An Operational Definition of Electric Charge

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Abstract : The paper focuses on the part of the Coulomb's Law that is just a definition and provides one possible mechanism for operationally defining electric charge based on the concept of force. Then a derivation of Coulomb's law from the definition is presented and the sign of the charges are defined. Finally, the paper concludes with a discussion on the conservation of charge.

Keywords : Operational Definition, Coulomb's law , Electric charge , replace

Introduction

Electric Charge can be defined for classical objects as the signed difference of the number of protons and the number of electrons. However, this definition does not enable us to know the charge of an object experimentally in the domain of classical electromagnetism. We have to use Coulomb's Law and its consequences for knowing the charge which means that we don't have any information about charge without appealing to the law. We will show in this paper how one can define and measure electric charge operationally and derive the Coulomb's Law from it.

When we define some concept using its properties, we say that the concept is defined operationally. Therefore, we will define electric charge using its properties whose verification lies upon experiment.

Charged Particle

Because of the force between charged and induced particle, we need some device to distinguish between charged and induced particle. The simplest way to do this is to use a third object and calculate the force of the test particles on this object. Some closer examination reveals that the third object can not have a net charge but it will be induced when a charged body is nearby with sufficient accuracy demanded by experimenter. *It is on experiment to find that in similar situation two identical bodies (intuitively) are charged in such a way that they repel.* Based on this, we can know that something is uncharged or charged. If we find an uncharged body, we just need to know whether there is force between this and some other object which we have found charged. If there is force between them, this uncharged body can be used as the third object. However, we can use "gold leaf electroscope" as a mean to know if something is charged or not.

Definition Of Electric Charge

Now, we are at the main objective of the present paper. We propose the following experiment: We have two charged particle P_A and P_1 both of them are used as the bob of some pendulum which we have hanged from two pivot points A and D as shown in the picture below.

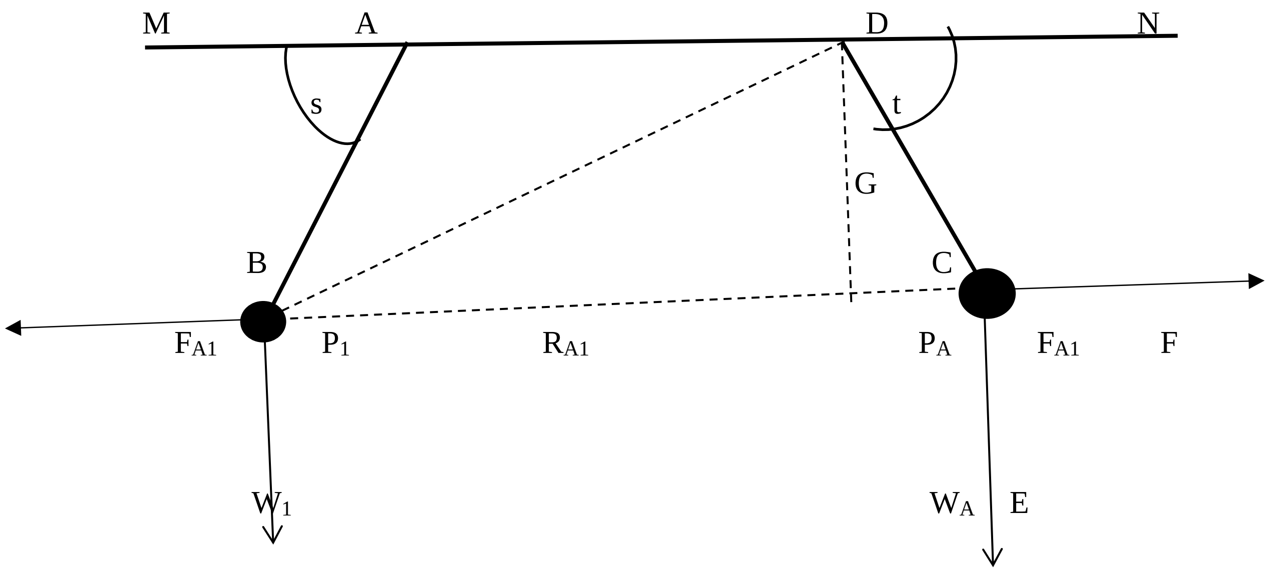

Fig 1 : Static Equilibrium of charges

Here, the distance AD > AB + DC , that is : the distance between the pivot points is greater than the sum of the length of the threads . It is necessary because we don't know whether the particles attract or repel and sum of the two sides of a triangle is greater than the third side ,since we don't want the particles to stick together if they attract . Also , the particles are in static equilibrium ,not necessarily stable, which can easily be done by changing the length of the threads and the distance between the pivots.

Now we assume that the particles repel. The other case is similar, so we don't present it here .In addition, we assume that we can calculate the weight , length of the threads and also the angles s and t mentioned in the Figure . *if we have length of AD, AB ,CD and the angle s , t , the quadrilateral ABCD is uniquely defined and we can find all the other angles and sides.*

Now, using Lami's theorem on statics , we arrive :

$$F_{A1} = W_A R_{A1} \cos t / a \sin < BDC \text{ …………… (i)}$$

A similar treatment allows one to find the force in the case of attraction ,as we have already said earlier. Hence, from now on we will assume that we can find the force in all cases which can be modeled using Fig-1 .

Now, we replace the charge P_A with another charge P_B in Fig-1 and we may even change the length of the threads and the distance between the pivots . We assume that we can arrange static equilibrium again . Then we have F_{A1} , F_{B1} ,R_{A1} , R_{B1} using (i) and (4) and we take the ratio $(F_{A1}R^2_{A1}) / (F_{B1}R^2_{B1})$ which we denote by q_1. Again, we replace P_1 by P_2 , P_3 , P_4 …………in each case we can change the length of the threads and pivots and measure the corresponding F_{A2},F_{A3},F_{A4},…. F_{B2} , F_{B3} , F_{B4} ……….., R_{A2}, R_{A3}, R_{A4}……., R_{B2},R_{B3},R_{B4} ……using (i) and (4) and using them the ratios q_2 ,q_3,q_4……

It is on experiment to conclude that $q_1=q_2=q_3=q_4=$………= q. Then we conclude that the ratios depends on P_B and P_A and not on P_1,P_2,P_3,P_4…….Therefore ,the ratio is a relative property. That is

$$(F_{A1}R^2_{A1}) / (F_{B1}R^2_{B1}) = q \;\ldots\ldots\ldots\ldots\text{(ii)}$$

Before proceeding , we need some discussion about the meaning of (ii) . The equation here means that *in Fig-1 if we replace only one charge with another charge ,we may even change the threads and pivots ,the ratio in the left side of the equation (ii) is independent of the unchanged particle* .

We will define charge relative to P_B. We define the charge , in accordance with the preceding sentence , of P_A to be q unit . Thus if P_A and P_B are the same ,in that case the substitution of P_B for P_A is in the Fig-1 is unnecessary, and also if we do not change the threads and pivots we find that P_B has an unit charge. We ,therefore, have *almost* completed our definition of electric charge

Derivation Of Coulomb’s Law

We take any charge P_A and P_1 in Fig-1 and then replace unit charge P_B for P_A, we may change the threads and pivots , we then have from (ii)

$$(F_{A1}R^2_{A1}) / (F_{B1}R^2_{B1}) = q \;\ldots\ldots\ldots\ldots\text{(1)}$$

The discussion after (ii) implies that in (1) k is the charge of P_A .Now, using charge P_C instead of P_A and P_1 in Fig-1 and then replacing P_C with P_B , we may change the threads and pivots, and using (ii) again, we have :

$$(F_{C1}R^2_{C1}) / (F_{B1}R^2_{B1}) = q^* \ldots\ldots\ldots\ldots\text{(2)}$$

Here again k* is the charge of P_C .Dividing (1) by (2) , we have

$$(F_{A1}R^2_{A1}) / (F_{C1}R^2_{C1}) = q/q^* \ \ldots\ldots\ldots\ldots\ldots\ldots\ldots (*)$$

The meaning of (*) is dependent on the meaning given for (ii). That is: *(*) means that the ratio ,independent of the unchanged particle ,threads and pivots, in (ii) is equal to the ratio of the charge that is changed (P_A in (ii)) and the charge which is used after replacement (P_B in (ii) and in (*) P_C) .*

Now, in (*) if we take all the particles (P_A, P_1, P_C) to be of unit charge and if we change the threads and pivots , allowed by the discussion after (ii) and also in the discussion above on (*) , we get in the right side of (*) 1 (one) . This suggests the following : *Mutual electric force times the square of the distance of two unit charge is a constant of nature.* This constant we denote by K_o.

We are now ready to get back to the derivation as the title of the section suggests . We take ,now, two charged particles S and T with charge q_S and q_T whose mutual force we want to calculate to establish Coulomb's law.

First, we take S and a unit charge u in Fig-1 . Then we replace S with v ,we may change the pivots and threads , in Fig-1. If the force and distance between S and u ,we denote by F_{Su} and R_{Su} respectively. We get , by (ii) and using the discussion about the constant K_o above in Italic .

$$(F_{Su}R^2_{Su})= K_o q_S \ldots\ldots\ldots\ldots (3)$$

In (7) we have used the ratio q in (ii) to be q_S , the charge of S , by the discussion after (ii).

Because of the meaning given here about (*) , if we replace u with T instead of replacing S with v , we get from (*) ,

$$(F_{Su}R^2_{Su}) / (F_{ST}R^2_{ST}) = 1/q_T \qquad \text{[u is unit charge]}$$

which implies $K_o q_S q_T = F_{ST}R^2_{ST}$ [using (3)]

which implies $F_{ST} = K_o q_S q_T / R^2_{ST} \ \ldots\ldots\ldots\ldots\ldots\ldots (**)$

(**) is the Coulomb's law by its very form which is easily recognized. However, one sees immediately after seeing the Coulomb Law in (**) , that there is some problem in our definition of charge . This is the very reason we have said earlier that our definition is almost complete. The problem is :It is known that there are two types of charge in Nature but our definition is not able to distinguish between them and as a consequence , we do not see modulus of any quantity in the right side of (**) ,although we are focusing on the magnitude of the force . For doing this , we will use a "algebraic trick" ,so to speak .

Completing the Definition

Suppose we have a charge X. We define two collections A and B where we assume that there are at least two elements in each of the collection other than X. We further assume that any member of A, other than X which also belongs to A, repels X and any member of B attracts X and this completes our definition of the collections. Now, we ask ourselves what will happen if we take any two members from the sets. It is on experiment to find that *if the two members belong to the same set, they repel whereas if they belong to different sets they attract.*

For formulating this mathematically, take the X to be the unit charge P_B. For any member of the collection B, we designate their charge by putting a negative sign in front of the magnitude of the charge we defined earlier. It should be noted that we could put the minus in front of the charge of the members of A instead of B and this is merely a matter of convention. After taking such convention granted, we are already done distinguishing between two types of charge in nature . But we have not yet outlined how to find ,in a simple way, whether two particle will repel or not .For doing this ,we mention that following is true how remarkable this may seem :

For any two particles S and T , if we designate the unit vector from S to T as $\boldsymbol{i}$,charge of S and T as q_S and q_T respectively , electric force on T by S as $\mathbf{F}_{ST}$, then form of (**) will be

$$\mathbf{F}_{ST} = (K_o q_S q_T / R^2{}_{ST})\boldsymbol{i} \quad \ldots\ldots\ldots\ldots\ldots.(***)$$

(***) is a vector equation and in this equation we have to take into consideration the sign of the charge. One can check by considering different cases , the consistency of the direction of force in (***) with the requirement of the sentence in Italic above. This last equation ,however , is the Coulomb's law in its general form.

Conservation of Charge

For the conservation of charge, we don't see any other way but to postulate it but we warn the reader that we have to take into account the sign of the charge here and this is the very portion of this law which makes it look artificial. If the law seems obvious to some reader , we should mention that in the case we did not use the minus sign for the collection B , the law can be stated thus : *In a closed system ,summation of initial total charges that belong to A and final total charges that belong to B is equal to the summation of final total charges that belong to A and the initial total charges that belong to B.* And it can safely be said that this form of the law does not seem obvious. Moreover, in the Italic sentence above we can interchange A and B and that also becomes a statement of the conservation of charge. This property of interchanging represents that the collection A and B are dual to each other.

Conclusion

We have tried here to give an elementary definition of electric charge and derive Coulomb's law with some other hypothesis at least for the systems that can be modeled using Fig-1 . The definition used here is operational and so is not just a mathematical truism rather it has some physical content which uses result from experiment. Although we have implicitly used Coulomb's law for predicting the result of the experiments, it is not a circularity in argument since we could proceed by performing the experiments.

References

The present paper is nearly self-contained. However, for a better understanding of the concept of operational definition, the reader can use the following books:

[1] Percy Williams Bridgman (1927)-"The Logic of Modern Physics" -published by THE MACMILLAN COMPANY

[2] Ernst Mach-"The Science Of Mechanics - A Critical and Historical Account of its Development" – Translated from German by Thomas J. McCormack