Epistemic "Holes" in Spacetime*

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1 Introduction

A number of models of general relativity seem to contain "holes" which are thought to be "physically unreasonable". One seeks a condition to rule out these models. We examine a number of possibilities already in use. We then introduce a new condition: epistemic hole-freeness. Epistemic hole-freeness is not just a new condition — it is new in kind. In particular, it does not presuppose a distinction between spacetimes which are "physically reasonable" and those which are not.

2 Preminilaries

We begin with a few preliminaries concerning the relevant background formalism of general relativity.[1] An n-dimensional, relativistic *spacetime* (for $n \geq 2$) is a pair of mathematical objects (M, g_{ab}). M is a connected n-dimensional manifold (without boundary) that is smooth (infinitely differentiable). Here, g_{ab} is a smooth, non-degenerate, pseudo-Riemannian metric of Lorentz signature $(+, -, ..., -)$ defined on M. Note that M is assumed to be *Hausdorff*; for any distinct $p, q \in M$, one can find disjoint open sets O_p and O_q containing p and q respectively. We say two spacetimes (M, g_{ab}) and (M', g'_{ab}) are *isometric* if there is a diffeomorphism $\varphi : M \to M'$ such that $\varphi_*(g_{ab}) = g'_{ab}$.

For each point $p \in M$, the metric assigns a cone structure to the tangent space M_p. Any tangent vector ξ^a in M_p will be *timelike* if $g_{ab}\xi^a\xi^b > 0$, *null* if $g_{ab}\xi^a\xi^b = 0$, or *spacelike* if $g_{ab}\xi^a\xi^b < 0$. Null vectors create the cone structure; timelike vectors are inside the cone while spacelike vectors are outside. A *time*

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[1] The reader is encouraged to consult Hawking and Ellis (1973), Wald (1984), and Malament (2012) for details. An outstanding (and less technical) survey of the global structure of spacetime is given by Geroch and Horowitz (1979).

orientable spacetime is one that has a continuous timelike vector field on M. A time orientable spacetime allows one to distinguish between the future and past lobes of the light cone. In what follows, it is assumed that spacetimes are time orientable.

For some open (connected) interval $I \subseteq \mathbb{R}$, a smooth curve $\gamma : I \to M$ is *timelike* if the tangent vector ξ^a at each point in $\gamma[I]$ is timelike. Similarly, a curve is *null* (respectively, *spacelike*) if its tangent vector at each point is null (respectively, spacelike). A curve is *causal* if its tangent vector at each point is either null or timelike. A causal curve is *future directed* if its tangent vector at each point falls in or on the future lobe of the light cone.

An *extension* of a curve $\gamma : I \to M$ is a curve $\gamma' : I' \to M$ such that I is a proper subset of I' and $\gamma(s) = \gamma'(s)$ for all $s \in I$. A curve is *maximal* if it has no extension. A curve $\gamma : I \to M$ in a spacetime (M, g_{ab}) a *geodesic* if $\xi^a \nabla_a \xi^b = \mathbf{0}$ where ξ^a is the tangent vector and ∇_a is the unique derivative operator compatible with g_{ab}. Let $\gamma : I \to M$ be a timelike curve with tangent vector ξ^b. The acceleration vector is $\alpha^b = \xi^a \nabla_a \xi^b$ and the magnitude of acceleration is $a = (-\alpha^b \alpha_b)^{1/2}$. The *total acceleration* of γ is $\int_\gamma a \, ds$ where s is elapsed proper time along γ.

For any two points $p, q \in M$, we write $p << q$ if there exists a future directed timelike curve from p to q. We write $p < q$ if there exists a future directed causal curve from p to q. These relations allow us to define the *timelike and causal pasts and futures* of a point p: $I^-(p) = \{q : q << p\}$, $I^+(p) = \{q : p << q\}$, $J^-(p) = \{q : q < p\}$, and $J^+(p) = \{q : p < q\}$. Naturally, for any set $S \subseteq M$, define $J^+[S]$ to be the set $\cup\{J^+(x) : x \in S\}$ and so on. A set $S \subset M$ is *achronal* if $S \cap I^-[S] = \varnothing$. A spacetime satisfies *chronology* if, for each $p \in M$, $p \notin I^-(p)$.

A point $p \in M$ is a *future endpoint* of a future directed causal curve $\gamma : I \to M$ if, for every neighborhood O of p, there exists a point $t_0 \in I$ such that $\gamma(t) \in O$ for all $t > t_0$. A *past endpoint* is defined similarly. A causal curve is *future inextendible* (respectively, *past inextendible*) if it has no future (respectively, past) endpoint.

For any set $S \subseteq M$, we define the *past domain of dependence of* S, written $D^-(S)$, to be the set of points $p \in M$ such that every causal curve with past endpoint p and no future endpoint intersects S. The *future domain of dependence of* S, written $D^+(S)$, is defined analogously. The entire *domain of dependence of* S, written $D(S)$, is just the set $D^-(S) \cup D^+(S)$. The *edge* of an achronal set $S \subset M$ is the collection of points $p \in S$ such that every open neighborhood O of p contains a point $q \in I^+(p)$, a point $r \in I^-(p)$, and a timelike curve from r to q which does not intersect S. A set $S \subset M$ is a *slice* if it is closed, achronal, and without edge. A spacetime (M, g_{ab}) which contains a slice S such that $D(S) = M$ is said to be *globally hyperbolic*.

3 A Condition to Disallow Holes?

Consider the following example (see Figure 1).

Example 1. Let (M, g_{ab}) be Minkowski spacetime and let p be any point in M. Consider the spacetime $(M - \{p\}, g_{ab})$.

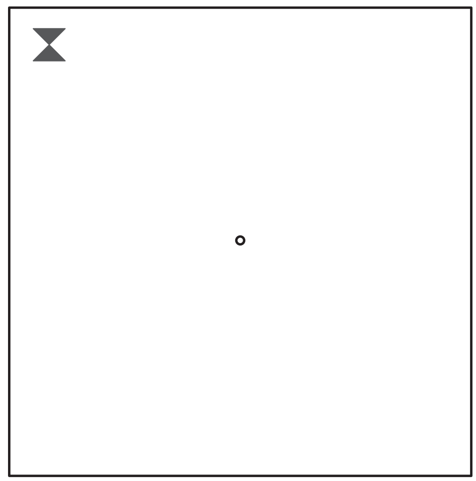

Figure 1: Minkowski spacetime with a point removed from the manifold.

The spacetime seems to have an artificial "hole". One seeks to find a (simple, physically meaningful) condition to disallow the example. (The condition need not be a sufficient condition for "physical reasonableness"; it need only be necessary.) But "although one perhaps has a good intuitive idea of what it is that one wants to avoid, it seems to be difficult to formulate a precise condition to rule out such examples" (Geroch and Horowitz 1979, 275).

Many of the conditions used to rule out the "hole" in Example 1 require that certain regions of (or curves in) spacetime be "as large as they can be". For example, geodesic completeness requires every geodesic to be as large as it can be in a certain sense. Hole-freeness essentially requires the domain of dependence of every spacelike surface to be as large as it can be. Inextendibility requires the entirety of spacetime to be as large as it can be. Let us examine each of these three conditions in more detail. First, consider geodesic completeness.

Definition . A spacetime (M, g_{ab}) is *geodesically complete* if every maximal geodesic $\gamma : I \to M$ is such that $I = \mathbb{R}$. A spacetime is *geodesically incomplete* if it is not geodesically complete.

If an incomplete geodesic is timelike or null, there is a useful distinction one can introduce (which we will need later on). We say that a future directed timelike or null geodesic $\gamma : I \to M$ without future endpoint is *future incomplete* if there is an $r \in \mathbb{R}$ such that $s < r$ for all $s \in I$. A *past incomplete* timelike or null geodesic is defined analogously. Next, consider inextendibility.

Definition A spacetime (M, g_{ab}) is *extendible* if there exists a spacetime (M', g'_{ab}) and an isometric embedding $\varphi : M \to M'$ such that $\varphi[M]$ is a proper subset of M'. Here, the spacetime (M', g'_{ab}) is an *extension* of (M, g_{ab}). A spacetime is *inextendible* if it has no extension.

Finally, consider hole-freeness. Initially, one defined (Geroch 1977) a spacetime (M, g_{ab}) to be *hole-free* if, for every spacelike surface $S \subset M$ and every isometric embedding $\varphi : D(S) \to M'$ into some other spacetime (M', g'_{ab}), we have $\varphi(D(S)) = D(\varphi(S))$. The definition seemed to be satisfactory. But surprisingly, it turns out the definition is too strong; Minkowski spacetime fails to be hole-free under this formulation (Krasnikov 2009). But one can make modifications to avoid this consequence (Manchak 2009).

Let (K, g_{ab}) be a globally hyperbolic spacetime. Let $\varphi : K \to K'$ be an isometric embedding into a spacetime (K', g'_{ab}). We say (K', g'_{ab}) is an *effective extension* of (K, g_{ab}) if, for some Cauchy surface S in (K, g_{ab}), $\varphi[K] \subsetneq int(D(\varphi[S]))$ and $\varphi[S]$ is achronal. Hole-freeness can then be defined as follows.

Definition. A spacetime (M, g_{ab}) is *hole-free* if, for every set $K \subseteq M$ such that $(K, g_{ab|K})$ is a globally hyperbolic spacetime with Cauchy surface S, if $(K', g_{ab|K'})$ is not an effective extension of $(K, g_{ab|K})$ where $K' = int(D(S))$, then there is no effective extension of $(K, g_{ab|K})$.

What is the relationship between the three conditions? There are only two implication relations between them (Manchak 2014).

Proposition 1. Any spacetime which is geodesically complete is hole-free and inextendible.

Now, any of the three conditions can be used to rule out the "hole" in Example 1. But due to the singularity theorems (Hawking and Penrose 1970), geodesic completeness is now considered to be much too strong a condition; it seems to be violated by "physically reasonable" spacetimes. In what follows, let us focus on the remaining two conditions which are usually taken to be satisfied by all "physically reasonable" spacetimes. Indeed, these two conditions are still in use (see Earman 1995). Might hole-freeness or inextendibility (or their conjunction) be the condition we are looking for? Consider the following example.

Example 2. Let (M, g_{ab}) be Minkowski spacetime and let p be any point in M. Let $\Omega : M - \{p\} \to \mathbb{R}$ be a smooth positive function which approaches zero as the point p is approached. Now consider the spacetime $(M - \{p\}, \Omega^2 g_{ab})$.

The spacetime in Example 2 is inextendible and hole-free. Nonetheless, it seems there is still an artificial "hole" in the spacetime. (The spacetime is geodesically incomplete.) One seeks a (simple, physically meaningful) condition to rule out even these holes.

4 A New Condition

Consider the following definition form.

Definition. A spacetime (M, g_{ab}) has an *epistemic hole* if there are two future inextendible timelike curves γ and γ' with the same past endpoint and which _______ such that $I^-[\gamma]$ is a proper subset of $I^-[\gamma']$.

The physical significance of the definition form is this: Suppose two observers are both present at some event. Now suppose (subject to the restrictions in the blank) they go their separate ways. If it is the case that one observer can eventually know everything the other can eventually know *and more*, then there is a kind of epistemic "hole" preventing the latter observer from knowing the extra bit. One might require the region of spacetime which an observer can eventually know to be "as large as it can be". In other words, one might require spacetime to be free of epistemic holes.

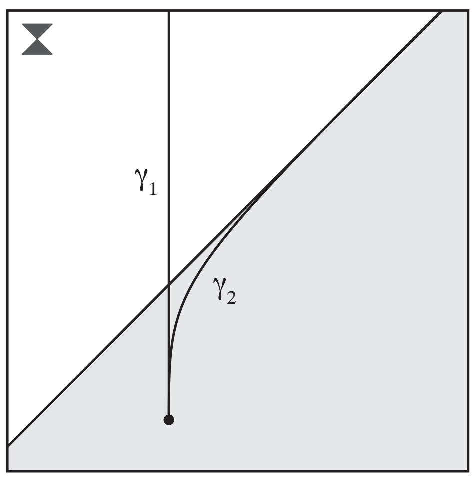

Figure 2: Observers γ_1 and γ_2 in Minkowski spacetime. The set $I^-[\gamma_2]$ (the shaded area) is a proper subset of $I^-[\gamma_1]$ (the entire manifold).

If no restrictions are given in the blank, Examples 1 and 2 count as having epistemic holes as we would hope. But, unfortunately, this version of the condition is too strong; it rules out spacetimes which are usually thought to be "physically reasonable" in some sense. Take Minkowski spacetime, for example. It counts as having epistemic holes. (Consider any point in the Minkowski spacetime. Now consider any observer at the point who, with infinite total acceleration, reaches "null infinity" and another observer at the point who does not. See Figure 2.)

In order to not count Minkowski spacetime as having epistemic holes, one seeks to fill the blank with reasonable restrictions. Let's consider two natural possibilities: "are geodesics" and "have finite total acceleration". Let EH(g) and EH(f) respectively denote these two versions of the epistemic hole definition. In addition, if a spacetime fails to have an EH(g), let us say it is EH(g)-*free* (and respectively for the EH(f) case).

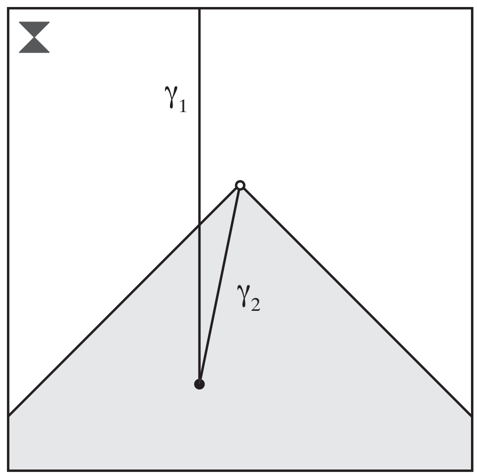

Figure 3: Geodesic observers γ_1 and γ_2 in Minkowski spacetime with one point removed. The set $I^-[\gamma_2]$ (the shaded area) is a proper subset of $I^-[\gamma_1]$ (the entire manifold).

Clearly, if a spacetime is EH(f)-free, then it also EH(g)-free.[2] And as we would hope, Examples 1 and 2 each have an EH(g) and therefore an EH(f) (see Figure 3). Indeed, acausal examples aside, it seems almost every artificially mutilated spacetime will have an epistemic hole of some type. Now, Minkowski spacetime is EH(f)-free and EH(g)-free by construction. What about other "physically reasonable" spacetimes? The Schwarzchild solution is a good test case; its future inextendible timelike curves have event horizons which might allow for epistemic holes.[3] But this is not the case; it and its Kruskal extension count as EH(f)-free and EH(g)-free (see Figure 4).

One can also show that the de Sitter, anti-de Sitter, and Gödel models all count as both EH(f)-free and EH(g)-free as well. On the other hand, Misner spacetime is neither EH(f)-free nor EH(g)-free (see Figure 5). However, Misner spacetime harbors "naked singularities" thought to be physically unreasonable. Consider the following influential definition (Geroch and Horowitz 1979, Earman 1995).[4]

[2]One wonders if the converse is also true. We conjecture that, due to its peculiar timelike geodesic structure, Reissner-Nordström spacetime is EH(g)-free but not EH(f)-free.

[3]For a complete treatment of event horizons, see Rindler (1956).

[4]The concept of terminal indecomposable past sets (TIPs), which can be shown to be

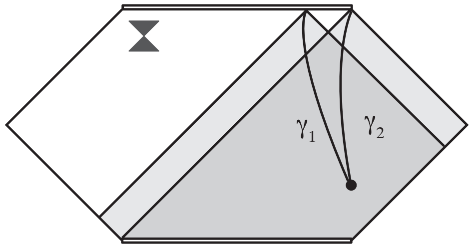

Figure 4: Conformal diagram of Kruskal-Schwarzchild spacetime. Any observers γ_1 and γ_2 with finite total acceleration are such that if $I^-[\gamma_2]$ and $I^-[\gamma_1]$ (shaded regions) are distinct, then they do not fully overlap.

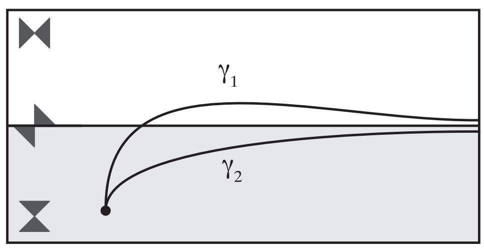

Figure 5: Unrolled Misner spacetime. Geodesic observers γ_1 and γ_2 are such that the set $I^-[\gamma_2]$ (the shaded area) is a proper subset of $I^-[\gamma_1]$ (the entire manifold).

Definition. A spacetime (M, g_{ab}) is *nakedly singular* if there is a point $p \in M$ and a future incomplete timelike geodesic γ such that $\gamma \subset I^-(p)$.

What is the relationship between naked singularities and epistemic holes? Consider the following examples.

Example 3. Let (M, g_{ab}) be Minkowski spacetime and let p be any point in M. Let $\Omega : M - \{p\} \to \mathbb{R}$ be a smooth positive function which approaches infinity as the point p is approached. Now consider the spacetime $(M - \{p\}, \Omega^2 g_{ab})$.

Example 4. Let (M, g_{ab}) be two dimensional Minkowski spacetime in standard t, x coordinates which is "rolled up" along the t direction. Let p be any point in M. Consider the spacetime $(M - \{p\}, g_{ab})$.

precisely the timelike pasts of future inextendible timelike curves, have been used to formulate a type of naked singularity definition. The definition turns out to be equivalent to the failure of global hyperbolicity (Penrose 1999). For details on the relationship between global hyperbolicity and epistemic holes, see below.

Example 3 is geodesically complete (and therefore contains no naked singularities). But it has both an EH(f) and an EH(g). On the other hand, Example 4 contains naked singularities but is EH(f)-free and EH(g)-free. Now, Example 4 is not causally well-behaved. If one were to limit attention to spacetimes satisfying chronology, can one still find examples with naked singularities and without epistemic holes? At least in the case of EH(f), no.[5]

Proposition 2. Any EH(f)-free, chronological spacetime is not nakedly singular.

Proof. Let (M, g_{ab}) be a chronological, nakedly singular spacetime. Let γ be a future incomplete timelike geodesic with past endpoint $q \in M$ such that $\gamma \subset I^-(p)$ for some $p \in M$. Let γ' be any timelike curve with finite total acceleration with past endpoint q which runs through p and is future inextendible. Clearly, $I^-[\gamma] \subseteq I^-[\gamma']$. Suppose $I^-[\gamma] = I^-[\gamma']$. Since $p \in I^-[\gamma']$, we have $p \in I^-[\gamma]$. So, there is a point $r \in \gamma$ such that $p \in I^-(r)$. But $r \in I^-(p)$. It follows that $p \in I^-(p)$ which is a violation of chronology: a contradiction. So, $I^-[\gamma] \neq I^-[\gamma']$. Thus, $I^-[\gamma]$ is a proper subset of $I^-[\gamma']$. So, there is an EH(f) in (M, g_{ab}). □

The proposition shows that, if one takes EH(f)-freeness as a necessary condition of physical reasonableness, then the weak causality assumption of chronology rules out naked singularities. Contrast this result with one (Geroch and Horowitz 1979) which shows that the strong causality assumption of global hyperbolicity is, by itself, enough to exclude naked singularities.[6] Now, what is the relationship between global hyperbolicity and epistemic holes? Consider the following example.

Example 5. Let (M, g_{ab}) be Minkowski spacetime and let p be any point in M. Let M' be the set $I^-(p)$. Let $\Omega : M' \to \mathbb{R}$ be a smooth positive function which approaches infinity as the boundary of $I^-(p)$ is approached. Now consider the spacetime $(M', \Omega^2 g_{ab})$. (See Figure 6.)

Example 5 shows that a globally hyperbolic spacetime, indeed even a globally hyperbolic spacetime which is geodesically complete, can nonetheless fail to be EH(f)-free and EH(g)-free. On the other hand, Example 4 shows that a spacetime which is non-globally hyperbolic, indeed even a non-chronological spacetime which fails to be inextendible and hole-free, can nonetheless be EH(f)-free and EH(g)-free. In sum: epistemic holes are very different from "holes" and "singularities" of various kinds.

[5] It is an open question whether the result holds for the EH(g) case as well.

[6] We know from anti-de Sitter spacetime that global hyperbolicity is neither equivalent to the conjunction of chronology and EH(f)-freeness nor equivalent to the conjunction of chronology and EH(g)-freeness.

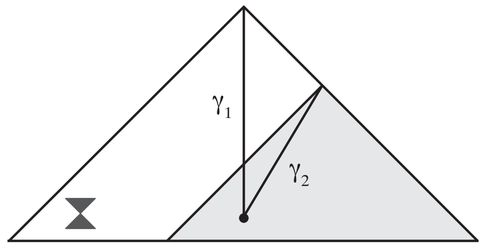

Figure 6: Geodesic observers γ_1 and γ_2 in a portion of conformal Minkowski spacetime. The set $I^-[\gamma_2]$ (the shaded area) is a proper subset of $I^-[\gamma_1]$ (the entire manifold).

5 A New Kind of Condition

Stepping back, one may ask: what justifies the use of epistemic hole-freeness? As we have seen, the condition rules out many intuitively "physically unreasonable" spacetimes (including those, like Example 2, which inextendibility and hole-freeness fail to rule out). On the other hand, no model with epistemic holes has yet been found which is clearly "physically reasonable". It is our position that this alone provides sufficient justification for the condition. In any case, the proper sorting of the intuitively "physically reasonable" and "physically unreasonable" examples is, at root, the justification for the widely used conditions of inextendibility and hole-freeness (Earman 1995, Manchak 2011). And, as with these other two conditions, one can hope to put epistemic hole-freeness to work in proving theorems of interest (such as Proposition 2).

In addition, we wish to highlight an important way in which the condition of epistemic hole-freeness is far superior to the conditions of inextendibility and hole-freeness. The definitions of the latter two conditions make reference to (and are functions of) the entire class of relativistic spacetimes; they require that certain regions of spacetime be "as large as they can be" in the sense that one compares them, from a God's eye point of view, to similar regions in *all possible spacetimes*. And whether or not a spacetime counts as inextendible or hole-free depends crucially on the makeup of this class of all possible spacetimes. But what is this class? We would like to emphasize that whatever the answer winds up being depends upon assumptions concerning what "physically reasonable" spacetimes are. (Surely, the "possible" here cannot be merely logical or mathematical.) And the fact that we have yet to pin down the class of "physically reasonable" spacetimes should give us pause. In essence, the fact implies that we have yet to pin down the class of inextendible spacetimes and the class of hole-free spacetimes.

An example might serve to illustrate the point. Following standard practice, we have assumed in the preceding that all manifolds are Hausdorff. Under this assumption, Minkwowski spacetime counts as inextendible. The spacetime is "as large as it can be" in the sense that it cannot be properly and isometrically

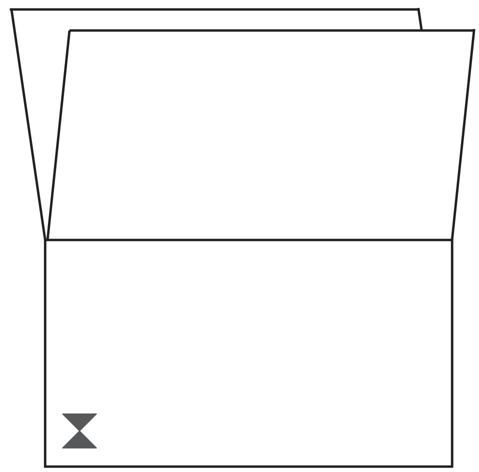

Figure 7: A non-Hausdorff extension of Minkowski spacetime.

embedded into another spacetime. But note that if the Hausdorff assumption is dropped, *Minkowski spacetime now counts as extendible* (see Figure 7). The example shows that, whatever else is the case, the class of "possible" spacetimes, as standardly interpreted, is not a class of merely logically or mathematically possible spacetimes; non-Hausdorff spacetimes are logically and mathematically well-defined (Earman 2008) and Minkowski spacetime is standardly interpreted as inextendible. Thus, the use of the condition of inextendibility has, all along, presupposed a distinction between spacetimes which are "physically reasonable" and those which are not. Once this important fact is clear, one is naturally "tempted to revise the principle of inextendibility" (Geroch 1970, 278). But how should one revise? Any revision is dubious given that we do not know, and arguably cannot know (Manchak 2011), the makeup of the class of "physically reasonable" spacetimes.

To see why this might be, consider another example: the bottom half of Misner spacetime (see Figure 5). It is globally hyperbolic and counts as extendible in the preceding. But suppose the cosmic censorship conjecture of Penrose (1979) is correct and all "physically reasonable" spacetimes are globally hyperbolic. Then *the bottom half of Misner spacetime now counts as inextendible*; it cannot be embedded properly and isometrically into a globally hyperbolic spacetime; the spacetime is "as large as it can be" if the possibility space is limited in just the way some experts think it is. It follows that whether or not the bottom half of Misner spacetime counts as inextendible depends crucially on the outcome of the cosmic censorship conjecture – a conjecture which is far from settled (Earman 1995, Penrose 1999). One is seemingly forced to conclude that the very content of the condition of inextendibility is unavoidably murky. (A similar argument can be given for the case of hole-freeness.)

Now consider epistemic hole-freeness. It requires that certain regions of spacetime be "as large as they can be" in the sense that one compares them to similar regions *within the very same spacetime*. The condition does not make reference to the class of all "possible" spacetimes. And thus, the condition does not presuppose that a distinction has been made between spacetimes which are "physically reasonable" and those which are not.[7] The content of the condition is perfectly clear. Among conditions used to rule out "holes" in spacetime, epistemic hole-freeness appears to be alone in possessing this desirable quality.[8]

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[7]Some distinctions may need to be made to "get things off the ground" as it were. For example, in order to articulate the defintion of epistemic hole-freeness, one must presuppose time orientability. But once off the ground, the definition of epistemic hole-freeness is not a function of the class of "physically reasonable" spacetimes (as are inextendibility and hole-freeness).

[8]Many other familiar conditions also possess this quality. For example: the Hausdorff condition, time orientability, chronology, the standard energy conditions. But these are not used to rule out "holes" in spacetime.

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