Depragmatized Dutch Book Arguments*

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Recently a number of authors have tried to avoid the failures of traditional Dutch book arguments by separating them from pragmatic concerns of avoiding a sure loss. In this paper I examine defenses of this kind by Howson and Urbach, Hellman, and Christensen. I construct rigorous explications of their arguments and show that they are not cogent. I advocate abandoning Dutch book arguments in favor of a representation theorem.

1. Introduction. Dutch book arguments have been a popular way of arguing that people's degrees of belief ought to satisfy the axioms of probability. Traditionally such arguments have purported to show that people who have degrees of belief that do not satisfy the axioms can be made to suffer a sure loss in a betting situation. Such arguments have been judged unsound by many authors, including me (Maher 1993, Section 4.6).

Recently a number of defenders of Dutch book arguments have conceded that this traditional kind of Dutch book argument is unsound. But instead of abandoning Dutch book arguments they have attempted to reinterpret the arguments to avoid the objections. A popular approach has been to say that the failures of traditional Dutch book arguments can be avoided if we separate the argument from pragmatic concerns of avoiding a sure loss. In this paper I examine defenses of this kind by Howson and Urbach, Hellman, and Christensen. I construct rigorous explications of their arguments and show that they are not cogent.

My own view is that the enterprise of defending the axioms of probability with a Dutch book argument is a good example of what Lakatos

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(1970) called a degenerating research programme. I would urge that this research programme be abandoned and the axioms of probability defended instead with a representation theorem, as is done in Maher 1993.

2. Howson and Urbach. In this section I will discuss the Dutch book argument for the axioms of (finitely additive unconditional) probability that Howson and Urbach present in their 1993 (75–89). I will begin by giving a formulation of their argument that is more precise and complete than their own. In doing this I have taken some liberties with what Howson and Urbach actually wrote, but I claim that my formulation of their argument is a maximally charitable one. The right way to dispute this claim would be to produce a more charitable formulation that is equally precise and complete.

2.1. Definitions and Premises. Let X be a set of states and $\mathcal{X}$ an algebra on X. The elements of $\mathcal{X}$ can be regarded as propositions, i.e., $A \in \mathcal{X}$ is the proposition that the true state is in A. A *random variable* is a real-valued measurable function defined on X. In this paper random variables usually measure the amount of money a person receives from some arrangement.

Definition 2.1 *For all $A \in \mathcal{X}$ and real numbers p and s, a* bet *on A with betting quotient p and stake s, denoted bet(A,p,s), is the random variable f such that $f(x) = (1 - p)s$ if $x \in A$ and $f(x) = -ps$ if $x \in \bar{A}$.*

Howson and Urbach assume that it is meaningful to talk about the *advantage* of a bet or, more generally, of a random variable. After presenting their Dutch book argument they say that the notion of advantage can be explicated as expected value relative to the person's subjective probability function (84f.). However, it is clear that Howson and Urbach do not intend their argument to depend on this explication, so I take Howson and Urbach to be assuming that the notion of advantage is pretheoretically intelligible. The explication that they later give is nevertheless useful for us in making clear that advantage is to be construed as relative to persons. I will use the notation $adv(r,f)$ to denote the advantage to person r of random variable f.

Howson and Urbach say that a betting quotient is *fair* if it gives "zero advantage to either side of a bet" (79). Since they say nothing about the stakes, I suppose the fair betting quotient to be independent of the stakes.

Definition 2.2 *p is a* fair betting quotient *on A for r iff, for all real numbers s,*

$$adv(r, bet(A,p,s)) = 0.$$

The "other side" of $bet(A,p,s)$ is $bet(A,p, -s)$, so Definition 2.2 agrees with Howson and Urbach's statement that a fair betting quotient gives "zero advantage to either side of a bet."

Howson and Urbach initially assume that "your" fair betting quotient for an arbitrary proposition exists and is unique. Taking "you" to be any rational person, this assumption can be expressed as:

> **Premise 2.1** *For all rational persons r and* $A \in \mathcal{X}$, *there is a unique number* $p_r(A)$ *which is a fair betting quotient on A for r.*

Howson and Urbach concede that Premise 2.1 is an idealization and that real people do not always have unique fair betting quotients. They propose to deal with this by supposing instead that fair betting quotients need only be determined to within some interval, rather than having a precise value (1993, 87ff.). I will follow Howson and Urbach in first discussing the idealized argument with Premise 2.1. Later I will show that their way of relaxing the idealization does not close the gaps I identify in their argument.

Howson and Urbach also assume that "if a particular betting strategy is *assured* of a positive net gain or loss for whoever adopts it, then the net advantage in betting at the odds involved cannot be zero" (79). In formalizing this assumption, I will use the notation $f > 0$ to mean that the random variable f is strictly positive, i.e., $f(x) > 0$ for all $x \in X$. Similarly for $f < 0$. Then the assumption can be stated as:

> **Premise 2.2** *For all rational persons r and random variables f, if* $f > 0$ *or* $f < 0$ *then* $adv(r,f) \neq 0$.

The third and last premise that I attribute to Howson and Urbach is the following:

> **Premise 2.3** *For all rational persons r and random variables f and g, if* $adv(r,f) = 0$ *and* $adv(r,g) = 0$ *then* $adv(r,f + g) = 0$.

Here $f + g$ is the random variable whose value, for any $x \in X$, is $f(x) + g(x)$.

Howson and Urbach do not seem to be aware of making any substantive assumption like Premise 2.3. They write: "the sum of finitely (or even denumerably) many zeros is zero; hence the net advantage of a *set* of bets at fair odds is zero" (79). In this argument they seem to be supposing that it is a mathematical truth that if several random variables have zero advantage then the sum of those variables also has zero advantage. However, this is not a mathematical truth, and I will later give examples in which it is false. Thus Premise 2.3 is a substantive assumption, contrary to what Howson and Urbach seem to think. I will also show that the assumption is necessary for their argument (in the sense that their conclusion does not follow from Premises 2.1 and 2.2 alone).

2.2. Theorems. In this section I will show that Premises 2.1 to 2.3 imply that the function p_r satisfies the axioms of (unconditional finitely additive) probability.

Theorem 2.1 *For all rational persons r and* $A \in \mathcal{X}$, $p_r(A) \geq 0$.

Proof: Suppose the theorem is false. Then there is a rational person r and $A \in \mathcal{X}$ such that $p_r(A) < 0$. From Definition 2.1 we have that $bet(A,p_r(A),1) > 0$. From Premise 2.1 we have that $adv(r,bet(A,p_r(A),1)) = 0$. But this contradicts Premise 2.2. ■

Theorem 2.2 *For all rational persons r,* $p_r(X) = 1$.

Proof: Suppose the theorem is false. Then there is a rational person r such that $p_r(X) \neq 1$. From Definition 2.1 we have that $bet(X,p_r(X),1) > 0$ if $p_r(X) < 1$ and $bet(X,p_r(X),1) < 0$ if $p_r(X) > 1$. From Premise 2.1 we have that $adv(r,bet(X,p_r(X),1)) = 0$. But this contradicts Premise 2.2. ■

Theorem 2.3 *For all rational persons r and* $A,B \in \mathcal{X}$, *if* $A \cap B = \varnothing$ *then* $p_r(A \cup B) = p_r(A) + p_r(B)$.

Proof: Suppose the theorem is false. Then there is a rational person r and A, $B \in \mathcal{X}$, such that $A \cap B = \varnothing$ and

$$p_r(A \cup B) \neq p_r(A) + p_r(B). \tag{1}$$

Let

$$g = bet(A,p_r(A),1) + bet(B,p_r(B),1) + bet(A \cup B,p_r(A \cup B),-1).$$

By Premise 2.1 all the bets on the right hand side have zero advantage to r and so by two applications of Premise 2.3 we have

$$adv(r,g) = 0. \tag{2}$$

From Definition 2.1 we have that, for all $x \in X$,

$$g(x) = p_r(A \cup B) - p_r(A) - p_r(B). \tag{3}$$

From (1) and (3) we have that $g > 0$ or $g < 0$, but this together with (2) contradicts Premise 2.2. ■

2.3. The Need for Premise 2.3. In this section I will show that Theorem 2.3 does not follow from Premises 2.1 and 2.2 alone. I will do this by describing a situation in which those two premises are satisfied but the theorem is false.

Let $\mathcal{X} = \{\emptyset, A, \bar{A}, X\}$, with all four of these sets distinct. Suppose Premise 2.1 is satisfied with

$$p\ (\emptyset) = 0; p_r(A) = 1/3; p_r(\bar{A}) = 1/3; p_r(X) = 1.$$

If f is a random variable, the requirement that random variables be measurable implies that $f(x)$ is the same for all $x \in A$; I will denote this common value as $f(A)$. Likewise for $f(\bar{A})$. For any random variable f, suppose

$$adv(r,f) = p_r(A)f(A) + p_r(\bar{A})f(\bar{A}) = [f(A) + f(\bar{A})]/3.$$

Then if $f > 0$ it follows that $adv(r,f) > 0$, and similarly, if $f < 0$ then $adv(r,f) < 0$. Thus Premise 2.2 is satisfied. However, Theorem 2.3 is not satisfied, since

$$p_r(A) + p_r(\bar{A}) = 2/3 \neq p_r(A \cup \bar{A}) = p_r(X) = 1.$$

2.4. What is Advantage? I have shown that Howson and Urbach's conclusion does follow from Premises 2.1 to 2.3. I will now consider whether those premises are true.

All three premises involve the notion of advantage. In Premise 2.1 this is because the notion of a fair betting quotient is defined using the notion of advantage, while in Premises 2.2 and 2.3 it is because the notion of advantage is used directly. So we can hardly be expected to know whether these three premises are true without knowing what is meant by 'advantage.'

As I remarked earlier, Howson and Urbach (84f.) say that $adv(r,f)$ can be explicated as the expected value of f relative to p_r, but they make it clear that their argument is not intended to depend on this explication. Furthermore, there is a good reason why the argument cannot make use of this explication. The reason is that to understand the explication we need to understand what p_r is. Presumably this would be defined by saying that $p_r(A)$ is r's fair betting quotient for A, as in Premise 2.1. But the notion of fair betting quotient is defined in terms of advantage in Definition 2.2. Thus to use this explication of advantage in the argument would be circular. The notion of advantage therefore needs to be understood in a way that does not depend on Howson and Urbach's proposed explication of it. Howson and Urbach apparently believe that there is an "informal notion of advantage" (85) which is sufficient for the purposes of their argument.

However, if this "informal notion of advantage" is meant to be the ordinary meaning of the word 'advantage', then advantage does not have the properties that Howson and Urbach require it to have. Of the various senses of 'advantage' reported in *Webster's Third New International Dictionary,* the one closest to what Howson and Urbach want is "benefit, profit, or gain." Now the "benefit, profit, or gain" to a person from *bet*(A,p,s) is not in general independent of s (even assuming $s \neq 0$). I will support this point with an example and then note its implications.

Let H be the proposition that a toss of a coin lands heads and suppose that I give H a probability of 1/2. Then for me an even money bet on H with a stake of \$10 has zero advantage, in the sense of 'advantage' we are presently considering. I would not regard receiving this bet as a "benefit, profit, or gain", and if I had such a bet I would not think that parting with it was a "benefit, profit, or gain." Thus if stakes are measured in dollars and m is me,

$$adv(m,\ bet(H,\ 1/2,\ 10)) = 0. \qquad (4)$$

On the other hand, an even money bet on H with a stake of \$10,000 has negative advantage for me. I would regard receiving this bet as a liability and, if I had been saddled with it, there would be a "benefit, profit, or gain" to me in ridding myself of the bet. Thus

$$adv(m,\ bet(H,\ 1/2,\ 10000)) < 0. \qquad (5)$$

Though I have reported these judgments as my own, they are normal judgments of a kind that many people would make. Furthermore, there is an intelligible rationale for these judgments. The disadvantage of losing a small amount of money is about equal to the advantage of gaining the same amount of money; but the disadvantage of losing a large amount of money is greater than the advantage of gaining the same amount of money. So these judgments cannot be condemned as irrational.

I have been arguing that the advantage of a bet depends in general on the size of the stake. Now I will note the consequences of this. First, it means that the ordinary notion of advantage cannot be correctly explicated as expected value. If the expected value of *bet*(H, 1/2, 10) is zero, then the expected value of *bet*(H, 1/2, 10000) must also be zero, but in my example the advantage of the first bet is zero while that of the second bet is negative.

A second consequence is that Premise 2.1 is false. For by (4), the only possible value of $p_m(H)$ is 1/2, and by (5) this is not the value of $p_m(H)$.

A third consequence is that Premise 2.3 is false. To see this suppose, what is not too far from the truth, that

$$adv(m, bet(H, 1/3, 10000)) = 0.$$

Since I think the coin equally likely to land heads and tails, I also have

$$adv(m, bet(\bar{H}, 1/3, 10000)) = 0.$$

However, $bet(H, 1/3, 10000) + bet(\bar{H}, 1/3, 10000)$ gives a sure gain of \$3333.33, so

$$adv(m, bet(\bar{H}, 1/3, 10000) + bet(\bar{H}, 1/3, 10000)) > 0$$

contrary to Premise 2.3.

I suppose it will be objected that I am understanding the notion of advantage in a way that ties it too closely to choice or preference, while Howson and Urbach (77) have insisted that a judgment of fairness does not imply willingness to accept a bet. In these terms, what I am saying is that the *ordinary* notion of perceived advantage is not as divorced from preference as Howson and Urbach need it to be. The notion of expected value is suitably divorced from the notion of preference, but this is not an understanding of 'advantage' that Howson and Urbach can use without circularity. So my question is: What is the notion of advantage that will make Howson and Urbach's premises true?

The problem that I am raising would be avoided if the outcomes of bets were expressed in terms of utility rather than money. However, it is clear that this is not what Howson and Urbach intend (77). Furthermore, such a use of utilities here would bring with it the obligation to defend the assumption that rational people do have utilities. This can be done with a representation theorem, but doing that without assuming probabilities (which is what is needed here) would require using a representation theorem like those of Ramsey or Savage or myself, and that kind of representation theorem also entails the existence of probabilities and so makes the Dutch book argument redundant. Thus the use of utilities is not a viable solution.

Perhaps it will be said that in Definition 2.2 s should be limited to small values, the thought being that for small amounts of money the advantage or disadvantage in gaining or losing is proportional to the amount of money gained or lost. However, the experimental literature in decision theory suggests that for many people the disadvantage of losing just a few dollars is greater than the advantage of gaining the same number of dollars, and I doubt that Howson and Urbach want to deem people irrational for this. Furthermore, in some circumstances all of us will violate the required assumption; for example, if I need \$1 to catch a bus home, acquiring \$1 will have a value for me that is out of proportion to the amount of money involved. Thus even with the restriction to small stakes, the basic problem is not avoided.

2.5. Upper and Lower Probabilities. I mentioned earlier that Howson and Urbach concede that Premise 2.1 is an idealization because degrees of belief may not be point-valued. They propose to accommodate this by replacing the notion of a fair betting quotient with upper and lower probabilities (87). These upper and lower probabilities are supposed to trichotomize bets in the following way:

- Bets with betting quotients below the lower probability give positive advantage.
- Bets with betting quotients above the upper probability give negative advantage.
- Bets with betting quotients between the lower and upper probabilities have indeterminate advantage (the person has no opinion regarding their advantage).

Premise 2.1 would then be weakened to say that, for all rational persons r and $A \in \mathcal{X}$, there are unique numbers $p^*(A)$ and $p_*(A)$ which are upper and lower probabilities on A for r.

This relaxation does not avoid the objection I made to Howson and Urbach's argument in the preceding section. In the counterexamples that I gave, I supposed that I had precise degrees of belief, and thus there is no call for introducing upper and lower probabilities in these cases. Likewise, I supposed that every bet had a definite advantage; there was no interval in which bets had indeterminate advantage.

2.6. Why Believe Premise 2.3? I've been objecting to Howson and Urbach's argument on the ground that there is no notion of advantage that will do what they require. I will conclude with another, independent, objection.

In Section 2.3 we saw an example of how Premise 2.3 can fail even when Premises 2.1 and 2.2 are true. Howson and Urbach have not given us any reason to think such an example involves any irrationality. Perhaps this oversight was due to their not seeing that such a violation is even mathematically possible. In any case, in the absence of any argument of this kind, Premise 2.3 stands entirely unsupported.

I myself think that the example of Section 2.3 does involve irrationality, but as far as I know this can only be demonstrated by using the principles that are used in deriving a representation theorem, so this kind of argument is not available to Howson and Urbach.

Since the example of Section 2.3 satisfied Premise 2.1, the difficulty here is not affected by allowing upper and lower probabilities.

3. Hellman. Hellman (1997, Section 2) endorses Howson and Urbach's argument, though without providing any support for its premises. Indeed, he writes at one point as if the argument has no premises; he says that a system of beliefs that violates a probability axiom "*reveals a logico-mathematical contradiction:* the believer has assessed as fair a set of odds that are provably not all fair, by logic and elementary mathematics" (p. 194). However, Hellman does not bother to show how this conclusion can be derived by logic and mathematics alone, and later he alludes to some of the substantive assumptions made by Dutch book arguments.

Hellman thinks that the virtues that he perceives in Howson and Urbach's argument can be realized even more fully in a modified version of that argument that he has devised and that he calls a *Dutch flow argument.* In this argument, Howson and Urbach's notion of a fair betting quotient (Definition 2.2) is replaced by the notion of a *fair belief-test quotient,* defined as follows.

Consider a random variable that has the value a if A is true and $-b$ if A is false. This random variable has an expected value of zero relative to probability function Pr iff

$$a\Pr(A) = b\Pr(\bar{A}). \qquad \text{(Zero Expected Flow)}$$

Hellman says that if this condition holds then the fair belief-test quotient for A is $b/(a + b)$.

Hellman appears to think that he is here defining a measure of degree of belief, analogous to Howson and Urbach's definition of a fair betting quotient. However, Hellman's definition differs from Howson and Urbach's in assuming that a probability function Pr already exists. Though Hellman never explains what Pr is, it is presumably meant to be a measure of degree of belief. Thus if Hellman's definition of the fair belief-test quotient is a definition of degree of belief, it is circular. The problem is the same one that I earlier noted when I considered the idea that the notion of advantage used by Howson and Urbach might be defined as expected value.

Let us see if we can nevertheless salvage something from Hellman's account of fair belief-test quotients. We might suppose that he is taking for granted the existence of a measure Pr of degree of belief and is using judgments of zero expected value to elicit (rather than define) the values of this measure. Thus when the person tells us that the random variable (a if A, $-b$ if $\bar{A}$) has zero expected value, we infer that $\Pr(A)$

$= b/(a + b$.) However, this last identity does not follow from (Zero Expected Flow) unless we assume that $\Pr(\bar{A}) = 1 - \Pr(A)$, and that is part of what a Dutch book argument should be proving, not assuming.

Alternatively, we might take the fair belief-test quotients $b/(a + b)$ to be a *second* measure of degree of belief, not necessarily identical to Pr, and the Dutch flow argument (which Hellman refers to but never presents) might be that judgments of zero expectation ought to satisfy conditions which imply that fair belief-test quotients satisfy the axioms of probability.

Let $q(A)$ denote a person's fair belief-test quotient for A. Depending on which of the preceding two interpretations of Hellman's argument is adopted, $q(A)$ may or may not be identical to $\Pr(A)$. Hellman's elusive Dutch flow argument is supposed to show that the function q ought to satisfy the axioms of probability. However it does that, it will need to assume that Pr has many of the properties of a probability function. I will illustrate this by demonstrating a couple of the assumptions that would need to be made (besides the assumption that $\Pr(\bar{A}) = 1 - \Pr(A)$).

Suppose that $\Pr(A) = -1/2$ and $\Pr(\bar{A}) = 1 - \Pr(A) = 3/2$. Then (Zero Expected Flow) holds with $a = -3$ and $b = 1$, whence $q(A) = -1/2$, violating the first axiom of probability. So it must be assumed that Pr cannot take on values like these.

Again, suppose that $\Pr(X) = 0$ and $\Pr(\emptyset) = \Pr(\bar{X}) = 1 - \Pr(X) = 1$. Then (Zero Expected Flow) holds with $a = 1$ and $b = 0$, whence $q(X) = 0$, violating the second axiom of probability. So it must be assumed that Pr cannot take on these values.

Hellman has not bothered to even articulate, much less defend, the properties that Pr must be assumed to have for his argument to reach its conclusion. And even if he were to articulate the required assumptions, there can be no reason to believe them in the absence of some interpretation of Pr, which Hellman has not provided. And if he were to provide an interpretation of Pr, and defend the assumptions using that interpretation, he would thereby have shown that the measure Pr of degree of belief has many or all of the properties of a probability function, so that it would then be largely if not completely redundant to argue that fair belief-test quotients also satisfy the laws of probability.

I conclude that Hellman's Dutch flow argument, whatever it is, is not only without any cogency as it stands, but also no argument of this kind could ever be useful for showing that degrees of belief ought to satisfy the axioms of probability.

4. Christensen. In this section I will discuss Christensen's (1996, 456–459) "depragramatized" interpretation of Dutch book arguments for the axioms of probability. As I did with Howson and Urbach, I will begin by giving a formulation of his argument that is more precise and complete than his own, though in doing so I have taken some liberties with what he actually wrote. I claim that the result is a maximally charitable formulation of Christensen's argument. The right way to dispute that would be to produce an equally rigorous explication of the argument that is more charitable.

4.1. Premises. Christensen begins thus:

> Putting aside any behaviorist or functionalist accounts of partial belief, it is initially quite plausible that a degree of belief of, for example, 2/3 that of certainty *sanctions as fair*—in one relatively pretheoretic, intuitive sense—a bet at 2:1 odds. The idea is not that any agent with 2/3 degree of belief in *P* is rationally obliged to agree to putting up $200.00 to the bookmaker's $100.00 on a bet the agent wins if *P* is true. Various factors—involving, for example, the nonlinear utility of money, or risk aversion, may make it irrational for him to accept such bets. But there does seem to me to be an intuitively appealing normative ceteris paribus connection: other things being equal, an agent should evaluate such bets as fair. Degrees of belief may in this way *sanction* certain betting odds, even if the degrees of belief do not *consist in* propensities to bet at those odds. (456f.)

Let p_r denote the degrees of belief of person *r*. Then I suggest that the premise Christensen is here invoking may be expressed as follows.

Premise 4.1 *For all* $A \in \mathcal{X}$ *and real numbers s,* p_r *sanctions judging fair* $bet(A,p_r(A),s)$.

Christensen continues:

> It is also intuitively plausible that, if a set of betting odds allows someone to devise a priori a way of exploiting those odds to inflict a sure loss, then there is something amiss with those betting odds. (457)

The following captures at least part of what Christensen is asserting here.

Premise 4.2 *For all random variables f, if* $f < 0$ *then the judgment that f is fair is defective.*

Next Christensen writes:

> And finally, if a single set of beliefs sanctions as fair each of a set of betting odds, and that set of odds is defective, then there is something amiss with the beliefs themselves. (457)

I will express this as

Premise 4.3 *If p_r sanctions judgments about fairness that are defective then p_r is defective.*

The conclusion Christensen wants to reach does not follow from the premises stated so far, but it does follow if we add the following further premise:

Premise 4.4 *If p_r sanctions judging f fair and p_r sanctions judging g fair then p_r sanctions judging $f + g$ fair.*

This premise is similar to Premise 2.3 in Howson and Urbach's argument.

4.2. Theorems. I will now show that Premises 4.1–4.4 imply that if p_r is not defective then it satisfies the axioms of (unconditional finitely additive) probability. The proofs are similar to the corresponding proofs for Howson and Urbach but there are differences in the details.

Theorem 4.1 *If p_r is not defective then, for all $A \in \mathcal{X}$, $p_r(A) \geq 0$.*

Proof: Suppose $p_r(A) < 0$. By Definition 2.1, $bet(A,p_r(A),-1) < 0$. So by Premise 4.2, the judgment that $bet(A,p_r(A),-1)$ is fair is defective. But by Premise 4.1, p_r sanctions judging fair $bet(A,p_r(A),-1)$. Hence by Premise 4.3, p_r is defective. ■

Theorem 4.2 *If p_r is not defective then $p_r(X) = 1$.*

Proof: Suppose $p_r(X) < 1$. By Definition 2.1, $bet(X,p_r(X),-1) < 0$. So by Premise 4.2, the judgment that $bet(X,p_r(X),-1)$ is fair is defective. But by Premise 4.1, p_r sanctions judging fair $bet(X,p_r(X),-1)$. Hence by Premise 4.3, p_r is defective.

If $p_r(X) > 1$, the same argument with the sign of the stake reversed shows again that p_r is defective. ■

Theorem 4.3 *If p_r is not defective then for all $A, B, \in \mathcal{X}$, if $A \cap B = \emptyset$ then $p_r(A \cup B) = p_r(A) + p_r(B)$.*

Proof: Suppose there exist $A, B \in \mathcal{X}$ such that $A \cap B = \emptyset$ and $p_r(A \cup B) < p_r(A) + p_r(B)$. Let

$$g = bet(A,p_r(A),1) + bet(B,p_r(B),1) + bet(A \cup B,p_r(A \cup B), -1).$$

By Premise 4.1, p_r sanctions judging each of the bets on the right hand side as fair so, by two applications of Premise 4.4, p_r sanctions judging g fair. By Definition 2.1, for all $x \in X$,

$$g(x) = p_r(A \cup B) - p_r(A) - p_r(B) < 0.$$

So by Premise 4.2, the judgment that g is fair is defective. Hence by Premise 4.3, p_r is defective.

If $p_r(A \cup B) > p_r(A) + p_r(B)$ the same argument with the signs of all stakes reversed shows again that p_r is defective. ■

4.3. The Need for Premise 4.4. In this section I will show that Theorem 4.3 does not follow from Premises 4.1–4.3 alone.

As in Section 2.3, let $\mathcal{X} = \{\emptyset, A, \bar{A}, X\}$, with all four of these sets distinct, and let

$$p_r(\emptyset) = 0; p_r(A) = 1/3; p_r(\bar{A}) = 1/3; p_r(X) = 1.$$

Define the expected value of random variable f to be

$$\mathscr{E}(f) = p_r(A)f(A + p_r(\bar{A})f(\bar{A}) = [f(A) + f(\bar{A})]/3.$$

Suppose that p_r sanctions judging f fair iff $\mathscr{E}(f) = 0$. If $f < 0$ then $\mathscr{E}(f) < 0$ and so p_r does not sanction judging f fair. Hence it is consistent to suppose both that Premises 4.2 and 4.3 hold and that p_r is not defective. But then Theorem 4.3 is false, since

$$p_r(A) + p_r(\bar{A}) = 2/3 \neq p_r(A \cup \bar{A}) = p_r(X) = 1.$$

4.4. Evaluation of the Premises. I will now go through Premises 4.1 to 4.4 in order, considering whether each is true.

What is it for a judgment of fairness to be *sanctioned?* Christensen says that "other things being equal, an agent should evaluate such bets as fair" (457). Thus Premise 4.1 can be interpreted as saying that ceteris paribus p_r justifies judging $bet(A,p_r(A),s)$ as fair, for all A and s. Christensen does not attempt to give an exhaustive account of what would violate the ceteris paribus condition, but he does mention nonlinear utility of money as an example. Given that the list of possible ceteris paribus conditions is open ended, it would be hard to argue that Premise 4.1 is false. On the other hand, I do not think it is obviously true either, unless the ceteris paribus condition is defined to include anything that would make one unjustified in judging $bet(A,p_r(A),s)$ fair. A representation theorem for expected utility enables one to argue that Premise 4.1 is in fact true, with the only ceteris paribus condition being that the utility of money be linear; but as I have noted before, such a representation theorem makes Dutch book arguments redundant.

Thus Christensen's first premise, while not obviously false, does lack support.

Premise 4.2 seems entirely uncontroversial, assuming that the random variable f measures something of value.

Turning to Premise 4.3, I will allow that if p_r were to justify a defective judgment then p_r would be defective. However, Premise 4.3 makes a stronger claim than that. On Christensen's account, to *sanction* a judgment is to provide a *ceteris paribus* justification for it; thus what Premise 4.3 asserts is that, if p_r provides a ceteris paribus justification for a defective judgment of fairness, then p_r is defective. However, it is possible that the defectiveness of the judgment of fairness is due to the failure of the ceteris paribus condition, not to the defectiveness of p_r. (Analogy: It is consistent, and not implausible, to hold that ethical principles provide a ceteris paribus justification for preserving life and yet that in some cases, where the ceteris paribus condition fails, it would be ethically defective to judge that life should be preserved.) I conclude that Premise 4.3, when understood as it must be for Christensen's argument, lacks plausibility.

To see what we should think of Premise 4.4, consider again the example that I used in Section 4.3, where p_r sanctioned two bets as fair but did not sanction their sum as fair. Christensen has presented no argument that would show there is something wrong with this example, and I see no prospects for a non-circular argument. The example depends on (i) sanctioning as fair bets with zero expected value and (ii) probabilities being non-additive. Christensen surely cannot object to (i) and it would beg the question to dismiss the example because of (ii). Thus Premise 4.4 requires some support but none has been provided and the prospects for providing it seem poor.

I conclude that Christensen's argument is not cogent because three of its premises need a support they have not received.

5. Concluding Comparison. In this paper I have examined three attempts to save Dutch book arguments by depragmatizing them. These attempts share certain features. First, in all three cases the authors have not identified all the premises that are required for their conclusion to follow. Second, the arguments make use of unclear concepts that have not been satisfactorily explicated. (Thus we have Howson and Urbach's notion of advantage, Hellman's function Pr, and Christensen's ceteris paribus condition.) Third, though the premises that these authors have stated are mostly not very plausible, the authors of these arguments have provided little or no discussion or defense of them.

I would like to conclude by briefly contrasting this sorry state of affairs with the defense of the axioms of probability via a representation theorem that I presented in Maher 1993. First, all the required premises were explicitly stated and it was rigorously proved that the conclusion follows from these premises. Second, the basic concept used in this representation theorem, namely preference, was explained in some detail. Third, reasons for all the required premises were given. The most important premises are that preferences satisfy transitivity and independence conditions and a chapter was devoted to critical discussion of arguments for and against each of these conditions. Further empirical evidence relevant to independence is reported and discussed in (Maher and Kashima 1997). So justification of the axioms of probability need not be based on inadequate arguments like those I have examined in this paper.

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