Fuzzy Epistemicism*

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Abstract

It is taken for granted in much of the literature on vagueness that semantic and epistemic approaches to vagueness are fundamentally at odds. If we can analyze borderline cases and the sorites paradox in terms of degrees of truth, then we don't need an epistemic explanation. Conversely, if an epistemic explanation suffices, then there is no reason to depart from the familiar simplicity of classical bivalent semantics. I question this assumption, showing that there is an intelligible motivation for adopting a many-valued semantics even if one accepts a form of epistemicism. The resulting hybrid view has advantages over both classical epistemicism and traditional many-valued approaches.

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It is taken for granted in much of the literature on vagueness that semantic and epistemic approaches to vagueness are fundamentally at odds. If we can analyze borderline cases and the sorites paradox in terms of degrees of truth, then we don't need an epistemic explanation. Conversely, if an epistemic explanation suffices, then there is no reason to depart from the familiar simplicity of classical bivalent semantics. Thus, while an epistemic approach to vagueness is not logically incompatible with the view that truth comes in degrees, it is usually assumed that there could be no motivation for combining the two.

My aim in this paper is to question this assumption. After describing the way in which many-valued theories are usually motivated in opposition to epistemicism (§1), I give an argument for degrees of truth that even an epistemicist should be able to accept (§2). Unlike traditional motivations for degree theories, this argument is compatible with the epistemicist's claim that we are irremediably ignorant of the semantic boundaries drawn by vague terms, and with nonsemantic (epistemicist and contextualist) approaches to the sorites paradox. Thus it opens up conceptual space for a hybrid between fuzzy and epistemic approaches, a "fuzzy epistemicism." According to fuzzy epistemicism, both uncertainty and partial truth are needed to understand our attitudes towards vague propositions. In §3, I consider how this hybrid theory can respond to some traditional objections to many-valued theories.

I do not think that this all adds up to a compelling case for fuzzy epistemicism as the best approach to vagueness. As I will indicate, there are a couple of non-epistemicist approaches that seem at least equally promising. My aim here is to show that *if* one is inclined towards epistemicism, then (contrary to the conventional wisdom) one has good reason to accept degrees of truth as well.

1 The standard dialectic

If "tall man" has a classical extension,[1] then there is a shortest tall man. Of course, we have no way of knowing how tall the shortest tall man is. And even if we could know, the placement of the line between the tall and the non-tall would appear arbitrary. Unlike "gold" and "water", "tall" does not seem to pick out any kind of natural property. Nor does anything about our use of "tall" make any particular cut-off point salient. So classical semantics is committed to unknowable and arbitrary-seeming semantic boundaries.

Epistemicism is an attempt to bite this bullet, by explaining on general epistemological grounds why we should *expect* to be ignorant in just this way, and by rejecting as verificationist the idea that we should be in a position to know exactly where the semantic boundaries lie. According to the epistemic approach, what distinguishes vague language from non-vague language has nothing to do with truth-conditions. Formally, then, epistemicism is compatible with both classical and nonclassical semantics. Typically, however, epistemicists defend classical semantics.

One popular alternative to classical semantics is to suppose that truth comes in degrees. The most common form of this view represents these degrees by real

[1]Relative to a context and an index of evaluation. I will not repeat this qualification in what follows.

numbers between 0 and 1, with 1 representing complete truth, 0 complete falsity, and the intermediate values various degrees of "partial truth." The extensions of predicates are then naturally understood as fuzzy sets, or mappings from objects to degrees of truth. Thus, "tall man" may map a 7-foot man onto 1, a 6-foot man onto 0.75, a 5-foot-11 man onto 0.68, and so on. Small differences in height will yield small differences in the degree to which the predicate is satisfied. So as we look at shorter and shorter men, we will see a slow, steady decline in the degree to which "tall man" applies, rather than a sudden, precipitous change from inclusion in the extension to non-inclusion.

Such a theory affords an attractive analysis of the sorites paradox. Suppose we have a line of 100 men of gradually increasing height. Man 0 satisfies "tall man" to degree 0, man 1 to degree 0.01, man 2 to degree 0.02, and so on up to Man 100, who satisfies "tall man" to degree 1. Now consider the following sorites argument:

(1) Man 100 is a tall man.

(C_{100}) If Man 100 is a tall man, Man 99 is a tall man.

(C_{99}) If Man 99 is a tall man, Man 98 is a tall man.

⋮

(C_1) If Man 1 is a tall man, Man 0 is a tall man.

(2) Therefore, Man 0 is a tall man.

On the Łukasiewicz semantics for the conditional, $[\![A \rightarrow B]\!] = 1$ if $[\![B]\!] > [\![A]\!]$ and $1 - ([\![A]\!] - [\![B]\!])$ otherwise (where $\ulcorner [\![\phi]\!] \urcorner$ denotes the degree of ϕ). So all of the conditionals $C_1 \ldots C_{100}$ have degree 0.99. That is, they are all *almost* completely true, and that, the degree theorist proposes, is why we are inclined to accept them. But although modus ponens is valid in the sense of preserving degree 1, it is not valid in the sense of preserving degree of truth in general. Thus, when the premises of a modus ponens inference do *not* all have degree 1, the conclusion can have a lower degree than any of the premises. With each application of modus ponens, then, we lose a little truth, so that by the end of the argument we have none left at all.

Notice how the degree theory is motivated as an alternative to epistemicism. By positing a smooth continuum of partial truth, we avoid the need to explain how our linguistic practices could fix a sharp boundary between the tall and the non-tall, and why we could not know where it lies. And by making it possible to say that the premises of the sorites are almost completely true, we avoid the need to explain why we should be inclined to accept a conditional that is just plain false (as one of $C_1 \ldots C_{100}$ must be, if classical semantics is correct).

The standard epistemicist response to such theories is to argue that they merely put off the pain, because the epistemicist's resources will be needed anyway, at a later stage of analysis. So if the point of degree theories is to avoid having to tell epistemic stories, these theories are unmotivated. Let us look at some arguments to this effect.

1.1 Hidden boundaries

One of the things that seemed objectionable about classical semantics was its commitment to unknowable, arbitrary-seeming semantic boundaries. But do degree theories do better? Just as on classical semantics, there will be a shortest man who falls into the extension of "tall man," so on a many-valued semantics, there will be a shortest man who satisfies "tall man" to degree 1. A man 1 mm shorter than this man will not satisfy "tall man" to degree 1. We have no way of knowing where this boundary lies, and even if we could know it, it would seem arbitrary. So the degree theory does not have any evident advantage over classical semantics in this respect. Roy Sorensen puts the point effectively:

> …advocates of alternative logics that use the sensitivity objection against the epistemic approach are guilty of special pleading. Given that the super-valuationists and many-valued theorists cannot use the sensitivity issue to claim an advantage over classical logic, what is left to recommend their positions? The central motive for appealing to these alternative logics was to avoid the commitment to unlimited sensitivity. Once it is conceded that this appeal cannot succeed, there is no longer any point in departing from classical logic. (Sorensen 1988, 247)[2]

[2]See also Keefe 2000, 115: "The best epistemic theorists offer detailed explanations of *why* we are ignorant in a borderline case…; a degree theorist taking option (i) similarly owes us an explanation of the ignorance it postulates, but one that does not at the same time justify the epistemic theorist's position about first-order vagueness. It is far from clear that this can be done."

Degree theorists standardly respond that their precise assignments of degrees are meant as *models* of something imprecise. The sharp boundaries, they say, are just artifacts of the numerical models being used (Edgington 1997, 297, 308–9; Cook 2002). This is a plausible response, but more must be said. Degree theorists ought to say which features of their models are artifacts, and which are meant to represent real features of degrees of truth (Keefe 2000). An obvious thought is that the *ordering* of the numerical degrees represents the real ordering of degrees of truth, even if it is an artifact which degree is represented by the number 0.5. But if the ordering is non-artifactual, so is the boundary between the maximal degree and all the others. So, also, is the question which which of a series of successively taller women satisfies "tall" to a greater degree than Sarah satisfies "short."

Indeed, as Rosanna Keefe points out, the degree theorist cannot coherently hold that only ordinal relations between numerical degrees represent relations between real degrees of truth. For the Łukasiewicz semantics for the conditional makes the *ordinal* position of conditionals depend on the *absolute* difference of the numerical degrees of their antecedents and consequents (Keefe 2000, ch. 5). So if we have conditional propositions, then the absolute distances between numerical degrees cannot be artifacts of the model unless some facts about ordering are also artifacts.

A natural proposal, explored by Cook 2002, is that only *large* differences in numerical degree represent real differences in degrees of truth (cf. Edgington 1997, 297–8). As Cook shows (244), this proposal is not strictly tenable: for example, on Edgington's theory, if there are n mutually independent propositions, there will be at least some non-artifactual differences in degree less than or equal to $1/2^n$, and for plausibly large values of n, these differences will be very small. Importantly, though, these small differences will be knowable in principle, since they can be predicted from the semantics of the connectives. So perhaps it is a sufficient reply to the epistemicist's *tu quoque* about unknown and arbitrary semantic boundaries to say that

> . . . truth (and falsity) do come in gradations, and both large differences in real number assignments and the logical relations between complex sentences and their constituents are indicative of real aspects of vague natural language. On the other hand, the assignment of particular real numbers to particular sentences, and the resulting sharp boundaries, are just conveniences, incorporated into the semantics for the sake of simplicity, but reflecting nothing actually present in the discourse being modeled. (Cook 2002, 245)

As Cook notes, to say this is not to make the semantics itself imprecise, since the word "large" is used not in the formal semantics, but in our informal description of how the semantics models linguistic reality. The fit between a formal model and the reality it models should not be expected to be precise.

I won't try to assess this response here. What's important for our purposes is that both sides in the debate assume that, if the numerical degrees are viewed in a strongly representational way, and not as models with many artifactual features, then degree theory is unmotivated. Both sides agree that if we are going to accept hidden and arbitrary-seeming semantic boundaries, we might as well stick with a bivalent semantics. That is why the degree theorist must parry the classicist's *tu quoque* by adopting the modeling perspective.

1.2 The sorites

It might be thought that the attractive many-valued analysis of the sorites paradox provides an independent reason for preferring many truth values to two. But on closer examination, this apparent advantage evaporates. As Weatherson 2005 observes, the sorites is no less compelling when run with negated conjunctions instead of conditionals:

(1) Man 100 is a tall man.

(NC_{100}) It's not the case that Man 100 is a tall man and Man 99 is not.

(NC_{99}) It's not the case that Man 99 is a tall man and Man 98 is not.

⋮

(NC_1) It's not the case that Man 1 is a tall man and Man 0 is not.

(2) Therefore, Man 0 is a tall man.

But with the usual many-valued semantics for the connectives,[3] (NC_{50}) gets degree 0.5—meaning that it is no more true than false. What this shows is that we

[3] $[\![P \,\&\, Q]\!] = max([\![P]\!], [\![Q]\!])$ and $[\![\neg P]\!] = 1 - [\![P]\!]$. This is the semantics that is usually discussed in the philosophical literature on degree theories (e.g. in Machina 1976, Williamson 1994, and Keefe 2000). Different choices are made in the fuzzy logic literature (see Hajek 2006). In "Łukasiewicz logics," *strong conjunction* is defined as follows: $[\![P \,\&\, Q]\!] = max(0, [\![P]\!] + [\![Q]\!] - 1)$. If the conjunctions in our sorites are understood this way, the NC_i's will all have degree 0.99. However, as we will see in the next section, there are strong reasons (independent of the sorites) for the degree theorist *not* to define conjunction this way. See note 5, below.

can't hope to explain the plausibility of the sorites argument solely by pointing to the very high degree of truth of its premises, since only in the conditional version of the argument do all the premises have a high degree of truth.

This is not to say that a degree theorist can't explain the plausibility of the sorites—just that the explanation cannot advert to the "near complete truth" of the premises. Weatherson endorses Kit Fine's suggestion that we are prone to confuse P with ⌜Determinately P⌝, even when P occurs as part of a larger sentence. So we take the (NC_{50}) to be true because we conflate it with

(NC^d_{50}) It is not the case that Man 50 is determinately tall but Man 49 is determinately not tall.

But as Weatherson notes, "Fine's hypothesis gives us an explanation of what's going on in Sorites arguments that is available in principle to a wide variety of theorists"—supervaluationists, classical semanticists, and degree theorists alike. As a result, a degree theorist who makes use of this explanation cannot claim to have an advantage over any of these other theories in explaining the plausibility of sorites arguments.[4]

Other explanations of the pull of the sorites are also possible. Perhaps we mistake our inability to give a counterexample to (NC_{50}) for evidence of its truth. Williamson 1994 argues that, because of general "margin of error" requirements on knowledge, we could never *know* that we had a counterexample (234). Contextualists argue that active consideration of a particular height changes the context

[4]Weatherson, who is arguing for a kind of degree theory himself, concedes that he doesn't "have a *distinctive* story about the Sorites in terms of *truer*."

so that the extension of "tall" draws no boundaries there (Raffman 1996; Soames 1999; Fara 2000). Either of these strategies might explain why we are unable to refute (NC_{50}), and hence why it seems plausible.

There is no reason why a degree theorist couldn't appeal to these explanations of the plausibility of the sorites. But then the degree theorist's *semantics* would not be doing any work in explaining the apparent force of sorites arguments. So, one wonders, why not just stick with the simpler classical semantics?

To sum up: the usual motivations for a degree-theoretic account of vague expressions assume that epistemic accounts of the sorites and of borderline cases are untenable. Both sides in the debate agree that if the degree theorist were to accept the epistemicist's explanation of our ignorance of the locations of sharp semantic boundaries, the game would be lost. They agree that there would be no point being an epistemicist *and* accepting a many-valued semantics, since the epistemicism would deprive the many-valued semantics of any useful job to do.

2 A new argument for degrees

Having brought this assumption into the open, I now want to question it. I will present a new argument for a many-valued semantics for vague discourse. Unlike the standard motivations for degree theories, this one is compatible with epistemicism and does not require a "modeling" perspective on numerical degree values. The core of the argument is an acute observation by Schiffer 2003. Though Schiffer himself rejects degree theories and argues instead for a complex "psychological" theory, I will argue that the position that Schiffer's observation really supports is a degree theory that accepts hidden semantic boundaries—a hybrid of traditional degree theories and traditional epistemic theories.

2.1 Combining uncertainties

Consider Borderline Jim. He's just short of six feet tall, with a small tuft of hair on his head, and he's pretty fast at solving sudoku puzzles, though not as fast as his brother Bill. He is, we might say, borderline tall, borderline bald, and borderline smart. Given Jim's borderline status, it would be wrong for us to flat-out believe that he is tall, bald, or smart. But it would also be wrong to flat-out believe that he is not tall, not bald, or not smart. The appropriate attitude is something between full acceptance and full rejection, though what kind of attitude is less clear.

Classical semantics would seem to commit us to a particularly simple answer to this question. Since according to classical semantics, there are facts of the matter as to whether Jim is tall, bald, or smart, our attitude toward each of these propositions should be one of *uncertainty*. If Jim is a paradigm borderline case—right in the middle between clear satisfiers of these predicates and clear non-satisfiers—we might take it to be 50% likely that Jim is bald, 50% likely that he is tall, and 50% likely that he is smart. Rather than full belief, we will have partial beliefs—credences of 0.5—in each of these propositions.

But what should our attitude be to the *conjunction* of these propositions? Assuming (harmlessly, I think) that these propositions are stochastically independent, our credence in the conjunction ought to be the product of our credences in the conjuncts: 0.125. Classical semantics, then, recommends that we should endorse conjunctions of independent borderline propositions much less strongly than we endorse the conjuncts individually.

But, as Schiffer observes, this just seems wrong (Schiffer 2003, 204). It seems perfectly appropriate to endorse the conjunctive proposition that Jim is tall and bald and smart to about the same (middling) degree as we endorse the conjuncts separately. Certainly it seems wrong that we should be quite confident (0.875) that Jim *doesn't* have all three properties.

If you don't have these intuitions, try increasing the number of independent properties. With seven independent properties, your credence that Jim has all of them should be less than 0.01, and your credence that Jim doesn't have all of them greater than 0.99. That is, if Jim is also borderline fat, borderline old, borderline rich, and borderline nice, you should be *very* confident that he is not tall, bald, smart, fat, old, rich, and nice. Are you?

The argument, then, runs as follows:

1. If classical semantics is correct for vague discourse, then borderline propositions are either true or false; no finer distinctions are made.

2. If borderline propositions are either true or false, then (since we don't know which truth value they have) our attitudes toward them must be attitudes of uncertainty-related partial belief.

3. If our attitudes towards borderline propositions are attitudes of uncertainty-related partial belief, they ought to obey norms of probabilistic coherence.

4. We regard the propositions *Jim is tall*, *Jim is bald*, and *Jim is smart* as independent. That is, we don't think Jim's being bald (or smart, or bald and smart) would make it any more likely that he is tall, and so on.

5. Probabilistic coherence demands that our credence in the conjunction of several propositions we take to be independent be the product of our credences in the conjuncts.

6. But it is not the case that we ought to have much less credence that Jim is bald and tall and smart than we have that he is bald.

7. Therefore, classical semantics is not correct for vague discourse.[5]

Unlike the usual arguments against classical semantics for vague discourse, this argument is not aimed at the classicist's commitment to unknowable and arbitrary-seeming semantic boundaries, and it has nothing to do with sorites arguments. Instead, it is aimed at the idea that our attitude toward borderline propositions is one of uncertainty as to whether they are true or false.[6]

[5]A similar argument can be used to rule out many-valued theories in which conjunction is understood as Łukasiewicz "strong conjunction" (see note 3, above). On such theories, $P \& Q \& R$ will have degree 0 when P, Q, and R each have degree 0.5. So this kind of fuzzy theorist will be even less well placed than the classical logician in accounting for our partial endorsement of the conjunction.

[6]Sorensen seems to reject the intuition that supports premise (6). He argues as follows against degree theories: "... suppose a speaker begins by describing Ted as short and then adds that he is also fat, bald, smart, athletic, and rich. We assign a degree of truth of 0.5 to 'Ted is short' and 0.6 to each of the remaining attributions. But contrary to the conjunction rule [of many-valued semantics], we do not believe that 'Ted is short, fat, bald, smart, athletic, and rich' equals the degree of truth of 'Ted is short.' Our uncertainties compound making us assign a much lower degree of truth to the claim that Ted exemplifies the conjunctive predicate. ... Also notice that 'Ted is fat, or bald, or smart' is less of a borderline attribution than 'Ted is fat.' " (Sorensen 1988, 235–6). Note that this argument just assumes that the degrees represent "uncertainties," which the

One might try to defend classical semantics by rejecting (2). This is essentially what Schiffer does. (Though he does not present his view as a way of defending classical semantics, he emphasizes that it is a *psychological* solution to the sorites, and is thus at least *consistent* with classical semantics.) Schiffer argues that our attitude to borderline propositions is not standard uncertainty-related partial belief (SPB), but a special kind of vagueness-related partial belief (VPB): "It is a *primitive and underived* feature of the conceptual role of each concept of a vague property that under certain conditions we form VPBs involving that concept, and *it is in this that vagueness consists*" (Schiffer 2003, 212). VPBs are distinguished from SPBs in the following ways (198–207):

- SPBs represent uncertainty, while VPBs represent ambivalence.

- SPBs generate corresponding likelihood beliefs, while VPBs do not. If one has a SPB of 0.5 that one left one's glasses at the office, one will take it to be 50% likely that one's glasses are at the office. But if one has a VPB of 0.5 that Jim is bald, one will not take it to be 50% likely that he is bald.

- Generally, if one has an intermediate SPB that *p*, one thinks that one is

degree theorist ought to deny.

An alternative way of rejecting (6), suggested by an anonymous referee, would be to acknowledge the intuitions that are taken to support it, but claim that they are misleading and not to be taken as normative. Psychologists have shown that ordinary intuitions about probabilities frequently violate even the most basic norms of probabilistic coherence: in one famous case, a majority of subjects took a conjunction to be more likely than one of its conjuncts (Kahneman and Tversky 1983; for a different interpretation of the data, cf. Crupi et al. forthcoming). Could it be that the intuitions to which Schiffer has drawn our attention are the result of the "conjunction fallacy" or something similar? That seems unlikely, since these intuitions can be found even in those who are not prone to probabilistic fallacies when vagueness is not in play. But there is room for further empirical investigation here.

not in the best possible epistemic position to pronounce on *p*. But one can have an intermediate VPB that *p* and think that one could not be in a better epistemic position to pronounce on *p*.

- SPBs are governed by norms of probabilistic coherence, whereas VPBs are governed by the Łukasiewicz many-valued truth tables. Thus, if one has a VPB of 0.5 that Jim is bald and a VPB of 0.5 that Jim is tall, one ought to have a VPB of 0.5 that Jim is bald and tall, even when the conjuncts are independent.

It's this last feature that allows Schiffer's theory to say that our degree of belief that Jim is tall and bald and smart shouldn't be less than our degree of belief in any of the conjuncts singly, when Jim is a borderline satisfier of each predicate.

Schiffer insists, reasonably, that

$$(*) \quad SPB(p) + SPB(\neg p) + VPB(p) + VPB(\neg p) = 1.$$

Where *p* is a complete borderline case, SPB(*p*) and SPB($\neg$*p*) will both be 0, and VPB(*p*) and VPB($\neg$*p*) will sum to 1; where *p* is fully determinate, the VPBs will be 0 and the SPBs will sum to 1. But mixed cases are also possible, and on these Schiffer's theory runs aground. Suppose, for example, that you think there's a 50% chance that Sam is completely hairless and a 50% chance that he has about 50 hairs on his head. (You can't remember which of two men he is.) If you knew he was completely hairless, you'd have an SPB of 1 that Sam is bald. If you knew that he had 50 hairs, you'd have a VPB of 0.8 that Sam is bald, and of 0.2 that he is not bald. But given your uncertainty, you're in a mixed state, with some SPB and some VPB in both the proposition that Sam is bald and its negation. Schiffer gives some plausible principles for computing SPBs and VPBs in cases like this, but as I show in MacFarlane 2006, they are inconsistent with (*).[7] The basic problem should be evident: the norms governing SPBs and VPBs are fundamentally different, so they are not going to march in the kind of lockstep that would be needed to keep them summing to 1.[8]

2.2 Taking-to-be-partially-true

Let us return to the problem Schiffer's theory was supposed to solve. Some kind of partial or qualified endorsement seems appropriate for borderline propositions. However, this partial endorsement does not seem to be standard uncertainty-related partial belief, since if it were, the degree of endorsement would drop dramatically as we added independent conjuncts. How, then, should we understand it?

Here, at last, we have a task degrees of truth are well suited to perform. My proposal, to simplify slightly, is that we understand this partial endorsement not

[7]The fix Schiffer proposes in his reply (Schiffer 2006) does not work. In fact, the first counterexample in MacFarlane 2006—$SPB(p) = VPB(p) = SPB(q) = VPB(q) = 0.3, SPB(\neg p) = VPB(\neg p) = SPB(\neg q) = VPB(\neg q) = 0.2$—is a counterexample to Schiffer's revised proposal as well, and it is easy to generate others.

[8]An alternative approach, due to Hartry Field (2003), is to avoid positing VPBs but allow $SPB(p) + SPB(\neg p) < 1$. In cases we take to be completely indeterminate, $SPB(p) + SPB(\neg p)$ will be 0. Field's approach agrees with Schiffer's in predicting that one should have the same degree of belief in the proposition that Jim is tall and tall and smart that one has in the conjuncts separately, but disagrees about what this degree should be—for Field, it is 0. Schiffer objects (210 n. 38) that agents should not have the same degree of belief (0) in propositions they take to be borderline as they do in propositions they take to be determinately false.

as partial belief in the truth of a proposition, but as belief in its partial truth. That is not quite the right thing to say, as it makes the attitude seem like a thought about a proposition, not about (say) Jim. In addition, it makes it seem as if the attitude requires deployment of a concept of degrees of truth—a concept many believers lack. But just as we might usefully understand first-order belief as taking-to-be-true, so we might understand the first-order partial endorsement appropriate in borderline cases as taking-to-be-partially-true (for example, taking-to-be-true-to-degree-0.5). In describing the attitudes this way, we identify them by their constitutive aims. Mark Sainsbury puts the point well:

> Truth is what we seek in belief. It is that than which we cannot do better. So where partial confidence is the best that is even theoretically available, we need a corresponding concept of partial truth or degree of truth. Where vagueness is at issue, we must aim at a degree of belief that matches the degree of truth, just as, where there is no vagueness, we must aim to believe just what is true. (Sainsbury 1995, 44)

An attitude towards *p* that a cognitive system normatively "aims" to be in just in case *p* is true can justly be called "taking-to-be-true," even if the possessor of this attitude lacks an explicit concept of truth. Similarly, an attitude towards *p* that a cognitive system normatively aims to be in just in case *p* is true to degree *N* can be justly be called "taking-to-be-true-to-degree-*N*," even if the possessor of the attitude lacks an explicit concept of partial truth.

Attitudes of taking-to-be-partially-true, I suggest, can do all of the work Schiffer aimed to do with his VPBs:

1. They can be clearly distinguished from attitudes of uncertainty. They reflect, rather, ambivalence: in a case where I take *p* to be partially true and partially false, I am ambivalent about whether *p*.

2. They fail to generate likelihood beliefs. To take *p* to be true to degree 0.3 is not to take it to be 30% likely that *p*.

3. Taking *p* to be partially true is consistent with taking oneself to be in the "best possible epistemic position to pronounce on *p*." Partial truth is an objective status, not a feature of the thinker's mental state or epistemic position.

4. Attitudes of taking-to-be-partially-true, unlike attitudes of partial belief, are not governed by norms of probabilistic coherence. If one takes the propositions that Jim is tall, that Jim is bald, and that Jim is smart to be true to degree 0.5, then one should take their conjunction to be true to degree 0.5 also. (On the Łukasiewicz semantics for continuum-valued logics, the degree of a conjunction is the minimum of the degrees of its conjuncts.)

Schiffer's VPBs look like a way of trying to get the benefits of a degree theory without accepting the idea that truth comes in degrees. But why not go for the original instead of this ersatz? Schiffer offers two arguments, neither of which is compelling.[9]

[9]Here I echo some of the discussion of MacFarlane 2006.

His first argument is that degree theories cannot capture what Crispin Wright calls "the absolutely basic datum that in general borderline cases come across as hard cases" (Wright 2001, 69–70). Schiffer argues that a degree theorist

> ... is evidently constrained to hold that *p* is true just in case *p* is T to degree 1 (or—allowing for the vagueness of ordinary language 'true'—to a contextually relevant high degree); false just in case *p* is T to degree 0 (or to a contextually relevant low degree); and neither true nor false just in case p is T to some (contextually relevant) degree greater than 0 and less than 1. But suppose Harry is borderline bald. Then, since it would be definitely wrong to say that 'Harry is bald' is T to degree 1 (or to some other contextually relevant high degree), the theory entails that it would also be definitely wrong to say it is true that Harry is bald. But if Harry is borderline bald, it would *not* be definitely wrong to say that he's bald, and thus not definitely wrong to say it's true that he's bald. (Schiffer 2003, 192)

In assuming that a degree theorist is "constrained to hold" that *p* is true simpliciter just in case its degree of truth exceeds some (perhaps contextually determined) threshold, Schiffer is thinking of a degree theory as a way of systematizing all-out truth and falsity assignments. That is one kind of degree theory. But on the more thoroughgoing degree theory recommended here, the degrees are given a significance *directly*, not indirectly through their role in systematizing "designatedness" or all-out truth.[10] According to this theory, when it is true to degree 0.5 that Harry

[10]Compare the discussion of M vs. M_D in Weatherson 2005, §1. The fact that normal talk of

is bald, it will be just as correct to believe that Harry is bald as it is to believe that Harry is not bald, and it will be just as correct to believe that it is true that Harry is bald as it is to believe that it is false that Harry is bald. This, I think, admirably captures the "ambivalence" we feel in borderline cases. Schiffer mischaracterizes this ambivalence in representing it as indecision about whether to *assert* the borderline proposition. It simply *isn't* correct to assert p when p is a borderline proposition, unless one is trying to effect some kind of "accommodation" (Lewis 1979) that would make it no longer count as borderline.

Schiffer's second argument against degree theories is that they allow that certain classically valid modes of inference (for example, *reductio ad absurdum*) can take one from premises that are true to degree 1 to a conclusion that is true to a degree very close to 0. His example is

> A person with \$50 million is rich.
>
> A person with only 37¢ isn't rich.
>
> Therefore, it's not the case that, for any n, if a person with \$$n$ is rich, then so is a person with \$$n - 1$¢.

which, on the degree-theoretic analysis, has premises true to degree 1 and a conclusion true to a degree slightly greater than 0. This, he says, is "apt to seem flat-out unacceptable" (193).

But why? If we agree that the premises are true and want to reject

---

truth and falsity does not include degree qualifiers is no obstacle for this view, since on a natural semantics for "true," "It is true that Harry is bald" will have exactly the same degree of truth as "Harry is bald." In fact, it *must* have the same degree of truth if the biconditional "Harry is bald iff it is true that Harry is bald" is to get degree 1 on the Łukasiewicz semantics.

(C) For some n, a person with \$$n$ is rich and a person with \$$n - 1$¢ is not rich,

then we have to give up *some* classically valid principle of reasoning. And a many-valued semantics gives an illuminating story about *why reductio* should fail in vague contexts. If we derive a contradiction from premises S_1, S_2, S_3 using valid (1-preserving) inference rules, then we know that at least one of them has degree less than 1. If we also know that S_1 and S_2 have degree 1, then we can infer that S_3 has degree less than 1. But all we can conclude about $\neg S_3$ is that it has degree greater than 0. We certainly cannot conclude that it has degree 1. That's why *reductio* fails in this context. Given that *something* needs to be done to block the reasoning that leads to (C), recognizing limits on the use of *reductio* seems well-motivated and at least as moderate as Schiffer's own solution, according to which it is *indeterminate* whether classical inference rules—including not just *reductio* but even *modus ponens*—are valid (Schiffer 2003, 224).

I suggest, then, that we explicate the kind of partial endorsement that is appropriate in borderline cases—what Schiffer calls "vagueness-related partial belief"—as taking-to-be-partially-true.

2.3 Combining partial truth with uncertainty

As we have seen, Schiffer's theory founders in its attempts to integrate two separate aspects of partiality of belief: the "ambivalence" that stems from vagueness and the uncertainty that stems from incomplete information. Can the present approach do better in integrating taking-to-be-partially-true with partially-taking-to-be-true?

This is a problem that any degree theorist must face in “mixed cases,” where the degree of truth of a vague proposition (say, *Sam is bald*) depends on some non-vague matter about which there is uncertainty (say, the number of hairs on Sam’s head). But the problem is especially acute for theorists who view all facts about the ordering of numerical degrees to be representationally significant (not artifacts of the model), since on their view *every* attitude towards a borderline proposition will combine ambivalence and uncertainty. We will never be in a position to know who is the shortest man who satisfies “tall man” to degree 1, and we will have no good basis for taking the proposition that Jim is tall to be true to degree 0.653 rather than 0.649. We may be confident that he satisfies “tall” to some intermediate degree, and perhaps we’d bet on 0.6 over 0.5, but there will remain some uncertainty. So, to model our attitudes towards borderline propositions, we will need to take into account both dimensions of partiality: ambivalence and uncertainty.

The most straightforward way to do this, I think, is to represent our attitudes to vague propositions as probability distributions over degrees of truth (strictly speaking, over an algebra of precise propositions that ascribe degrees of truth to the vague propositions at issue). So, for example, your attitude towards the proposition *Jim is tall* might be depicted by Figure 1, where the horizontal axis represents degrees of truth and the height of the curve over any given degree represents the probability that *Jim is tall* has that degree of truth. This picture combines both dimensions of partial endorsement, taking-to-be-partially-true and partially-taking-to-be-true, in a unified representation.[11]

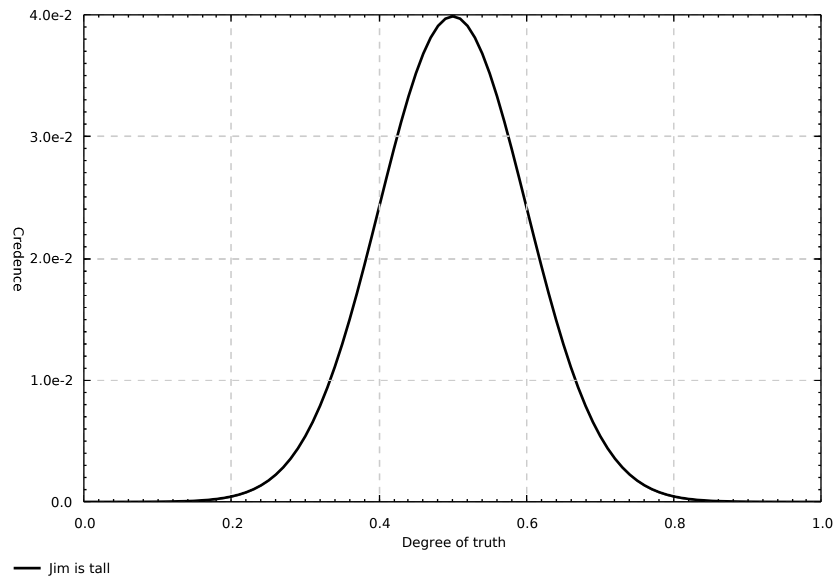


Figure 1: $Pr([\![Jim\ is\ tall]\!] = x)$.

This approach can deal straightforwardly with the "mixed cases" that proved troublesome for Schiffer's theory. Suppose you aren't sure exactly how many hairs Tom has on his head. Your credence function is represented by Figure 2, where the vertical axis represents probabilities and the horizontal axis the number of hairs.

For each possible number of hairs x, there will be a probability distribution

[11]This graph, and those that follow, was generated by a custom Haskell program using Tim Docker's Charts library and Martin Erwig's Probabilistic Functional Programming library (Erwig and Kollmansberger 2006). To simplify the calculations in these charts, I use a *finite* set of degrees, $\{0, 0.01, 0.02, \ldots, 0.99, 1\}$. This allows us to do probability calculations using simple algebra. Everything said here should generalize to real-valued degrees, but more complex methods would be needed to handle probability distributions over the degrees (see Zadeh 1968).

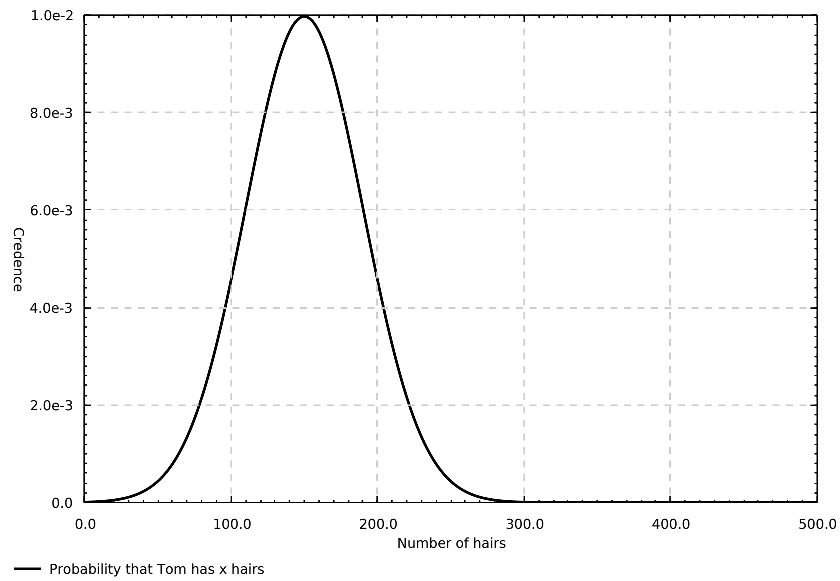


Figure 2: *Pr*(*Tom has x hairs*).

over degrees of truth that represents the attitude you would have towards the vague proposition *Tom is bald* if you knew that Tom had exactly *x* hairs. Three of these distributions are plotted in Figure 3.

Taking into account your uncertainty about the number of hairs Tom has on his head, what should be your attitude towards the vague proposition *Tom is bald*? Since Figures 2 and 3 both represent probability distributions, the solution is a simple application of probability theory. We construct a probability distribution over assignments of degrees of truth to *Tom is bald* as follows:

$$Pr([\![\textit{Tom is bald}]\!] = x) = \sum_{0 \leq n < 500} Pr(\text{Tom has } n \text{ hairs}) \times Pr([\![\textit{Tom is bald}]\!] = x \,|\, \text{Tom has } n \text{ hairs})$$

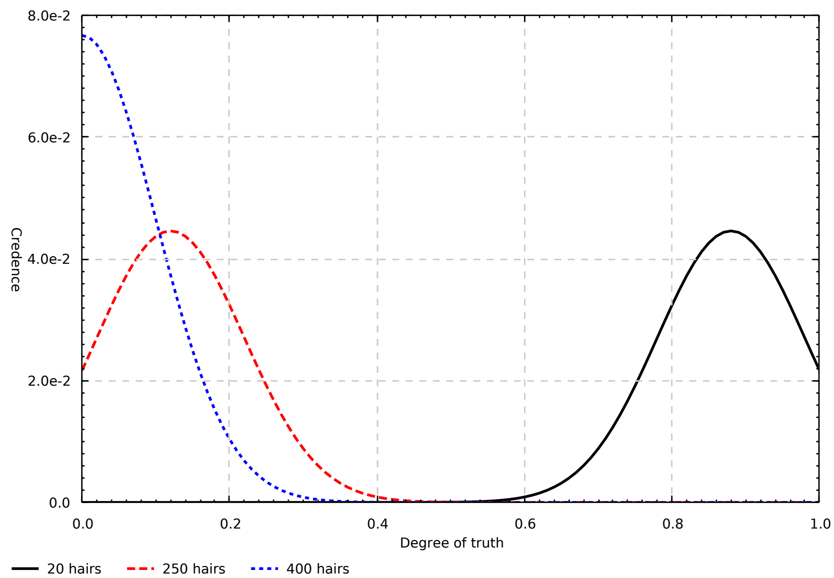


Figure 3: $Pr([\![Tom\ is\ bald]\!] = x \,|\, \text{Tom has } n \text{ hairs})$, for $n = 20, 250, 400$.

The resulting curve, which represents your composite attitude of partial endorsement toward *Tom is bald*, is displayed in Figure 4.

Given probability distributions over degree-assignments for atomic propositions, we can easily calculate distributions for truth-functional compounds. This is most straightforward in cases where the conjuncts are degree-independent:

> *Definition:* $P_1, \ldots, P_n$ are *mutually degree-independent* iff for all subsets $\{P_j, \ldots, P_k\}$ of $\{P_1, \ldots, P_n\}$ containing two or more elements, and for all $0 \geq d_j, \ldots, d_k \leq 1$,
>
> $$Pr([\![P_j]\!] = d_j \,\&\, \ldots \,\&\, [\![P_k]\!] = d_k) = Pr([\![P_j]\!] = d_j) \times \cdots \times Pr([\![P_k]\!] = d_k).$$

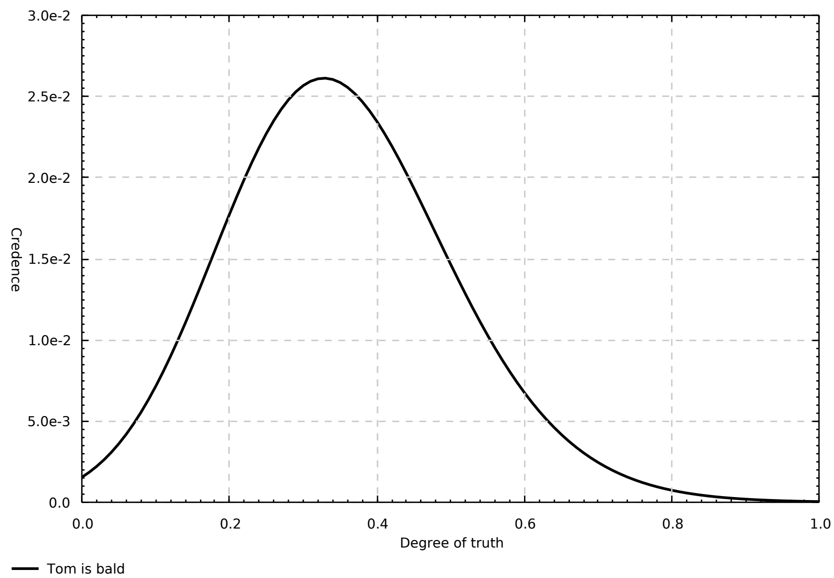


Figure 4: $Pr([\![Tom\ is\ bald]\!] = x)$.

If the propositions *Jim is tall*, *Jim is bald*, and *Jim is smart* are degree-independent (as seems plausible), then the probability that they will have degrees d_1, d_2, and d_3 respectively is just

$$Pr([\![Jim\ is\ tall]\!] = d_1) \times Pr([\![Jim\ is\ bald]\!] = d_2) \times Pr([\![Jim\ is\ smart]\!] = d_3).$$

And the degree of the conjunction *Jim is tall* & *Jim is bald* & *Jim is smart* on this assignment of degrees to the conjuncts is just the minimum of $\{d_1, d_2, d_3\}$. So we can compute the probability that the conjunction has degree x by summing the probabilities of combinations $d_1, \ldots, d_n$ of degrees whose minimal member = x.

The result is given in Figure 5.[12]

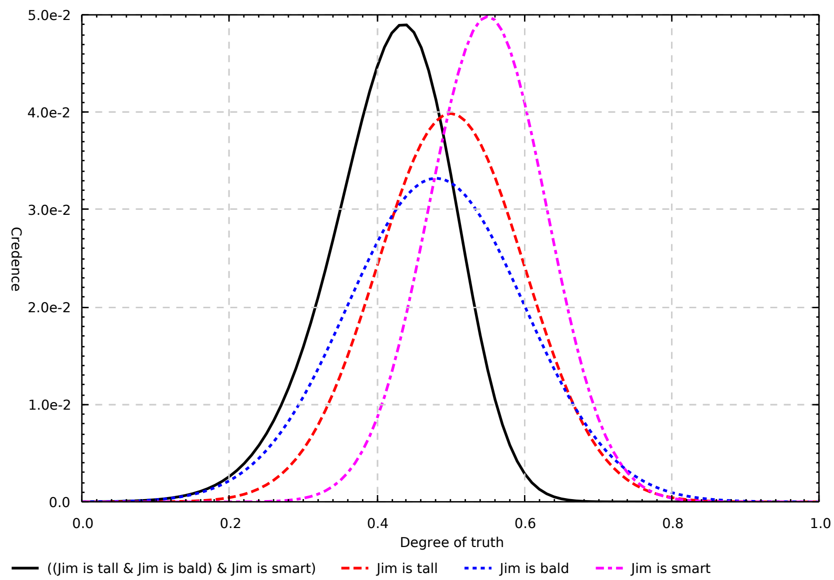


Figure 5: $Pr(\llbracket \textit{Jim is tall and bald and smart} \rrbracket = x)$.

Our complaint about classical semantics was that it predicts that one should have far less confidence in the conjunction *Jim is tall and bald and smart* than in any of the conjuncts. The view now being considered, by contrast, predicts that one should have *a little* less confidence in the conjunction than in the conjuncts. The more uncertainty there is about the degrees of the conjuncts, the larger the drop in confidence will be (see Figure 6). This seems to me about the right result: midway between Łukasiewicz and Williamson.[13]

[12]Cases in which the conjuncts are not independent can also be handled, though with added complexity. We consider such a case below (§3.3).

[13]A similar view is defended by Nicholas Smith in his contribution to this volume. The main difference is that Smith defends the view that one's "degrees of belief" in a proposition p should

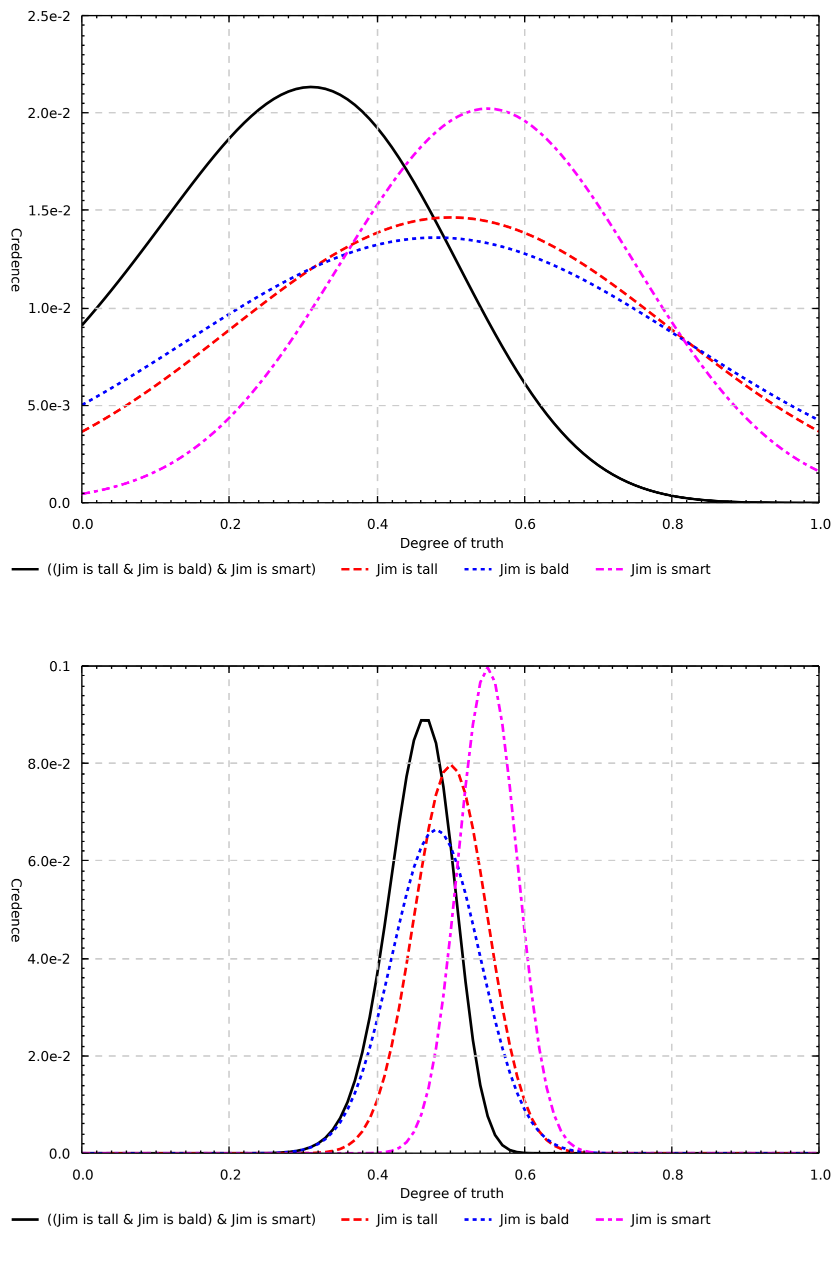


Figure 6: Conjunction with more and less uncertainty.

3 Traditional objections reconsidered

Traditionally, one of the most serious problems for degree theories has been the absence of any compelling motivation. Degrees of truth do not seem to be needed, as some thought they were, to understand the semantics of graded adjectives, hedges, or the ordinary predicate "true" (see Lakoff 1973, Williamson 1994, Haack 1996, Kennedy 2007). Nor is it clear that degree theories provide a better diagnosis of the sorites paradox than is available to the classical semanticist. Finally, it is not clear that degree theories can avoid a commitment to arbitrary-seeming and unknowable semantic boundaries, so if *that* was what was objectionable about classical semantics, degree semantics fares no better. If we need epistemicism anyway, why should we abandon the elegant simplicity of classical semantics?

We can now answer this question. Classical semantics should be rejected for vague discourse because it forces us to think of the partial endorsement appropriate in borderline cases as a kind of uncertainty. As Schiffer observed, this conception yields implausible recommendations about our attitudes towards conjunctions of independent borderline propositions. A many-valued semantics provides

be identified with the *expected value* of the degree of truth of p—that is, with the average degree of truth weighted by probability—while I do not attempt to arrive at a single-number degree of belief. While it might be useful for some purposes to measure beliefs by the expected value of degree of truth, it seems rash to suppose that all the interesting quantitative differences between partial belief states can be boiled down to this one number. For example, compare (a) a belief state that assigns equal credence to each degree of truth (a flat line on our graphs), (b) a belief state that assigns near certainty to degree 0.5 (a sharp spike), and (c) a belief state in which credence clusters around two points, degree 0.2 and degree 0.8 (two humps with a dip in the middle). In all three cases the expected value of degree of truth will be 0.5, but these belief states can be expected to have different effects on behavior and inference. Hence I prefer to represent states of partial belief in two dimensions, rather than attempting to integrate the uncertainty and partial-truth aspects into one number. That said, Smith and I agree about much more than we disagree about.

an elegant way to represent and work with the two kinds of partiality that can characterize our attitudes to borderline propositions: ambivalence (taking-to-be-partially-true) and uncertainty (partially-taking-to-be-true). It can be motivated on these grounds *even if* we accept hidden semantic boundaries and a diagnosis of the sorites paradox that is compatible with classical semantics.

But not all of the worries people have had about degree theories concern motivation. It has been alleged that such theories run into problems with higher-order vagueness, and that their use of numerical degrees involves an implausible commitment to the comparability of degrees of truth. In addition, many criticisms have been raised against degree-functional semantics for the connectives, and specifically against the *min* rule for calculating the degree of a conjunction. In this section, I consider how a fuzzy epistemicist might respond to these objections.

3.1 Higher-order vagueness

A classic objection to degree theories is that, even if they do give a nice story about borderline cases and the sorites paradox, all the problems come back at a higher level. For example, it is alleged that someone could be borderline between satisfying "tall" to degree 1 and satisfying "tall" to a degree less than 1. If so, we could construct a new sorites on the predicate "satisfies 'tall' to degree 1," and the degree theory would have no distinctive diagnosis of this higher-order sorites.

This is clearly not a problem for fuzzy epistemicism, which does not make use of degrees of truth to give a diagnosis of the sorites. As we saw in section 1.2, the many-valued analysis does not work well for sorites arguments using negated conjunctions, so some other story about the sorites is needed anyway. The motivation for degrees of truth offered in the last section is consistent with a number of different possible accounts of the sorites paradox, including epistemic and contextualist accounts.

Indeed, I'm not convinced that the higher-order sorites poses a serious worry even for standard degree theories. The predicate "satisfies 'tall' to degree 1" is sufficiently theoretical that it's not clear why we should accept a sorites premise formulated with it. Perhaps that is why the objection is usually run using a sentential operator D (for "definitely"), stipulated to have the following semantics:

$$\begin{aligned} [\![D\phi]\!] &= 1 \text{ if } [\![\phi]\!] = 1 \\ & \quad\ 0 \text{ otherwise.} \end{aligned}$$

We do have a strong inclination to accept a sorites premise for "definitely tall." But it's not clear that the ordinary meaning of "definitely" matches that of D as defined above. More plausibly, "definitely ϕ" means something like "ϕ is true enough, by a good margin, for present purposes," or "ϕ has degree 1 by a good margin," and on this understanding we should expect "definitely ϕ" to take non-extremal degrees, since "enough" and "good margin" are vague. If that is right, then a degree theory can say exactly the same thing about a sorites with "definitely tall" that it says about a sorites with "tall."

3.2 Comparability

Any theory that represents degrees of truth as (real or rational) numbers, and takes the ordering of these numbers to be significant, not an artifact of the model, is committed to the degrees being *totally ordered*: for any sentences *A* and *B*, the degree to which *A* is true will be either less than, equal to, or greater than the degree to which *B* is true. Both critics and friends of degree theories have suggested that "multidimensional predicates" pose a problem for the idea that degrees are totally ordered.[14] Here is Williamson's version of the complaint:

> Comparisons often have several dimensions. To take a schematic example, suppose that intelligence has both spatial and verbal factors. If *x* has more spatial intelligence than *y* but *y* has more verbal intelligence than *x*, then '*x* is intelligent' may be held to be truer than '*y* is intelligent' in one respect but less true in another. Moreover, this might be held to be a feature of the degrees to which *x* and *y* are intelligent: each is in some respect greater than the other. How can two real numbers be each in some respect greater than the other? On this view, degrees are needed that preserve the independence of different dimensions, rather than lumping them together by an arbitrary assignment of weights. (Williamson 1994, 131)

Multidimensional predicates do pose a problem for degree theorists who understand degrees of truth in terms of comparatives. But the present theory explicates

[14] In addition to Williamson, quoted below, see Goguen 1969, 350–1, Forbes 1985, 175, Keefe 2000, 129, and Weatherson 2005.

and motivates degrees of truth in terms of their role in explaining our attitude of declining partial endorsement of successive members of a sorites series, not by appeal to comparatives. *A is F-er than B* can be true to degree 1 even when *A is F* and *B is F* have the same degree of truth (Williamson 1994, 126). Multidimensional predicates just give us one more reason not to tie our understanding of degrees of truth to comparatives.

The same considerations provide a response to this argument, from Keefe 2000:

> ...consider the case where p = '*a* is tall' and q = '*b* is red'. Here we have no single comparative on which 'true to a greater degree' can piggy-back. The comparison may be read as '*a* is more clearly tall than *b* is red' and if, for example, *a* is clearly tall and *b* is clearly not red, then this will be true. But in a wide variety of cases (e.g. with a 5-foot 10-inch man and a reddish-orange patch), neither disjunct of (C_T) ["either $p \geq_T q$ or $q \geq_T p$"] will be true. (129)

If our sole grip on the notion of degrees of truth came from comparatives, then in cases like these, where there are no comparatives to appeal to, there would be a case for saying that there is no fact of the matter about whether *Joe is tall* is truer than *Patch #50 is red*. But we have rejected the close tie between comparatives and degrees of truth. So why think that the difficulty in determining which proposition is truer is anything other than epistemic? Indeed, an epistemic difficulty is to be expected on the present theory, since our attitudes towards these propositions are represented as probability distributions over degrees. When their curves overlap, we will not be in a position to know which proposition has the higher degree.

Of course, I am not opposed to the development of non-numerical degree theories that relax the requirement that degrees be totally ordered. I am just expressing skepticism about the usual motivations for such theories. There are, in addition, technical reasons for wanting degrees to be totally ordered. One is that it is unclear how to define negation with partially ordered degrees (see Williamson 1994, 133). Weatherson 2005 ends up with Boolean negation, but this is patently unsuited to the purposes of a degree theory, as it allows the degree of A to be greater than the degree of $\neg A$ only when A is determinately true and $\neg A$ determinately false.[15] Surely a degree theory ought to allow that, say, *Man 60 is tall* can be truer than it is false—truer than *Man 60 is not tall*—even if it is not completely true.

3.3 Degree functionality

Another standard group of objections to degree theories is directed at the degree-functional semantics for the connectives. These objections are often regarded as devastating, even by those sympathetic with degrees (most prominently, Edgington 1997, who argues for a degree theory with non-degree-functional connectives). Although the motivation for fuzzy epistemicism does not assume that the connectives are degree-functional, it does assume that the degree of a conjunction of independent propositions whose degrees are 0.5 is 0.5, and none of the

[15]Proof: Suppose $A \geq_T \neg A$. Then $A \,\&\, \neg A \geq_T \neg A$, because $\neg A \geq_T \neg A$, and the degree of a conjunction is the greatest lower bound of the degrees of its conjuncts under the $\geq_T$ ordering. But then, since $A \,\&\, \neg A$ is determinately false, so is $\neg A$. See Weatherson 2005, 67.

non-degree-functional degree theories I know of deliver that result.[16] But I am not convinced that the arguments against degree-functionality are compelling, especially when one takes account of the fact that our attitudes toward borderline propositions will generally be mixed with uncertainty.

I will focus here on negation, conjunction, and disjunction, ignoring the conditional. This is fair, I think, for a couple of reasons. First, there are lots of worries about *classical* logic's truth-functional conditional, so if there are problems with the Łukasiewicz conditional, it is not clear that they should reflect badly on the multivalued framework. Whatever improved technologies are devised to handle conditionals in a two-valued framework can presumably be ported to the multivalued framework as well. Second, the argument I've used to motivate degrees does not rely on any particular semantics for the conditional. (It may be contrasted, in this respect, with motivations for degree theory that focus on its treatment of the sorites paradox.)

Perhaps the most obvious objection to degree functionality is that it forces us to accept that some contradictions are not completely false. Assuming degrees of truth make sense, it should be possible for there to be a sentence that is just as true as it is false. Let P be such a sentence:

(1) $[\![P]\!] = [\![\neg P]\!]$.

Now, plausibly,

(2) $[\![P \,\&\, P]\!] = [\![P]\!]$.

[16]On Edgington's theory, the degree of a conjunction of degree-independent propositions is the product of the degrees of the conjuncts.

It follows immediately from these premises that

(3) If & is degree-functional, then $[\![P \,\&\, \neg P]\!] = [\![P]\!] = [\![\neg P]\!] \neq 0$.

It is hard to see how one could have a meaningful degree theory while rejecting every instance of (1),[17] and rejecting (2) would be very strange.[18] Our choice, then, is clear. We can have a degree-functional semantics for conjunction, at the price of allowing some contradictions to be partly true, or we can ensure that all contradictions have degree 0, at the price of rejecting degree functionality.

So can we grasp the nettle of half-true contradictions? Note, first, that a contradiction that is half-true is also half-false. In accepting such things, then, one is not committing oneself to the assertibility of any contradictions, since it is presumably not appropriate to assert half-falsehoods. One is merely accepting that some kind of "ambivalent" attitude is appropriate towards certain contradictions. And this doesn't seem wrong. Indeed, "It is, and it isn't" is just how we express our ambivalence in borderline cases.[19]

Moreover, if Jim is a borderline tall man, it seems wrong to assert, "Either he's a tall man or he isn't." This is explained nicely on the hypothesis that such disjunctions are only half-true. Granted, there are other possible explanations: for example, we may be construing "Either he's tall or he isn't" as "Either he's definitely tall or he's definitely not tall," or the assertion may carry some implicature

[17]Though Weatherson 2005 seems to do just that. See note 15, above.

[18]Though Goguen 1969, 347 does reject it, defining the degree of a conjunction as the product of the degrees of the conjuncts. (2) is also rejected in Łukasiewicz logics with strong conjunction, where $[\![P \,\&\, P]\!] = [\![P]\!]$ only when $[\![P]\!] = 1$ (see note 3, above).

[19]When we say this, I take it, we are not *asserting* a contradiction; we're finding words to express our attitude of partial endorsement. Note also that such uses cannot be construed as "It is in one sense, and it isn't in another sense."

about definiteness. But it should be registered that there is at least a *prima facie* case for assigning intermediate degrees of truth to some instances of excluded middle (the flip-side of assigning them to some contradictions).

Finally, although many philosophers seem to think it's obvious that a contradiction cannot have any positive degree of truth,[20] it's not clear what the argument is supposed to be. Timothy Williamson writes:

> By what has just been argued, the conjunction 'He is awake and he is asleep' also has that intermediate degree of truth. But how can that be? Waking and sleeping by definition exclude each other. 'He is awake and he is asleep' has no chance at all of being true. (Williamson 1994, 136)

But to say that a contradiction has degree 0.5 is not to say that it has a *chance* of being true. Indeed, if we are certain that it has degree 0.5, then we will take it to have *no* chance of being completely true. To say that waking and sleeping exclude one another is to say that if "x is awake" is true to degree 1, "x is asleep" is true to degree 0, and vice versa. Perhaps it is also to say that "x is awake" is as true as "x is asleep" is false. But on neither understanding does it rule out the possibility that both are true to some intermediate degree.

So much for the objections to half-true conjunctions. It might be thought, however, that there are other, more powerful grounds for objecting to degree functionality. Here is an argument that can be found (in different versions) in Williamson

[20] For example, Williamson asks, "how can an explicit contradiction be true to any degree other than 0?" (1994, 136).

1994, Edgington 1997, and Keefe 2000. Let Jim be a borderline satisfier of "tall," so that *Jim is tall* is true to degree 0.5, and let Tim be just a bit shorter than Jim, so that *Tim is tall* is true to degree 0.45. Then, since *Jim is tall* has the same degree as *Jim is not tall*, by degree functionality we can conclude that *Tim is tall and Jim is not tall* has the same degree as *Tim is tall and Jim is tall*. This is counterintuitive, since Jim is taller than Tim. So we should reject degree functionality.

But *why* does it seem counterintuitive that *Tim is tall and Jim is not tall* and *Tim is tall and Jim is tall* have the same degree of truth? It might be argued that when Jim is taller than Tim, *Tim is tall and Jim is not tall* must be completely false (degree 0). But then we must presumably say that *Tim is not tall or Jim is tall* is completely true (degree 1), and that seems odd in a case where, although nothing is hidden from us, we would not assert either that Tim is not tall or that Jim is tall. The issues here are, I think, very similar to the issues we considered above in connection with half-true contradictions. If we are willing to accept half-true contradictions, it does not seem too bad to accept that *Tim is tall and Jim is not tall* might be just slightly more false than true.

Even if we do accept this, though, the feeling persists that we should have different attitudes toward *Tim is tall and Jim is not tall* and *Tim is tall and Jim is tall*, and that the former is bad in a way that the latter is not. That is why this objection is deeper than the bare rejection of half-true contradictions. To resist it, we need to explain why different attitudes are appropriate towards these propositions, despite the fact that they have the same degree of truth.

I think that fuzzy epistemicism can provide a kind of answer to this question.

Our attitudes towards the propositions in question will not amount to certainty that they have such-and-such degree of truth. Rather, they will be representable as distributions of probability over a *range* of possible degrees. And, as we saw in section 2, above, attitudes toward logically compound propositions will be determined by these distributions in a way that factors in both the uncertainty and the ambivalence that they reflect. So let's see what happens when we add a bit of uncertainty about the degrees (using a normal probability distribution centered on 0.5 for *Jim is tall* and 0.45 for *Tim is tall*).[21]

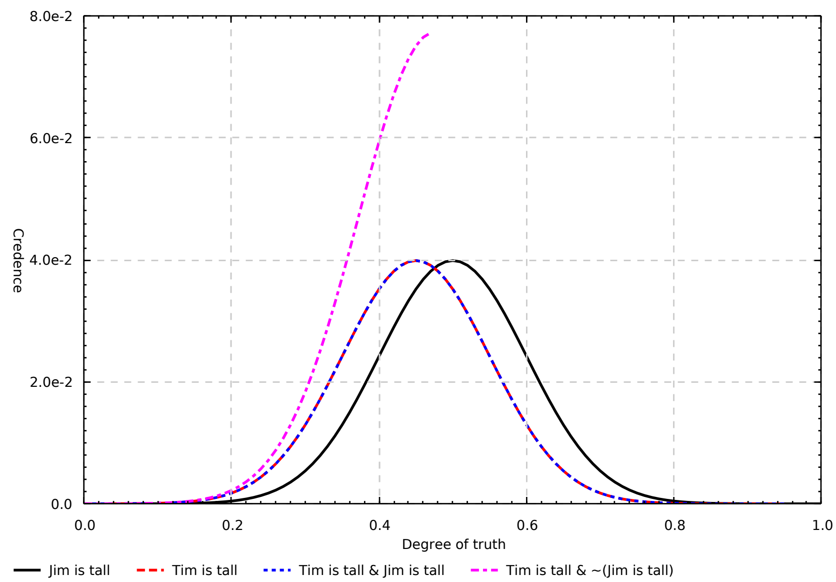


Figure 7: Differences between *Tim is tall and Jim is tall* and *Tim is tall and Jim is not tall* when Tim is slightly shorter than Jim.

As Figure 7 reveals, there is a clear difference in the recommended distribution

[21]Note that *Jim is tall* and *Tim is tall* are not degree-independent. It is assumed here that ⟦*Tim is tall*⟧ = ⟦*Jim is tall*⟧ - 0.05.

of credences over degrees for our two propositions. Particularly salient is the fact that *Tim is tall and Jim is not tall* cannot be truer than 0.5, while *Tim is tall and Jim is tall* has a tail that goes up beyond 0.8. This, I suggest, may be enough to account for the persistent feeling that our attitudes toward the two propositions should differ, and that the latter is more strongly endorsable than the former.

3.4 Edgington's argument against the *min* rule

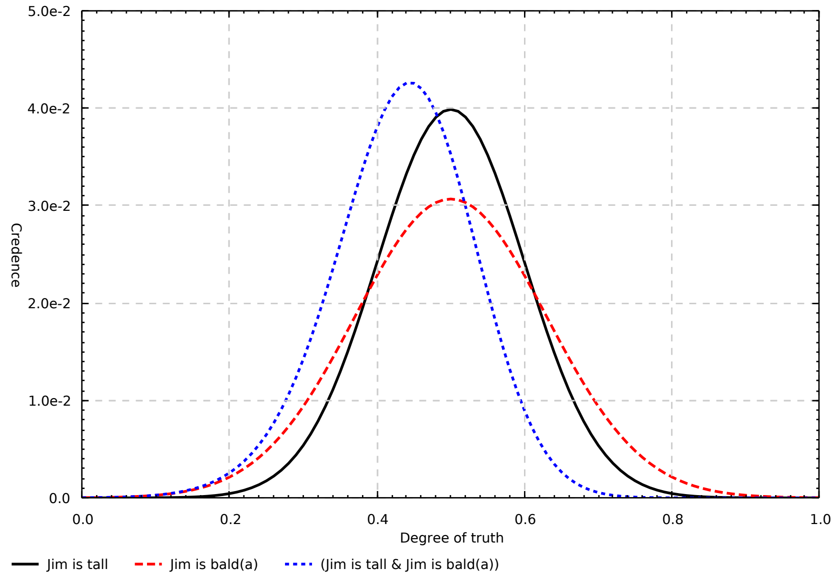
The same resources can help defend against an argument, put to me by Dorothy Edgington, that the degree of the conjunction of two independent propositions each with degree 0.5 should be less than 0.5. The argument goes as follows.[22] Suppose the degree of *Jim is tall* is 0.5. As we vary the degree of *Jim is bald* from 0 to 1, we should expect the degree of *Jim is tall and bald* to vary gradually from 0 to a maximum of 0.5. So the conjunction should have a degree less than 0.5 when *Jim is bald* has degree 0.5, contrary to the *min* rule.

Edgington's assumption that the conjunction is truer when *Jim is bald* has degree 0.7 than when it has degree 0.5 should, I think, be resisted. It draws intuitive support from the fact that, if asked to point to the "tall, bald man," I will point to the balder of two equally tall men, even if both are borderline tall and borderline bald. But this fact can be explained pragmatically; we need not conclude that the balder man satisfies the description "tall, bald man" to a greater degree than the thinner one. If Yao Ming and Shaquille O'Neal are both on the court, every-

[22]Here I draw on Edgington's comments on an earlier version of this paper at the Arché Vagueness Conference. For related arguments, see Edgington 1997, 304.

one will understand who we mean if we talk about "the tall man," even though presumably both of them satisfy "tall man" to degree 1.

We can do more justice to the intuitions underlying Edgington's argument if we bring in the dimension of uncertainty and represent the attitudes at issue by probability distributions over degrees. As Figure 8 shows, fuzzy epistemicism recommends a clear difference in attitude towards *Jim is tall and bald* as the degree of *Jim is bald* goes from 0.5 to 0.7. The effect, as before, is due to the uncertainties, and will be more prominent the greater the uncertainty about the precise degree of truth of *Jim is bald*. Thus we can vindicate the intuitions Edgington deploys against the *min* rule for conjunction without giving up the *min* rule itself—another nice illustration of the way in which the insights of epistemicism and degree theory can complement each other, once we give up trying to motivate degree theory as a way to avoid hidden semantic boundaries or solve the sorites paradox.



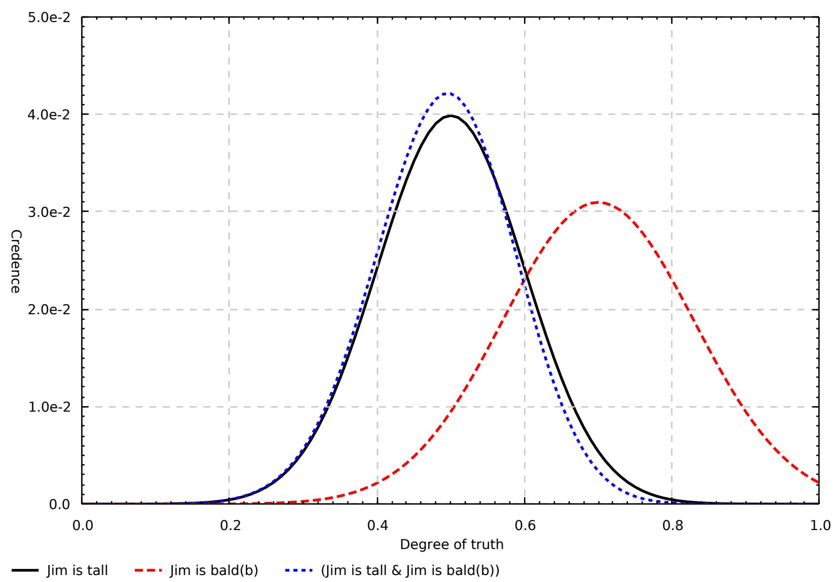


Figure 8: The degree of *Jim is tall and bald* as the (expected) degree of *Jim is bald* varies from 0.5 (top) to 0.7 (bottom).

References

Roy Cook. Vagueness and mathematical precision. *Mind*, 111:225–47, 2002.

Vincenzo Crupi, Branden Fitelson, and Katya Tentori. Probability, confirmation, and the conjunction fallacy. *Thinking and Reasoning*, forthcoming.

Dorothy Edgington. Vagueness by degrees. In Rosanna Keefe and Peter Smith, editors, *Vagueness: A Reader*, pages 294–316. MIT Press, Cambridge, 1997.

Martin Erwig and Steve Kollmansberger. Probabilistic functional programming in Haskell. *Journal of Functional Programming*, 16:21–34, 2006.

Delia Graff Fara. Shifting sands: An interest-relative theory of vagueness. *Philosophical Topics*, 28:45–81, 2000.

Hartry Field. The semantic paradoxes and the paradoxes of vagueness. In JC Beall and Michael Glanzberg, editors, *Liars and Heaps*, pages 262–311. Oxford University Press, Oxford, 2003.

Graham Forbes. *The Metaphysics of Modality*. Oxford University Press, Oxford, 1985.

J. A. Goguen. The logic of inexact concepts. *Synthese*, 19:325–73, 1969.

Susan Haack. *Deviant Logic, Fuzzy Logic: Beyond the Formalism*. University of Chicago Press, Chicago, second edition, 1996.

Petr Hajek. Fuzzy logic. In Edward N. Zalta, editor, *Stanford Encyclopedia of Philosophy*. Fall 2006 edition, 2006. URL `http://plato.stanford.edu/archives/fall2006/entries/logic-fuzzy/`.

Daniel Kahneman and Amos Tversky. Extensional versus intuitive reasoning: the conjunction fallacy in probability judgment. *Psychological Review*, 90:293–315, 1983.

Rosanna Keefe. *Theories of Vagueness*. Cambridge University Press, Cambridge, 2000.

Christopher Kennedy. Vagueness and grammar: the semantics of relative and absolute gradable adjectives. *Linguistics and Philosophy*, 30:1–45, 2007.

George Lakoff. Hedges: A study in meaning criteria and the logic of fuzzy concepts. *Journal of Philosophical Logic*, 2:458–508, 1973.

David Lewis. Scorekeeping in a language game. *Journal of Philosophical Logic*, 8:339–359, 1979.

John MacFarlane. The things we (kinda sorta) believe. *Philosophy and Phenomenological Research*, 73:218–224, 2006.

K. F. Machina. Truth, belief and vagueness. *Journal of Philosophical Logic*, 5: 47–78, 1976.

Diana Raffman. Vagueness and context relativity. *Philosophical Studies*, 81:175–192, 1996.

R. M. Sainsbury. *Paradoxes*. Cambridge University Press, Cambridge, second edition, 1995.

Stephen Schiffer. *The Things We Mean*. Oxford University Press, Oxford, 2003.

Stephen Schiffer. Replies. *Philosophy and Phenomenological Research*, 73:233–243, 2006.

Scott Soames. *Understanding Truth*. Oxford University Press, Oxford, 1999.

Roy A. Sorensen. *Blindspots*. Oxford University Press, Oxford, 1988.

Brian Weatherson. True, truer, truest. *Philosophical Studies*, 123:47–70, 2005.

Timothy Williamson. *Vagueness*. Routledge, London and New York, 1994.

Crispin Wright. On being in a quandary: Relativism, vagueness, logical revisionism. *Mind*, 110:45–98, 2001.

L. A. Zadeh. Probability measures of fuzzy events. *Journal of Mathematical Analysis and Applications*, 23:421–427, 1968.