

Understanding Defective Theories: The case of Quantum Mechanics and non-individuality

Moisés Macías-Bustos (UMass-Amherst) &
María del Rosario Martínez-Ordaz (UFRJ)

Forthcoming in: Jonas Rafael Becker Arenhart and Raoni Wohnrath Arroyo (eds)
*Non-Reflexive Logics, Non-Individuals and the Philosophy of Quantum Mechanics:
Essays in honor of the philosophy of Décio Krause.*

Abstract

Here, we deal with the question of under which circumstances can scientists achieve a legitimate understanding of defective theories *qua* defective. We claim that scientists understand a theory if they can recognize the theory's underlying inference pattern(s) and if they can reconstruct and explain what is going on in specific cases of defective theories as well as consider what the theory would do if non-defective –even before finding ways of fixing it. Furthermore, we discuss the implications of this approach to understanding the meta-metaphysics of Quantum Mechanics, specifically with regard to Quasi-set theory. We illustrate this by employing Quasi-set theory to structure a defective scientific theory and make possible the understanding of the theory.

Keywords— Scientific Understanding, Structuralism, Ideology, Meta-metaphysics, Quasi-Set Theory, Non-Individuality in Quantum Mechanics.

1 Introduction

If a sign of the maturity in a particular philosophical research program is that it begins to confront itself with some of the major topics in traditional or mainstream philosophy, in this paper, we would like to contribute to moving forward some of Krause's views in the philosophy of science and logic towards discussion of the implications of the tolerance of *defective* (partial, vague, conflicting, inconsistent, and false) information for broader concerns in the philosophy and the metaphysics of science. Specifically, we consider what role Quasi-set theory might play *vis a vis* rational agent's understanding of the scientific and metaphysical elements of quantum mechanics.

Broadly speaking, *scientific understanding* is considered to be knowledge of relations of dependence. When one understands a theory, one can build a comprehensive picture of that theory as well as of the relations that hold within it. Understanding a

theory allows scientists to find new domains of application for it, and understanding an empirical domain makes it possible to build new theoretical approaches to that domain. Science is generally concerned with explanation, prediction, manipulation, and actual knowledge of what the world is like. This last factor is metaphysical in nature, for metaphysics is concerned ultimately with the question of how the world is fundamentally like.¹ Therefore, it is undeniable that scientific understanding is a fundamental component of any successful scientific enterprise.

So far, understanding has been considered to be *factive* and *explanatory*, meaning that its content should only include true propositions and that it should come only after the achievement of explanatory knowledge.² Unfortunately, if this were the case, however, we wouldn't be able to legitimately understand any theories, models, or phenomena that are formulated in a defective manner. At least we wouldn't be able to do understand them *qua defective*—yet, if there was no need for understanding defective theories, this wouldn't be a problem.

However, many of our most successful scientific theories, at some point in their development, are or have been *defective*. Some of them, like Bohr's model of the atom, have been, allegedly, inconsistent. Some others have conflicted significantly with observation, like Newtonian dynamics. And some others, like Quantum Mechanics, are conceptually vague and imprecise, as well as (depending on the philosophical reconstruction) inconsistent (Cf. Arenhart and Krause 2016; da Costa and Krause 2014). This shows that much scientific practice has used and uses defective theories and models. And even more importantly, these theories, even when defective, have grounded and shaped our current science. And yet, while philosophers of science scrutinized the rationality behind using defective theories, they have significantly struggled when explaining how, if possible, to achieve any legitimate understanding of them.

Here, we deal with the question of under which circumstances can scientists achieve a legitimate understanding of defective theories *qua defective*. We claim that scientists understand a theory if they can recognize the theory's underlying inference pattern(s) and if they can reconstruct and explain what is going on in specific cases of defective theories as well as consider what the theory would do if not defective—even before finding ways of fixing it. Moreover, we claim that understanding the inferential structure of the theory involves understanding the structure of its domain. Furthermore, this understanding is modal in nature,³ in that the domain might not actually instantiate that structure, the structure need only be possible. This last point we illustrate with specific reference to quantum mechanics.

In order to do so, we proceed in five steps. First, in Sec. 2, we introduce the generalities of scientific understanding and we use some illustrations from the philosophy of science and the history of analytic philosophy. Second, Sec. 3 is devoted to analyzing the challenges around the legitimate understanding of defective theories; here we also introduce our case study. In Sec. 4, we sketch a structuralist approach to understanding and furthermore elaborate on what sort of presuppositions from metaphysics and meta-metaphysics are required by this type of approach. In Sec. 5, we explain in

¹We are presupposing some form of scientific realism is true as part of their working assumptions.

²More specifics about the components of scientific understanding are discussed in Sec. 2 and Sec. 3.

³It is important to notice that it's no requirement of ours that this type of epistemic modality should be metaphysically primitive.

which way the detection of specific inferential patterns and logical constraints allows for the promotion of scientific understanding in the case of the quantum theory with non-individuality (Cf Krause and French 1995; Arenhart and Krause 2014). Sec. 6 is devoted to drawing some conclusions.

2 The generalities of scientific understanding

In this section, we introduce some of the most common assumptions about scientific understanding and its role in scientific development.

2.1 Preliminaries

There is a shared intuition of science as being mostly motivated by our need of *making sense* of the world. This idea of making sense goes beyond only attaining bits of disconnected knowledge; as a matter of fact, making sense of the world is not about discovering new facts alone, it is not even about providing specific explanations for these individual facts but it is about going to a broader more general level: understanding.

Understanding has been traditionally considered to “consist of knowledge about relations of dependence. When one understands something, one can make all kinds of correct inferences about it” (Ylikoski 2013: 100). Understanding a theory allows scientists to find new domains of application for it, and understanding an empirical domain makes it possible to build new theoretical approaches to that domain. The fact that science is significantly motivated by the possibility of satisfactorily connecting our different scientific beliefs about the world makes understanding a fundamental component of any successful scientific enterprise. Scientific understanding is a relational phenomenon. It consists of combining doxastic bodies for the building of comprehensive pictures of a particular domain as well as for integrating theoretical frameworks that could, initially, look disconnected. Understanding then is a task of building networks that successfully connect our scientific beliefs about X and that allow us to get a better grasp of X .

This notion of understanding is very modern but has predecessors in the historical records. To take an influential precursor, Russell (1913: “Logical Data”) claimed that understanding a proposition involved a subject’s acquaintance with the constituents of the proposition: objects, properties, and relations, together with their logical form and hence the ultimate role of the proposition and their objects in logical space. Furthermore, understanding for Russell precedes truth, for to understand a proposition only means to be acquainted with the logical form relating its objects, properties, and relations, and these need not be instantiated. A proposition is defined as true only if the object’s properties and relations are actually logically related in such a way. Clearly, this intuition can be extended to whole theoretical structures when considering the Ramsey sentence of a theory, i.e., the existential generalization of the conjunctive proposition specifying the theoretical predicates and relations realized by the objects in the theory’s domain (Cf. Ramsey 1929, Lewis 1970).

Furthermore, such a notion of understanding involves theoretical elements inasmuch as it aims at characterizing the structure of the world according to our scientific

theories, under the assumption that scientific success does involve at the very least a successful grasp of the world's underlying structures and their corresponding logical space i.e., the underlying logic capturing valid inferences in the domain.

2.2 The ten elements of understanding

Scientific understanding has been seen as the combination of ten elements: (1) its structural character, (2) its standing state, (3) a subjective (psychological) element, (4) an objective element, (5) its coherence-constrain, (6) its order requirement, (7) the intelligibility component –that emerges from (5) and (6); (8) its epistemic robustness, (9) its gradability and (10) its praiseworthy character.⁴

First, the *structural character* of scientific understanding refers to the general idea of what it means to understand something. When one understands a particular phenomenon, one is capable of *making sense of* that phenomenon by connecting epistemic bodies (beliefs, knowledge, among others) in such a way that the inferences around the understood phenomenon allow us to portray it in the clearest possible way. Thus, understanding consists in selecting a particular way to accommodate and relate epistemic bodies, this is, in building a structure that can connect satisfactorily sets of information around a specific object or phenomenon.

The *standing state* of understanding refers to the fact that understanding seems to be of a different class of epistemic objects than belief and knowledge. Understanding can affect epistemic bodies by reinforcing them (or the relations between them) or by weakening them; when one understands something one can also make sense of how certain epistemic bodies hang together in order to build a more cohesive picture of what has been understood. This is, through understanding one can see how relevant or legitimate certain epistemic bodies are (or aren't) with respect to what is understood. In this sense, understanding circulates between beliefs and different types of knowledge, and it provides and distributes the epistemic force within the bodies to which it relates.

The *subjective*, psychological, component of understanding refers to the feeling of *grasping* what is being understood, and it is stronger presentation is often reported as the so-called “eureka effect”. This sensation reveals that we are aware of having acquired a new competence for putting together bits of information that make more sense when together than when separated. The *objective component* of scientific understanding results from considering that this feeling of grasping depends solely on the individual agent's experiences and reports and that agents often get mistaken about how reliable these experiences are. Epistemologists have required that understanding requires also the grasping of a fragment of reality (Cf. Elgin 2007: 35).

The *coherent constraint* and the *order requirement* are direct results of the structural spirit of understanding. To understand something is to be able to order the components of what has been understood in a coherent way (Cf. Bengson 2018: 19). The *intelligibility* component of the understanding can be generally characterized as the epistemic virtue that reflects a harmony between the content of an agent's beliefs contained in the understanding of *X* and the agent's actions around *X* (Cf. Chang 2009). There is

⁴For a similar characterization of the components of understanding, see [Bengson 2018: 19-20].

a sense in which intelligibility is the result of the combination of coherence and order; however, it also extends the scope of understanding into a performative area.

The *epistemic robustness* of understanding consists “in a way that mere acquaintance with particular deeds or facts –even if intelligent and objective– is not” (Bengson 2018: 19). This supports the idea that once legitimate understanding is achieved, it is very hard to find reasons to give it up.⁵ Nonetheless, while it is difficult to give up successful understanding already achieved, that is compatible with the possibility of improving or deepening our understanding; this feature has come to be known as the *gradability* of understanding. Finally, the *praiseworthy* character of understanding results from the combination of all of the above points; “attributing understanding to an individual is not merely to credit her with some kind of success (...) but to compliment, or praise her for it” (Bengson 2018: 19).

While there are still important ongoing philosophical debates around ways to expand this view on scientific understanding, I believe that what has been said in this section reflects the current most common agreements about scientific understanding and will suffice for the purposes of the paper.

3 Defects, theories, and understanding

Here, we address what defective theories are, why is it important to achieve a legitimate understanding of them, as well as which are the challenges that traditional views can face when trying to explain the achievement of scientific understanding of defective theories.

The section is divided in two main parts: Sec. 3.1. explains very briefly what defective scientific theories are and what has been said in the literature about them. Sec. 3.2. introduces the case study of Quantum Mechanics being a defective theory and argues in favor of the need of it to be understood *qua defective*. Finally, Sec. 3.3., provides an overview of the challenges around the understanding of defective theories.

3.1 Defectiveness in science

Defective information is an umbrella term that covers cases of partial, vague, conflicting, inconsistent, and false information. These types of information carry a negative connotation given the fact that, when present, they make reasoning more challenging in different respects. For instance, when the information that we are reasoning from is defective, it is harder to determine which are the legitimate consequences of this information, as well as to estimate the reliability of the inferences carried out with such data. This makes the problem of the use of defective information inherently both epistemological and logical.

Let empirical theories be formulated based on the following theoretical model: $T = \langle D, R_i^n \rangle$ “where D is a particular domain (a set of objects to which the theory is supposed to apply) and R_i is a family of n -place relations holding between the elements of D ” (Bueno, 1997: 588). While the domain could be selected and individuated

⁵What happens in cases where agents might think they understanding, but they are wrong about that will be discussed in Sec. 3.

depending on the methodological preference of the research program in which the theory is being used and vary from time to time; the set of relations, R_i , work in a very different way: first, they are what helps to order, classify, and evaluate the objects in the domain (and the propositions through which they are described). Second, they close under specific logical consequence relations the objects of D , allowing and forbidding certain interactions between them. And third, as they regulate the behavior of D , they will not necessarily change if D increases or decreases.⁶

Defective theories are theoretical constructs that operate on a defective basis—either assuming incompatible commitments, accepting defective procedures or characterizing defective entities, etc. When these theories are or have been successfully employed in scientific practice, this reveals that they can preserve and stress particular inference patterns between propositions—and it is expected that such patterns are what warrant the applicability of the theories in different contexts.

A scientific theory (or model) can be defective in different ways: *conceptually*, meaning that some of the concepts at the core of the theory are defective; *empirically*, this is, the relations that hold between the theory and its intended domains of applications are defective—the most common cases of this are those of either descriptive gaps or conflicts between the theory's predictions and relevant observational reports. Furthermore, a theory can be *intertheoretically* defective, this is, with respect to other relevant theories, by conflicting with them, or by being extremely partial even though complementarity between these theories was initially expected. Moreover, a theory can be *metaphysically* defective, inasmuch as it is not clear what picture of the world the theory paints either because it involves elements of indeterminacy, contradiction, incompatibility with other fundamental theories, lack of clarity with respect to its theoretical ontology and ideology and so forth.⁷

In recent decades, the rational use of defective theories has significantly captured the attention of epistemologists and logicians of science. First of all, it is a common intuition that scientific methods are a sort of extremely sophisticated epistemic filter that can and should guarantee the high quality of the information that is used and obtained through science. And when they do not succeed at doing so, we find ourselves puzzled by this—arguably, especially so in the so-called "hard" sciences. Nonetheless, if we pay close attention to the history of science, we would realize that scientific theories are and have been most of the time defective. Sometimes, observational reports conflict with one another, theories are conceptually vague or models are extremely partial; and in spite of this, science is still carried out as one of our most rational enterprises. More importantly, the defects of some theories are crucial components of either the theories themselves or the phenomenon that they are representing, and therefore an adequate understanding of them needs to grasp them *qua* defective.

⁶We are aware of the fact that there is an ongoing philosophical debate about the status of the different characterizations of scientific theories (see [Halvorson, 2016] for a comprehensive revision of the different views on scientific theories); however, we think this will suffice for the purposes of the paper.

⁷In the literature we can find examples of metaphysicians defending that possibility that reality itself is indeterminate, inconsistent, disjoint and so on (Cf. Cartwright 1999, Priest 1985, Torza 2021). However, on the methodologically conservative picture reality is consistent or coherent; fully determinate and unified or integrated into a whole.

3.2 A case study: Quantum Mechanics and Non-Individuality

Nowadays, Quantum Mechanics is considered to be our most successful and fundamental scientific framework. A significant amount of important physical phenomena have been explained by using the theory, it has provided us with valuable epistemic access to the nature of light, electricity, and elementary particles, among other objects of scientific study. The theory in general allows us to draw important connections between different areas of physics and other disciplines like (some subareas of) chemistry, in order to build an image of the world that is cohesively integrated and epistemically robust.⁸ Moreover, the theory has found numerous applications in engineering and the high-tech economy. And the combination of all of this can only speak about the richness, success, and trustworthiness of the theory.

It seems even obvious that any scientist should aim at understanding the theory in order to later, understand the physical world through it. However, understanding might not be so straightforward in this case. In the vast literature concerning the foundational and conceptual issues of Quantum Mechanics, one of the most salient issues is the metaphysical status of the entities posed by the theory, in particular, quantum entities can be considered as individuals (Cf. Saunders 2016, Krause and Arenhart 2016). Regarding this point, three main options have been entertained: either they can and should be considered as individuals, they should be considered as non-individuals or one should neglect particular objects and endorse a kind of ontic structural realism. Now, this discussion is rooted in the origins of Quantum Mechanics,

[H]istorically, the issue has been treated from a very naturalistic point of view. That is, the choice should be made bearing always in mind what quantum mechanics itself dictates us concerning those matters. In that case, the adoption of a metaphysics of non-individuals seems to have at least historical precedence over the other two options. Really, right from the beginning of the theory it was seen by some of the founding fathers of quantum mechanics that it dealt with items without identity, in the sense of having no individuality. That is, it seemed to follow from the strange statistical behavior of quantum particles that they had no individuality, no identity, and so were a very strange-behaved kind of thing. That view was called *the Received View* on quantum non-individuality. (Arenhart and Krause 2014:2).

Individuals are taken to be the intuitive values of the bound variables of classical first-order logic. Furthermore, for first-order languages that have individual constants (as opposed to predicate constants), individuals will be those objects that will be unique referents of constants. As stressed by Krause (2012: 3) classical logic has Leibniz's *Identity of Indiscernibles* as a theorem:

$$\text{II: } \forall x, \forall y (\Phi(x) \text{ iff } \Phi(y) \rightarrow x = y)$$

Individuals which are qualitatively indiscernible are identical. In order to avoid trivializing the identity of indiscernibles the qualities under discussion should not include the

⁸For a comprehensive critical analysis of the epistemic robustness of quantum mechanics see [Hofer, 2020].

property of being identical to some object o , for in such a case if both x and y are identical to o , then by the transitivity of identity they would be identical to each other. The identity of indiscernibles was used by Leibniz for a number of purposes, for example in the Newton-Clarke correspondence (Cf. AG, *Philosophical Writings*) Leibniz makes liberal use of the principle to prove that it is impossible to shift every material object in some direction in space; that it is impossible to boost the whole material world such that the relative motion of every particle remains the same; that it is impossible for there to be absolute space (a substratum existing independently of material objects) and analogously that it is impossible for there to be a world temporally indiscernible to ours but that being existing either earlier or later. Famously Max Black (1952) argued that the identity of indiscernibles entailed the impossibility of what he took to be an obviously possible world, a world containing two indiscernible spheres differing *solo numero* –if the identity of indiscernibles is true, then there cannot be objects differing numerically without differing qualitatively. Therefore, the identity of indiscernibles must be false.

We have to distinguish between properties in the world and predicates in our logical language. If our language lacks an identity two-place predicate it can in fact be introduced derivatively as shorthand for any two individuals satisfying the same predicates (as in Whitehead and Russell’s *Principia Mathematica*).⁹ Restricting ourselves to the first-order case, since predicates are sets in first-order model theory, by the extensionality axiom of ZFC (classical first-order Zermelo-Fraenkel set theory with choice) any two sets are identical whenever they have the same elements. So far so good, suppose we allow ourselves the hypothesis that there are individuals whose only particularizing property is being that unique individual, a haecceity, and in doing so we reject the identity of indiscernibles. We can always introduce names for these objects differing *solo numero* to distinguish them, even if in name only. We can thus define “indistinguishables” as those individuals which are indiscernible with respect to every non-haecceitistic property. Allowing ourselves, per impossibile, the possibility of having names for every non-haecceitistic property i.e., predicate letters standing for subsets of the domain and furthermore presupposing quantification over the all-inclusive domain, such that no larger structure can discern our objects, we can say that indiscernibles are those individuals which are invariant with respect to n –place predicate permutations, such that they satisfy the same formulas of this ideal language.

As Krause (2012: 2) and Krause and Arendhart (2016: 2) point out however this framework is *prima facie* in tension with the standard interpretation of quantum mechanics inasmuch as standard Quantum Mechanics allows for the possibility of fundamental indistinguishables, but needn’t require that these be individuals. The basic idea is that quantum objects are fully invariant with respect to predicate permutations, so much so that we can intuitively regard them as the same object, while at the same time speaking of these plurally. According to Krause (2012) classical, intuitively macroscopic objects are indeed individuals for they cannot be substituted for each other with

⁹Russell gives a metaphysical defense of the principle in his *An Inquiry Into Meaning and Truth* (1959). On the interpretation of *Principia Mathematica*’s metaphysical logic see Landini (1998), Linsky (1999), and Klement (2018). It is interesting, in this connection, to consider that philosophers such as Russell and Frege felt that there had to be a philosophical elucidation of the systems of higher-order logic they were working on in spite of them being systems of logic and not applied mathematics. We think this bolsters our view about the generality of understanding as an epistemic activity.

everything remaining qualitatively the same, but quantum objects can.

An example cited in the literature is that of fermions and bosons with regard to certain states (Cf. Ladyman and Bigaj 2010). In this context, the claim that quantum particles are indistinguishables boils down to the fact that if particles are in some state S then permuting those particles within S produces a state that is physically indiscernible from S . In standard Quantum Mechanics, physical systems are represented by vectors in a Hilbert space with specific states corresponding to vectors of length 1 in the space. Properties of such systems are represented by Hermitian operators on the vector space, mappings of the vector space onto itself (Cf. Okon 2014) where the expectation value of a measurement for a property (e.g., spin, position) corresponds to the eigenvalue of the eigenvector of that Hermitian operator. As pointed out by Ladyman and Bigaj (2010) the expectation value of a Hermitian operator is the same for all indistinguishable particles, quantum states involving fermions and bosons are permutation invariant and this property is retained even if further particles are added to the system, unlike what happens in classical systems (e.g., enantiomorphs¹⁰ which are distinguished when further objects are added to the space).

To take an example of what a fundamental physical theory should involve considering Maudlin's (2018, p. 2) discussion of this issue. On his view a fundamental physical theory is a theory about "matter in motion", hence it should involve: (1) local beables (matter); (2) non-local beables (if any), such as the quantum state; (3) a space-time structure and (4) the dynamical laws. Parts (1), (2), and (3) tell us what there is according to the theory, the ontology whereas part (4) tells us what it does. Standard quantum mechanics is muddled about the dynamics, by having two radically different types of evolution (deterministic Schrodinger evolution and non-deterministic collapse according to the Born rule).

Standard quantum mechanics is defective inasmuch as it lacks clarity with respect to the status of the nature of the quantum state, and involves a notion of "measurement" which, though it doesn't prevent the use of the theory in experimental settings, is mysterious, imprecise, vague and in no way allows us to understand how the world is supposed to be according to it, hence the "interpretations" of quantum mechanics, which really are ways of fleshing out the theory in a manner compatible with (1) - (4) above. In spite of its success as an instrument, there are conceptual and inter-theoretic difficulties reconciling the theory with our experience of the world, with other fundamental physical theories, such as general relativity, as well (allegedly) as with the fundamental logical frameworks that underlie traditional mathematics, physics, and metaphysics, on account of the failure of the identity of indiscernibles for quantum objects: classical logic and ZFC set theory.

We take this to have shown that there is an important sense in which Quantum Mechanics should be considered to be a successful, yet defective, theory. The question that we address in the following paragraphs is whether the achievement of scientific understanding of defective theories is really possible, and if so, under which conditions this can occur.

¹⁰Such as the left-hand and the right-hand.

3.3 The challenges for understanding defective theories

There are two broad types of scientific understanding: a theoretical one and a practical one. The former consists of making sense of either theories or phenomena (by using those theories); while the latter refers to having the ability to perform complex tasks in a systematically successful way. In sciences these two types of understanding are often seen as closely linked; the expert scientist is expected to understand the theories that she works with, as well as the phenomena that she studies by implementing such theories, and the procedures that are followed in her daily practice.

(Theoretical) scientific understanding has been traditionally characterized as *explanatory* and *factive*. On the one hand, the explanatory requirement means that understanding comes only after having obtained causal explanatory knowledge (Cf. Grimm 2006, 2014; Lawler 2016, 2018). In this sense, understanding is the most demanding epistemic good that we can attain. On the other hand, the factivity requirement means that the content of understanding includes only true propositions. This is, we legitimately understand only propositions that we know are true and that adequately refer to facts of the world.

The explanatory requirement has been justified by the (epistemological) grounding role that causal explanations seem to play in the sciences and the factivity requirement has been motivated by the aim of truth preservation. This gives the impression that understanding cannot be attained in absence of causal explanations and more importantly, that the content of understanding cannot include any defective (vague, incomplete, conflicting, inconsistent, impossible) data. In what follows, we focus only on the factivity condition of understanding and the challenges that it poses for the understanding of defective theories.

The satisfaction of the factivity condition plays a crucial role in determining whether a case of alleged scientific understanding is *legitimate*. A case of understanding is legitimate when it is robust towards updates of information; while it can upgrade consistently, it should never decrease in quality —the content of understanding shouldn't go from being consistent, complete, and precise, to being inconsistent, incomplete, and vague or partial. Having the impression of understanding something that is knowingly imprecise, incoherent, or false is called the *illusion of depth of understanding* (Cf. Ylikoski 2013).

The upshot of the factivity condition the factivity condition is that the reliability and the legitimacy of understanding depend on the truth of its content. The factivity requirement is false though, considering that even if some cases of legitimate understanding fully satisfy it, it is not clear that this should be a necessary condition for understanding. In particular, considering cases of theoretical understanding of superseded theories. If one takes seriously this requirement when a scientist reports having understood theory *a* from the past, which we already know is partially false, one should accept that this is a case of the mere illusion of depth of understanding —or at least that some of the main features of the theory, those that are false, cannot be understood. Nonetheless, systematically, scientists and philosophers more generally have a strong feeling of understanding abandoned theories. This rises the question of how can we explain cases of understanding theories that are vague, incomplete, conflicting, inconsistent, or even false consistently with having a normative epistemological approach to

understanding.

When trying to satisfactorily combine the normative elements of understanding with the actual cases that we find in scientific practice, epistemologists have adopted at least three different standpoints: *factivism*, *quasi-factivism* and *non-factivism*.

Factivism. The content of understanding can only include true propositions (or at least approximations to the truth) that are known to be so.

Factivism accounts for the clearest cases of epistemic success. Indirectly, it also encompasses the clearest cases of error; as understanding is extremely hard to achieve, in the majority of cases in which we thought we had understood something, we were very much mistaken, and we discover this only when faced with the falsehood of our beliefs. Unfortunately, this standpoint fails at addressing the gray area that exists between radical success and radical error.

Quasi-factivism. The content of understanding might include elements that are known to be non-true, but these elements are to be located in the periphery of the content of understanding.

Quasi-factivism addresses the cases in which understanding is only achievable thanks to certain epistemic tools, which might go from bits of logical rules to sophisticated idealizations, abstractions, and fictions, among others. Nonetheless, as the epistemic role of these non-true elements is only to ease the reasoning, they are part of the content of understanding only by being elements of its periphery, but they are not located at its core, this is, they are not part of what has been understood.

Non-factivism. The content of understanding can include non-true propositions that are known to be so; and, when they are essential for the achievement of understanding in virtue of being false, they are located in its core.

This standpoint deals with the issue of the way in which non-true elements can be part of the content of understanding; but in doing so, it loses track of the warrant that truth provided the other two theses with. And for the supporter of non-factivism errors become a real challenge, there is no way to make a clear division between cases of error as cases of understanding in the gray area concerning the use of false statements.¹¹ So far, these standpoints capture different features of real-life scientific understanding, the cases of impressive epistemic success, the cases in which scientists use tools to ease their grasping of a theory or a phenomenon, and those in which these tools are indispensable. Nonetheless, there is an important difference between saying that one can include some idealizations in the content of understanding and saying that one can legitimately understand a defective theory.

Understanding a theory that is defective, especially if it is knowingly defective, requires paying attention not only to its successes but also to its defects. If one, for

¹¹It is important to notice that for the non-factivist, the non-true propositions that can be included into the content of understanding are exclusively those that lead to (empirical) success when being used. These propositions have been called *felicitous falsehoods* and are falsehoods that facilitate understanding by virtue of being the falsehoods they are and whose “divergence from truth or representational accuracy fosters their epistemic functioning” (Elgin 2017: 1).

instance, tries to separate the phlogiston theory from the falsehoods that we now know were part of it, one would end up with a different theory and lacking understanding of the one that one was initially trying to grasp. Thus, for the case of the superseded theories, the inclusion of their defects in the content of our understanding is crucial for agents to be able to see the relations that hold between the elements of the theory, their connections with the domains of application as well as the reasons for which they were abandoned. Furthermore, for those theories that despite their defects are still in use, understanding them as defective allows scientists to interpret their defects in novel ways –not only to solve them but also to tolerate them or even accept them. And more importantly, for those theories that are defective because the phenomenon that they are representing is essentially defective, understanding them *qua* defective becomes a crucial task for explaining their success and furthering scientific development.

Furthermore, when it comes to the commitments of the scientific realist *vis a vis* knowledge there is a metaphysical commitment to specifying what a possible world instantiating the structure of the theory would be like.¹²

Summing up, while epistemologists have shed light on the ways in which certain non-true propositions can be included in the content of understanding (either at its core or its periphery), they have failed to explain the ways in which agents can legitimately understand defective theories *qua* defective. In the following sections, we explain how this is possible and we illustrate this in more detail with the case of Quantum Mechanics and non-individuality.

4 Understanding Defective Theories: A structuralist approach

Here we provide the generalities of a structuralist approach to scientific understanding that, on the one hand, remains neutral with respect to the debates about the truth value of the content of understanding; and on the other hand, allows us to explain the legitimacy of some cases of understanding of defective theories.

The section is divided in four parts: Sec. 4.1. is devoted to summarizing the defective-theories motivation behind this account. Sec. 4.2. addresses the structuralist roots of the proposal and Sec. 4.3. sketches its epistemological and metaphysical import.

4.1 The motivations defectiveness-wise

Scientific theories are epistemic vehicles that help scientists to filter, order and relate the varied information that they get about the world in order to provide accurate descriptions, predictions, and explanations of the domain that they are talking about. Broadly speaking, theories are clusters of information which are initially incomplete but that, in the long run, tend to incorporate new data in order to improve the picture

¹²We do not take a stance on the nature of possible worlds in this paper, we use the concept as shorthand for possibilities. For a contemporary sympathetic and systematic approach to possible world realism, however, see [Bricker 2020].

of the world that they provide. In that sense, it is not surprising that theories are, at least initially, vague and incomplete—in some cases, this also causes the presence of contradictions.

Much scientific practice makes use of defective theories; some of the most famous examples of this are: the early calculus, Bohr's theory of the atom and Frege's foundations of arithmetic, among others. And despite the fact that some of these theories are knowingly defective, scientists still report having 'understood' both the theories as well as the phenomena that they describe. Yet, according to traditional accounts of scientific understanding, these reports should be considered to be illusions.

There are broadly two very general ways to go about explaining what is going on when we grasp or communicate some defective theory: either claim that in none of those cases do we understand the aforementioned theories, which is on its face exceedingly implausible, or to say that we do understand them and then offer an explanation about what understanding comes down to in those cases. In the rest of the paper, we adopt the latter. For doing this, we adopt a strategy similar to the one already employed by structural realists for salvaging the continuity and preservation of science in light of pessimistic meta-induction style arguments.

4.2 The proposal and its structuralist roots

First of all, we take the notion of *structure* in the sense of mathematical structures (Cf. Bricker, 1992/2020). As pointed out by Shapiro (2000) mathematical structures are often interpreted in terms of set theory, Frigg and Votsis (2011; pp. 229-230) illustrate this for the case of ontic and epistemic structural realism in their wide-ranging survey of that research program. However, there is underdetermination at the level of what set-theoretical structures or metaphysical structures (e.g., pluralities of tropes, universals, possibilia) should be posited (if any) as the ontic ground of these mathematical structures. We take no commitment here on this question.¹³

Part of the appeal of scientific realism is the claim that this view can explain why more mature scientific theories are more successful than their predecessors. However, on the one hand, there are substantial changes in the ontologies and explanatory relations between any pairs of predecessor-successor theories, even those very close in time. Among theories we now consider false, there are those which are strikingly successful, e.g. Newtonian mechanics. So it's not clear why, assuming our newest theories are also successful, their success is explained via their truth, since the earlier cases it was not.

The structural realist research program claims to be able to explain why reference is irrelevant for success, Getting the right referents does not suffice for getting at the right structure. Furthermore, structural realists claim that they can explain why a successful theory does not need a genuine reference: it does not matter if the relationship relating the terms is some specific relation R or if the property had by the terms related by R is some specific property P , the abstract description in terms of some set of objects in the

¹³Relatedly, the notion of "structure" plays an important role in debates about scientific realism, structural realism, and so on (Russell 1927, Frigg and Votsis 2011, French 2014). We assume that the notion of a mathematical structure e.g., the natural number structure, the real number structure, is robust enough that there is no methodological need to dive further here given our aims.

domain (the objects having P) and some relation relating these objects in the right way will suffice to deliver truths in as much as the domain instantiates this structure (Cf. Russell 1927; Votsis 2004, 2018). If structural realism is to deliver on these promises, it needs a more robust notion of structure than the set-theoretic one, as we will discuss below, but for all that, we believe the view is correct in its intuitive formulation: structure is what matters to science.

Analogously, we want to say something similar when it comes to understanding. What is understood in cases of defective theories is, broadly speaking, that some structure is being posited of some objects in some domain for the purposes of saying explanatory things about them given the posited structure. This works in cases where the theories are contingently false since we can consider some possible structure instantiating the pattern with the ontology of the theory just so related and more importantly, where the theories are necessarily false: for example, whenever they are inconsistent and their underlying logic is classical i.e. they are explosive. For those cases, one can consider impossible structures: where the ontology of the theory would be related in some patterned way if it were not for the inconsistent elements. In a similar way, agents would be able to deal with theories that are either extremely partial, vague, or incomplete.

When scientists report having understood a defective theory, even if clearly false or impossible, their claim might be legitimate. We argue that scientists understand a defective theory if they can recognize the theory's underlying pattern(s) and if they can reconstruct and explain what is going on in specific cases of defective theories as well as consider what the theory would do if not defective –even before finding ways of fixing it. An important remark is that, while the purposes of this paper concern the accommodation of defects in the content of understanding, in general, our approach is neutral with respect to the corresponding debate.

4.3 The epistemological and metaphysical value

If what has been said here is along the right lines, one still might wonder what type of understanding is gained in cases of defective theories; this is, which is its epistemic status. This concern comes from the fact that the explanatory and factivity conditions of understanding are the result of aiming at a factual understanding of the empirical world. So when we decide to include non-true elements in the content of understanding, there is an important sense in which we might be driving away from that goal. Responding to this issue, we take the type of understanding that agents gain of defective theories *qua* defective is *modal understanding*. “One has some modal understanding of some phenomena if and only if one knows how to navigate some of the *possibility space* associated with the phenomena” (Le Bihan 2017: 112. Our emphasis).

The notion of possibility space is meant to be comprehensive. First, we consider the set S of possible worlds in which P , or some subset of P in the sense above, is the case. Next we consider the set of dependency structures that, when appropriately associated together, give rise to P , or to some subset of P , within S . The possibility space for P will be the set of dependency structures in those possible worlds that give rise to any subset

of P and the relations between those structures. Note that the possibility space does not only include the set of possible dependency structures for P and the subsets of P : it also includes the relationships between these structures. (Le Bihan 2017: 114)¹⁴

In the case of defective theories, to achieve modal understanding would be to determine the set of possible worlds that correspond to the generic structural features assumed by the theory, broadly speaking, as well as by its most salient models. This is, if the theory and its components were to be true, which type of domain would they describe. But, what is the value of modal understanding? For the case of scientific theories, it grounds any further type of understanding that an agent would gain. Modal understanding reflects the expertise needed to identify and explain the multiple relations of dependence that hold within a theory and that a theory posits for its intended domain.

As we have argued throughout this paper, understanding is non-factive in the sense that it does not presuppose truth. This is a subtle point, for we have also argued that understanding is structural and pattern-guided. In grasping the mathematical structures of theories we acquire understanding of their possible nature and their logical space, there is a factive element there involving this notion of structure. We wish to remain non-committal as to the nature of structure here, for there are many plausible candidates that are faithful to our core intuitions, however it is important to clarify two things. First, owing to model-theoretic considerations, any notion of structure must be distinguished structure, there will be variable elements but also fixed points, specific patterns or relations that are causally or naturally special in an objective way, this is a question of metametaseantics (Cf. Sider 2011, Bricker 2020). Second, the notion of structure pertaining to the ideological primitives of fundamental theories introduced by Sider (2011) is not the notion of mathematical structure but a more general notion involving the fundamental acceptance of the logical primitives of our most fundamental theories as part of ultimate reality.¹⁵ This notion of structure is relevant to the problem of understanding, not only for metaphysicians, those persecuted but noble beings, but also for philosophers of science, since scientific theorizing is greatly concerned with representation and inference and these involve ideological choices in the above sense.

We take this section to have explained that when agents understand a defective theory *qua* defective, they only can do so by incorporating to the content of their understanding defective elements as well as the structural relations that allow them to remain well-behaved when leading to successful outputs (predictions, descriptions, explanations, among others). Here we have also explained that the type of understanding that is gained through doing so is modal understanding. For the purposes of this discussion, in the next sections we focus illustrating this with a case from Quantum Mechanics.

¹⁴The notion of modal understanding has been used by Le Bihan (2017) to address the way in which we understand theories and models that misrepresent the actual world by not being true. Here, we extend its scope in two directions: we cover other cases of defects, besides falsehood, and we explain its structuralist grounds.

¹⁵This point is discussed in more detail in Sec. 5.2.

5 Understanding via Quasi-set theory

This section aims at two main things, first, introduce the technical and philosophical basics of Quasi-set theory, and second, to illustrate what has been said in the previous section considering the case study from Sec. 3.2.

In order to do so, we proceed in three steps, first, we introduce the technical basics of a paraconsistent Quasi-set theory. Second, we discuss its metaphysical value, focusing on the issue of non-individuality. Third, we explain the way in which the use of Quasi-set theory can play a crucial role for the understanding of Quantum Mechanics as a defective theory.

5.1 The basics of the (paraconsistent) Quasi-set theory Ω_P

Quasi-set theories are mathematical systems that allow us to deal with indiscernible elements. As the reader might imagine, the main motivation for these theories is the presence of indiscernible entities in quantum physics. Here we describe the basics of one Quasi-set theory, Ω_P , which is paraconsistent; this, taking into account that in one of the most problematic scenarios, the non-individuality of some quantum objects can be understood as the root for inconsistency within the theory.

Let \mathcal{L} be the language of Ω_P , the paraconsistent Quasi-set theory. \mathcal{L} includes the logical constants: negation \neg , conjunction \wedge , disjunction \vee , and material implication \rightarrow , and the bi-conditional, \leftrightarrow ; all defined as usual. It also includes quantifiers, \forall and \exists , and auxiliary symbols of punctuation. The specific symbols of \mathcal{L} are:

- * four unary predicates: m , M , Z and C ,
- * two binary predicates: \equiv and \in ,
- * a unary functional symbol qc .

”The terms of \mathcal{L} are the individual variables and the expressions of the form $qc(x)$, where x is an individual variable” (Krause 2012: 6); $qc(x)$ indicates ’the quasi-cardinal of x ’. That said, $m(x)$ indicates that ’ x is a m -atom, a quantum object; $M(x)$ says that ’ x is a M -atom’, which acts as ZFU’s ur-elements. Furthermore, $Z(x)$ says that x is a set, and $x \equiv y$ that x is indistinguishable (or indiscernible) from y . Finally, $x \in y$ says that x is an element of y .

Now, these are the crucial concepts around Ω_P are:

1. $\alpha^\circ := \neg(\alpha \wedge \neg\alpha)$ We say that α is *well-behaved*; otherwise, it is *ill-behaved*.
2. $\neg * \alpha := \neg\alpha \wedge \alpha^\circ$ This is the *strong negation*. It will have all the properties of standard negation.
3. $x \approx y := [Q(x) \wedge Q(y) \wedge \forall z(z \in x \leftrightarrow z \in y)] \vee [(M(x) \wedge M(y) \wedge \forall_Q z(z \in x \leftrightarrow z \in y))]$ This is the *strong equality*, or *identity*. It will have all the properties of classical equality. For simplicity, we shall write $x = y$ and read it “ x is certainly identical to y ”.
4. $x \neq y := \neg * (x = y)$ we read “ x is certainly distinct from y ”.

5. $Q(x) := \neg m(x) \wedge (x)$ (x is quasi-set, or qset for short).
6. $E(x) := Q(x) \wedge \forall y(y \in x \rightarrow Q(y))$ (x is a qset whose elements are also qset, or x as no atoms as elements).
7. $x \subseteq y := \forall z(z \in x \rightarrow z \in y)$ (subqset) Remark: since the notion of identity (\cong does not hold for m -atoms, in general we don't have effective means to know either a certain m -atom belongs or does not belong to a certain qset. But the definition works in the conditional form.)
8. $D(x) := M(x) \vee Z(x)$ (x is a *Ding*, a “classical object” in the sense of Zermelo’s set theory, namely either a set or a macro-ur-element). (Krause 2012: 6-7)

It is important to mention that the underlying logic of Ω_P is da Costa’s paraconsistent calculus \mathcal{C}_1^* . We think that this would suffice for the purposes of the paper but if interested in a comprehensive description of the theory, see [Krause 2012].

5.2 Ideology, Ontology and Quantum Mechanics

Metaphysics, broadly understood, is the philosophical inquiry into the ultimate structure of reality (Van Inwagen 1998: 11). This study crucially involves ontological questions such as “what exists according to our best metaphysical theories?”. Metaphysicians would like to find out what entities populate the world as part of this broader inquiry into the nature of reality, this is ontology. Tackling such questions requires in turn that we possess a reliable methodology for extracting ontological commitments from our best theories. In formulating theories about anything it is inevitable that we will presuppose primitive, undefined notions and assumptions which are not part of the ontology, in the sense of corresponding to objects, properties, or relations within the theory’s domain, but instead are required for even formulating it. Those primitive notions and assumptions in any theory we can characterize as the ideology of the theory (Cf. Quine 1951, Cowling 2019).

Supposing we are metaphysical realists, that is, we believe the world has some structure that is mind-independent, then following Quine (1948, 1951) there are two interrelated questions we might ask from the standpoint of our theories: *what is the theory’s ontology?* and *what is the theory’s ideology?* These inquiries inevitably lead to metametaphysics, the philosophical study of the concepts, methods, and principles of metaphysics. Ontology is about what exists according to our theories, ideology is about the primitive concepts and notions, logical and non-logical, expressible within our theories that enable them to represent the domains they are about (Bricker 2016).

Since [Quine 1948] the popular response to the question of ontology is inextricably linked to the metaphysical status of quantifiers, bits of logical ideology.¹⁶ Recently some realist metaphysicians have considered that we should *go beyond the predicate* (Sider 2011), that is they have defended the claim that we need a distinction between distinguished structure and gerrymandered structure, where “structure” stands

¹⁶For a detailed discussion on this topic, see [Macías-Bustos 2022a.]

for the ideological primitives of our most fundamental metaphysical theory. This distinguished structure Sider (2011: 5) calls “metaphysical structure”. In his view, metaphysical structure: the operators, quantifiers, logical consequence relations, and logical connectives of our fundamental theory, are also metaphysically committing. More successful theories of fundamental science get at nature’s joints better than less successful ones. For Sider, this is evidence that the primitive ideology of these theories is about the ultimate nature of reality. It is a realism that takes such primitive ideology as both irreducible and worldly.

According to Sider, metaphysics is not only about ontology but about metaphysical structure, where “metaphysical structure” is to be read off from the primitive ideology of our most fundamental theory. Metaphysical structure, on this view, is not ontic structure but it is nevertheless about the world. Consider for example the difference between someone who believes in primitive modality and embraces the operators “possible” and “necessary” as distinguished (or joint-carving) as opposed to another philosopher who believes modality can be eliminated or reduced to facts about possible worlds and the objects within them. To be sure, modal operators are given an analysis in terms of possible worlds by some who embrace primitive modality, but they do not think this analysis gets the structure of reality right. Furthermore, some philosophers of primitive modality might simply refuse to give even an elucidatory analysis of their primitive modal ideology (Cf. Finocchiaro, 2021).

On Sider’s (2011) view, the world will have individuals if the fundamental quantifiers of the fundamental theory range over elements of a domain.¹⁷ There will be a further question about anti-haecetism and haecetism, the views that all facts supervene on qualitative facts and its negation respectively. In his “Individuals”, Dasgupta (2009) further argued that individualistic facts were shown by physics to be redundant and empirically undetectable and proposed changing our fundamental logical ideology to that of Quine’s logic of functors, which has the same expressive power as classical first-order logic but avoids commitment to individual entity variables by using variable binding and instantiation operators for qualitative n-place predicates and relations.

We want to highlight the compatibility of this research program with [Krause 2012] and [Krause and Arenhart 2016] motivation for introducing quasi-set theory. The ideology of quasi-set theory, its primitive predicates, logical constants, and logical consequence relation constitute a logical framework whose main aim is to allow for the representation of a different kind of ontic structure, quantum ontic structure. The classical ontic structure has been described by Turner (2010, 2011) in his influential papers on ontological nihilism and ontological pluralism as a pegboard structure. Think of reality, according to the ontological realist, as consisting of a pegboard with a series of pegs representing the variables quantifiers range over. We can attach a rubber band to the pegs, one rubber band for monadic predicates to an individual peg, one rubber band between two pegs for dyadic relations, three for triadic, and so on. To say the world has an ontological structure is to say it is structured like the pegboard and this structure is distinguished and mind-independent, the pegs are individuals.

Seen in this light Krause’s (2012) Quasi-set theory is in the business of quantum

¹⁷For the sake of argument suppose you take all axioms of a plurality of fundamental theories as the axioms of the one fundamental metaphysical theory, as in the Best System Accounts of Laws according to which the laws of nature are the axioms of our best theory that best balance simplicity and strength.

metametaphysics in the sense of Torza (2021), i.e., it aims to determine the nature of quantum logical space. The motivation is thoroughly metaphysical in that it presupposes that quantum mechanics is broadly true about the world. If we follow these methodological guidelines and quantum mechanics is a fundamental theory, its acceptance should involve a reconceptualization of the structure of logical space and hence a transition from classical to quantum logical space.

If this is right, Quasi-set theory is a contender for a metametaphysics of quantum mechanics taken seriously as a fundamental framework for objects in the most general possible sense, that of logic and set theory.¹⁸ In quasi-set theory there's no individualistic presupposition at the level of the fundamental ontic structure unrestrictedly (as holding for all objects), but quasi-set theory can nevertheless recover the classical framework's logical behavior for the objects of the domain under certain conditions: this is highlighted in the distinction between quantum objects which are indistinguishables and the M-objects, which are the classical emergent objects in this mathematical framework. In Krause's quasi-set theory, the fundamental notion is that of the indistinguishable instead of that of the individual (Krause and Arendhart, 2016). Indeed, Krause's quasi-set theory can recover ordinals and cardinals, the fundamental set-theoretic structures of order-types and sizes of sets respectively as special cases of quasi-set structures that hold for the classical part of the theory.

Krause's quasi-set theory and its metaphysical applications, as we've argued above, could be profitably investigated from Sider's (2011) perspective. From this perspective, the world has more than ontic structure, it has metaphysical structure where which is a term of art introduced by him denoting the primitive ideology of our metaphysically fundamental theories: their primitive logical connectives, consequence relations, quantifiers, operators, notions of object, predicates and so on. For Sider, metaphysical structure is as much about the world as ontic structure, but it does not correspond to objects, our understanding of what those metaphysical structures are about results partly from adopting those ideological frameworks and using them for representation and inference. Successful scientific theories employ primitive ideology and hence scientific success is a guide to structural truth, both ontic and ideological, a guide to metaphysical structure.

5.3 Quantum Mechanics, Measurement and the Crisp Axiom

All is not right with the world, however. There is a serious difficulty in Krause's (2012) and Krause and Arendhart's (2016) proposal for adopting quasi-set theory as our fundamental logical theory. To be clear, we have no objections to the formal details of the theory and it is certainly of great interest logically and mathematically, however its main motivation, as we've argued above, springs from the metametaphysics of quantum mechanics. It takes standard quantum mechanics (and its extension to quantum field theory) as a fundamental physical theory. This is captured in their system by the introduction of the Crisp axiom. Whenever an object is "crisp" then it has a classical structure, it becomes an M-object. Indeed, there is a "Crisp" predicate such that

¹⁸Quasi-set theory is also not a classical theory at the level of its logical consequence relation, which is paraconsistent i.e., it is inconsistency tolerant.

if a quantum object satisfies the predicate it becomes an M-object. Krause (2012) is explicit that intuition comes from the measurement postulate of Quantum Mechanics: upon measurement objects become Crisp. This is captured by the following axiom:

$$(C) \quad \forall x(m(x) \rightarrow (C(x) \rightarrow M(x)))$$

The standard quantum mechanical formalism has two crucial postulates meant to guide scientists whenever they are applying them to some physical system: the Schrodinger equation and the Born rule (Albert, 1992). Physical systems however will evolve in vastly different ways depending on whether we take them to be evolving only according to the Schrodinger equation or the Schrodinger equation plus the Born rule at some time. More precisely, our formal descriptions of the evolution of the system will be radically different depending on whether we consider the system evolving only according to the Schrodinger equation or both the Schrodinger equation and the Born Rule for a given interval of time.

Generally speaking, when considering the spin of a particle, the vector space associated with the spin-properties will be a two-dimensional complex vector space.¹⁹ Associated with the different spin properties there will be linear operators, mappings of the vector space onto itself which preserve the underlying structural properties of the space i.e., the ones specified by the axioms for vector spaces. We interpret the eigenvector-eigenvalue rule as telling us that the system is in a given state with some value for the property *iff* the vector associated with that state is an eigenvector of the operator associated with the property with a specific eigenvalue. The dynamical Schrodinger equation of standard quantum mechanics tells us that the system will evolve linearly between any two different times so long as no measurement of its physical properties is performed. Whenever we want to measure a property and the vector corresponding to the state of the system is not an eigenvector of the corresponding operator, we apply the Born Rule to the state vector of the system. It will tell us the probability that the system will be found to have some value or other.

Schrodinger evolution and evolution that involves collapse upon measurement are the two types of dynamical evolutions at the root of standard quantum mechanics, however, these two types of evolution are completely at odds: deterministic and linear versus indeterministic and non-linear. It is not that there is some sort of formal inconsistency here (Cf. Albert 1992, Maudlin 2019, Okon 2014) rather it is extremely puzzling that the physical systems which are allegedly represented in the standard Quantum Mechanics formalism behave in ways that seem almost magical: evolving deterministically when not measured and collapsing nondeterministically when measured.

But the measurement problem is not only that it is hard to make sense of the Quantum Mechanics formalism and its radically distinct laws of evolution as a physical theory: rather, it is that there is no physical theory specified here at all! Why? Because a crucial notion, that of “measurement” is completely vague. There is simply no formal counterpart in the theory for the act of “measuring”. Furthermore, there is no consensus in the community of physicists about when measurements take place or their nature. Fundamentally, it is useful to contrast the rest of the Quantum Mechanics

¹⁹Infinite dimensional when we consider the position, even for a single particle as there will be an infinite number of mutually orthogonal eigenvectors associated with the position operator.

formalism with the imprecise notion of measurement.²⁰ The Quantum Mechanics formalism represents a mathematical structure: the structure of complex Hilbert spaces, a sort of generalization of Euclidean spaces. The mathematical objects that we can define there, such as operators or tensor products, behave in precise ways; it is possible to tell about the formal theory whether some proposition couched in its language is an axiom, a theorem, an application of an operation defined within the structure and so on. In contrast “measurements” aren’t specified in any sort of formally rigorous way; they are understood contextually and have no clear conditions of individuation or applicability.

The crux of the issue however is this: since standard Quantum Mechanics can only be successfully applied if it combines the Schrodinger equation and the Born rule in ways that invoke the notion of “measurement” and since that notion is obscure in ways that go beyond lack of precision e.g. conditions of applicability, causal mechanisms, etc; then it has to be said that standard Quantum Mechanics doesn’t in fact qualify as a theory of the physical world. The issue is not that standard Quantum Mechanics isn’t scientifically respectable as a predictive tool, its astounding success shows otherwise, rather it is that any theory of the physical world should give a clear and precise specification of what its objects are (its ontology) and how they behave (the dynamics): otherwise there is simply no fact of the matter as to how the theory says the world is.

Quantum Mechanics is, metaphysically, a defective theory absent a solution to the measurement problem. Indeed some physicists (see for example, Hance and Hossenfelder 2022) have wondered whether further progress in theoretical physics has been slowed down by these methodological considerations, if so then the measurement problem poses a difficulty that goes beyond understanding how quantum mechanics is meant to represent the world fundamentally, regardless of whatever instrumental and engineering successes it has had so far. Metametaphysical approaches such as Krause’s (2012) help in the project of understanding quantum mechanics from the logical side, but the defectiveness in this approach is a result of taking an already defective theory as metaphysically fundamental. It might be that we need to reconceptualize logical space if we take seriously the quantum theory as a theory about the world’s ultimate metaphysical structure, but arguably that will require a solution to the measurement problem and hence a reconceptualization of logical space that starts from a better foundation e.g., GRW, Bohmian Mechanics, Wave Function Realism, State Space Realism or Many Worlds.

For these reasons, it would be a mistake to ignore the role that the identification and selection of specific structures play in enabling our understanding of scientific theories and their domains; especially when any of those are considered to be defective. As we hope to have shown for the case of Quantum Mechanics and non-individuality, is the identification of specific inferential constraints and patterns that allows for the intelligibility of the theory *qua* defective in a non-problematic way.

²⁰For a more comprehensive discussion on this issue, see [Macías-Bustos 2022b].

6 Final remarks

Here, we addressed the question of under which circumstances can scientists achieve a legitimate understanding of defective theories *qua* defective. And as a response to it, we introduced a structuralist approach to scientific understanding according to which, scientists understand a theory if:

- they can recognize the theory's underlying inference pattern(s) and
- if they can reconstruct and explain what is going on in specific cases of defective theories as well as consider what the theory would do if not defective –even before finding ways of fixing it.

Understanding the inferential structure of the theory involves understanding the structure of its domain. Furthermore, this understanding is modal in nature, in that the domain might not actually instantiate that structure, the structure need only be possible.

We illustrated the above with a case from Quantum Mechanics for which the theory is seen as defective, due to the non-individual metaphysical status of some of the entities of the theory. We contended that the identification of a structure that allows making the theory intelligible even if defective, but especially, *qua* defective, this structure should be included in the content of the understanding of the theory. For our case study, this structure was provided by the (paraconsistent) Quasi-set theory Ω_P (Cf. Krause 2012).

Acknowledgments.

The first author wants to thank Phil Bricker and Kevin Klement for valuable discussions on metaphysics and the philosophy of logic relevant to portions of this work. We thank the reviewer for the suggestions and critical comments. Also, thanks to Jonas Arenhart and Raoni Arroyo for their assistance during the process of getting the paper ready: they kindly dealt with our delays and other difficulties still inherent to editorial processes. For the second author, this research was supported by the Programa Nacional de Pós-Doutorado PNPd/CAPES (Brazil).

References

1. Albert, D. Z. (1992): *Quantum Mechanics and Experience*, Harvard University Press.
2. Arenhart, J.R.B., and D. Krause (2014): “Why Non-individuality? A Discussion on Individuality, Identity, and Cardinality in the Quantum Context”, *Erkenntnis* 79: 1–18.
3. Bengtson, J. (2018): “The Unity of Understanding”, in Grimm, S. (ed.) *Making Sense of the World: New Essays on the Philosophy of Understanding*, OUP.
4. Black, M. (1952): “The Identity of Indiscernibles”. *Mind* 61 (242):153-164.

5. Bueno, O. (1997): "Empirical Adequacy: A Partial Structures Approach", *Studies in History and Philosophy of Science* 28: 585–610.
6. Bueno, O. and S. French (2011): "How theories represent", *The British Journal for the Philosophy of Science* 62 (4): 857-894.
7. Bricker, P. (2014). "Ontological commitment". *Stanford Encyclopedia of Philosophy*.
<https://plato.stanford.edu/entries/ontological-commitment/>
8. Bricker, P. (1992/2020). "Realism without parochialism". *Modal matters: Essays in metaphysics*, OUP.
9. Bricker, P. (2020): *Modal Matters: Essays in Metaphysics*, OUP.
10. Cartwright, N. (1999): *The Dappled World: A Study of the Boundaries of Science*, Cambridge University Press.
11. Chiara, M.L.D, R. Giuntini and D. Krause (1998): "Quasiset theories for microobjects: A comparison", in Elena Castellani (ed.), *Interpreting Bodies*. Princeton University Press. pp. 142–52.
12. Chang, H. (2009): "Ontological Principles and the Intelligibility of Epistemic Activities" in de Regt, H. W., S. Leonelli and K. Eigner (eds.) *Scientific understanding : philosophical perspectives*, University of Pittsburgh Press.
13. Cowling, S. (2020): "Ideology and Ontology". *The Routledge Handbook of Metametaphysics*, Routledge, 376-386.
14. Cowling, S. (2013): "Ideological parsimony". *Synthese*, 190(17), 3889-3908.
15. da Costa N. C. A. and D. Krause (2014): "Physics, inconsistency, and quasi-truth", *Synthese* 191 (13):3041-3055.
16. Dasgupta, S. (2009): "Individuals: An essay in revisionary metaphysics". *Philosophical Studies*, 145(1), 35-67.
17. De Regt, H. W. (2009): "Understanding and Scientific Explanation" en *Scientific Understanding: Philosophical Perspectives*, H. W. de Regt, S. Leonelli, and K Eigner(eds.), University of Pittsburgh Press: 21–42.
18. De Regt, H. W. (2015): "Scientific Understanding: Truth or Dare?" *Synthese* 192: 3781–97.
19. De Regt, H. W., and D. Dieks. (2005): "A Contextual Approach to Scientific Understanding." *Synthese* 144: 137–70.
20. De Regt, H. W., and V. Gijsbers (2017): "How False Theories Can Yield Genuine Understanding", in S. R. Grimm, C. Baumberger and S. Ammon(eds.), *Explaining Understanding: New Perspectives from Epistemology and the Philosophy of Science*, Routledge: 50-75.

21. Elgin, C. Z. (2004): “True Enough”, *Philosophical Issues* 14: 113-131.
Elgin, C. Z. (2007): “Understanding and the Facts”. *Philosophical Studies*, 132(1): 33–42.
22. Elgin, C. Z. (2017): “Exemplification in Understanding”, in S. R. Grimm, C. Baumberger and S. Ammon(eds.), *Explaining Understanding: New Perspectives from Epistemology and the Philosophy of Science*, Routledge: 76-91.
23. Finocchiaro, P. (2021): “Ideology and its role in metaphysics”, *Synthese* 198, 957–983.
24. French, S. (2007): “The Limits of Structuralism”, BSPS Presidential Address: http://www.oxfordjournals.org/society/bsps/french_talk.
25. French, S. and D. Krause (1995): “Vague identity and quantum non-individuality”, *Analysis* 55 (1), 20-26.
26. Grimm, S. R. (2006): “Is understanding a species of knowledge?”, *British Journal for the Philosophy of Science*, 57(3): 515–535.
27. Grimm, S. R. (2014): “Understanding as knowledge of causes”, in A. Fairweather (Ed.), *Virtue epistemology naturalized*. New York, NY: Synthese Library: 329–345.
28. Halvorson, H. (2016): “Scientific Theories” in Paul Humphreys (ed.) *The Oxford Handbook of Philosophy of Science*, Oxford University Press.
29. Hofer, C. (2020): “Scientific Realism without the Quantum”. In: French, S. and J. Saatsi (eds) *Scientific Realism and the Quantum*: 19-34.
30. Khalifa, K. (2013): “Is understanding explanatory or objectual?”, *Synthese*, 190(6), 1153–1171.
31. Kelp, C. (2014): “Knowledge, understanding and virtue”, in A. Fairweather (Ed.), *Virtue epistemology naturalized*. New York, NY: Synthese Library. 366: 347–360.
32. Klement, K. (): “Russell on Ontological Fundamentality and Existence”, in *The Philosophy of Logical Atomism*, Palgrave Macmillan, Cham.: 155-179.
33. Krause, D. (2005): “Structures and Structural Realism”, *Logic Journal of the IGPL*, Volume 13, Issue 1: 113–126.
34. Krause, D. (2010): “Logical Aspects of Quantum (Non-)Individuality”, *Foundations of Science* 15:79–94.
35. Krause, D. (2012): “Paraconsistent Quasi-Set Theory” Retrieved from: <http://philsci-archive.pitt.edu/id/eprint/9053>.

36. Krause, D. and J.R.B. Arenhart (2016): “Individuality, Quantum Physics, and a Metaphysics of Nonindividuals: The Role of the Formal” in Guay, A. and T. Pradeu (eds.) *Individuals Across the Sciences*, OUP.
37. Krause, D., and J.R.B. Arenhart (2019): “Is Identity Really so Fundamental?”, *Foundations of science* 24: 51–71.
38. Krause, D., and A.M.N. Coelho (2005): “Identity, Indiscernibility, and Philosophical Claims”, *Axiomathes* 15: 191–210.
39. Krause, D., and S. French (1995): “A formal framework for quantum non-individuality”, *Synthese* 102, 195–214.
40. Kvanvig, J. (2003): *The value of knowledge and the pursuit of understanding*. Cambridge University Press.
41. Ladyman, J. and T. Bigaj (2010): “The principle of the identity of indiscernibles and quantum mechanics” *Philosophy of Science* 77 (1):117-136.
42. Landini, G. (1998): *Russell’s hidden substitutional theory*. Oxford University Press, USA.
43. Lawler, I. (2016): “Reductionism about understanding why”, *Proceedings of the Aristotelian Society*, 116(2): 229–236.
44. Lawler, I. (2021): “Scientific understanding and felicitous legitimate falsehoods”, *Synthese* 198 (7):6859-6887.
45. Le Bihan, S. (2017): “Enlightening Falsehoods: A Modal View of Scientific Understanding”, in S. R. Grimm, C. Baumberger and S. Ammon(eds.), *Explaining Understanding: New Perspectives from Epistemology and the Philosophy of Science*, Routledge: 111-136.
46. Leibniz, G. W., S. Clarke (1646-1716) (2000 ed.): *Leibniz and Clarke: Correspondence*, R. Ariew (ed.), Hackett Publishing Company, Inc. USA.
47. Lewis, D. (1970): “How to define theoretical terms”, *Journal of Philosophy* 67 (13):427-446
48. . Linsky, B. (1999): *Russell’s metaphysical logic*.
49. Maudlin, T. (2019): *Philosophy of Physics: Quantum Theory*, Princeton University Press.
50. Maudlin, T. (2018): “Ontological Clarity via Canonical Presentation: Electromagnetism and the Aharonov–Bohm Effect”. *Entropy* 20, 465.
51. Macías-Bustos, M. (2022a): “Metaphysical Realism and The Ideology of Logic”. *Manuscript in preparation*. Retrieved from: <https://umass.academia.edu/Moiss%3%A9sMac%3%ADasBustos>.

52. Macías-Bustos, M. (2022b): “Quantum Mechanics, Measurement and Scientific Realism”. *Manuscript in preparation*.
53. Morris, K. (2012): “A Defence of Lucky Understanding”, *British Journal for the Philosophy of Science*, 63(2): 357–71.
54. Okon, E. and D. Sudarsky (2014): “Measurements according to consistent histories”, *Journal Studies in History and Philosophy of Science Part B: Studies in History and Philosophy of Modern Physics* 48: 7-12.
55. Okon, E. (2014): “El problema de la medición en mecánica cuántica”, *Revista mexicana de física E* 60 (2): 130-140.
56. Pettit, D. (2002): “Why Knowledge is Unnecessary for Understanding Language.” *Mind* 111: 519–549.
57. Priest, G. (1985): “Inconsistencies in Motion”, *American Philosophical Quarterly* 22 (4):339 - 346.
58. Ramsey, F. (1929): ”Theories”, in R.B. Braithwaite (ed.), in *The Foundations of Mathematics and Other Logical Essays* (1931) with a preface by G.E. Moore, London: Kegan Paul, Trench, Trubner, & Co.
59. Russell, B. (1913): *Theory of Knowledge*.
60. Russell, B. (1927): *The Analysis of Matter*, London: Kegan Paul, Trench, Trubner; New York: Harcourt, Brace.
61. Russell, B. (1959): *An Inquiry into Meaning and Truth*.
62. Saunders, S. (2016): ”On The Emergence of Individuals in Physics”, in Guay, A. and T. Pradeu (eds.) *Individuals Across the Sciences*, OUP.
63. Sider, T. (2011): *Writing the Book of the World Get access Arrow*, OUP.
64. Sliwa, P. (2015): “Understanding and knowing”, *Proceedings of the Aristotelian Society*, 115(1): 57–74.
65. Strevens, M. (2013): “No Understanding without Explanation.” *Studies in the History and Philosophy of Science* 44: 510–5.
66. Strevens, M. (2017): “How Idealizations Provide Understanding”, in S. R. Grimm, C. Baumberger and S. Ammon(eds.),*Explaining Understanding: New Perspectives from Epistemology and the Philosophy of Science*, Routledge: 37–49.
67. Torza, A. (2021): “Quantum metametaphysics”, *Synthese* 199 (3-4):1-25.
68. Turner, J. (2010): “Ontological pluralism”. *The Journal of Philosophy*, 107(1), 5-34.
69. Turner, J. (2011): “Ontological nihilism”. *Oxford studies in metaphysics*, 6, 3-54.

70. Votsis, I. (2004): *The Epistemological Status of Scientific Theories: An Investigation of the Structural Realist Account*, Doctoral Dissertation, London School of Economics (University of London).
71. Votsis, I. (2018): "Structural Realism and its Variants", in J. Saatsi (ed.), *The Routledge Handbook of Scientific Realism*, London: Routledge: 108-119.
72. Wilkenfeld, Daniel A. 2017. "Understanding without Believing", in S. R. Grimm, C. Baumberger and S. Ammon(eds.), *Explaining Understanding: New Perspectives from Epistemology and the Philosophy of Science*, Routledge: 318–333.
73. Wilkenfeld, D. A., D. Plunkett, and T. Lombrozo. 2018. "Folk Attributions of Understanding: Is there a Role for Epistemic Luck?" *Episteme* 15(1): 24-49.
74. Ylikoski, P. (2013): "The Illusion of Depth of Understanding in Science", in H. De Regt, S. Leonelli & K. Eigner (eds.) *Scientific Understanding: Philosophical Perspectives*, Pittsburgh University Press: 100-119.