

Research Article

Self-Organized Temporal Criticality: Bottom-Up Resilience versus Top-Down Vulnerability

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We propose a social model of spontaneous self-organization generating criticality and resilience, called Self-Organized Temporal Criticality (SOTC). The criticality-induced long-range correlation favors the societal benefit and can be interpreted as the social system becoming cognizant of the fact that altruism generates societal benefit. We show that when the spontaneous bottom-up emergence of altruism is replaced by a top-down process, mimicking the leadership of an elite, the crucial events favoring the system's resilience are turned into collapses, corresponding to the falls of the leading elites. We also show with numerical simulation that the top-down SOTC lacks the resilience of the bottom-up SOTC. We propose this theoretical model to contribute to the mathematical foundation of theoretical sociology illustrated in 1901 by Pareto to explain the rise and fall of elites.

1. Introduction

The recent book of Haidt [1] aims at explaining the psychological reasons for the conflicts between parties with arguments ranging from psychology to evolutionary biology and from religion to theoretical sociology. There exists a connection between these conflicts and the societal resilience that is supposed to be sufficiently robust as to prevent either societal collapses or rapid social changes. These important sociological issues were addressed in 1901 by Pareto [2], who discussed the capability that elites should develop in order to adapt themselves to changing circumstances. The main goal of the present paper is to contribute to the discussion on the resilience issue, with a simplified model that was recently proposed by our group to resolve the altruism paradox, namely, the emergence of cooperation from the social interaction of individuals who make their choice between cooperation, C , and defection, D , on the basis of their self-interest [3].

The new model of spontaneous organization [3] is based on the conjecture that there are close connections between resilience and information transport, resilience and consciousness, and between consciousness and criticality.

1.1. Criticality and Temporal Complexity. Phase transitions and critical phenomena occur frequently in nature and have been widely studied by physicists; see, for instance, [4]. The Ising model [5] originally introduced to explain ferromagnetic phase transition is well known, and the exact solution found by Onsager [6] for the occurrence of phase transition in the two-dimensional case is widely recognized as an example of outstanding theoretical achievement. In the last few years some scientists have used the Ising model to shed light on biological and neurophysiological collective processes [7–10]. More precisely, the authors of [7] used the Ising model to explain the collective behavior of biological networks and the authors of [8–10] adopted the Ising model for the purpose of supporting their hypothesis that the brain works at criticality, but without establishing a clear distinction between phase transition and self-organized criticality [11]. Finally, we have to mention that the Ising model is frequently used, see, for instance, [12, 13], to model neurophysiological data subject to the constraint of maximal entropy. The term criticality is used to denote the physical condition corresponding to the onset of a phase transition, generated by the adoption of a suitable value of the control parameter K .

The Decision-Making Model (DMM) [14], which is used in Sections 2 and 3, was proved [15] to generate phase transition as a function of its control parameter K identical to that of the Ising model, where the control parameter is the temperature. In other words, the DMM belongs to Ising universality class [14].

At criticality, namely, when the dynamics of the system are determined by the control parameter generating phase transition, the mean field $x(t)$, which in this paper is defined as the ratio of the difference between the number of cooperators and the number of defectors to the total number of units, fluctuates around the vanishing value. The occurrence of a vanishing value is a *crucial event*. The crucial events are defined as follows. The time interval between consecutive crucial events is described by the waiting-time probability density function (PDF) $\psi(\tau)$ that in the long-time limit $\tau \rightarrow \infty$ has the inverse power law (IPL) structure:

$$\psi(\tau) \propto \frac{1}{\tau^\mu}, \quad (1)$$

with $\mu < 3$. The crucial events are renewed thereby making the correlation function $\langle \tau_i \tau_j \rangle$ vanish if $i \neq j$.

In the case of the brain dynamics there is wide consensus on the connection between consciousness and criticality. See, for instance, [9, 16–18] and the recent review paper [19]. The electroencephalogram (EEG) signals are characterized by abrupt changes, called rapid transition processes (RTP), which are proved [20, 21] to be renewal non-Poisson events, with $\mu \approx 2$. This means that the brain in the awake state is a generator of crucial events.

The crucial events are responsible for the information transport from one system at criticality to another system at criticality [22]. Furthermore, the emergence of crucial events requires that the size of the complex system is finite. In this paper M is the total number of units within the system. The intensity of the fluctuations of the mean field $x(t)$ obeys the general prescription

$$\Delta\zeta \propto \frac{1}{M^\nu}, \quad (2)$$

where

$$\Delta\zeta = A(t) - \bar{A}. \quad (3)$$

When working with DMM at criticality, A is the mean field x , with $\bar{x} = 0$, $\nu = 0.25$ [23]. In the case of SOTC [3], with $A = K$, see Section 2, we find $\nu = 0.5$. These criticality-induced fluctuations, becoming visible for finite values of M , are referred to as an expression of *temporal complexity*.

1.2. Swarm Intelligence and Resilience. We may afford an intuitive interpretation of crucial (complex) events, using the example of a flock of birds flying in a given direction, as an effect of self-organization. A crucial (complex) event is equivalent to a complete rejuvenation of the flock that after an organizational collapse may freely select any new flying direction. An external fluctuation of even weak intensity can force the complex system to move in a given direction, if it occurs at the exact instant of the free will of the SOTC model system.

It is important to stress that the organizational collapse is not the fall of an elite, which will be discussed subsequently, because the flock self-organization occurs spontaneously and does not rest on the action of a leader. The choice of a new flying direction is thus determined by an external stimulus of even weak intensity occurring at the same time as the collapse, thereby implying the property of *complexity matching* between the perturbed and the perturbing complex system [14].

As mentioned earlier, the crucial events favor the transport of information from one complex system to another [22]. Crucial events are generated by criticality and consequently the transport of information becomes maximally efficient at criticality [24].

However, criticality may also be Achilles' heel of a complex system, if criticality is generated by a fine tuning control parameter. In fact, committed minorities acting when a crucial event occurs in the case of DMM can make the system jump from the state C to the state D [25]. Herein we show that this lack of resilience is not shared by the bottom-up approach to SOTC modeling; in fact, starting from the bottom generates a very resilient social organization.

1.3. From Criticality Generated by the Fine Tuning of a Control Parameter to Self-Organization. The model of [3] is a form of spontaneous transition to criticality, revealed by the emergence of events with the temporal properties of crucial events, thereby explaining the adoption of the name Self-Organized Temporal Criticality (SOTC) to define it. We show that the bottom-up SOTC modeling is resilient and that the top-down SOTC modeling is not. We believe that the SOTC model may help to contribute to the discussion of the sociological issues of Haidt [1] with the tools of Complexity Science. In fact, Haidt emphasized that the political conflict between conservatives and liberals is due to cultural and religious influences that have the effect of creating divisions. We believe that the top-down SOTC approach may be used to model these cultural influences. This is an extremely difficult problem, made even more difficult by the philosophical controversies on definition of morality [26]. According to the brilliant picture of Haidt, the philosophy of Hume and Menciu may be compatible with the bottom-up origin of cooperation, while the hypothesis that *morality transcends human nature*, an interpretation moving from Plato to Kant [1], may justify a top-down perspective. We make the extremely simplified assumption that the top-down SOTC, undermining social resilience, explains the fall of elites, if they represent only limited groups, a phenomenon that may be explained by noticing that “our minds were designed for groupish righteousness” [1]. The source of social conflict seems to be that cultural evolution differs from life evolution. These culturally induced conflicts may overcome the biological origin of cooperation.

1.4. Bottom-Up versus Top-Down Approach to Morality. For clarity in Sections 2 and 3 we provide a review of the SOTC model [3], while stressing some properties of SOTC model that were not discussed in the original paper, for instance, the behavior of single units with their frequent regression

to the condition of independence of the other units, for the bottom-up process and the Pareto cycles of the top-down version of the model. The original results of this earlier paper indicated a lack of resilience of the top-down SOTC model and a robustness of the bottom-up SOTC model, which are illustrated in Section 4. We devote Section 5 to balancing the results of the present paper against the open problems that we propose to study in future research.

2. Bottom-Up Approach to Self-Organized Temporal Criticality

The decisions of single individuals in our model are made in accordance with the criterion of bounded rationality [27, 28], expanded by Kahneman [29] and more recently discussed from within the perspective of evolutionary game theory (EGT) [30, 31]. The nonrational component of the decision-making process is stressed also by the work of Gigerenzer [32, 33]. Herein individuals make decision using DMM. The individuals of the social network aiming at increasing their payoff make the control parameter K_r , for individual r , evolve towards criticality, thereby creating an intelligent group mind [34], perhaps connected to the intelligent unconscious of Gigerenzer [33], leading the individuals to fast but essentially wise decisions. As we shall see, the time evolution of K_r is slow, because it depends on the payoffs of the individual at earlier time, corresponding to the slow thinking mechanism discussed by Kahneman [29].

2.1. The Intuitive and Emotional Level. In this paper we use the DMM on a two-dimensional lattice of size L , with $M = L \times L$ individuals, and we set $L = 10$. The DMM is based on individuals imperfectly imitating the majority opinion of their four nearest neighbors, thereby biasing the probability of making a transition from being a cooperators (C) to being a defector (D):

$$g_{CD}^{(r)} = g_0 \exp \left\{ -K_r \frac{(N_C^{(r)} - N_D^{(r)})}{N} \right\}, \quad (4)$$

where $N_C^{(r)}$ is the number of nearest neighbors to individual r that are cooperators, $N_D^{(r)}$ is the number of defectors, and each individual on the simple lattice has $N = 4$ nearest neighbors. In the same way the transition rate from defectors to cooperators $g_{DC}^{(r)}$ is

$$g_{DC}^{(r)} = g_0 \exp \left\{ K_r \frac{(N_C^{(r)} - N_D^{(r)})}{N} \right\}. \quad (5)$$

The unbiased transition rate is $g_0 = 0.01$ throughout the calculations, and $1/g_0$ defines the time scale for the process. The DMM has been shown [14] to undergo critical phase transitions and to be a member of the Ising universality class in which all the members of the network can act cooperatively, depending on the magnitude of the interaction strength K [14]. However, this important result is obtained by assigning to all the individuals the same degree of attention to the opinions of their nearest neighbors, called K . Herein

TABLE 1: The payoffs of Prisoner's dilemma game. The first value of each pairs is the payoff of player X and the second value is the payoff of the player Y.

Player X	Player Y	
	C	D
C	(1, 1)	(0, 1.9)
D	(1.9, 0)	(0, 0)

each individual may have a different degree of attention and this degree of attention does not fit the reciprocity principle. The degree of attention that the individual r devotes to the individual r' may differ from the degree of attention that the individual r' devotes to the individual r . To explain how the individual r is influenced by her nearest neighbors, let us consider, for instance, (4). The individual we are considering is a cooperator and (4) establishes the rate of her transition to the defection state. If $N_C^{(r)} > N_D^{(r)}$ the rate decreases and will vanish in the extreme limit $K_r = \infty$. Of course, this will have the effect of favoring the cooperation state.

2.2. The Rational Level. This decision-making process is fast and emotional and does not involve any direct reasoning about the payoff. The connection with the self-interest, according to the slow thinking mechanism discussed by Kahneman [29], is established over a more extended time scale, where the single individual exerts an influence on the process aiming at maximizing her payoff. To define the payoff we adopt the prisoner's dilemma game (PDG) [35]. Two players interact and receive a payoff from their interaction adopting either the defection or the cooperation strategy. If both players select the cooperation strategies, each of them receives the payoff R and their society receives the payoff $2R$. The player choosing the defection strategy receives the payoff T . The temptation to cheat is established by setting the condition $T > R$. However, this larger payoff is assigned to the defector only if the other player selects cooperation. The player selecting cooperation receives the payoff S , which is smaller than R . If the other player also selects defection, the payoff for both players is P , which is smaller than R . The PDG is based on the crucial payoffs $T > R > P > S$ and $S + T < 2R$.

We adopt the choice of parameter values made by Gintis [35] and set $R = 1$, $P = 0$, and $S = 0$. The maximal possible value of T is 2, and we select the value $T = 1.9$, which is a very strong incentive to cheat. These choices are summarized in Table 1. We evaluate the social benefit for the single individual, as well as for the community as a whole as follows. We define the payoff P_r for individual r as the average over the payoffs from the interactions with its four nearest neighbors. If both players of a pair are cooperators, the contribution to the payoff of the individual r is $B_r = 1$. If one of the two playing individuals is a cooperator and the other is a defector, the contribution to the payoff of r is $B_r = T$. If both players are defectors, the contribution to the payoff of r is $B_r = 0$. The payoff P_r to individual r is the sum over the four B_r 's.

Each individual receives a total payoff from the game with the four nearest neighbors and adjusts her imitation strength as follows:

$$K_r(t) = K_r(t - \Delta t) + \chi \frac{P_r(t - \Delta t) - P_r(t - 2\Delta t)}{P_r(t - \Delta t) + P_r(t - 2\Delta t)}, \quad (6)$$

where the parameter χ determines the intensity of interest of the individuals to the fractional change in their payoffs in time. The key equation (6) is based on the assumption that the intuitive decision-making process is so fast that at both times $t - \Delta t$ and $t - 2\Delta t$ it is possible to evaluate the corresponding payoffs on the basis of fast decisions made by each individual and by her 4 nearest neighbors. The decision of adjusting the social sensitivity $K_r(t)$ requires the time interval $2\Delta t$, while the intuitive decision is virtually instantaneous.

The second term on the right-hand side of (6) is the ratio between two quantities that for special cases vanish. In these cases we set the condition

$$K_r(t) = RK_r(t - \Delta t), \quad (7)$$

with $R < 1$. We selected $R = 0$ but for other values we get the same result. When $K_r(t)$ goes to negative values we set it equal to zero.

Note that in the limit of vanishing time intervals (6) relates the time rate of change of an individual's imitation strength to the time rate of change of the logarithm of the local payoff to that individual. On the global scale, the mean benefit to society of all the individuals is given by the average over all the P_r 's:

$$\Pi(t) = \frac{1}{N} \sum_{r=1}^N P_r(t), \quad (8)$$

whereas the mean imitation strength is given by the average over all the $K_r(t)$:

$$K(t) = \frac{1}{N} \sum_{r=1}^N K_r(t). \quad (9)$$

For the bottom-up case discussed in this section, the calculation is done with the parameters $M = 100$, $g_0 = 0.01$, $T = 1.9$, and $\chi = 1$, with the social benefit, imitation strength, and mean field starting from zero.

The results of Figure 1 are used to establish the bottom-up origin of altruism, rather than interpreting it, as it is frequently done, to be the result of a religion-induced top-down process. The calculations show that the top-down process generating altruism weakens the system's resilience, whereas the genuinely bottom-up approach makes the emergence of altruism robust against external perturbation. Figure 1 shows that the time evolution of the individual social sensitivity K_r is characterized by abrupt jumps that from time to time may also bring the single individual back to a behavior totally independent of the choices made by her nearest neighbors. This is a healthy social condition that has the effect of making the global properties $x(t)$, $K(t)$, and $\Pi(t)$ host crucial events favoring the transmission of information between different social systems, either countries or parties.

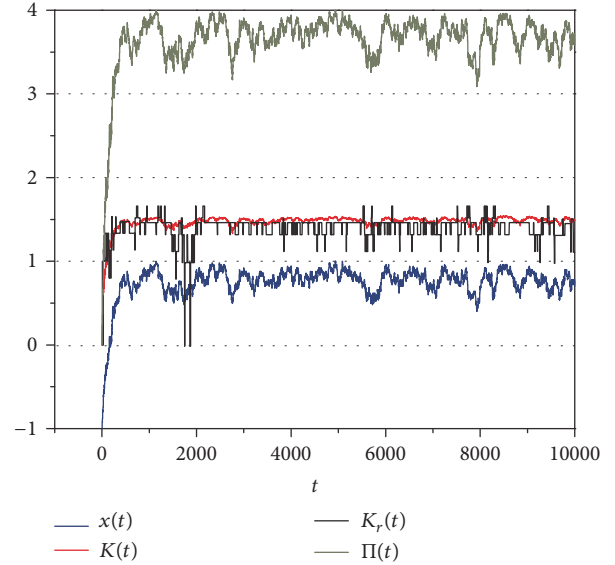


FIGURE 1: For the bottom-up SOCT: time evolution of the average social benefit $\Pi(t)$, the average imitation strength $K(t)$, the mean field $x(t)$, and the imitation strength of one of the units $K_r(t)$ plotted versus time. The mean field fluctuates around 0.8 which means about 90 percent of individuals are cooperators and 10 percent are defectors. Mean field of 1 has all individuals in the state of cooperation, and the maximum average social benefit has the value 4, because this is the number of nearest neighbors on the two-dimensional lattice.

To stress the occurrence of crucial events in a social system resting on the bottom-up emergence of altruism, we have to extend the method used for criticality generated by the fine tuning of the control parameter K . In that case, at criticality the mean field fluctuates around the vanishing value and the crucial events correspond to the occurrence of this vanishing value [15, 23]. We follow [36] and evaluate the fluctuations around the proper nonvanishing mean value of $K = 1.5$. To explain this choice notice that in the conventional case of criticality, generated by the choice of a proper control parameter K , with $M = 100$, $K = 1.5$ is the value at which the onset of phase transition occurs. This is the value making the mean field $x(t)$ of the conventional DMM fluctuate around $x = 0$ with complex fluctuations and which generates criticality-induced intelligence [37, 38]. In the case of the SOTC model this condition of criticality-induced intelligence, with fluctuations of $K(t)$ around 1.5, is spontaneously generated. When the criticality condition is reached the complex fluctuations of $x(t)$ do not occur any longer around $x = 0$, but around a positive value of the order of 0.8. The time intervals τ between consecutive crossings of the 1.5 level are monitored and the corresponding waiting-time PDF $\psi(\tau)$ is illustrated in Figure 2.

Figure 2, similar to Figure 4 of [3], is compared in Section 4 with Figure 4, illustrating the perturbation by independents. We limit ourselves to noticing that temporal complexity shows up in the intermediate asymptotics regime [3] and it is characterized by the IPL index $\mu = 1.3$, a property shared by other systems at criticality; see, for instance, [22].

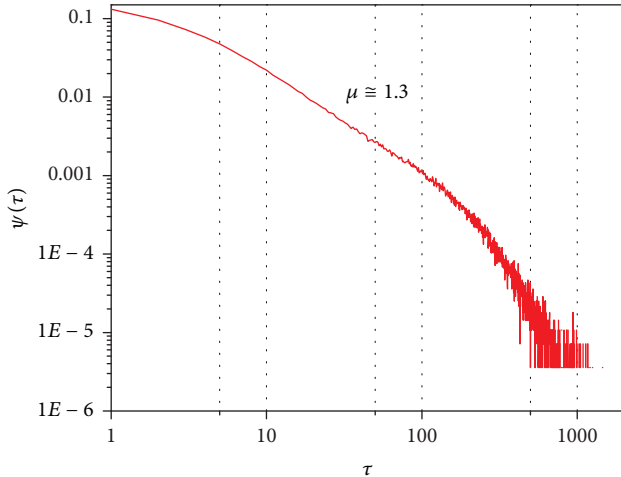


FIGURE 2: For bottom-up SOTC: the waiting-time PDF of the time interval between two consecutive crossings of the average value of the mean value of $K(t)$, which is ≈ 1.5 , is graphed. $\psi(\tau)$ is exponentially truncated and has an intermediate asymptotic regime with an index of $\mu \approx 1.3$. This figure is similar to Figure 4 of [3].

This is evidence that the spontaneous transition to criticality also generates the crucial events responsible for information transport. Earlier work [3] confirms that the crucial events facilitate the transport of information from one complex social system to another, if the two systems are at criticality as a result of the spontaneous process of self-organization.

In spite of the exponential truncation that may lead to the misleading conclusion that the Poisson-like character of the long-time regime quenches the manifestations of complexity, the transport of information is determined by the intermediate asymptotics. In the earlier work of [3] we studied the efficiency of transport of information from one bottom-up SOTC to another identical SOTC and we found that the maximal efficiency of the information transport corresponds to about $M = 100$, which is the condition studied in this paper. The explanation of these interesting effects is the following. The intensity of fluctuations generating crucial events decreases with the size increase, according to the following formula (see Eq. (14) of [3]):

$$\Delta\zeta \propto \frac{1}{M^\nu}, \quad (10)$$

with $\nu = 0.5$. Note that $\Delta\zeta$ denotes the intensity of the fluctuations of the variables K, Π , and $x(t)$ around their mean values. Therefore the systems with $M > 100$ have crucial fluctuations of smaller intensity, thereby explaining the reduction of the process of information transport. For values $M < 100$, the role of the exponential truncation becomes more important and the time extension of the complex intermediate asymptotics is reduced and eventually the intermediate asymptotics regime vanishes, turning the system into a Poisson system, with no complexity. This has the effect of significantly reducing the efficiency of the process of information transport. As far as the resilience of the bottom-up SOTC is concerned, the theory of this paper rests

on the connection between resilience and the efficiency of information transport. As a consequence the results on the resilience of the bottom-up SOTC for $M = 100$ automatically correspond on the condition of maximal resilience [39].

3. Top-Down Approach to Self-Organized Temporal Criticality

The top-down approach to self-organization is done using again (4) and (5). The adoption of the top-down perspective is realized by replacing (6) with

$$K(t) = K(t - \Delta t) + \chi \frac{\Pi(t - \Delta t) - \Pi(t - 2\Delta t)}{\Pi(t - \Delta t) + \Pi(t - 2\Delta t)}. \quad (11)$$

The top-down origin of this process is made evident by the fact that all individuals in the network are forced to adopt the same time-dependent imitation strength. Furthermore, rational choice is made on the basis of the collective payoff $\Pi(t)$, using PDG. The conceptual difference with the bottom-up approach of Section 2 is impressive. In fact, with (11) all the individuals of this society must change their social sensitivity at the same time and the information about the increase or decrease of the global payoff implies that all the individuals are given this information from a central source such as the government, suggesting that a form of organization already exists and is not created by the interaction between the individuals. In [3] the assumption was made that a benevolent dictator exists and leads such a process. Using Pareto's social theory we make the assumption that this process implies the leadership of an elite [2].

The second term on the right-hand side of (11) is the ratio between two quantities that, similarly to the bottom-up model, for special cases vanishes. In these cases, as done in Section 2, we set the condition

$$K_r(t) = RK_r(t - \Delta t), \quad (12)$$

with $R < 1$. We selected $R = 0$. When $K_r(t)$ goes to negative values we set it equal to zero.

For the top-down case discussed in this section, the calculation is done with the parameters $M = 100$, $g_0 = 0.01$, $T = 1.9$, and $\chi = 4$, with the social benefit, imitation strength, and mean field starting from zero. χ is chosen to be larger than in the bottom-up case because of the fact that transition to criticality in the top-down case is much slower (see Figures 1 and 3).

Under the leadership of an elite, see Figure 3, the control parameter $K(t)$ shows a behavior totally different from that of Figure 1. In this section we focus on the behavior of $K(t)$ in the absence of perturbation and discuss the effects of perturbation in Section 4. With no perturbation there is a transient from $t = 0$ to $t \approx 40000$, after which time a sequence of rises and falls occurs. The value of K adjusts according to (11) from small values around 0.2 to a maximal value of 1.8, which is known to correspond to a supercritical condition in the case of the conventional DMM. When using the fine tuning control parameter approach we set $K = 1.8$; the social system is far from the intelligence condition that according to a widely accepted opinion [9, 16–19] requires criticality.

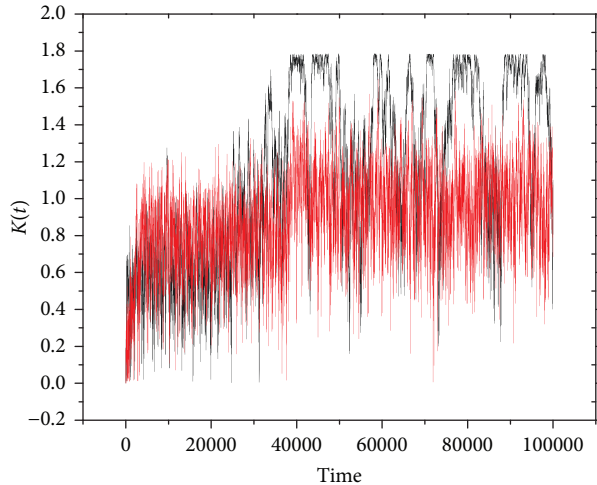


FIGURE 3: Black line: the time evolution $K(t)$ of (11) for the same regular two-dimensional network of Figure 1; red line: the time evolution of $K(t)$ of the self-organized system described by the black under the influence of the weak noisy perturbation described in Section 4.

The mean field $x(t)$ has very fast fluctuations around a mean value close to 1, but these fluctuations are Poisson and the conventional DMM system loses its complexity [22].

The falls to the small values of K are interpreted as falls of elites. The subcritical condition, as well as the supercritical, is characterized by a lack of intelligence. We have to remark also that values of K significantly smaller than $K \approx 1.5$ indicate that there are many units with $K_r = 0$, like the single unit of Figure 1 at a time close to $t \approx 2000$. In conclusion, both small and maximal values of K are affected by a lack of consciousness, and the transitions through $K \approx 1.5$ are too fast for the social system to benefit from the intelligence of the critical condition. This lack of intelligence is responsible for the lack of resilience. The sojourn times in the supercritical state correspond to the time durations of elites. We do not have to confuse the fluctuations of $K(t)$ with those of the mean field $x(t)$ that are not shown here. The fluctuations of $x(t)$ are always Poisson, around mean values close to 1, when $K(t)$ is close to 1.8 and around the vanishing mean value when $K(t)$ drops.

It is interesting to notice that also the time interval between consecutive falls of elite is a complex dynamical process characterized by an IPL, with $\mu \approx 2$ in this case, as shown by Figure 4. However, the system is not resilient. The sojourn in a supercritical state with a K significantly larger than $K \approx 1.5$ is characterized by fast Poisson fluctuations and an external perturbation can easily affect the time duration of this regime [40]. In fact, the big difference between Poisson events and crucial events is that the former events obey conventional linear response theory and any forms of perturbation can deeply affect their dynamics, thereby undermining the social resilience, as we show in the next section.

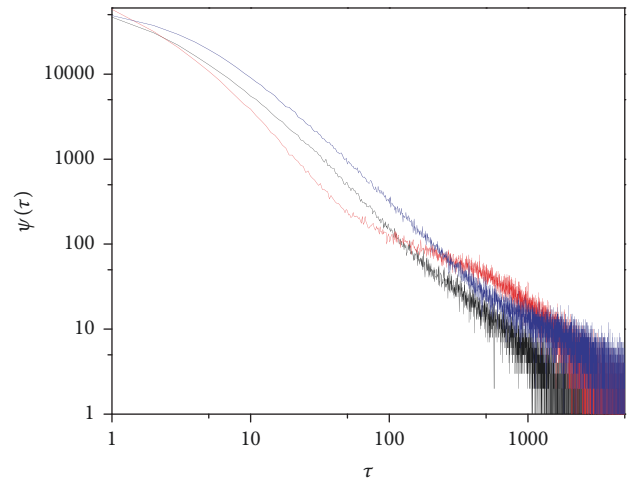


FIGURE 4: For bottom-up SOTC, the waiting-time PDF of the time intervals between two consecutive crossings of the horizontal line with $K = 0.7$ (black and red line) and $K = 1.7$ (blue line) for the top-down SOTC. The blue and the black lines describe the unperturbed case and the red line describes the perturbation described in Section 4.

4. Perturbing the Self-Organized Society

To substantiate the arguments of the earlier section with the results of a numerical simulation we devote this section to illustrating some numerical experiments on the effects of a perturbation on the process of societal self-organization.

First of all let us define two different sources of perturbation, the *independent* and the *committed* minorities. We assume that a minority of independent individual exists. An independent individual is a unit that is characterized by $K_r = 0$. As a consequence this unit does not adopt (6) and is completely insensitive to the connection between individual and societal benefit that yields the emergence of cooperation [3]. The perturbing nature of this independent individual is realized by the fact that, while the independent keeping $K_r = 0$ is completely independent of the choices made by the other units, her nearest neighbors are influenced by the choices of the independent through the DMM and through the evaluation of the payoff $P_r(t)$ of (6).

In the top-down SOTC model the independent influences the process through his vanishing contribution to $K(t)$ and through his contribution to the global payoff of (11). The perturbation of independents is made more devastating when the independents are allowed to move randomly through the social network.

The other kind of perturbation, produced by committed minorities, has already been studied elsewhere [25]. These are minorities that keep selecting the state D . The committed minorities are also called zealots and have been the subject of many publications; see [41] for a wide set of references. These publications emphasize the dramatic consequences that the zealots have on their societies, thereby implying that their models of organization are not resilient. The experiment on the perturbation of zealots done herein shows that the top-down SOTC model shares the lack of resilience observed in

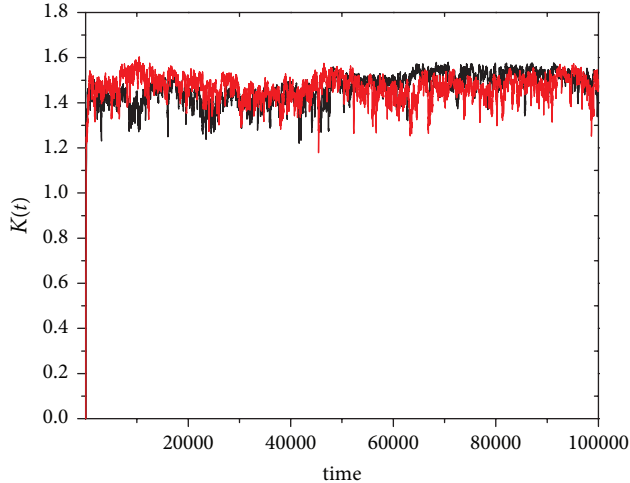


FIGURE 5: $K(t)$ as a function of time for the bottom-up SOTC, in the no perturbation (black line) and perturbed case (red line).

these earlier studies on the social influences of zealots. The bottom-up SOTC model seems to be the only fully resilient model.

Let us discuss first the strongest source of perturbation, the randomly moving independents. At any time step one of the $M = 100$ is randomly selected to play the role of independent; namely, we force her to adopt the value $\xi = 1$ or the value $\xi = -1$, with equal probability. In the case of the bottom-up SOTC this perturbation does not have significant effect on the time evolution of $K(t)$, as shown by Figure 5.

In the case of the top-down SOTC the effects of this perturbation are impressive. The black line of Figure 3 shows the rise and the fall of an elite. The weak noisy perturbation makes $K(t)$ evolve as illustrated by the red line of the same figure, which shows that the sojourn times of unperturbed elites are filled with many falls that are a clear manifestation of the lack of societal resilience.

Important information on the lack of resilience of the top-down SOTC is afforded by Figure 4, showing $\psi(t)$ for different values of the threshold used to find the statistics of crucial events. When the threshold is 1.7, close to the top supercritical region reached by the system, the intermediate asymptotics has a power index $\mu \approx 1.45$, larger than that of the bottom-up SOTC. The adoption of the threshold $K = 0.7$ as an effect of the collapse of elites cancels the intermediate asymptotic temporal complexity and favors the birth of a Poisson shoulder. The noisy perturbation of independents makes this behavior even more pronounced. This strong exponential shoulder is a signature of the death of dynamical complexity and of the transition from non-Poisson to Poisson behavior [3].

The perturbing action of independents is weaker if the independent individuals do not move. This response to this form of perturbation is illustrated in Figure 6, showing that even in this case the bottom-up SOTC model is more robust than the top-down.

We establish the perturbation of independent individuals in a different way. We assume that all the units are

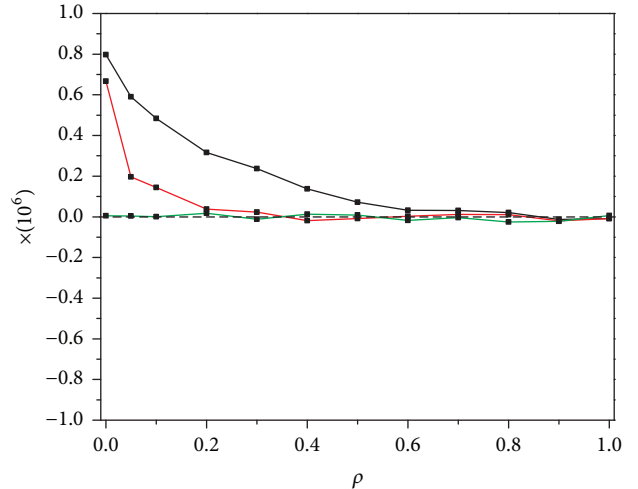


FIGURE 6: Dependence of mean field (at time 10^6) on the ratio ρ of independent units (which are fixed on the lattice) for the bottom-up SOTC model (black line), top-down SOTC model (red line), and ordinary DMM with tuned control parameter $K_c = 1.5$ (green line). Ensemble average is done over 10 experiments.

independent for a fraction η of their time. The results are depicted in Figure 7. It is clear from the figure that the bottom-up SOTC model is more resilient than the top-down, even to this most violent form of disruption.

Finally in Figure 8 we show the action of committed minorities. We see that only the bottom-up SOTC model is resilient. The top-down SOTC model shares the same lack of resilience shown by ordinary DMM at criticality.

5. Conclusions

It is remarkable that according to SOTC the crucial events may be harmful as well as beneficial. If the global parameter $K(t)$, fluctuating around the long-range correlation generating mean value, returns to the vanishing value, the temporary collapse is turned into a societal disaster. The collapse into $K = 0$ would correspond to a new initial condition and, as shown by Figure 1, $K(t)$ would start increasing again generating a new organization led by a new elite [2]. However, the genuinely bottom-up process leading the time evolution illustrated by Figure 1 is expected to keep forever the social system in the condition of weak fluctuations around $K \approx 1.5$. In other words, there is an impressive difference between the crucial events hosted by the weak fluctuation around $K \approx 1.5$ and the regressions of $K(t)$ to values of $K \ll 1.5$, generated by the adoption of a top-down process led by an elite.

The main conclusion of this paper concerning resilience is that criticality is necessary for resilience, but it is not sufficient. The top-down SOTC model generates criticality, but it is not resilient. Therefore information transport from one top-down SOTC model system to another top-down SOTC model system is expected to occur by means of complexity matching, in spite of the fact that the two systems are not resilient and the information transport may be easily quenched by stray perturbing noises.

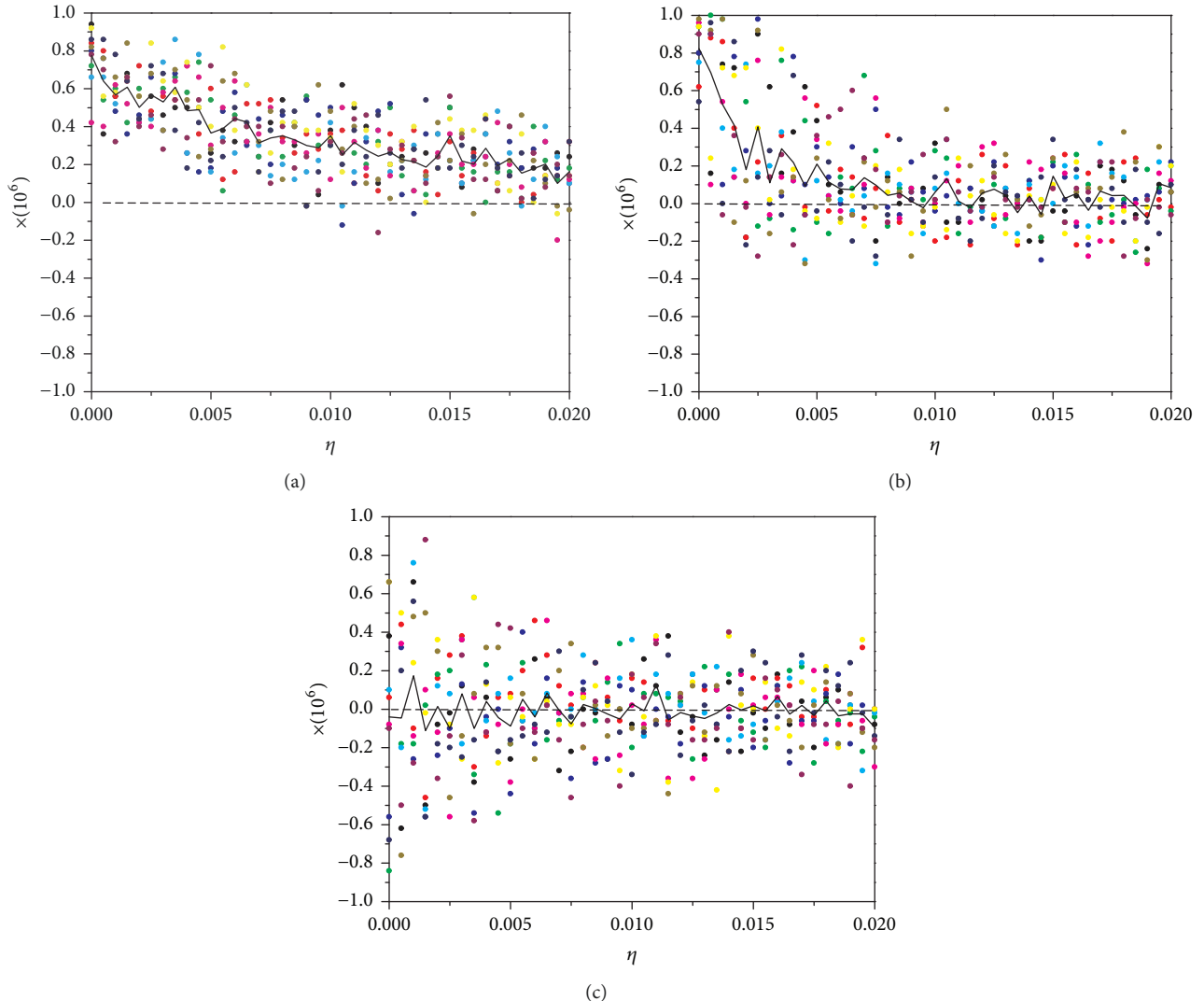


FIGURE 7: Dependence of mean field (at time 10^6) on η (the ratio of time in which units behave individually) for bottom-up SOTC model (a), top-down SOTC model (b), and ordinary DMM with tuned parameter $K_c = 1.5$ (c). Each set of colored dots corresponds to one experiment and the black solid lines are the average of them.

An attractive interpretation of the resilient nature of the bottom-up SOTC model is that the ideal condition of full democracy is the most robust form of social organization.

In this paper the social payoff is evaluated using the PDG [35]. The prisoner's dilemma game is frequently used in the field of EGT [42, 43]. EGTs aim at solving the altruism paradox using the concept of network reciprocity [43]. A game is played many times on a network where each individual is surrounded by a set of nearest neighbors and adopts the strategy of the most successful nearest neighbor. Since the clusters of cooperators are richer than the clusters of defectors it is plausible that the most successful nearest neighbor is a cooperator. However, this attempt at mimicking the action of a collective intelligence failed because the social activity of the units, being subcritical, disrupts the beneficial effects of network reciprocity [44, 45]. We note that SOTC modeling represents an attempt to amend the field of EGT

by the limitations preventing, for instance, the concept of network reciprocity from yielding a satisfactory resolution of the altruism paradox.

The human inclination to cooperate is the result of biological evolution and of the spontaneous evolution towards criticality. The time appears ripe to unify the models of biology and physics made necessary to reach the ambitious goal of achieving a rigorous scientific foundation of this important human characteristic [46, 47]. The spontaneous transition to criticality of SOTC contributes to bypassing the current limitations of the field of EGT. SOTC models, as shown in this paper, can be adapted to take into account the top-down processes connected with the nonresilient action of elites. It is possible to supplement the nonrational decision-making process based on (4) and (5) with self-righteous biases [1] taking into account the influence of religion or other polarizing influences. We expect that such generalizations

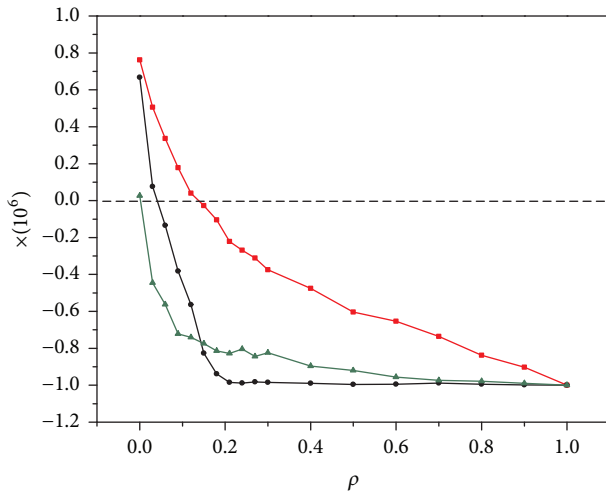


FIGURE 8: Dependence of mean field (at time 10^6) on the ratio ρ of fanatics (having fixed position on the lattice) for the bottom-up SOTC model (red line), top-down SOTC model (black line), and ordinary DMM with tuned control parameter $K_c = 1.5$ (green line). Ensemble average is done over 10 experiments.

of SOTC theory will lead to a lack of societal resilience. However, this is left as a subject for future research.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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