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Penrose's Square-Root Rule and the EU Council of Ministers: Significance of the Quota

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ABSTRACT

In a two-tier decision-making system such as the EU Council of Ministers, if the number of constituencies (member-states) is sufficiently large (say, 15 or more), Penrose's Square-Root rule can be implemented to a high level of approximation by a simple weighted decision rule at the top level (the Council) with any given quota q smaller than the total weight. This leaves one degree of freedom: the value of q as a free parameter, to be determined by some additional condition. I propose to survey and discuss critically the most important considerations for fixing this value – some of which have actually been used by theoreticians or practitioners: efficiency, transparency, sensitivity (total voting power of citizens), mean majority deficit, giving certain 'privileged' coalitions blocking status. Some of these considerations are reasonably compatible; others less so. Some kind of compromise is clearly needed. But which? This is essentially a political matter; but a political decision ought to be made in a theoretically enlightened way.

Penrose's Square-Root Rule and the EU Council of Ministers: Significance of the Quota

1 Introductory remarks

In 1946, Lionel S Penrose [13] (see also [14]) proposed his Square-Root Rule (SQRR) for the General Assembly of the UN. Like many people at that time, he looked forward to the UN evolving into a kind of federal world government. That hope was soon abandoned with the onset of the cold war; and the SQRR – along with the rest of Penrose's scientific theory of voting power – was ignored and forgotten by mainstream social-choice research.

In 1965, John F Banzhaf [1] – who re-invented the basic idea of that scientific theory – proposed the SQRR in the following year in connection with US local (county) government (Banzhaf [2]); and in 1968 he discussed the SQRR in connection with the Electoral College used for the (indirect) election of the US President (Banzhaf [3]).[1]

By then the EU had come into being: the Treaty of Rome, establishing what was then called the European Economic Community, was signed in 1957 and came into force on 1 January 1958. But although the logic of the SQRR is clearly applicable to the EU's main decision-making body, the Council of Ministers (CM), it took quite a long time for this to be argued and proposed. As far as I know, the first paper to do so in an academic journal was published ten years ago, in 1997, by Dan Felsenthal and me [8]. This was soon followed by others, and by now the idea of applying the SQRR to the main decision rule of the CM has been urged by many academic experts.

However, EU practitioners – politicians and civil servants – have on the whole shown little enthusiasm for applying the SQRR to decision-making in the CM.

During preparatory discussions before the the December 2000 Nice Conference, Sweden proposed something that looked similar to the SQRR: a weighted rule in which the *weight* of every member-state was *approximately* proportional to the square root of its population size. This scheme was not really conceived as an implementation of the SQRR, but as a convenient way of implementing the principle of so-called 'degressive' allocation of voting weights, which had hitherto been practised by the EU. Indeed, the actual

[1] Banzhaf's discussion of the SQRR is somewhat roundabout: he presents what amounts to a proof of the rule, but does not actually state the rule itself.

effect of the Swedish scheme would have been only a rather rough and imperfect approximation to the SQRR. The main reason for this is that the quota proposed by Sweden, 71.57% of the total weight, was – as I shall show – too high for close proportionality between voting weights and voting powers.[2] But in any case, the Swedish scheme did not get much support. Instead, the Nice Treaty [5], adopted by the Nice conference, prescribes for the CM a decision rule modelled roughly on the degressive weighted rules used in previous phases of the EU, but with a couple of 'epicycles' added on to it as extra complications.

More recently, at the June 2007 Council of the European Union (the 'EU Summit'), held in Brussels, Poland argued very insistently for adopting a decision rule for the CM based explicitly on the SQRR in the form known as the 'Jagelonian Compromise' (Słomczyński and Życzkowski [16]). However, Poland won little support for this position and remained virtually isolated. Instead, the decision rule adopted at that meeting (see [6]) for inclusion in the forthcoming Reform Treaty – which is the same rule as the one contained in the failed EU Constitution – is much further away from the SQRR than is the Nice rule (or, for that matter, the weighted rules used by the CM during earlier phases of the EU).

In view of this past experience, I – as an advocate of the SQRR – feel there is not much ground for optimism. The experience of the United States also does not bode well.[3]

Circumstances may of course change and encourage greater political receptiveness for the SQRR.[4] However, it would be naïve to expect that our task would be easy.

In the present talk I am not going to present any new mathematical result or analysis. My aim here is to propose a politically flexible tactic in our advocacy of the SQRR. Briefly, I argue that we lose little and may well gain by not insisting on a too-precisely determined quota. There are various

[2]The Swedish scheme, along with other schemes that were discussed at the time, was analysed by Dan Felsenthal and me in [10]. The Swedish scheme is labelled there as 'Proposal D'. Another reason for the not-so-good fit between this scheme and the SQRR is that in the former even the weights were only *roughly* proportional to population square roots.

[3]For an account of the US experience concerning the SQRR, see Felsenthal and Machover [9, Ch. 4].

[4]This could be brought about by a coalition of all the middle-sized EU member-states, who would gain by adopting a decision rule based on the SQRR instead of the one prescribed by the Reform Treaty. In fact, the greatest gain would be made by those whose population is about 11 million: Greece, Portugal and Belgium. See Felsenthal and Machover [11].

considerations that have been or may be used in determining the quota; and they do not necessarily lead to exactly the same conclusion. It may therefore be easier to gain acceptance for the SQRR if we do not insist on packaging it together with a specific formula for the quota. In other words, the argument in favour of the SQRR should be separated from arguments in favour of this or that quota.

2 Various effects of the quota

As is well known, the set of all simple voting games with a given set of voters is finite. The same applies *a fortiori* to the set of proper weighted voting games (WVGs) with a given set of voters.[5] However, if the number n of voters is sufficiently large (say $n > 15$) then these WVGs are rather densely distributed in the appropriate n-dimensional space. What this means is that for given $q \in [\frac{1}{2}, 1)$ and non-negative $\beta_1, \ldots, \beta_n$ such that $\sum_{i=1}^{n} \beta_i = 1$, it is possible to find non-negative weights $w_1, \ldots, w_n$ such that $\sum_{i=1}^{n} w_i = 1$, and in the WVG $[q; w_1, \ldots, w_n]$ the relative powers (the values of the relative Banzhaf index) of the voters are quite close to the respective β_i.

Dennis Leech has written an algorithm that performs this 'inverse computation'. We used it in our 2003 paper [12], where we took the β_i to be proportional to the square root of the respective populations of the 27 present member-states of the EU.[6] For each value of q under consideration,[7] we obtained a system of weights $w_1(q), \ldots, w_{27}(q)$, which we called 'equitable weights' – a terminology which I will also use here – because the WVG $[q; w_1(q), \ldots, w_{27}(q)]$ produced the desired relative voting powers (values of the Banzhaf index) $\beta_1, \ldots, \beta_{27}$ to an excellent approximation.

Thus, for any quota $q \in [\frac{1}{2}, 1)$ the SQRR can be implemented in the CM using the WVG $[q; w_1(q), \ldots, w_{27}(q)]$ as a decision rule. The required equitable weights can be readily computed.[8] In other words, the WVGs that would implement the SQRR in the CM – and so would result in what we regard as an equitable distribution of voting power – constitute a one-parameter family, with the quota q as a parameter whose value can be chosen freely from a purely mathematical viewpoint.

[5] Of course, a given WVG has infinitely many different *representations* by means of a system of weights and quota, and this is still the case for *normalized* representations, in which the weights are non-negative and add up to 1. For convenience I shall only consider weighting systems that are normalized in this sense.

[6] At that time 12 of them were of course only *prospective* members.

[7] In fact we did the computation for values of q from 0.51 to 0.99 at 0.01-intervals.

[8] The same can be done for a future somewhat enlarged EU.

The question I wish to address here is what considerations may – or should – be used in making the choice.

2.1 Transparency

Strong heuristic arguments put forward by Słomczyński and Życzkowski [15] show that for WVGs $[q; w_1, \ldots, w_n]$ with sufficiently many voters (say $n > 15$), the relative voting powers of the voters tend to be most closely proportional to their respective weights when the quota q has an 'optimal' value that, to a very good approximation, is given by

$$q := \frac{1}{2}\left\{1 + \sqrt{\sum_{i=1}^{n} {w_i}^2}\right\}. \tag{1}$$

This is also confirmed by quite extensive simulations (Chang et al [4]).

Thus, equitable weights for the CM can be obtained *directly* from population data, by taking weights w_i proportional to the square root of the respective population size of the member-states – provided q is then determined by (1). For the present 27-member EU, using the most recent (2006) population data provided by EUROSTAT [7], we obtain $q = 61.57\%$.[9]

This recipe for implementing the SQRR is known as the 'Jagelonian Compromise'.

As a mathematician, I find the elegance of this scheme extremely appealing. It is – in a somewhat loose, but nonetheless obvious, sense – a *fixed point* solution,[10] which is always an admirable thing. Moreover, the actual value of q that it comes up with is in my opinion very reasonable by various substantive yardsticks, which I shall mention later on.

But the main selling point of the Jagelonian Compromise is its *transparency*: the relative ease of explaining it to those who are not mathematically minded – which include the vast majority of the general public, as well as most EU practitioners. Most people can easily understand the concept of *voting weight*, but are baffled by the scientific concept of *voting power*, and by the distinction between these two concepts. In the Jagelonian scheme, the weights are calculated directly, in a fairly simple way, from population figures, while the concept of voting power is, as it were, hidden behind the scenes and does not enter explicitly into the calculation. To follow the calculation, you only need to know the definition of *square root*, which most

[9] See Felsenthal and Machover [11].

[10] The mapping of which it is a fixed, or almost fixed, point is of course the one that maps weighting systems to distributions of voting power.

people can recall (possibly with a bit of help to jog their memory) from their school days. Of course, most people will be puzzled as to *why* weights are allocated in proportion not to population size but to its square root. But they may accept it as a reasonable application of the degressive allocation of weights, which has been EU practice hitherto. (Recall that this was how it was justified by Sweden before the Nice conference.)

Personally, I regard the Jagelonian scheme as ideal; and I know that many, probably most, voting-power experts share this view.

However, the determination of the quota in this scheme is by no means transparent. In order to justify Equation (1) one must make use of sophisticated mathematical concepts and results from the theory of voting power, which the vast majority of people (including EU practitioners) certainly find totally opaque.

This surely constitutes a serious obstacle to the acceptance of the Jagelonian Compromise – in addition, of course, to the general reluctance to adopting the principle of the SQRR as such.

But it is not just a matter of failure to understand the justification for Equation (1). The point is that there are various independent considerations, some of which I shall now discuss, that may be applied in determining the quota, and they may conflict to a lesser or greater extent with one another and with the value determined by the Jagelonian scheme.

All the WVGs considered below are assumed to be equitable as decision rules for the CM – in other words, they implement the SQRR – but with various values of the quota. In particular, I will use the WVGs $[q; w_1(q), \ldots, w_{27}(q)]$ computed in the paper mentioned above (Leech and Machover [12]), and I will draw heavily on the results reported there.

2.2 Sensitivity, MMD and efficiency

Equitability is a democratic desideratum of the two-tier decision-making structure in which the citizens are the indirect voters at the lower tier, acting through their representatives at the CM, which constitutes the upper tier. It is desirable because it equalizes the (indirect) voting powers of the citizens, thus implementing the democratic principle of equal suffrage, encapsulated in the slogan *one person, one vote*.

But equitability is not the *only* democratic desideratum. Two other principles are empowerment of the people (slogan: *power to the people*) and majoritarianism (slogan: *majority rule*).

In the present context, empowerment of the people means that the (indirect) voting power of the citizens should be as great as possible. Obviously, I am referring here to *absolute* voting power, which we quantify by the Penrose measure. The *sensitivity* of the EU two-tier structure is the sum of the indirect voting powers of all EU citizens.[11]

Majoritarianism prescribes that majority deficits should be minimized. A majority deficit occurs when a decision taken at the upper tier (the CM) is opposed by a majority at the lower tier (the citizens of the EU at large).

Actually, these two desiderata are equivalent, because the mean majority deficit (MMD) of a decision-making structure (the a priori expected value of the size of the majority deficit, regarded as a random variable) is a negative linear function of its sensitivity.[12]

Now, as can be seen from Fig. 3 and the last column of Table 1 in [12], the MMD is a monotone non-decreasing function of q. For the present 27-member EU, the MMD first increases quite rapidly with q, but remains constant from about $q = 77\%$ on.

So the democratic desideratum of majoritarianism (as well as the equivalent one of empowering the citizens) would suggest choosing a very low value of q.

Another consideration that points in the same direction is that of *efficiency.* This is an attribute of the CM as a stand-alone decision-making body (rather than as the upper tier of a two-tier structure). This is conveniently measured by Coleman's index A ('the power of a collectivity to act'), which is defined as the a priori probability that a proposed bill will be adopted (as opposed to blocked) by the CM. As we can see from Fig. 1 and the penultimate column of Table 1 in [12], A is a monotone non-increasing function of q. For the present 27-member EU, it decreases rapidly as q increases from 51%, and becomes dangerously small once q reaches about 73%.

2.3 Stability and blocking power

There are of course other considerations, pointing in the opposite direction and suggesting a high value of the quota. These have to do with political stability and the member-states' interest in blocking power.

For obvious reasons of political stability, the status quo should arguably be given a privileged position, and changing it not be made too easy. This argument has special force because the EU is supposed to be not only a union of Europe's people but also a union of states. This is why certain

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[11] This should not be confused with the sensitivity of the CM considered as a stand-alone decision-making body, which is the sum of the direct absolute voting powers of its members.

[12] For details, see [9, p. 60–61].

issues are decided by the CM by the unanimity rule, which is extremely inefficient – so inefficient that it threatens paralysis. The danger of this is generally recognized. But it would be quite reasonable if some issues of special importance were to be decided by a quota of two-thirds, say $q = 67\%$.

Blocking power of individual member-states is another important issue. From a detached scientific viewpoint, there is complete symmetry between both components of absolute voting power: the positive power to help getting a resolution adopted, and the negative power to help getting it blocked – Coleman's 'power to initiate action' and 'power to prevent action', respectively. Indeed, Penrose's measure of voting power is a harmonic mean of these two Coleman measures. But for well-known political reasons member-states are considerably more interested in the negative component of voting power. (The fact that most practitioners do not know how to quantify blocking power does not reduce their motivation to secure it for their respective governments.) Assuming an equitable weighted decision rule, the only way to increase the blocking powers of the individual member-states is to increase the quota.[13] For the interesting behaviour of blocking powers as functions of q, see Figs. 4a, 4b in [12].

There is also a special insistence on the part of the present four largest member-states to fix the quota at a level that would make the four of them a blocking coalition. Assuming an equitable weighting system, this would imply a quota of about 67% for the present 27-member CM.

2.4 Room for flexibility

In the end, the 'right' value of the quota must be determined by political considerations, balancing between the various arguments – some of which conflict with others. Although I believe that the quota proposed by the Jagelonian scheme is ideal or near-ideal, I think it would be counter-productive to insist on it as an inseparable part of the SQRR.

Fortunately, the behaviour of the equitable weights $w_i(q)$ leaves considerable room for flexibility. This is so because although the Jagelonian quota is indeed optimal in securing proportionality between voting weights and relative voting powers, this optimality is by no means 'sharp'. As can be seen from Figs. 2a–2e in [12], the $w_i(q)$ vary very little indeed – their graphs are very nearly flat – as q varies from 51% to about 70%. Thereafter they begin to vary quite rapidly, and from about $q = 76\%$ onwards vary as steep (and

[13] This is true more generally, for any weighted rule in which the weights are a non-decreasing function of population size. It was this fact that led to fixing the quota at a dangerously high value in the Nice Treaty.

very nearly linear) functions of q.

What this means is that if q is chosen anywhere within a reasonable interval – say between 55% and 69%, the resulting equitable weights will not differ very significantly from the transparent weighting of the Jagelonian scheme. Alternatively, if the weights will be fixed *exactly* as in the Jagelonian scheme – that is, proportional to population square root – and the quota chosen anywhere in that reasonable interval, then the resulting decision rule will be almost as equitable as the Jagelonian scheme in terms of Penrose's SQRR.

References

[1] Banzhaf J F 1965: 'Weighted voting doesn't work: a mathematical analysis', *Rutgers Law Review* **19**:317–343.

[2] —— 1966: 'Multi-member electoral districts – do they violate the "one man, one vote" principle', *Yale Law Journal* **75**:1309–1338.

[3] —— 1968: 'One man, 3.312 votes: a mathematical analysis of the Electoral College', *Villanova Law Review* **13**:304–332.

[4] Chang P-L, Chua V C H and Machover M 2006: 'L S Penrose's limit theorem: Tests by simulation', *Mathematical Social Sciences* **51**:90–106. Also downloadable from `http://eprints.lse.ac.uk/535/`

[5] Conference of the Representatives of the Governments of the Member States, Brussels, 28 February 2001: *Treaty of Nice amending the Treaty on European Union, the Treaties establishing the European Communities and certain related Acts*; EU document CONFER 4820/00.

[6] Council of the European Union: Brussels European Council 21/22 June 2007 – Presidency Conclusions, Brussels, 20 July 2007. EU Document 11177/1/07 REV 1.

[7] EUROSTAT (the statistical office of the European Commission): `http://europa.eu.int/comm/eurostat/`.

[8] Felsenthal D S and Machover M 1997: 'The weighted voting rule in the EU's Council of Ministers, 1958–95: intentions and outcomes', *Electoral Studies* **16**:33–47.

[9] Felsenthal D S and Machover M 1998: *The Measurement of Voting Power: Theory and Practice, Problems and Paradoxes*; Cheltenham: Edward Elgar.

[10] —— 2000: *Enlargement of the EU and Weighted Voting in its Council of Ministers*; London: London School of Economics and Political Science, Centre for Philosophy of Natural and Social Science, VPP 01/00. Downloadable from `http://eprints.lse.ac.uk/407/`

[11] —— 2007: *Analysis of QM Rule adopted by the Council of the European Union, Brussels, 23 June 2007*, Project Report. London School of Economics and Political Science, London, UK. Downloadable from `http://eprints.lse.ac.uk/archive/2531/`

[12] Leech D and Machover M 2003: 'Qualified majority voting: The effect of the quota' in Holler M J, Kliemt H, Schmidtchen D and Streit M E, (eds), *European Governance* (Jahrbuch für Neue Politische Ökonomie Vol 22), Tübingen: Mohr Siebeck 2003, pp. 127–143. Also downloadable from `http://eprints.lse.ac.uk/archive/435/`

[13] Penrose L S 1946: 'The elementary statistics of majority voting', *Journal of the Royal Statistical Society* **109**:53–57.

[14] —— 1952: *On the Objective Study of Crowd Behaviour*; London: H. K. Lewis & Co.

[15] Słomczyński W and Życzkowski K 2007 *From a toy model to the double square root voting system*, downloadable from `http://arxiv.org/abs/physics/0701338`

[16] —— 2007: *Jagiellonian Compromise: An alternative voting system for the Council of the European Union*. Downloadable from `http://chaos.if.uj.edu.pl/~karol/pdf/JagCom07.pdf`