normal and $\tilde{\mathfrak{E}}$-normal structures are isomorphic, $\tilde{\mathfrak{P}}$ and $\tilde{\mathfrak{E}}$ are not partially isomorphic either by claims 1 and 7 (§A). Electromagnetic theory as described by Maxwell's equations is partially empirically adequate, however, for partial empirical adequacy generalizes empirical adequacy and electromagnetic theory is empirically adequate.

C.2 Quasi empirical adequacy

The requirements for the pragmatic empirical adequacy of a theory are almost trivially weak. This has the intended effect of avoiding Suárez's counterexample *if* the relation described by the Londons' equation is considered to determine the structure of the data (the lowest level of the models of the phenomena), not the appearance at the highest level: While the relation between Λ, c, and $\vec{H}$ described by the Londons' equation has to hold for the structure of the data, the structure of the appearances can contain any relation between Λ, c, and $\vec{H}$ under one further condition: At least one element in the domain of the highest level must have been measured. Apart from this condition, one can choose any structure with the acceleration equation as relation between Λ, c, and $\vec{H}$. Therefore the classic electromagnetic theory is pragmatically empirically adequate.

In the formalization of the classic electromagnetic theory that I have sketched,

[22] That is, the tuple $\langle\Lambda, c, \vec{H}\rangle$ clearly fulfills relation $R_i^{\tilde{\mathfrak{P}}}$.

the case is similar: The domain of the Londons' equation contains the spacetime volumes that contain superconductors, and its terms are Λ, c, and $\vec{H}$. (To apply Bueno's definitions for relational structures, the theory first has to be reformulated to contain only relations rather than functions and constants.) Since the domain of the appearances, the spacetime volume of the superconductors, is infinite, there is no restriction on the domain of the highest level of the models of the phenomena. One can now choose a structure with the same terms and at least one measured object in its domain in which the classic electromagnetic theory is true, so that the theory is pragmatically empirically adequate. Of course, this strength of Bueno's definition is also a huge weakness, since any other theory with the same relation symbols is pragmatically empirically adequate as well.

In summary, the notion of partial empirical adequacy does not avoid Suárez's criticism, and the notion of pragmatic empirical adequacy solves it, but only on pain of triviality. Neither failure poses a problem for empirical adequacy or the project of constructive empiricism in general, however, because Suárez's criticism is an artifact of an ill-chosen formalization of the theory of electromagnetism.

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