

Research Article

Adaptive Neural Networks Control Using Barrier Lyapunov Functions for DC Motor System with Time-Varying State Constraints

Lei Ma¹ and Dapeng Li ²

¹College of Science, Liaoning University of Technology, Jinzhou, Liaoning 121001, China

²School of Electrical Engineering, Liaoning University of Technology, Jinzhou 121001, China

Correspondence should be addressed to Dapeng Li; li_dapengsir@163.com

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This paper proposes an adaptive neural network (NN) control approach for a direct-current (DC) system with full state constraints. To guarantee that state constraints always remain in the asymmetric time-varying constraint regions, the asymmetric time-varying Barrier Lyapunov Function (BLF) is employed to structure an adaptive NN controller. As we all know that the constant constraint is only a special case of the time-varying constraint, hence, the proposed control method is more general for dealing with constraint problem as compared with the existing works on DC systems. As far as we know, this system is the first studied situations with time-varying constraints. Using Lyapunov analysis, all signals in the closed-loop system are proved to be bounded and the constraints are not violated. In this paper, the effectiveness of the control method is demonstrated by simulation results.

1. Introduction

Due to the requirements of practice and the development of theory, the controller design of uncertain system has become a new research direction and attracted more and more scholars' attention. The uncertainty of the actual engineering system has been studied in many works [1–4]. The neural networks [5] and fuzzy logic systems [6] have become the two main tools which can effectively deal with the unknown functions in the systems. In [7, 8], these are studies of some actual engineering systems with uncertain parameters. In [9, 10], the NN is used to approximate several random perturbations and unknown functions. In [11–16], several nonlinear system solutions are studied based on neural networks and fuzzy logic systems. In [17], adaptive control schemes based on neural networks were proposed for nonlinear systems with unknown functions. Based on neural networks and fuzzy logic systems, the significant studies proposed the novel adaptive tracking control methods for nonlinear SISO systems in [18–20] and MIMO systems in [21–23]. However,

it is worth noting that the constraint problem is worth noting in the above approaches, which lead to the inaccuracy or oscillations of the engineering systems and even cause control systems instability.

In fact, there are constraints in most physical systems with various forms, for example, physical stoppages, saturation, performance, and safety specifications, such as restricted robot manipulation system [24], application to chemical process [25], networked surveillance robots systems [26], and nonuniform gantry crane [27]. In recent years, the barrier Lyapunov functions become the main tools to solve the constrained problem which was proposed for the first time in [28]. Based on BLF, some adaptive control methods were presented for nonlinear systems with output constant constraint in [29, 30] and state constant constraint in [31–34]. As we known, the constant constraint is the special case of the time-varying constraint. Subsequently, the authors in [35, 36] proposed some adaptive control approaches to address the stability problem of nonlinear systems with time-varying constraints.

Motor is the most important electromechanical energy conversion device, which has been widely used in the industrial and agricultural production, transportation, aerospace, and so on. In particular, the motor system with unknown uncertainties has attracted the attention of many scholars, and the control problem of the motor system becomes more and more important. In [37, 38], the authors proposed two adaptive control methods for systems with unknown functions. The authors in [39, 40] presented an adaptive control with time-varying output constraints for DC motor systems. According to the above descriptions, the urgent problem is how to address the stability problem of the DC motor system with time-varying state constraints.

This paper presents an adaptive NN tracking control method for DC motor systems with time-varying state constraints. As far as we know, there is no work dealing with such DC motor systems in the literature at present stage. The contributions of this paper are summarized as follows. (1) The time-varying state constraints are first considered in the DC motor systems; comparing with the existing on DC motor systems, the proposed control method is more general and extensive in the engineering field. (2) To guarantee that the state constraints always remain in the time-varying constrained sets, the asymmetric time-varying BLF is utilized. (3) A novel adaptive tracking controller based on the neural networks and backstepping technique is structured to guarantee that all signals in the closed-loop system are bounded, the tracking errors converge to a small neighborhood of zero and the time-varying state constraints are not transitioned.

2. Problem Formulation and Preliminaries

Consider the dynamic system with the DC motor without vibration mode as the following form:

$$\begin{aligned} \dot{\alpha}_1 &= \alpha_2 \\ J\dot{\alpha}_2 + f\dot{\alpha}_1 + T_f + d &= u \\ y &= \alpha_1, \end{aligned} \quad (1)$$

where $\alpha_1(t)$ is the motor angular position; α_2 stands for motor angular velocity; J is a known inertia, f is an unmeasured viscous friction, and T_f is an unmeasured nonlinear friction; $d(t)$ represents the unknown disturbance but bounded with $\|d(t)\| \leq d_M$; $y \in R$ is the system output; and u represents the motor torque. In particular, output $y(t)$ is required as follows:

$$\underline{k}_{c_1}(t) < y(t) < \bar{k}_{c_1}(t), \quad \forall t \geq 0, \quad (2)$$

where $\bar{k}_{c_1} : R_+ \rightarrow R$ and $\underline{k}_{c_1} : R_+ \rightarrow R$ such that $\bar{k}_{c_1}(t) > \underline{k}_{c_1}(t)$, $\forall t \in R_+$.

Remark 1. From (2), the states of DC systems are constrained by the considered time-varying functions. In [35, 36], the constraint problem is omitted, which is the main factor of the oscillations of the engineering systems. The authors

in [39] addressed the stability problem of DC motor systems with constant constraint which is the special case of the time-varying constraint. Comparing with the [40], the authors only consider time-varying output constraint; the proposed adaptive control method tries to stabilize the DC motor systems with time-varying state constraints, which cause the difficulty of controller design.

In this paper, the control objective is to design an adaptive NN tracking controller u which adjusts the output of DC motor systems y to track desired trajectory of the reference signal $y_d(t)$ in the range of time-varying constraint functions. Meanwhile, all signals in the closed-loop systems are bounded and the time-varying state constraints are not violated.

Assumption 2 (see [35]). There exist constants \bar{K}_{c_i} and \underline{K}_{c_i} , $i = 0, 1, \dots, n$, such that $\bar{k}_{c_i}(t) \leq \bar{K}_{c_i}$, $\underline{k}_{c_i}(t) \geq \underline{K}_{c_i}$, and $|\bar{k}_{c_i}^{(i)}(t)| \leq \bar{K}_{c_i}$, $|\underline{k}_{c_i}^{(i)}(t)| \leq \underline{K}_{c_i}$, $\forall t \geq 0$, $i = 1, \dots, n$.

Assumption 3 (see [32]). There exist functions $\bar{Y}_0 : R_+ \rightarrow R_+$ and $\underline{Y}_0 : R_+ \rightarrow R_+$ satisfying $\bar{Y}_0(t) < \bar{k}_{c_1}(t)$ and $\underline{Y}_0(t) > \underline{k}_{c_1}(t)$ $\forall t \geq 0$, and positive constants Y_i , $i = 1, \dots, n$, such that the desired trajectory $y_d(t)$ and its time derivatives satisfy $\underline{Y}_0(t) \leq y_d(t) \leq \bar{Y}_0(t)$ and $|y_d^{(i)}(t)| \leq Y_i$, $i = 1, \dots, n$, $\forall t \geq 0$.

The following lemma is represented for the establishment of binding compensation and performance limits.

Lemma 4 (see [28]). Let $Z := \{\xi \in R : |\xi| < 1\} \subset R$ and $N := R^l \times Z \subset R^{l+1}$ be open sets. Take into account the system

$$\dot{\alpha} = g(t, \alpha), \quad (3)$$

where $\alpha := [\omega, \xi]^T \in N$ and $g : R_+ \times N \rightarrow R^{l+1}$ in t is piecewise continuous and in α is locally Lipschitz, united in t , on $R_+ \times N$.

Suppose that there are functions $H : R^l \times R_+ \rightarrow R_+$ and $V_1 : Z \rightarrow R_+$. In their respective domains, they are continuously differentiable and positive definite, such that

$$\begin{aligned} V_1(\xi) &\rightarrow \infty, \quad |\xi| \rightarrow 1 \\ \lambda_1(\|\omega\|) &\leq H(\omega, t) \leq \lambda_2(\|\omega\|), \end{aligned} \quad (4)$$

where λ_1 and λ_2 are class K_∞ functions. Let $V(\alpha) := V_1(\xi) + U(\omega, t)$, and $\varepsilon(0) \in Z$. If the inequality is established:

$$\dot{V} = \frac{\partial V}{\partial \alpha} g \leq 0 \quad (5)$$

in $\xi \in Z$, $\xi(t) \in Z \forall t \in [0, \infty)$.

Lemma 5 (see [35]). For all $|\xi| < 1$ and positive integer p , the inequality $\log 1/(1 - \xi^{2p}) < \xi^{2p}/(1 - \xi^{2p})$.

Proof. For $|\xi| < 1$, the term $\xi^{2p}/(1 - \xi^{2p})$ can be rewritten as

$$\begin{aligned} \frac{\xi^{2p}}{1 - \xi^{2p}} &= \log\left(e^{\xi^{2p}/(1 - \xi^{2p})}\right) \\ &\geq \log\left[1 + \frac{\xi^{2p}}{1 - \xi^{2p}} + \sum_{n=2}^{\infty} \frac{(\xi^{2p}/(1 - \xi^{2p}))^n}{n!}\right] \\ &\geq \log\left(1 + \frac{\xi^{2p}}{1 - \xi^{2p}}\right) = \log\frac{1}{1 - \xi^{2p}}. \end{aligned} \quad (6)$$

The proof is completed. \square

3. State Feedback Adaptive Controller Designs

This paper presents an adaptive tracking controller based on a backstepping technique with the asymmetric time-varying BLF for the DC motor systems. The detailed designs process is shown in this section.

Denote $z_1 = \alpha_1 - y_d$, $z_2 = \alpha_2 - \sigma_1$, where σ_1 is the virtual controller which will be given later on. We consider the time-varying asymmetric BLF:

$$\begin{aligned} V_1 &= \frac{q(z_1)}{2p} \log \frac{k_{b_1}^{2p}(t)}{k_{b_1}^{2p}(t) - z_1^{2p}} \\ &\quad + \frac{1 - q(z_1)}{2p} \log \frac{k_{a_1}^{2p}(t)}{k_{a_1}^{2p}(t) - z_1^{2p}}, \end{aligned} \quad (7)$$

where p is a positive integer.

The time-varying barriers are chosen as

$$k_{a_1}(t) = y_d(t) - \underline{k}_{c_1}(t) \quad (8)$$

$$k_{b_1}(t) = \bar{k}_{c_1}(t) - y_d(t) \quad (9)$$

and $q(\cdot)$ is defined as

$$q(\cdot) = \begin{cases} 1, & \text{if } \cdot > 0 \\ 0, & \text{if } \cdot \leq 0. \end{cases} \quad (10)$$

Based on Assumptions 2 and 3, there are positive constants \underline{k}_{b_1} , \bar{k}_{b_1} , \underline{k}_{a_1} , and \bar{k}_{a_1} , such that

$$\begin{aligned} \underline{k}_{b_1} &\leq k_{b_1}(t) \leq \bar{k}_{b_1}, \\ \underline{k}_{a_1} &\leq k_{a_1}(t) \leq \bar{k}_{a_1}, \quad \forall t \geq 0 \end{aligned} \quad (11)$$

Through the change of error coordinates,

$$\begin{aligned} \xi_{a_i} &= \frac{z_i}{k_{a_i}}, \\ \xi_{b_i} &= \frac{z_i}{k_{b_i}}, \\ \xi_i &= q(z_i) \xi_{b_i} + (1 - q(z_i)) \xi_{a_i}, \quad i = 1, 2. \end{aligned} \quad (12)$$

According to (12), (7) can be rewritten as

$$V_1 = \frac{1}{2p} \log \frac{1}{1 - \xi_1^{2p}}. \quad (13)$$

Remark 6. According to (10), we know that when $z_1 > 0$, we obtain $q(z_1) = 1$, $\xi_1 = \xi_{b_1}$, and $V_1 = (1/2p) \log(1/(1 - \xi_{b_1}^{2p})) = (1/2p) \log(1/(1 - \xi_1^{2p}))$. When $z_1 < 0$, we obtain $q(z_1) = 0$, $\xi_1 = \xi_{a_1}$, and $V_1 = (1/2p) \log(1/(1 - \xi_{a_1}^{2p})) = (1/2p) \log(1/(1 - \xi_1^{2p}))$. From the above, we can get (13) based on (12).

Obviously, under the premise of $|\xi| < 1$, V_1 is definite continuously differentiable. The time derivative of V_1 is

$$\begin{aligned} \dot{V}_1 &= \frac{q(z_1) \xi_{b_1}^{2p-1}}{k_{b_1} (1 - \xi_{b_1}^{2p})} \left(z_2 + \sigma_1 - \dot{y}_d - z_1 \frac{\dot{k}_{b_1}}{k_{b_1}} \right) \\ &\quad + \frac{(1 - q(z_1)) \xi_{a_1}^{2p-1}}{k_{a_1} (1 - \xi_{a_1}^{2p})} \left(z_2 + \sigma_1 - \dot{y}_d - z_1 \frac{\dot{k}_{a_1}}{k_{a_1}} \right). \end{aligned} \quad (14)$$

Choose the virtual controller σ_1 as

$$\sigma_1 = -(\kappa_1 + \bar{\kappa}_1(t)) z_1 + \dot{y}_d - \frac{2p-1}{2p} z_1. \quad (15)$$

The time-varying gain is given as

$$\bar{\kappa}_i(t) = \sqrt{\left(\frac{\dot{k}_{a_i}}{k_{a_i}}\right)^2 + \left(\frac{\dot{k}_{b_i}}{k_{b_i}}\right)^2} + \beta_i, \quad (16)$$

where β_i and κ_i , $i = 1, 2$ are any positive constants. Make sure that the time derivative α_1 is bounded, when \dot{k}_{a_1} and \dot{k}_{b_1} are both zero. Substituting (15) and (16) into (14) and noting that

$$\bar{\kappa}_i + q(z_i) \frac{\dot{k}_{b_i}}{k_{b_i}} + (1 - q(z_i)) \frac{\dot{k}_{a_i}}{k_{a_i}} \geq 0 \quad (17)$$

we obtain

$$\begin{aligned} \dot{V}_1 &= \left(\frac{q(z_1) \xi_{b_1}^{2p-1}}{k_{b_1} (1 - \xi_{b_1}^{2p})} + \frac{(1 - q(z_1)) \xi_{a_1}^{2p-1}}{k_{a_1} (1 - \xi_{a_1}^{2p})} \right) z_2 \\ &\quad - z_1 \left(\bar{\kappa}_1(t) \frac{q(z_1) \xi_{b_1}^{2p}}{(1 - \xi_{b_1}^{2p})} + \bar{\kappa}_1(t) \frac{(1 - q(z_1)) \xi_{a_1}^{2p}}{(1 - \xi_{a_1}^{2p})} \right. \\ &\quad \left. + \frac{\dot{k}_{b_1}}{k_{b_1}} \frac{q(z_1) \xi_{b_1}^{2p}}{(1 - \xi_{b_1}^{2p})} + \frac{\dot{k}_{a_1}}{k_{a_1}} \frac{(1 - q(z_1)) \xi_{a_1}^{2p}}{(1 - \xi_{a_1}^{2p})} \right) \\ &\quad - \kappa_1 z_1 \left(\frac{q(z_1) \xi_{b_1}^{2p}}{(1 - \xi_{b_1}^{2p})} + \frac{(1 - q(z_1)) \xi_{a_1}^{2p}}{(1 - \xi_{a_1}^{2p})} \right) - \frac{2p-1}{2p} \mu_1 z_1^{2p}, \end{aligned} \quad (18)$$

where

$$\mu_1 = \frac{q(z_1)}{k_{b_1}^{2p} - z_1^{2p}} + \frac{1 - q(z_1)}{k_{a_1}^{2p} - z_1^{2p}}. \quad (19)$$

After finishing it, we get

$$\begin{aligned} \dot{V}_1 = & \left(\frac{q(z_1)}{(k_{b_1}^{2p} - z_1^{2p})} + \frac{(1 - q(z_1))}{(k_{a_1}^{2p} - z_1^{2p})} \right) z_1^{2p-1} z_2 \\ & - \frac{\xi_1^{2p}}{(1 - \xi_1^{2p})} \left(\frac{\dot{k}_{b_1}}{k_{b_1}} q(z_1) + \frac{\dot{k}_{a_1}}{k_{a_1}} (1 - q(z_1)) \right. \\ & \left. + \bar{\kappa}_1(t) \right) - \frac{\kappa_1 \xi_1^{2p}}{(1 - \xi_1^{2p})} - \frac{2p-1}{2p} \mu_1 z_1^{2p}. \end{aligned} \quad (20)$$

Based on (12), we obtain

$$\dot{V}_1 \leq -\frac{\kappa_1 \xi_1^{2p}}{1 - \xi_1^{2p}} + \mu_1 z_1^{2p-1} z_2 - \frac{2p-1}{2p} \mu_1 z_1^{2p}. \quad (21)$$

Using Young's inequality, the following inequality holds:

$$\mu_1 z_1^{2p-1} z_2 \leq \mu_1 \left(\frac{2p-1}{2p} z_1^{2p} + \frac{1}{2p} z_2^{2p} \right). \quad (22)$$

Substituting (22) into (21), \dot{V}_1 can be further rewritten as

$$\dot{V}_1 \leq -\frac{\kappa_1 \xi_1^{2p}}{1 - \xi_1^{2p}} + \frac{1}{2p} \mu_1 z_2^{2p}. \quad (23)$$

The Barrier Lyapunov Function V_2 is given as

$$\begin{aligned} V_2 = & V_1 + \frac{q(z_2)}{2p} \log \frac{k_{b_2}^{2p}(t)}{k_{b_2}^{2p}(t) - z_2^{2p}} \\ & + \frac{1 - q(z_2)}{2p} \log \frac{k_{a_2}^{2p}(t)}{k_{a_2}^{2p}(t) - z_2^{2p}}. \end{aligned} \quad (24)$$

Then, differentiating of V_2 with respect to time is given by

$$\begin{aligned} \dot{V}_2 = & \dot{V}_1 + \frac{q(z_2) \xi_{b_2}^{2p-1}}{k_{b_2} (1 - \xi_{b_2}^{2p})} \left(\dot{z}_2 - z_2 \frac{\dot{k}_{b_2}}{k_{b_2}} \right) \\ & + \frac{(1 - q(z_2)) \xi_{a_2}^{2p-1}}{k_{a_2} (1 - \xi_{a_2}^{2p})} \left(\dot{z}_2 - z_2 \frac{\dot{k}_{a_2}}{k_{a_2}} \right). \end{aligned} \quad (25)$$

From the definition of the tracking error $z_2 = \alpha_2 - \sigma_1$, it is easy to obtain $\dot{z}_2 = \dot{\alpha}_2 - \dot{\sigma}_1$, and the derivative of the virtual controller is given as

$$\dot{\sigma}_1 = \frac{\partial \sigma_1}{\partial \alpha_1} \alpha_2 + \sum_{j=0}^1 \frac{\partial \sigma_1}{\partial \zeta_1} \zeta_1^{(j+1)}, \quad (26)$$

where $\zeta_1 = [y_d, k_{a_1}, k_{b_1}]^T$.

According to (26), (25) can be rewritten as

$$\begin{aligned} \dot{V}_2 = & (\dot{\alpha}_2 - \dot{\sigma}_1) \left(\frac{1 - q(z_2)}{k_{a_2}} \frac{\xi_{a_2}^{2p-1}}{1 - \xi_{a_2}^{2p}} + \frac{q(z_2)}{k_{b_2}} \frac{\xi_{b_2}^{2p-1}}{1 - \xi_{b_2}^{2p}} \right) \\ & - z_2 \frac{\dot{k}_{b_2}}{k_{b_2}} \frac{q(z_2) \xi_{b_2}^{2p-1}}{k_{b_2} (1 - \xi_{b_2}^{2p})} \\ & - z_2 \frac{\dot{k}_{a_2}}{k_{a_2}} \frac{1 - q(z_2) \xi_{a_2}^{2p-1}}{k_{a_2} (1 - \xi_{a_2}^{2p})} + \dot{V}_1. \end{aligned} \quad (27)$$

Based on (23), we get

$$\begin{aligned} \dot{V}_2 \leq & -\frac{\kappa_1 \xi_1^{2p}}{1 - \xi_1^{2p}} + \frac{1}{2p} \mu_1 z_2^{2p} + (\dot{\alpha}_2 - \dot{\sigma}_1) \frac{\xi_2^{2p}}{z_2 (1 - \xi_2^{2p})} \\ & - z_2 \frac{\dot{k}_{b_2}}{k_{b_2}} \frac{q(z_2) \xi_{b_2}^{2p-1}}{k_{b_2} (1 - \xi_{b_2}^{2p})} \\ & - z_2 \frac{\dot{k}_{a_2}}{k_{a_2}} \frac{1 - q(z_2) \xi_{a_2}^{2p-1}}{k_{a_2} (1 - \xi_{a_2}^{2p})}. \end{aligned} \quad (28)$$

Substituting (26) into the above formula, we obtain

$$\begin{aligned} \dot{V}_2 = & -\frac{\kappa_1 \xi_1^{2p}}{1 - \xi_1^{2p}} \\ & + \frac{\xi_2^{2p}}{z_2 (1 - \xi_2^{2p})} \left(J^{-1} (-f \dot{\alpha}_1 - T_f + u) - \dot{\sigma}_1 \right) \\ & - \frac{q(z_2) \xi_{b_2}^{2p-1}}{k_{b_2} (1 - \xi_{b_2}^{2p})} \left(z_2 \frac{\dot{k}_{b_2}}{k_{b_2}} \right) \\ & - \frac{1 - q(z_2) \xi_{a_2}^{2p-1}}{k_{a_2} (1 - \xi_{a_2}^{2p})} \left(z_2 \frac{\dot{k}_{a_2}}{k_{a_2}} \right) - \frac{J^{-1} \xi_2^{2p}}{z_2 (1 - \xi_2^{2p})} d \\ & + \frac{1}{2p} \mu_1 z_2^{2p}. \end{aligned} \quad (29)$$

Using Young's inequality and noting $\|d(t)\| \leq d_M$, we obtain

$$-\frac{J^{-1} \xi_2^{2p}}{z_2 (1 - \xi_2^{2p})} d \leq \frac{1}{2\gamma_1} \left(\frac{J^{-1} \xi_2^{2p}}{z_2 (1 - \xi_2^{2p})} \right)^2 + \frac{1}{2} \gamma_1 d_M^2, \quad (30)$$

where γ_1 is a positive constant.

Based on (30), (29) can be rewritten as

$$\begin{aligned} \dot{V}_2 = & -\frac{\kappa_1 \xi_1^{2p}}{1 - \xi_1^{2p}} + \frac{1}{2p} \mu_1 z_2^{2p} + \frac{\xi_2^{2p}}{z_2 (1 - \xi_2^{2p})} \\ & \times (J^{-1}(-f\dot{\alpha}_1 - T_f + u) - \dot{\sigma}_1) + \frac{q(z_2)}{k_{b_2}} \frac{\xi_{b_2}^{2p-1}}{1 - \xi_{b_2}^{2p}} \\ & \times \left(-z_2 \frac{\dot{k}_{b_2}}{k_{b_2}} \right) + \frac{1 - q(z_2)}{k_{a_2}} \frac{\xi_{a_2}^{2p-1}}{1 - \xi_{a_2}^{2p}} \left(-z_2 \frac{\dot{k}_{a_2}}{k_{a_2}} \right) \\ & + \frac{1}{2\gamma_1} \left(\frac{J^{-1}\xi_2^{2p}}{z_2(1 - \xi_2^{2p})} \right)^2 + \frac{1}{2}\gamma_1 d_M^2. \end{aligned} \quad (31)$$

For convenience, we define

$$M(Z) = J^{-1}(T_f + f\dot{\alpha}_1) + \dot{\sigma}_1. \quad (32)$$

In fact, since the parameters of f and T_f are not available, M is unknown in practice. In order to solve the uncertainty of this parameter, we designed NN, as shown below to estimate

$$M(Z) = W^{*T}S(Z) + \varepsilon(Z), \quad (33)$$

where $Z = [\alpha_1^T, \alpha_2^T]^T \in \Omega_z \subset R^3$, and similar to [28], we assume that the approximate error $\varepsilon(Z)$ satisfies $|\varepsilon(Z)| \leq \varepsilon^*$ with the constant $\varepsilon^* > 0$.

Substituting (33), (31) can be rewritten as

$$\begin{aligned} \dot{V}_2 \leq & -\frac{\kappa_1 \xi_1^{2p}}{1 - \xi_1^{2p}} + \frac{1}{2p} \mu_1 z_2^{2p} + \frac{J^{-1}\xi_2^{2p} \widehat{W}^T S(Z)}{z_2 (1 - \xi_2^{2p})} \\ & - \frac{J^{-1}\xi_2^{2p}}{z_2 (1 - \xi_2^{2p})} \varepsilon(Z) + \frac{1}{2\gamma_1} \left(\frac{J^{-1}\xi_2^{2p}}{z_2 (1 - \xi_2^{2p})} \right)^2 \\ & + \frac{1}{2}\gamma_1 d_M^2 + \bar{\kappa}_2 \frac{\xi_2^{2p}}{1 - \xi_2^{2p}} + \frac{J^{-1}\xi_2^{2p}}{z_2 (1 - \xi_2^{2p})} u. \end{aligned} \quad (34)$$

According to Young's inequality, we can easily obtain

$$-\frac{J^{-1}\xi_2^{2p} \varepsilon(Z)}{z_2 (1 - \xi_2^{2p})} \leq \frac{1}{2\gamma_2} \left(\frac{J^{-1}\xi_2^{2p}}{z_2 (1 - \xi_2^{2p})} \right)^2 + \frac{1}{2}\gamma_2 \varepsilon^{*2}, \quad (35)$$

where γ_2 is a positive constant.

Based on (35), (34) can be rewritten as

$$\begin{aligned} \dot{V}_2 \leq & -\frac{\kappa_1 \xi_1^{2p}}{1 - \xi_1^{2p}} + \frac{1}{2p} \mu_1 z_2^{2p} + \frac{J^{-1}\xi_2^{2p} \widehat{W}^T S(Z)}{z_2 (1 - \xi_2^{2p})} \\ & + \frac{1}{2\gamma_2} \left(\frac{J^{-1}\xi_2^{2p}}{z_2 (1 - \xi_2^{2p})} \right)^2 + \frac{1}{2}\gamma_2 \varepsilon^{*2} + \bar{\kappa}_2 \frac{\xi_2^{2p}}{1 - \xi_2^{2p}} \end{aligned}$$

$$\begin{aligned} & + \frac{1}{2\gamma_1} \left(\frac{J^{-1}\xi_2^{2p}}{z_2 (1 - \xi_2^{2p})} \right)^2 + \frac{1}{2}\gamma_1 d_M^2 \\ & + \frac{J^{-1}\xi_2^{2p}}{z_2 (1 - \xi_2^{2p})} u. \end{aligned} \quad (36)$$

The actual controller is given as

$$\begin{aligned} u = & -\frac{1}{2p} J \mu_1 z_2 (k_{b_2}^{2p} - z_2^{2p}) - \frac{J^{-1}\xi_2^{2p}}{2\gamma_1 z_2 (1 - \xi_2^{2p})} \\ & - \frac{J^{-1}\xi_2^{2p}}{2\gamma_2 z_2 (1 - \xi_2^{2p})} + J z_2 \widehat{W}^T S(Z) \\ & - J z_2 (\kappa_2 + \bar{\kappa}_2). \end{aligned} \quad (37)$$

Substituting (37), we obtain

$$\begin{aligned} \dot{V}_2 \leq & -\frac{\kappa_1 \xi_1^{2p}}{1 - \xi_1^{2p}} - \kappa_2 \frac{\xi_2^{2p}}{(1 - \xi_2^{2p})} + \frac{1}{2}\gamma_1 d_M^2 \\ & + \frac{\xi_2^{2p}}{1 - \xi_2^{2p}} \widehat{W}^T S(Z) + \frac{1}{2}\gamma_2 \varepsilon^{*2}. \end{aligned} \quad (38)$$

Design the Lyapunov function candidate V_3 :

$$V_3 = V_2 + \frac{1}{2} \widehat{W}^T \Gamma^{-1} \widehat{W}, \quad (39)$$

where $\Gamma = \Gamma^{-1} > 0$ is a constant matrix and $\widehat{W} = \widehat{W} - W^*$.

The time derivative of V_3 is

$$\dot{V}_3 = \dot{V}_2 + \widehat{W}^T \Gamma^{-1} \dot{\widehat{W}}. \quad (40)$$

Based on (38), we obtain

$$\begin{aligned} \dot{V}_3 \leq & -\frac{\kappa_1 \xi_1^{2p}}{1 - \xi_1^{2p}} - \kappa_2 \frac{\xi_2^{2p}}{(1 - \xi_2^{2p})} + \frac{1}{2}\gamma_1 d_M^2 + \frac{1}{2}\gamma_2 \varepsilon^{*2} \\ & + \widehat{W}^T \Gamma^{-1} \dot{\widehat{W}} + \frac{\xi_2^{2p}}{1 - \xi_2^{2p}} \widehat{W}^T S(Z). \end{aligned} \quad (41)$$

The adaptive law is given as follows:

$$\dot{\widehat{W}} = \Gamma \left(-\frac{\xi_2^{2p}}{1 - \xi_2^{2p}} S(Z) - \eta \widehat{W} \right), \quad (42)$$

where η is a positive constant.

Substituting (42) into (41), we get

$$\begin{aligned} \dot{V}_3 = & -\frac{\kappa_1 \xi_1^{2p}}{1 - \xi_1^{2p}} - \kappa_2 \frac{\xi_2^{2p}}{(1 - \xi_2^{2p})} + \frac{1}{2}\gamma_1 d_M^2 + \frac{1}{2}\gamma_2 \varepsilon^{*2} \\ & - \widehat{W}^T \eta (\widehat{W} + W^*). \end{aligned} \quad (43)$$

Using Young's inequality,

$$\begin{aligned} \dot{V}_3 \leq & -\frac{\kappa_1 \xi_1^{2p}}{1 - \xi_1^{2p}} - \kappa_2 \frac{\xi_2^{2p}}{(1 - \xi_2^{2p})} + \frac{1}{2} \gamma_1 d_M^2 + \frac{1}{2} \gamma_2 \varepsilon^{*2} \\ & - \frac{\eta}{2} (\|\widehat{W}\|^2 - \|W^*\|^2). \end{aligned} \quad (44)$$

After finishing it, we get

$$\begin{aligned} \dot{V}_3 \leq & -\kappa_1 \log \frac{1}{1 - \xi_1^{2p}} - \frac{\eta}{2} \|\widehat{W}\|^2 + \frac{\eta}{2} \|W^*\|^2 \\ & - \kappa_2 \log \frac{1}{1 - \xi_2^{2p}} + \frac{1}{2} \gamma_1 d_M^2 + \frac{1}{2} \gamma_2 \varepsilon^{*2}. \end{aligned} \quad (45)$$

Then, the above inequality can be rewritten as

$$\dot{V}_3 \leq -\rho V_3 + C, \quad (46)$$

where

$$\begin{aligned} \rho &= \min \{2\kappa_1, 2\kappa_2, \eta\Gamma^{-1}\}. \\ C &= \frac{1}{2} \gamma_1 d_M^2 + \frac{1}{2} \gamma_2 \varepsilon^{*2} + \frac{\eta}{2} \|W^*\|^2 \end{aligned} \quad (47)$$

Theorem 7. Consider the unknown DC motor control system (1), based on the assumptions of Assumptions 2 and 3, Lemma 4, actual controller (37), and the adaptive law (42). The following properties guaranteed that the tracking error singles will remain in a compact neighborhood of zero, that is, $\lim_{t \rightarrow \infty} |y(t) - y_d(t)| = 0$, all signals of the closed-loop system are bounded, and all state constraints are never violated.

Proof. With both sides of (46) multiplied by $e^{\rho t}$, we obtain

$$\frac{d}{dt} (V_3 e^{\rho t}) \leq C e^{\rho t}. \quad (48)$$

Integrating (48) over $[0, t]$, we have

$$0 \leq V_3(t) \leq V_3(0) e^{-\rho t} + \frac{C}{\rho}. \quad (49)$$

Based on (7), (24), and (39), we can obtain

$$\begin{aligned} V_3 &= \frac{1}{2} \log \frac{k_{b_1}^{2p}(t)}{k_{b_1}^{2p}(t) - z_1^{2p}} + \frac{1}{2} \log \frac{k_{b_2}^{2p}(t)}{k_{b_2}^{2p}(t) - z_2^{2p}} \\ &+ \frac{1}{2} \widehat{W}^T \Gamma^{-1} \widehat{W}. \end{aligned} \quad (50)$$

Then, we have

$$\frac{1}{2} \log \frac{k_{b_1}^{2p}(t)}{k_{b_1}^{2p}(t) - z_1^{2p}} \leq V_{3A}(t) \leq V_{3A}(0), \quad (51)$$

where

$$\begin{aligned} V_{3A}(0) &= \frac{1}{2} \log \frac{k_{b_1}^{2p}(0)}{k_{b_1}^{2p}(0) - z_1^{2p}} \\ &+ \frac{1}{4} \lambda_{\max}(\Gamma^{-1}) \|\widehat{W}(0) - W^*\|^2 + \frac{C}{2\rho}. \end{aligned} \quad (52)$$

Therefore, we know that

$$\left(\frac{z_1}{k_{b_1}(t)} \right)^{2p} \leq 1 - e^{-2V_{3A}(0)}. \quad (53)$$

Based on the above inequality, the following inequality is obtained:

$$|z_1(t)| \leq D_1(t), \quad (54)$$

where

$$D_1(t) = k_{b_1}(t) \sqrt[2p]{1 - e^{-2V_{3A}(0)}}. \quad (55)$$

Similar to the derivation of z_1 , we can obtain the conclusion that

$$|z_2(t)| \leq D_2(t), \quad (56)$$

where

$$D_2(t) = k_{b_2}(t) \sqrt[2p]{1 - e^{-2V_{3B}(0)}}. \quad (57)$$

From Assumption 2, we can be known that $|\alpha_1(0)| < K_{c_1}(0)$, and from the definition of $k_{c_1}(t)$, we have $|z_1(t)| < k_{b_1}(t)$. In fact, from $\alpha_1 = z_1 + y_d$ and $\alpha_2 = z_2 + \sigma_1$, we obtain

$$|\alpha_1(t)| < k_{b_1}(t) + y_d(t). \quad (58)$$

Based on the above inequality, we know $|y(t)| \leq k_{c_1}(t)$, $\forall t \geq 0$. Therefore, the output signals are bounded.

Obviously, we can clearly obtain that the virtual controller σ_1 is bounded in (15). Based on $z_2 = \alpha_2 - \sigma_1$ and (56), α_2 is bounded. In addition, from (37) and (42), we know the actual controller u and the adaptive law \widehat{W} are bounded. Therefore, all the closed-loop system signals are bounded.

The proof is completed. \square

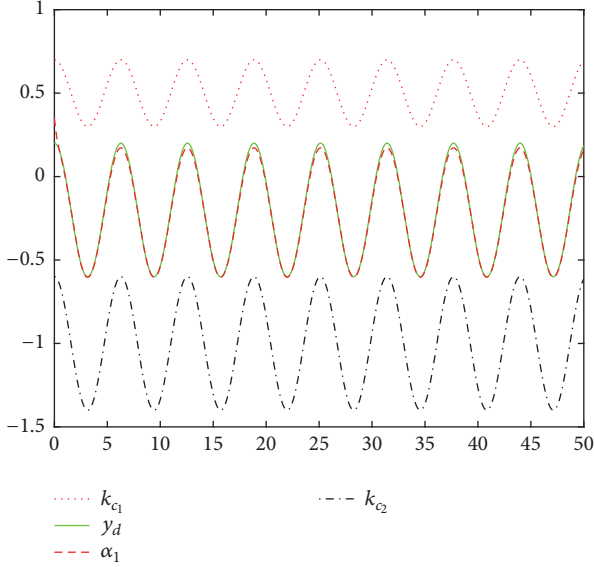
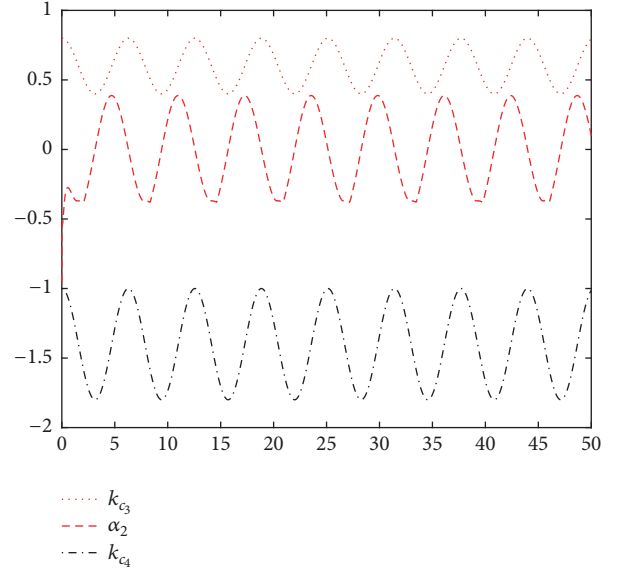
Remark 8. In the above analysis, it is apparent that the boundedness of z_1 lies on the design parameters γ_1 , γ_2 , η , d_M , ε^* , W^* , κ_1 , κ_2 , and Γ^{-1} . If we fix $\eta > 0$, it is clear that decreasing γ_i might result in small C and increasing κ_i might result in large ρ ; thus, it will help to reduce C/ρ . This represents that the tracking errors can be made arbitrarily small by selecting the design parameters appropriately.

4. Simulation Results

To illustrate the validity of the proposed adaptive NN control method, a simulation example is provided. Specifically, the following the DC motor system is described by

$$J\ddot{\alpha}_2 + f\dot{\alpha}_1 + T_f + d = u, \quad (59)$$

where the inertia is $J = 0.018 \text{ kg}\cdot\text{m}^2$, f denotes an unmeasured viscous friction with $f = \sin(t) + 1.82$, T_f is an unmeasured nonlinear friction with $T_f = 0.987$, and d is the external interference with $d = 0.05 \sin t$. The desired reference signal is given as $y_d = 0.4 \cos t - 0.2$. The virtual

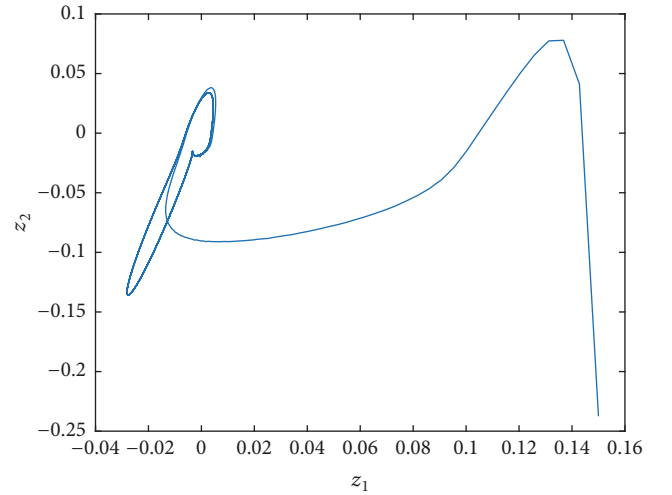
FIGURE 1: The trajectories of output α_1 and the reference signal $y_d(t)$.FIGURE 2: Trajectory of state α_2 .

controller, the actual controller, and the adaptation law are chosen as follows:

$$\begin{aligned}
 \sigma_1 &= -(\kappa_1 + \bar{\kappa}_1(t))z_1 + \dot{y}_d - \frac{2p-1}{2p}z_1 \\
 u &= -\frac{1}{2p}J\mu_1z_2(k_{b_2}^{2p} - z_2^{2p}) - J(\kappa_2 + \bar{\kappa}_2)z_2 \\
 &\quad - \frac{J^{-1}\xi_2^{2p}}{2\gamma_1z_2(1-\xi_2^{2p})} - \frac{J^{-1}\xi_2^{2p}}{2\gamma_2z_2(1-\xi_2^{2p})} \\
 &\quad + Jz_2\widehat{W}^TS(Z) \\
 \dot{\widehat{W}} &= \Gamma \left(-\frac{\xi_2^{2p}}{1-\xi_2^{2p}}S(Z) - \eta\widehat{W} \right).
 \end{aligned} \tag{60}$$

The angular position and the angular velocity of motor systems are bounded by $k_{c_1} < \alpha_1 < k_{c_2}$ and $k_{c_3} < \alpha_2 < k_{c_4}$ with $k_{c_1} = 0.5 + 0.2 \cos(t)$, $k_{c_2} = -1 + 0.4 \cos(t)$, $k_{c_3} = -1.4 + 0.4 \cos(t)$, and $k_{c_4} = 0.6 + 0.2 \cos(t)$. The NN $W^TS(z)$ contains 20 nodes and the centers μ_i , $i = 1, \dots, 20$. The design parameters of the proposed control method are chosen as $\Gamma = 2.5I$, $p = 2$, $\beta_1 = 4$, $\beta_2 = 4$, $\kappa_1 = 2$, $\kappa_2 = 2$, $c = 1$, and $T_f = 0.987$ and the initial condition of the system state is chosen as $\alpha_1(0) = 0.35$, $\alpha_2(0) = -0.95$, and $\widehat{W}(0) = 0$.

For the DC motor system, using a method of controlling the program can be obtained by the simulation results shown in Figures 1–5. Figures 1 and 2 show the output trajectory. Figure 1 shows the output and the reference signal tracking effect; the figure shows that the two curves almost coincide; that is to say, the tracking error converges to zero. Figure 3 shows the tracking error trajectory of $z_1(t)$ initially from the boundaries $k_{b_1}(t)$ and $-k_{a_1}(t)$ repulsion, but eventually converging to zero. Figure 4 shows a bounded and adaptive law of locus. According to Figure 4, we can see that the track

FIGURE 3: Phase portrait of z_1 and z_2 .

adaptation law is bounded. Thus, we can conclude that a good tracking performance can make all the signals in the closed-loop system bounded. From Figure 5, it can be observed that the control input is bounded by a bounded back and forth reciprocate.

5. Conclusion

In this paper, we propose an adaptive tracking control method for a DC system with full state constraints. The asymmetric time-varying BLF is employed to guarantee that the states always remain in the time-varying constrained sets. In the asymmetric system, neural networks and a backstepping technique are used to construct an adaptive control and adaptation laws to ensure that all signals in the closed-loop system are bounded and the state constraints are

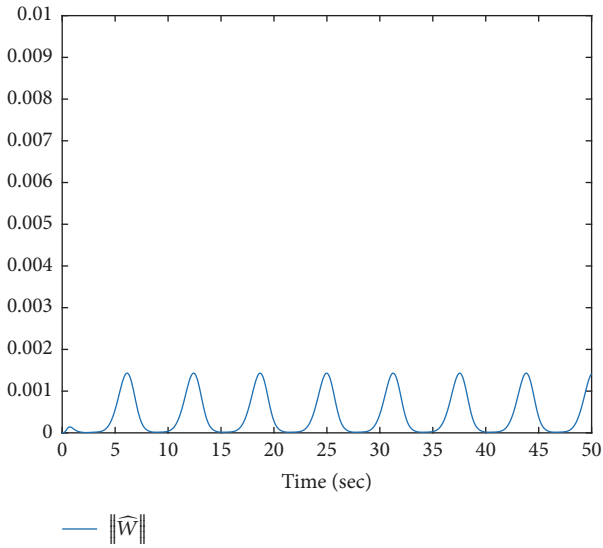


FIGURE 4: The trajectory of $\|\hat{W}\|$.

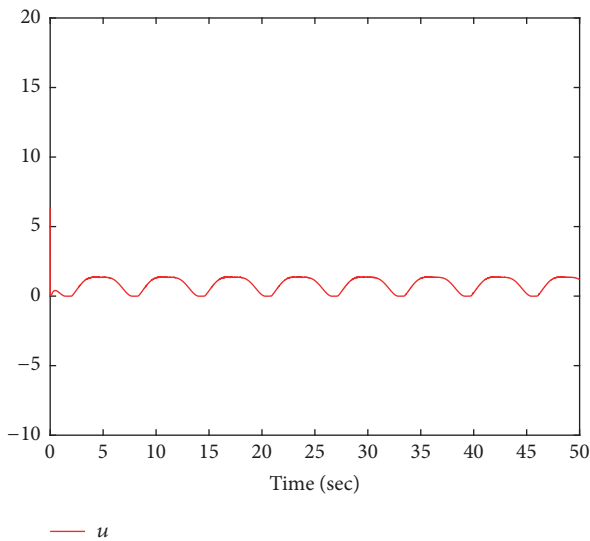


FIGURE 5: The trajectory of input u .

not transitioned. The performances of the adaptive control method based asymmetric time-varying BLF are verified by a simulation example.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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