On Feyerabend, General Relativity, and 'Unreasonable' Universes*

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Abstract

I investigate the principle *anything goes* within the context of general relativity. After a few preliminaries, I show a sense in which the universe is unknowable from within this context; I suggest that we 'keep our options open' with respect to competing models of it. Given the state of affairs, proceeding counter-inductively seems to be especially appropriate; I use this method to blur some of the usual lines between 'reasonable' and 'unreasonable' models of the universe. Along the way, one is led to a useful collection of variant theories of general relativity – each theory incompatible with the standard formulation. One may contrast one variant theory with another in order to understand foundational questions within 'general relativity' in a more nuanced way. I close by sketching some of the work ahead if we are to embrace such a pluralistic methodology.

1 Introduction

In what follows, I will investigate the principle *anything goes* within the context of general relativity. It is of some interest that even after one restricts attention in this way, one can still carry out a sort of Dadaist "joyful experiment" (Feyerabend [1975] 2010, xiv) demonstrating the chimerical nature of various distinctions between 'reasonable' and 'unreasonable' models of the universe.[1] Here, I intend to sketch the contours of such an undertaking without presupposing any prior familiarity with general relativity on the part of the reader.

In the first portion of the paper, I will walk through some of the basic structure of general relativity. I then consider a remark made by Feyerabend in the introduction to *Against Method*. There, we are told that a pluralistic approach

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[1]From 1970 until 1988, both the paper and book versions of *Against Method* were given a subtitle – *Outline of an Anarchistic Theory of Knowledge* – and an accompanying footnote in which Feyerabend distanced himself from the 'seriousness' of anarchism and asked that he be remembered as a Dadaist instead. For more on Feyerabend and Dada, see the introductory note by Ian Hacking in the fourth edition of *Against Method* (Feyerabend [1975] 2010, xiii-xvi).

to epistemology seems to be appropriate given that "the world which we want to explore is a largely unknown entity" (Feyerabend [1975] 2010, 4). The meaning of the claim is not clarified and no argument is ever given for it. But, if true, it does seem to lead naturally to the position that we ought "keep our options open" with respect to competing models of the universe (Feyerabend [1975] 2010, 4). In the second portion of the paper, I will articulate a sense in which the claim is true with respect to the models of the universe compatible with general relativity; any idealized person represented in virtually any such model will not have the epistemic resources – even with a robust type of inductive reasoning – to know which model she inhabits. For her, the universe is (and will always remain) a largely unknown entity. I will then consider the esteemed property of *inextendibility* which requires that a model universe be 'as large as it can be' in a natural sense. This property is almost universally thought to be satisfied by all 'reasonable' models of the universe. But I will emphasize that any idealized person in virtually any model cannot know that her universe is inextendible. The upshot is this: within this context, there is (and will always remain) the epistemic possibility that our universe is best represented by an 'unreasonable' model.

Given the state of affairs, proceeding counter-inductively seems to be especially appropriate.[2] In particular, the practice of making "the weaker case the stronger" (Feyerabend [1975] 2010, 14) can be used to blur some of the usual lines between 'reasonable' and 'unreasonable' models of the universe. In the third portion of the paper, I will return to the property of inextendibility and work to cast doubt on the idea that a 'reasonable' model universe must be 'as large as it can be.' I will do this in two steps. First, I will argue that the usual distinctions between 'reasonable' and 'unreasonable' models can be upheld only if the definition of inextendibility is radically modified; the now standard formulation allows for extendible models to nonetheless be 'as large as they can be' in the sense that they cannot be 'reasonably' extended. Second, I will call into question the metaphysical underpinnings of the position that our universe is 'as large as it can be' in light of the need to modify the definition of inextendibility. A celebrated foundational result states that every extendible model of the universe has at least one corresponding inextendible extension. But I will show that under some 'reasonable' revisions to the definition of inextendibility, the analogous results do not hold; it is not always possible for a 'reasonable' model of the universe to be 'as large as it can be.'

Along the way, we seem to be led to consider an "ocean of mutually incompatible alternatives" to the standard formulation of general relativity (Feyerabend [1975] 2010, 14). I will close with an articulation of such alternatives –

[2]One may proceed counter-inductively by using "hypotheses that contradict well-confirmed theories and/or well-established facts" (Feyerabend [1975] 2010, 13). In what follows, I will only consider hypotheses that contradict well-confirmed theories rather than well-established facts. In this sense, I will be also be following Feyerabend's earlier *principle of proliferation*: "Invent, and elaborate theories which are inconsistent with the accepted point of view, even if the latter should happen to be highly confirmed and generally accepted" (Feyerabend 1965, 101). For more on the principle of proliferation, see Shaw (2017).

each a variant of general relativity incompatible with the standard formulation. I hope to show that by contrasting the situation in one variant theory with another, we can come to understand foundational questions within 'general relativity' in a more nuanced way. I will sketch some of the work ahead if we are to embrace such a pluralistic methodology.

2 Preliminaries

Let us begin with a few preliminaries.[3] A (relativistic) *model of the universe* is an ordered pair (M, g) where M is a smooth four-dimensional 'manifold' representing the shape of the universe and g is a smooth relativistic 'metric' encoding the geometry of the universe. Each point in the manifold represents a possible event in space and time. Experience seems to tell us that any event (e.g. the moon landing) can be characterized by four numbers – one temporal and three spatial coordinates. Accordingly, the local structure of a manifold 'looks like' a four-dimensional Cartesian coordinate system. But the global structure can be quite different. Many two-dimensional manifolds are familiar to us: the plane, the sphere, the torus, and so on.

Manifolds are good for representing events in the universe. But the metric tells us how these events are related to one another. In particular, the 'causal structure' between events is of special interest to us here. Consider a model of the universe (M, g) and a given event p in M. Which events in M can be causally influenced by p? The metric tells us. We can think of g as a kind of smooth function which assigns lengths to all vectors at all points in M – either positive, negative, or zero. This partitions the vectors at each point into a cone structure where zero length vectors make up the the boundary of the cone while positive and negative length vectors fall, respectively, inside and outside of that boundary. Physically, vectors at a point represent velocity vectors. Since light always moves with a velocity vector of length zero, the cone structure determined by g can be thought of as demarcating the 'speed of light' in all directions. Central to general relativity is the idea that "nothing can travel faster than light" and that includes any causal influences. So physically, velocity vectors must fall on or inside the boundary of the light cone. A velocity vector with this property is said to be *causal*; if a causal vector falls strictly inside the light cone, it is *timelike* (see Figure 1).

Let us follow standard practice and suppose that, ranging over the entire manifold M, one can label the two lobes of the cone structure as 'past' and 'future' in a continuous way. If a model has this property, we say it is *time-orientable*. Physically, such a model can be given a 'direction of time' in a global sense. (A model which is not time-orientable can be constructed by using a Möbius strip as the underlying manifold.) And let us also suppose that such a labeling has been carried out at every point in M. Now we are in a

[3]For details on general relativity, see Hawking and Ellis (1973), Wald (1984), and Malament (2012). For less technical introductions to global spacetime structure, see Geroch and Horowitz (1979) and Manchak (2013).

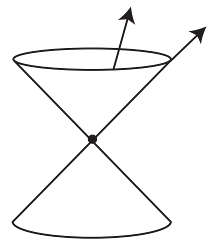

Figure 1: An event with associated cone structure is depicted along with a pair of causal vectors. (One spatial dimension has been suppressed.)

position to answer the question from above: an event q in M can be causally influenced by event p in M only if there is a smooth curve on M which starts at p and ends at q and whose tangent vector is always (i) causal and (ii) pointed in the future direction. A curve such as this is a *future-directed causal* curve. A future-directed causal curve is a *future-directed timelike* curve if all of its tangent vectors are timelike. Let us say that the collection of all points q in M such that there is a future-directed causal curve from p to q is the *causal future* of p. Analogous constructions yield the *causal past*, *timelike past*, and *timelike future* of p.

Let us now consider four properties which will form a simplified hierarchy of causal conditions. Let (M, g) be a time-orientable model of the universe. If there is an event p in M such that the causal past of p is all of M, we say it has a *God point*.[4] We can think of the causal past as representing the region of the universe that can possibly be observed by an idealized person at p. (After all, if q is an event located outside of the causal past of p, then how can q be observed at p if no causal influence can go from q to p?) So a model universe has a God point if it has an event from which an idealized person can possibly observe the entire universe.

Our next two causal properties preclude 'causal loops' of certain types. Let (M, g) be a time-orientable model of the universe. If the model contains a future-directed timelike curve which intersects itself, we say it is not *chronological*. Models which fail to be chronological allow for 'time travel' in the sense that an idealized person in the model can both begin and end a journey at the very same event (see Figure 2). A slight variant of the chronology property rules out self-intersecting future-directed causal curves; if a model satisfies this condition, then we say has the *causality* property. It is immediate that every causal model is chronological; one can show the converse does not hold.

Our final causal property ensures that a model universe is so causally 'well-behaved' that it might be considered 'deterministic' in some sense. Let (M, g) be any model of the universe satisfying causality. If it is the case that for any events p, q in M, the intersection of the causal past of p and the causal future

[4]Thanks to Zvi Biener and Chris Smeenk for the terminology.

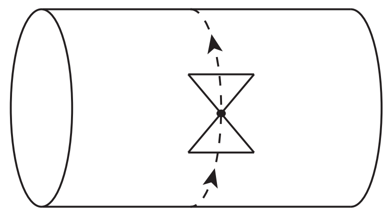

Figure 2: A non-chronological model of the universe is depicted with a self-intersecting future-directed timelike curve. (Two spatial dimensions have been suppressed.)

of q is a 'compact' region, then the model is said to be *globally hyperbolic*. A globally hyperbolic model can be split into a 'stack' of three-dimensional 'spatial universes' along a one-dimensional 'time'. In such a model, information about the structure of the spatial universe at one time can be used to determine that structure at all times. By definition, every globally hyperbolic model is causal. The converse is not true: for a counterexample, take any globally hyperbolic model and remove one point from the manifold. For a sense of the relative strengths of the four causal properties examined here, consider that the first three (non-existence of a God point, chronology, causality) are usually taken to be satisfied by all 'reasonable' models of the universe while global hyperbolicity is only sometimes taken that way (Earman 1995). More on this below.

Now let us turn to a pair of relations on the collection of models of the universe. Let us say that the models (M, g) and (M', g') are *isometric* if there is a smooth one-to-one correspondence between the points in M and the points in M' which preserves all metric structure. Isometric models of the universe are physically identical. Because the isometry relation is an equivalence relation, we can partition the collection of all models into corresponding isometry equivalence classes. A similar relation is also useful: Let us say that the models (M, g) and (M', g') are *locally isometric* if (i) for every point p in M, there is a 'local neighborhood' around p, a point p' in M', and a 'local neighborhood' around p' such that the neighborhoods are isometric and (ii) likewise with the roles of (M, g) and (M', g') interchanged. If two models of the universe are locally isometric, they share the same 'local' physics. Local isometry is also an equivalence relation; indeed, the partition given by the isometry relation is just a 'refinement' of the partition given by the local isometry relation; any two isometric models are locally isometric but not the other way around. Let us say that a property of a model is a *local* property if, for any two locally isometric models, one model has the property if and only if the other does as well. We say a property is a *global* property if it is not local. It turns out that each of the three causal properties considered above count as global. On the other hand, the 'energy conditions' which limit the distribution and flow of matter can be used to define a family of local properties (see Curiel 2017).

3 On the Unknowability of the Universe

We are now ready to consider the claim that

(#) "the world which we want to explore is a largely unknown entity" (Feyerabend [1975] 2010, 4).

It seems natural to interpret (#) as a type of defense for the pluralistic methodology outlined by Feyerabend; if (#) is true, we ought to "keep our options open" so that that we do not prematurely close down alternatives which seem 'unreasonable' now but which may, at some later time, point the way to some "deep-lying secrets of nature" (Feyerabend [1975] 2010, 4).[5] Even though (#) is not elaborated upon in the introduction to *Against Method*, the claim does not seem to be an off-hand remark; much of Feyerabend's historical work concerning case studies (in *Against Method* and elsewhere) serves to bolster the position that certain (e.g. empirical) methodologies would have closed off alternatives we now judge to be quite 'reasonable'.[6] This "keep our options open" defense of Feyerabend's methodological pluralism is distinct from the "benefits of competition" defense and, from what I understand, has yet to become a focus in the literature so far.[7]

Let us now work to show a sense in which (#) in true within the context of general relativity. First, consider that, in any model, empirical observations at an event must be confined to the casual past of that event for the reasons mentioned above. Following Clark Glymour (1977) and David Malament (1977), let us say that a model (M, g) is *observationally indistinguishable* from a model (M', g') if, for each point p in M there is a point p' in M' such that the causal pasts of p and p' are isometric (i.e. share the same physical structure). The physical significance of the definition is this: If one model is observationally indistinguishable from another model, then no idealized person represented in the first model has the epistemic resources to tell the difference between the first and second models; in other words, an idealized person in the first model cannot know that she inhabits the first model. It turns out that models can be

[5]The "keep our options open" defense of pluralism is often called the "hedging our bets" defense. For example, here is Kitcher (1993, 344): "Intuitively, a community that is prepared to hedge its bets when the situation is unclear is likely to do better than a community that moves quickly to a state of uniform opinion."

[6]Consider the following (Feyerabend [1975] 2010, 115): "Now, what our historical examples seem to show is this: there are situations when our most liberal judgements and our most liberal rules would have eliminated a point of view which we regard today as essential for science, and would not have permitted it to prevail – and such situations occur quite frequently. The ideas survived and they now are said to be in agreement with reason. They survived because prejudice, passion, conceit, errors, sheer pigheadedness, in short because all the elements that characterize the context of discovery, opposed the dictates of reason and because these irrational elements were permitted to have their way. To express it differently: Copernicanism and other 'rational' views exist today only because reason was overruled at some time in their past."

[7]For an exception along these lines, see Oberheim (2005) on the role of incommensurability in Feyerabend's thought – especially as it relates to Duhem. For excellent discussions of the "benefits of competition" defense, see Lloyd (1997) and Bashir (2015).

different (i.e. non-isometric) and yet observationally indistinguishable. For example, start with the 'de Sitter' model where the manifold is cylindrical and the vertical cone structures narrow ever more rapidly the more distant they are in the 'past and 'future' directions. The upshot is that the causal past of any event in the model does not 'wrap all the way around' the cylinder. Now consider a similar model where the cylinder in 'unrolled' but the metric structure remains the same. The two models are observationally indistinguishable since, for any event in either model, one can find a similar event in the other model such that the two causal pasts have the same metric structure (see Figure 3). Note that the relation of observational indistinguishability, although reflexive and transitive, is not symmetric; there are situations where one model is observationally indistinguishable from another but not vice versa. Consider, for example, the 'bottom half' of the De Sitter model – it is observationally indistinguishable from the de Sitter model but not the other way around.

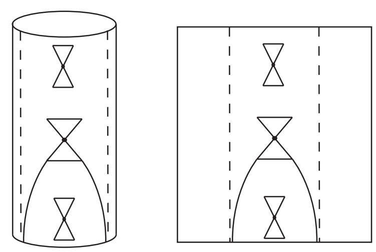

Figure 3: The de Sitter and 'unrolled' de Sitter models are depicted, both with a representative causal past bounded by the dotted lines. Each model is observationally indistinguishable from the other. (Two spatial dimensions have been suppressed.)

What is the relation of observationally indistinguishability like? Which models of the universe have a non-isometric but observationally indistinguishable counterpart? Malament (1977) conjectured that *any* model without a God point must be related to another model in just this way. This conjecture turns out to be true but even more can be said: the result goes through even if one requires any collection of local properties to be satisfied. So even under a robust type of inductive reasoning – that the local structure of universe is the same everywhere – the underdetermination remains. Consider the following (Manchak 2009).

Proposition 1. Consider a model of the universe with any collection of local properties. If the model fails to have a God point, it is observationally indistinguishable from some other (non-isometric) model with all of the same local properties.

The proposition shows a sense in which any idealized person represented in virtually any model will not have the epistemic resources to know which model she inhabits. For her, the universe is (and will always remain) a largely unknown entity. But how serious is this epistemic predicament? Perhaps it is the case that, although the full structure of a model universe is unknowable from within, partial knowledge can be obtained – again, with a robust type of inductive reasoning – concerning some global properties of interest. It turns out even this cannot be done for many important global properties (e.g. global hyperbolicity) thought to be satisfied by some 'reasonable' models of the universe. Here, we will focus attention on one property in particular: inextendibility.[8] Roughly, this property requires that a model universe be 'as large as it can be' in a natural sense. We say a model of the universe (M, g) is *extendible* if there is another model (M', g') such that M is isometric to a proper subset of M'; here (M', g') is a (proper) *extension* of (M, g). A model (M, g) is *inextendible* if it is not extendible (see Figure 4).

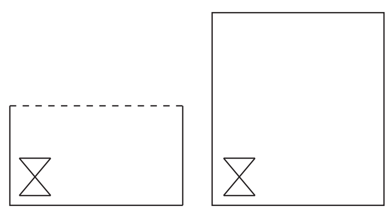

Figure 4: The first model is extendible. The second model is an extension of the first and is inextendible. (Two spatial dimensions have been suppressed.)

The idea that all 'reasonable' models of the universe must be inextendible is more or less taken for granted within the community. The reasoning behind this position is summarized by John Earman (1995, 32-33).

> Metaphysical considerations suggest that to be a serious candidate for describing actuality, a spacetime should be [inextendible]. For example, for the Creative Force to actualize a proper subpart of a larger spacetime would seem to be a violation of Leibniz's principles of sufficient reason and plenitude. If one adopts the image of spacetime as being generated or built up as time passes then the dynamical version of the principle of sufficient reason would ask why the Creative Force would stop building if it is possible to continue...Some readers may be shocked by the introduction of metaphysical considerations in the hardest of the "hard sciences." But in fact leading workers in relativistic gravitation, though they don't invoke the name of Leibniz, are motivated by such principles.

We will return to the metaphysical justification of inextendibility in the next portion of the paper. For now, let us turn to the strengthened underdetermina-

[8]See Geroch (1970) and Clarke (1976) for details concerning this property. The appendix of Geroch (1970) is an especially good resource; it contains a list of precise foundational questions concerning inextendibility – many of which remain open.

tion result mentioned above (Manchak 2011).

Proposition 2. Consider a model of the universe with any collection of local properties. If the model fails to have a God point, it is observationally indistinguishable from some other (non-isometric) model with all of the same local properties which is also extendible.

The proposition shows a sense in which any idealized person in virtually any model cannot know – even with a robust type of inductive reasoning – that her universe is inextendible. And since inextendibility is taken to be a necessary property of all 'reasonable' models, the upshot is this: within general relativity, there is (and will always remain) the epistemic possibility that the universe is best represented by an 'unreasonable' model.

4 On 'Unreasonable' Universes

The underdetermination results just presented seem to lead naturally to the position that we ought to "keep our options open" with respect to competing models of the universe. In particular, we ought to be suspicious about various distinctions made between the 'reasonable' and 'unreasonable' models – especially in cases where the 'unreasonable' ones remain live epistemic possibilities. And yet, as mentioned above, it seems to be taken for granted by the community that

(†) "any reasonable space-time should be inextendible" (Clarke 1993, 8).

Just how entrenched is the (†) position? Like inextendibility, there are a number of other properties which are often considered necessary for all 'reasonable' models of the universe; as mentioned above, chronology is one such. But research is still routinely conducted on models which fail to have many of these properties – consider the vast and flourishing literature on 'time travel' and 'time machines' for example (Earman et al. 2016). It is of some interest that this is not the case with respect to the failure of inextendibility; the subject has given rise to almost no literature at all.[9] Apparently, the negation of (†) is an especially 'unreasonable' position. Still, I do not think it has been "given all the chances it deserves" (Feyerabend [1975] 2010, 29).

In what follows, I will proceed according to the counter-inductive suggestion to "introduce and elaborate hypotheses which are inconsistent with well-established theories" (Feyerabend [1975] 2010, 13). In particular, I will introduce and elaborate the negation of (†). Why work to proliferate an especially 'unreasonable' position in this way? One reason has already been emphasized

[9]Indeed, work has shifted somewhat dramatically toward an exploration of even stronger definitions of inextendibility; an 'inextendible' model can be extended if the metric is not required to be smooth. See Galloway and Ling (2017) and Sbierski (2018) for the latest twists and turns.

above: if (†) is wrong, we do not want to prematurely settle on it. Another reason is this: even if (†) were 'right' in some sense, the development of its negation via a "process of competition" (Feyerabend [1975] 2010, 14) only serves to improve our understanding of the (†) position itself. One is guided by the "hope that working without the rule, or on the basis of a contrary rule we shall eventually find a new form of rationality" (Feyerabend 1977, 368). It is this prospect of a 'new form of rationality' which amounts to an additional defense of Feyerabend's methodological pluralism (see Shaw 2017). As we will soon see, this prospect is realized in the present case. As a result of elaborating of the negation of (†), the usual lines between 'reasonable' and 'unreasonable' models of the universe will seem to blur. In addition, entirely new ways of looking at the situation will present themselves. The entire undertaking proceeds in the spirit of Feyerabend as one might expect: one "plays the game of Reason in order to undercut the authority of Reason" (Feyerabend [1975] 2010, 16).

We begin by drawing a distinction between the 'reasonable' and 'unreasonable' models of the universe; let $\mathscr{U}$ be the collection of all models and let $\mathscr{R} \subset \mathscr{U}$ be a working collection of 'reasonable' ones. Such a collection proves useful to consider even if we have yet to pin down its make up. It is crucial in what follows that that we do not suppose from the outset that that every model in $\mathscr{R}$ is inextendible since this is the very question under investigation. But this opens the way for a curious possibility: perhaps there is an 'extendible' model of the universe in $\mathscr{R}$ which nonetheless cannot be 'reasonably' extended in the sense that all of its extensions fail to be in $\mathscr{R}$. Would not such a model be 'as large as it can be' in the only sense that mattered? Would it not be 'reasonable' to demand that such a model be called 'inextendible'?

It turns out that scenarios like the one just mentioned not only can be constructed – they arise quite naturally when various 'reasonable' hypotheses are entertained. For example, consider the so-called (strong) 'cosmic censorship conjecture' of Roger Penrose (1979). The content of the conjecture can be expressed as the position that

(††) all "reasonable spacetimes are globally hyperbolic" (Wald 1984, 304).

It should be noted that, unlike (†), the position (††) is not uncontroversial (Earman 1995). But whatever else is the case, it is not 'unreasonable' to consider its consequences. Suppose (††) is true; suppose the collection $\mathscr{G}$ of globally hyperbolic models is such that $\mathscr{R} \subset \mathscr{G}$. Now consider the 'Misner' model where the manifold is cylindrical and the cone structures 'tip over' as they go up the cylinder (see Figure 5). The 'bottom half' of the Misner model – call it the 'lower Misner' model – counts as extendible when taken as a model in its own right (the Misner model itself being just one of its many extensions). But aside from its extendibility, the lower Misner model checks all of the usual boxes required of all 'reasonable' models. In particular, it is both globally hyperbolic and a 'vacuum solution' of Einstein's equation. This latter fact ensures that the model satisfies all of the energy conditions mentioned above which guarantee a 'reasonable' distribution and flow of matter. Indeed, the lower Misner model is about as ‘reasonable’ as an extendible spacetime can be. Given that we have deliberately left open the possibility that $\mathscr{R}$ contain extendible models, it seems ‘reasonable’ to consider the case where the lower Misner model is found in that collection. But now observe: one can show that every extension of the lower Misner model fails to be in $\mathscr{G}$ (Manchak 2017). By (††), every such extension fails to be in $\mathscr{R}$ as well. The situation seems to be this: we have a ‘reasonable’ model of the universe which cannot be ‘reasonably’ extended and yet counts as ‘extendible’ according to the standard definition.

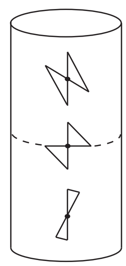

Figure 5: The Misner model. The region below the dotted line is a globally hyperbolic vacuum solution when taken as a model it its own right. (Two spatial dimensions have been suppressed.)

Due to examples like the one given above, Bob Geroch has considered the possibility of revising the standard definition of inextendibility. For example, a variant definition can be constructed for each property $\mathscr{P} \subset \mathscr{U}$: Let us say that a model is a *$\mathscr{P}$-model* if it is in the collection $\mathscr{P}$; a $\mathscr{P}$-model is *$\mathscr{P}$-extendible* if it has an extension in $\mathscr{P}$ – such an extension is a *$\mathscr{P}$-extension* – and *$\mathscr{P}$-inextendible* otherwise. But such “unpleasant modifications” to the standard definition of inextendibility seem to be unecessary if, for a variety of ‘reasonable’ properties $\mathscr{P} \subset \mathscr{U}$, the following is true (Geroch 1970, 278).

(*) Every $\mathscr{P}$-inextendible $\mathscr{P}$-model is inextendible.

Let us try out some properties $\mathscr{P}$ in an attempt to get a grip on the situation. It is immediate from the lower Misner example that (*) is false if $\mathscr{P}$ is the collection $\mathscr{G}$ of globally hyperbolic models. What about other ‘reasonable’ properties of interest? A simple example shows (*) to be false if $\mathscr{P}$ is the collection $\mathscr{C}$ of models which satisfy causality. And we have recently learned that (*) is false if $\mathscr{P}$ is the collection $\mathscr{E}$ of models satisfying the ‘weak’ energy condition (Manchak 2017). More work is certainly needed. Still, at present no significant ‘reasonable’ property $\mathscr{P}$ has been found which renders (*) true. It seems ‘reasonable’ to explore revisions to the standard definition of inextendibility.

Let us take a step back. Recall that the primary justification for (†) seemed to rest on Leibniz’s principles of sufficient reason and plenitude. Here is a representative statement along these lines (Geroch 1970, 262): “Why, after all, would Nature stop building our universe...when She could just as well have carried on?" Underpinning the metaphysical views expressed by Geroch and others is this central fact (Geroch 1970).

Proposition 3. Every extendible model has an inextendible extension.[10]

To be sure, the proposition is beautiful in its simplicity and power. But given the need to consider revisions to the definition of inextendibility, is it not 'reasonable' to investigate whether the following is true for a variety of 'reasonable' properties $\mathscr{P} \subset \mathscr{U}$?

(**) Every $\mathscr{P}$-extendible $\mathscr{P}$-model has a $\mathscr{P}$-inextendible $\mathscr{P}$-extension.

Very little is known concerning the status of (**) with respect to 'reasonable' properties of interest.[11] We do know that (**) is true if $\mathscr{P}$ is the collection $\mathscr{K}$ of chronological models (Manchak 2017). Perhaps this gives comfort to those wishing to defend (†). But there are examples which also go the other way. Let us say that a model has the *big bang* property if every 'straight' causal curve in the model ends in a 'singularity' in the past direction. It turns out that (**) is false if $\mathscr{P}$ is the collection $\mathscr{B}$ of big bang models (Manchak 2016). We see that under some 'reasonable' revisions to the definition of inextendibility, it is not always possible for a 'reasonable' model of the universe to be 'as large as it can be.' Reason seems to have led us here: perhaps (†) is wrong. (See Table 1 for a synopsis of the situation so far.)

	$\mathscr{U}$	$\mathscr{K}$	$\mathscr{C}$	$\mathscr{G}$	$\mathscr{E}$	$\mathscr{B}$	...
(*)	T	?	F	F	F	?	...
(**)	T	T	?	?	?	F	...

Table 1: A synopsis of the situation so far.

5 Final Remark

By proceeding counter-inductively with respect to (†), we have stumbled upon an even more fundamental way to "introduce and elaborate hypotheses which are inconsistent with well-established theories" (Feyerabend [1975] 2010, 13). Let me explain. For a number of 'reasonable' properties $\mathscr{P} \subset \mathscr{U}$, the following position concerning general relativity seems natural.

> "One might now modify general relativity as follows: the new theory is to be general relativity, but with the additional condition that only [$\mathscr{P}$] space-times are permitted" Geroch (1977, 87).

[10]It is of some interest that this proposition is one of the few results in general relativity which seems to depend crucially on the axiom of choice for its proof. See Clarke (1976).

[11]Presumably, the axiom of choice (in the form of Zorn's lemma) can be applied straightforwardly to obtain positive results for some 'reasonable' properties (e.g. the energy conditions). But this route will not work in other cases (Low 2012).

So for each property $\mathscr{P}$, we have a variant theory of general relativity – call it GR($\mathscr{P}$). At once we find ourselves swimming in an "ocean of mutually incompatible alternatives" (Feyerabend [1975] 2010, 14). We know that the property of inextendibility 'works differently' in some of these variant theories than it does in the standard one; if (*) is false for some $\mathscr{P} \subset \mathscr{U}$, then there is a model universe in $\mathscr{P}$ which is 'inextendible' according to GR($\mathscr{P}$) but 'extendible' according to GR($\mathscr{U}$). So a question like "is the model inextendible?" can be interpreted in as many ways as there are variant theories permitting the model under consideration. Given the state of affairs, a study of the property of inextendibility from within each alternative would seem to be quite appropriate. From the work mentioned above, we find that foundational claims like the following come out as true in some variant theories and false others.

($\star$) Every extendible model has an inextendible extension.

Other statements of interest could be explored in this way. For example, some precise statements concerning 'singularities' and 'time machines' depend on the definition of inextendibility (Geroch 1970; Earman et al. 2016). Statements concerning the 'stability' of various properties (e.g. causal properties) could also be investigated (cf. Hawking 1969). A table presents itself in which such statements are the rows and the columns are variant theories of general relativity (see Table 2).

	GR($\mathscr{U}$)	GR($\mathscr{K}$)	GR($\mathscr{C}$)	GR($\mathscr{G}$)	GR($\mathscr{E}$)	GR($\mathscr{B}$)	...
($\star$)	T	T	?	?	?	F	...
⋮	⋮	⋮	⋮	⋮	⋮	⋮	

Table 2: Results concerning variant theories of general relativity.

By contrasting one variant theory with another, we come to understand 'general relativity' in a more nuanced way. In particular, this pluralistic methodology awakens us to the fact that positions like (†) cannot possibly be settled – if they are to be settled at all – before the requisite work is completed. Just imagine the number of question marks implicit in the table above![12]

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[12]Even if one restricts attention only to 'reasonable' properties consisting of combinations of standard causal and energy conditions, dozens of 'reasonable' variant theories of general relativity can be easily constructed. If non-standard causal conditions are permitted, the number of 'reasonable' variant theories becomes infinite (see Carter 1971). This is to say nothing of the 'unreasonable' alternatives.

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