Against Reducing Newtonian Mass to Kinematical Quantities

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Abstract

It is argued that Newtonian mass cannot be reduced to kinematical quantities—distance, velocity and acceleration—without losing the explanatory and predictive power of Newtonian Gravity.

Could mass be reduced to kinematical quantities—here defined as distances, velocities, accelerations and higher-order time derivatives? This paper considers the case-study of mass as it features in Newtonian Gravity (NG), where we will not distinguish between gravitational and inertial mass, but take them to be equivalent. I am interested in reducing mass *within* NG, rather than reducing NG as a whole to a more fundamental theory and identifying a counterpart of Newtonian mass in that reduced theory. I will argue that, in a sense to be specified below, Newtonian mass is *not* so reducible to kinematical quantities.

One, modest motivation for this intra-theoretical reduction is the desideratum of ontological parsimony. If a specific notion does no work—in some sense to be further specified—at all, Ockham urges us to expunge it from our theory. This is to be distinguished from the much more radical motivation that drives many empiricists: the urge to get rid of all unobservables, regardless of any other virtues they might have, such as explanatory power. It will become important later on that it is merely the more modest motivation—a methodological principle that I believe everyone should apply, regardless of any sympathies for empiricism—that drives the specific project of this paper[1].

[1] Another, more specific motivation, under the general banner of ontological parsimony, is worth mentioning. In the debate between comparativism and absolutism about mass (defined below), the comparativist claims that absolute masses 'do no work' over and above the mass ratios. Hence, it would be ontologically more parsimonious to get rid of them. As argued elsewhere [1], the only comparativist arguments that have some chance at succeeding in fact throw out (or risk throwing out) the massive baby with the bathwater: they eliminate mass altogether, rather than merely the absolute mass scale. It therefore becomes interesting to look at independent arguments against eliminating mass altogether, as is done in this paper.

1 The Project

Why did we ever introduce mass in the first place? We cannot observe (a particle having the property of having) mass as directly as, for instance, the location of a massive object (relative to us). What we do observe however—*pace* Barbour [2]—is that some trajectories (i.e. relative locations over time) do occur in nature, say two celestial bodies approaching each other at ever increasing speed, and other trajectories never occur, say two celestial bodies executing the Argentinian tango. We postulate the primitive notion of mass because it explains why certain patterns are allowed by nature and other patterns are not. Mass therefore becomes indirectly observable. (Or so the standard story goes. It is the aim of this paper to explore whether the same explanatory power can be obtained without primitive masses.) More specifically, if we include the values of the primitive masses in the initial state of our models of NG, and postulate laws that refer to this notion of mass, it turns out that we can find *unique* solutions to the corresponding initial value problems and thereby fix the evolution of the system up to infinity. Including mass in our theory thus allows us to 1) **predict** future states of the world based on past data. But not only that. It turns out that in addition we can 2) **explain** the observed particle trajectories. Why this is the case is most easily illustrated by showing why the reductionist theory is lacking in this respect, as will be done in several places below. In a nutshell, in the mass theory the initial variables and parameters can take on all[2] possible values, whereas the reductionist theory exhibits ad-hoc, holistic, brute (i.e. *unexplained!*) constraints on the initial values (which cannot even be formulated without piggy-backing on the mass theory). Finally, if we range over all the values the initial variables and parameters could take, and solve the initial value problems for each of these cases, we obtain the correct[3] set of empirically possible models. In other words, including mass gives us the correct 3) counterfactuals and other **modal claims** of the Newtonian Theory of Gravity. What more could we want from a physical theory?

The three virtues above provide a rough characterisation of the 'work' that the notion of mass seems to do within our theory despite not being directly observable (or more precisely, as directly observable as relative distances). If we can show that kinematical quantities could do that work all by themselves, then it was never really (i.e. fundamentally) the notion of mass which did all that work (contrary to the standard story above), and we could and should get rid of a primitive notion of mass on grounds of ontological parsimony. More specifically, for our specific reductionist project to succeed, we would need to purge mass from our initial state, leaving only kinematical quantities, and have our laws refer only to those kinematical quantities. If the corresponding initial value problems give the correct, unique solutions, we can 1) *predict* future data

[2]Except perhaps for negative mass values, but these can be made irrelevant if we take it as a fundamental feature of the gravitational law that it is attractive and thus only cares about the magnitudes of the masses. (See Jammer [3, Ch.4] for a historical overview of the search for negative (gravitational) masses.)

[3]In the regime of applicability of the theory.

using past data that is purely kinematical. Provided there are no unexplained constraints on the initial data, we can also 2) *explain* the observed or allowed particle trajectories. Finally, if we then range over all the possible initial kinematical states, and again get the correct set of empirically possible models, the 3) counterfactuals and other *modal claims* are also accounted for without having invoked the notion of mass.

The most obvious way to proceed with this reductionist project seems to be to find an operational definition of mass in terms of kinematical quantities. We can then directly substitute[4] the notion of mass in the initial state and the laws with these kinematical quantities, and our work is done. Mach's famous operational definition of mass immediately springs to mind. Indeed, we will shortly start our discussion with Mach. However, it is important to flag at this point once more that Mach's more radical, empiricist project is substantially different from the more specific project just outlined, as will be discussed in more detail below. Thus, although it would be imprudent not to start off our discussion with Mach's famous operational definition, Mach exegesis is not the aim of this paper; as soon as our project diverges from his, we will leave Mach behind.

Before moving on we need to make more explicit what was already implicit in the previous story: how far do we want to go? One current strand of research considers a moderate version of reducing mass. It considers taking *some* notion of mass to be fundamental, namely the mass determinates—i.e. it is a matter of fact whether two massive particles are equally massive or not—but aims to derive its further quantitative structure—ordering[5], metric[6] and additive structure[7]—from for instance the dynamics[8]. Both the current and the Machian project are interested in reducing mass altogether[9], not merely its quantitative structure. Keeping in mind the desideratum of ontological parsimony, it is simply not clear what work the fundamental mass determinates are still supposed to do once their quantitative structure has already been reduced away.

In the next section I discuss a bad argument against reductionism by McKinsey, Sugar & Suppes, before proceeding to Mach's operational definition of mass in Section 3. Section 4 treats historical responses to Mach, culminating in my main argument against reductionism in Section 4.3. Section 5 responds to two loopholes in the argument. The final section teases out a different line of argument that has been looming in the background, resulting once more in the conclusion that Newtonian mass is not reducible to kinematical quantities without loss of explanatory and predictive power.

[4] Zanstra [4] adopts a similar 'substitution approach' in the analogous debate on relationalism about space.

[5] Whether a massive particle is less or more massive than another particle with a different mass determinate.

[6] The ratio between the masses of two massive particles.

[7] How the mass of one massive particle compares to the combined mass of two other massive particles.

[8] Dees [5] advocates a position like this. Perry [6] considers a more varied range of reductionist projects, including the Machian project.

[9] See also Esfeld & Deckert's project [7] and my paper on regularity comparativism [8].

2 A bad argument against reductionism

Before properly starting our main discussion with Mach in the next section, we will quickly discuss and even more quickly dismiss a famous argument against reducing mass to kinematical quantities. McKinsey, Sugar & Suppes [9] famously provide an axiomatization of Newtonian mechanics in terms of the primitive notions of mass, position and force. They point out that for mass to be reducible it would have to be definable in terms of the other notions. Hence, it should be impossible to find two distinct models of the theory which differ solely with respect to the primitive mass, but not with respect to the other primitives. They then claim to provide two such models: both models contain one particle only, which is at rest at all times, and has zero force acting upon it, but the mass values differ between the models.

As mentioned, we are here not interested in eliminating mass by reducing it to an alternative unobservable, metaphysical primitive such as force. Nevertheless, if an axiomatization of Newtonian Gravity were to be provided in terms of the primitives mass, position, velocity and acceleration, an analogous argument to the one above might be generated. Presumably such a theory would include two models each with one particle at eternal rest, but with different masses.

Both arguments deserve the same response. Although both models are indeed distinct (metaphysical) models of the theory, they fall in the same empirical equivalence class—in both cases the particle is simply at rest and its property of being massive is trivially irrelevant, observationally speaking. In so far as we only care about models to the extent that they get the observables right, this pair of models does not provide an interesting counter-example to reductionism.

A fortiori, the reductionist may insist that McKinsey, Sugar & Suppes' argument backfires, since it seems to in fact work against the mass theory that it acknowledges metaphysical distinctions that are empirically indistinguishable. This violates the Principle of the Identity of Indiscernibles (which is very similar to our principle of ontological parsimony, i.e. Ockham's razor). But this would be an overreaction. It is not at all surprising that a property which is supposed to manifest itself by influencing other particles is empirically idle or dormant in a model where there are no other particles to interact with. What would count against mass is if it were to have no observable consequences in any model of the theory, or at least not in the model that represents the actual world. The pair of models discussed in this section does nothing to support this stronger claim.

3 Mach

Having put aside this argument against reductionism, we turn to the most famous positive attempt to reduce mass. Mach is well-known for providing the first operational definition of inertial mass [10]. He vehemently opposed employing 'hidden' metaphysical notions such as mass in physics in order to *explain* observable phenomena. The task of physics is merely the "abstract quantitative expression of facts" [10, p.502] concerning the relations between observable phenomena. He defines inertial mass (relations) in terms of observable accelerations (or more correctly, acceleration relations) only, a feat that so inspired the logical empiricists.

Consider two particles, which are either alone in the universe or approximately dynamically isolated from any other matter. If F_{12} is the force exerted on particle 1 by particle 2, and F_{21} the force exerted on particle 2 by particle 1, then Newton's third law gives $F_{12} = -F_{21}$. But Newton's second law also gives $F_{12} = m_1 a_{12}$ and $F_{21} = m_2 a_{21}$, where a_{12} denotes the acceleration of particle 1 due to particle 2, and vice versa. Combining this to eliminate the (directly) unobservable notion of force, we obtain[10]:

$$\frac{m_1}{m_2} = -\frac{a_{21}}{a_{12}}. \tag{1}$$

We have operationally defined the mass ratio of these two isolated particles, or simply "named" [10, p.266] their acceleration relations as Mach would put it.

Of course our actual universe does not consist merely of (subsystems of) two isolated particles. Perhaps we could manually approximately isolate such systems (in turns) on the surface of the Earth where we dwell—or, since we can never perfectly isolate any system from gravity, obtain the mass ratios via the limit of a series of better and better isolated two-particle subsystems—but this would not be an option when considering celestial objects [12]. It definitely will not be an option in the generic messy, crowded worlds that we consider in this paper[11]. This raises the important question of whether the previous procedure can be consistently generalised to a larger system of interacting particles. Two issues arise. Firstly, the operational definition depends on the component of the acceleration of a particle that is induced by a *single* other particle, whereas we only have empirical access to the *total* acceleration. In Section 4.1 we will consider whether these individual components can be retrieved from the total acceleration.

[10] Note that in general $\frac{a_{21}}{a_{12}}$ depends on the reference frame, although it will be constant across inertial reference frames. Mach's definition therefore also depends on the first law [3, p.15], which provides the notion of an inertial frame. However, operationalising the inertial frames brings with it its own problems. Pendse [11] proves that there exists an infinite set of special non-inertial frames such that observers at rest with respect to those frames obtain positive and constant values for the mass ratios via Mach's operational definition, which nevertheless differ from the corresponding values found in the inertial frames. Importantly, those observers will not be able to tell that they are not in an isolated, inertial frame. Note that these problems are irrelevant to the project in this paper: we take the initial kinematical quantities *with respect to some inertial frame* to be given, and use them to attempt to calculate the emergent masses and the evolution of the system. The Machian project—"the heuristic aspect" in Pendse's terminology [11, p.55]—on the other hand starts out with observed trajectories only, and needs to somehow reconstruct the *inertial* kinematical quantities before one may derive the mass (ratios).

[11] Here I sympathise with Barbour's warning not to be "misled by the special circumstances of our existence. ... Take a billion of particles and let them swarm in confusion - that is the reality of 'home' almost everywhere in the universe. The stars do seem to swarm... We must master celestial [determination of mass ratios] and not be content with the short cuts that can be taken on the Earth, for they hide the essence of the problem" [2, p.137-8].

The second issue is prior to the first, since it arises even if we could (*per impossibile*, in general) isolate each pair of particles (in turns). Consider a universe with three or more particles. Step 1: take particle 1 and 2 away from the other matter, such that they form an effectively isolated subsystem. Obtain their mass ratio from the acceleration ratio via Mach's protocol. Step 2: repeat for particle 2 and 3. What can we now expect if we repeat this for particle 1 and 3? Will it satisfy the following consistency check:

$$\left(\frac{m_1}{m_3}\right)_{s_3} = \left(\frac{m_1}{m_2}\right)_{s_1} \cdot \left(\frac{m_2}{m_3}\right)_{s_2}, \tag{2}$$

where s_i indicates the instance of the operational procedure (i.e. step) used to determine that mass ratio? If absolute[12] masses are primitive properties of the particles, this condition is satisfied as a matter of *logical neccesity*. In Mach's framework, this equation is satisfied only if

$$\left(-\frac{a_{31}}{a_{13}}\right)_{s_3} = \left(-\frac{a_{21}}{a_{12}}\right)_{s_1} \cdot \left(-\frac{a_{32}}{a_{23}}\right)_{s_2}. \tag{3}$$

But the accelerations which particle 1 and 2 induce in each other and which particle 2 and 3 induce in each other place no (logical) constraints on the accelerations which particle 1 and 3 induce in each other. Mach acknowledges this. For him it is just a *brute empirical fact* that it does not matter which particle we use as a standard to compare every other particle to; any standard will provide the same mass ratios. But this is just another way of saying that the reductionist assumes a highly mysterious and holistic fact without any explanation whatsoever. A fact that is trivially explained—as a matter of logical necessity!—if absolute masses are taken to be fundamental. We here encounter the first loss of explanatory power for the reductionist.

If this empirical fact is nevertheless assumed, one can then proceed by *chosing* one of the particles as the standard unit of mass, say 1kg or 1lb, in order to fix all the other masses via the consistently determined mass ratios. (Note that knowing the absolute accelerations would not by itself help to fix the absolute masses.) Mass thus seems to have been reduced to acceleration relations.

3.1 Mach & Comparativism

Before evaluating Mach's definition *qua* reductionist project, it is worthwhile pointing out that this definition also makes him a comparativist about mass (as opposed to an absolutist). Absolutism about mass is the view that the most fundamental facts about material bodies vis-à-vis their masses are facts about which intrinsic masses they possess. Mass ratios are grounded in those intrinsic masses. Comparativism, on the other hand, is the view that those most fundamental facts are the mass ratios[13] [15]. Absolute masses have no empirical

[12] For comparativism about mass—defined below—an analogous issue of consistency or transitivity arises, as discussed in [13, Ch.3].

[13] Mass relations, more generally. But I will follow Baker [14] in focusing on mass ratios.

meaning; they are merely a convention. Thus, since Mach operationally defines only mass *ratios*, if anything, and arrives at *absolute quantities* merely via a convention, this makes him a comparativist.

Is any justification given for stopping at this point and not continuing to provide a further operational definition of the intrinsic masses? Before responding to this question, it should be pointed out that the absolutist should rejoice in Mach's achievement, as far as it goes. The absolutist acknowledges mass ratios of course, and has never claimed that masses (either the absolute masses or mass ratios) are 'directly observable' (or more correctly, as directly observable as relative distances). If mass were 'directly observable' the whole debate between absolutism and comparativism would not exist in the first place—so the absolutists always admitted the need for a method of measuring those mass relations. And this can of course only be done via (more directly) observable, kinematical notions, such as acceleration. Once we have this operational definition of mass ratios though, do we also need a further operational definition of the absolute mass scale?

Since I have argued elsewhere in detail that Newtonian mass is absolute, let me only briefly rehearse the core of that argument here [1, 13, 14]. Consider a Newtonian world with two equally massive particles a distance r apart, with a relative positive initial velocity v and zero angular momentum. How will this world evolve?

Whereas this description corresponds to a unique choice of initial values and parameters for the comparativist, the absolutist will demand that more information is needed: this description is compatible with uncountably infinitely many intrinsic masses. And, she claims, this choice is important, because for some choices of intrinsic masses the particles will escape each other and for other choices they will collide[14]—two evolutions that are obviously empirically distinct—depending on whether the following inequality is satisfied:

$$v > v_e = \sqrt{\frac{2Gm}{r}}. \tag{4}$$

It is clear from this inequality that the evolution depends on the initial intrinsic masses of the particles, over and above their mass ratios. (Besides, once the initial masses are fixed, the corresponding absolutist initial value problem has a unique solution: absolutism is deterministic[15].) The comparativist initial state lacks the resources to distinguish between these two categories of evolutions, with indeterminism between empirically distinct evolutions as a result. Absolute masses are empirically relevant.

Could we supplement Mach's project with an operational definition of the absolute mass scale? It seems that attempting to do so would not violate the spirit of the original project. The main thrust of the Machian project was the

[14] In the case of non-zero angular momentum, the set of solutions that features coinciding particles is of measure zero. In that case we need to turn to the evolution of shapes/angles to empirically distinguish the models, rather than coincidence.

[15] Modulo some well-known exotic counter-examples [16, 17].

reduction of mass. Mach incorrectly interpreted 'mass' to refer to mass ratios only. If we manage to additionally reduce the absolute mass scale, this would complete the original project of reducing mass (now correctly understood as both mass ratios and an absolute mass scale).

The obvious candidate for such an operational definition is exactly the escape velocity scenario that was used to prove the empirical relevance of absolute masses in the first place. The escape velocity inequality[16] can be reformulated in terms of kinematical quantities only: $v^2 > v_e^2 = 2ar$. This suggests that the absolute mass scale could be defined in terms of some ratio of r, v and a[17]. Although this seems unproblematic for the case of two particles, we will see below (Section 5) that this does not in fact generalise to more particles.

4 Beyond Mach

4.1 Generalising to more particles

Let us now evaluate Mach's project *qua* reductionism. Pendse famously points out that Mach's definition depends crucially on the simple two-particle scenario—which initially seemed like a mere pedagogical simplification—and does not generalise to any number of particles [20]. Mach's definition requires the *separate contributions* induced by every other particle to the acceleration of a specific particle, whereas we only have empirical access to the *total* acceleration of that particular particle. In systems with too many particles the total acceleration underdetermines the individual contributions. More specifically, Pendse argues that, if we use only acceleration relations at one instant, the mass-ratios are not uniquely determined for systems of more than four particles. Moreover, even if we consider acceleration relations at *any* number of instants, systems with more than seven particles will not give a unique set of mass ratios. I will briefly outline the first argument here [3].

Let n be the number of particles. $\mathbf{a}_k$ is the observed, induced total acceleration of the kth body at t_0, and $\hat{\mathbf{u}}_{kj}$ the unit vector in the direction from body k to body j at t_0. Then

$$\mathbf{a}_k = \sum_{j=1}^{n} a_{kj}\hat{\mathbf{u}}_{kj}, \qquad (k = 1, \ldots, n) \tag{5}$$

where we solve for a_{kj} ($a_{kk} = 0$), the $n(n-1)$ unknown coefficients in $3n$ linear equations, which represent the induced acceleration on particle k by particle

[16]This inequailty governs the special case where the mass ratio is one, but this could easily be generalised

[17]We may call this a (spatiotemporally) local operational definition. See Martens [18] for an (unsuccessful) attempt at a global definition, namely a reduction of the intrinsic masses to the full 4D mosaic of particle trajectories (and perhaps their mass ratios). Dasgupta provides an example of an alternative global definition. He introduces a notion of plural grounding, and argues that the totality of kilogram facts is plurally grounded in the totality of mass ratios [19].

j at t_0. It is these coefficients that Mach needs to fix the mass ratios. They are uniquely determined only if their number does not exceed the number of equations, $n(n-1) \leq 3n$, and this is not the case for systems with more than 4 particles. QED. The reader is referred to Pendse's paper for the proof concerning acceleration data at any number of instants.

4.2 Including other kinematical quantities

Narlikar responds by echoing the thought that underlies the suggested operational definition of the absolute mass scale: accelerations might be insufficient, but we have other kinematical notions at our disposal [21]. In particular, we can measure inter-particle distances as well as accelerations, and insert them into the Gravitational Law[18]. Setting Newton's constant to one for convenience, we get the following equation for the (arbitrarily chosen[19]) x-component of the acceleration of particle 1 due to the gravitational interaction of all the other particles, at t_0:

$$a_{1,x}(t=t_0) = \frac{m_2(x_2-x_1)}{r_{12}^3} + \frac{m_3(x_3-x_1)}{r_{13}^3} + ... + \frac{m_n(x_n-x_1)}{r_{1n}^3}, \quad (6)$$

where it is understood that the positions and distances are measured at $t = t_0$ also. These, together with $a_{1,x}$, can be observed, resulting in a linear equation of the form

$$A_{12}m_2 + A_{13}m_3 + ... + A_{1n}m_n = X_1, \quad (7)$$

where only the m's are unknown. Repeating this procedure for a total of $(n-1)$ different instants, we get $(n-1)$ (supposedly)[20] linearly independent equations, allowing us to solve for $m_2, m_3, ...m_n$. Observing in addition a single acceleration-component of any of the other particles at $t = t_0$ only is sufficient to determine the remaining m_1.

4.3 The main argument

It is here that we diverge from Mach's project. Mach's project was of a reconstructive, *descriptive* and epistemological/empiricist nature. It is the project of humans reconstructing (after the fact!) the masses from the appearance of the four-dimensional mosaic generated by God[21]. Therefore, using kinematical

[18] Pendse [22] objects that we do not have independent empirical access to the Gravitational Law. However, in the context of the project in this paper we simply take the laws as given. In fact, Mach and Pendse's own projects take Newton's Laws as given, so why could Narlikar not add the Gravitational Law to this?

[19] The arbitrariness of this choice will be discussed in Section 6.

[20] Although these equations may be linearly independent in general, presumably not all specific instances will be so. What to do with those deviant cases? Perhaps it will turn out that these specific systems are of measure zero in the space of solutions, and that that gives us some reason to ignore them. Or perhaps these cases result in infinitely many solutions which are all empirically equivalent. Or perhaps choosing a different set of instants to measure the distances suffices to restore linear independence. All of this remains to be shown though.

[21] See Martens [18] for a discussion of reconstructing absolute masses using this approach.

data at any number of instants is perfectly acceptable; we are here not in the business of explaining part of the data (the future data) from other parts of the data (the initial state). And this project had better work! We have been applying Newtonian physics succesfully for over three centuries now. We have modeled and predicted the behaviour of the planets in our solar system, based on presumed knowledge of the masses of those planets. Thus, there had better be some response to the potential problems with Narlikar's argument as elaborated upon in footnote 20, unless we want to invoke some error theory[22] about the way we have been doing Newtonian physics for the past three centuries.

In this paper we are however interested in the much more specific, metaphysical project of *explaining* our actual world by deterministically generating it from the initial conditions. That is, we are 'playing God', rather than reconstructing some true, after-the-fact statements about God's creation. Hence, we are only allowed to use kinematical data at the initial time. The future data is part of the explanandum, not the explanans. Using it would be explanatorily circular. The tools used by Narlikar (and by Pendse when proving his second claim) are not available in the context of this project[23].

Does this mean that the reductionist project is doomed? No. We can retain Narlikar's insight—that we have more kinematical data at our disposal than merely accelerations—but restrict ourselves to that additional data *at the initial time only*.

If we could find an operational definition of the masses in terms of the *initial* kinematical notions, then this would guarantee that these initial kinematical notions would suffice (via some law which is obtained by substituting all references to mass by its operational definition) to generate a unique evolution, since this is guaranteed by the initial masses (plus distances positions and relative velocities). As we have seen that initial accelerations are insufficient, we might follow Narlikar's lead by including distances and inserting them into the gravitational law. We start of with his Eq. 6 for the x-component of the acceleration of particle 1 at t_0, but instead of supplementing it with similar equations at different instants, we consider the analogous equations for the other particles *at the same instant*. For instance:

$$a_{2,x}(t=t_0) = \frac{m_1(x_1-x_2)}{r_{21}^3} + \frac{m_3(x_3-x_2)}{r_{23}^3} + \frac{m_4(x_4-x_2)}{r_{24}^3} + ... + \frac{m_n(x_n-x_2)}{r_{2n}^3}. \tag{8}$$

We obtain the matrix equation $G\mathbf{m} = \mathbf{a}$, where G is the following $n \times n$ matrix:

$$G = \begin{pmatrix} 0 & \alpha_{12} & \cdots & a_{1n} \\ \alpha_{21} & 0 & & \vdots \\ \vdots & & \ddots & \vdots \\ \alpha_{n1} & \cdots & \cdots & 0 \end{pmatrix} \tag{9}$$

[22] Beyond of course the obvious errors in the quantum and relativistic regimes.

[23] For similar reasons Schmidt's reduction of mass [23, 3] is disqualified.

where $\alpha_{ij} = \frac{x_j - x_i}{r_{ij}^3}$. Since $\alpha_{ij} = -\alpha_{ji}$[24], G is an antisymmetric matrix. But the determinant of an antisymmetric matrix with odd dimensions is singular! Recall that it is a property of the determinant that $|G| = |G^T|$ and $|-G| = (-1)^n |G|$. For an antisymmetric matrix ($G^T = -G$) these properties combine to give $|G| = |G^T| = |-G| = (-1)^n |G|$. For odd n then $|G| = -|G| = 0$. Since a unique solution requires a non-zero determinant, this proves that there is no unique solution of masses. QED.

4.4 Unpacking the argument

What exactly follows from this? If the determinant had been non-zero, then reductionism would have been straightforwardly successful. It is less straightforward whether the vanishing of the determinant rules out reductionism. A vanishing determinant (for systems with an odd number of particles) proves that *either* there are no solutions *or* there are infinitely many solutions. Given that standard Newtonian Gravity has some solutions, we know that there are at least some sets of initial kinematical quantities that fall into the latter category. Are there any sets that fall into the former?

Horn 1: No solutions

One might think that the following set of initial kinematical quantities does not correspond to any (physical) solution. Consider a simple example of a system consisting of three particles. Figure 1 depicts three collinear particles, with the middle particle being one meter away from each of the outer particles. The middle particle has zero acceleration, and the outer particles each an acceleration of 1 m/s^2 *outwards*. Since gravity is supposed to be attractive, one might think that there are no mass solutions corresponding to this scenario, but there are in fact two categories of (mathematical) solutions: one in which the middle particle has a negative mass and the other two a positive mass, and vice versa. Although there is in fact a solution, this seems 'unphysical', since standard NG with mass includes the postulate that masses are always positive[25]. Standard NG thus does not contain these types of solutions. What should we do with such 'non-physical' solutions? Perhaps the reductionist could respond by claiming that we can somehow throw away these mathematical solutions since they are non-physical. We discuss such moves below. Instead I will now move on to a more decisive example, where there are not even any mathematical solutions.

[24] This is true only because Newtonian Gravity contains both Newton's third law and the principle of equivalence of gravitational and inertial mass—without for instance the latter Eq. 6 would not have been as simple. This seems to suggest that if we were to go beyond Newtonian Gravity by adding other forces which, for instance, do not obey a similar equivalence principle (such as the Coulomb Force), the argument against reductionism would collapse. This cannot be true however, since this would only introduce more unknowns (i.e. the electric charges) without extra 'knowns' to determine those unknowns (unless perhaps the additional force depended on velocity and we could measure the velocities to aid us).

[25] Or, that the gravitational law does not care about the sign of the masses.

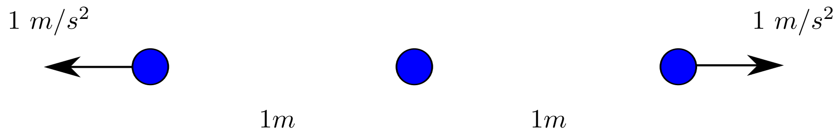

Figure 1: First example of a set of accelerations to which no (positive) mass distribution corresponds.

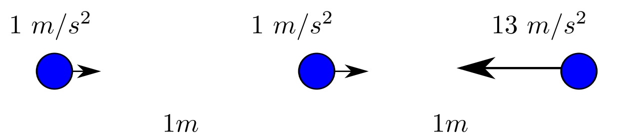

Figure 2: Second example of a set of accelerations to which no mass distribution (either positive or negative) corresponds.

In the second example all accelerations are 'inwards', which seems *prima facie* compatible with the attractive nature of gravity. In Figure 2 the two particles on the left accelerate with 1 m/s^2 to the right, and the third particle accelerates in the opposite direction with 13 m/s^2. It is easy to show that there is no solution in terms of masses, not even negative masses.

Could the reductionist just choose to (*a priori*) rule out those deviant sets of initial kinematical quantities? Especially Humeans about laws of nature might be tempted by this approach. For instance, in the Mill-Ramsey-Lewis Best Systems approach [24, 25] any true statement that is part of the best system to axiomatise the data counts as a law. Thus, if the statement that rules out these deviant sets of initial kinematical quantities is part of the best system, we could just postulate it as a law of our reductionist theory. Compare this to the Humean solution to the problem of the arrow of time: if the Past Hypothesis (i.e. the claim that the initial entropy of the universe was sufficiently low [26]) forms part of the best sytem, this allows us to promote it to the status of law.

Apart from the standard complaints that such statements are not at all the type of beast that we normally consider as a candidate for law-hood, it is important that any such postulated constraint on the initial conditions is neither ad hoc, nor *unexplained*. Moreover, this constraint should be formulatable without (implicitly) referring to masses, that is without piggy-backing on the theory that takes masses to be primitive[26]. There are several reasons to believe that

[26] Pooley [27, Ch.5.4] discusses analogous issues (concerning Sklar's relationalist manoeuvre

these conditions are not satisifed.

Whereas the restriction on entropy was straightforward—the initial entropy had to be below a certain value—the restrictions that would rule out the deviant set of initial kinematical quantities—or more specifically initial accelerations—that do not correspond to any mass solution are much more complicated. In fact, no value of initial acceleration for any individual particle is ruled out from the start; the constraint takes on a holistic form instead. Only if a particle is located 'on the outside', do we all of a sudden require that its acceleration is not directed 'outwards'. (Notice that this also holds for systems with an even number of particles.) Similarly, once the initial accelerations of all but one particle have been chosen, this can restrict the allowed values of the acceleration of the 'final' particle (even if that particle was on the 'inside'). Leaving out the acceleration of a single particle from the initial conditions is not an option since even when we do include this piece of acceleration the mass solutions are already underdetermined in some cases (see below), nor would this solve the former problem regarding outward accelerations. The choices of the initial accelerations of a particle thus depend on the choices for the initial accelerations of the other particles. It is as if the laws determine, after the fact, in a holistic sense, which initial accelerations were allowed in the first place. Namely, exactly those that correspond to initial masses. Inference to the best explanation suggests that that is the case exactly because there are fundamental (initial) masses. There is no non-ad-hoc, reductionist *explanation* for ruling out the deviant sets of initial accelerations, especially not one that does not piggy-back on the concept of mass. In contrast, these constraints are trivially explained (by the attractive nature of gravity) if we do take masses to be primitive.

Horn 2: Infinite solutions

Let us turn to the sets of initial kinematical quantities that correspond to an infinite set of solutions in terms of (initial) masses. These sets underdetermine the masses, and since different masses correspond, in general, to different (metaphysical) evolutions of the system, an initial state that contains only kinematical data leads to an indeterministic evolution (if there is any well-defined evolution in the first place). Such a reductionist theory will not provide the explanatory and predictive power that NG with primitive masses does.

The first, most obvious line of responses consists of variations on the theme that perhaps each set of infinite solutions is similar enough, *in some sense*, to 'count as one' and to therefore *effectively* form a single unique solution.

Variation 1: it might be the case that, even though each of these sets of initial kinematical quantities corresponds to several distinct possible sets of masses each of which lead to *metaphysically distinct* evolutions, these respective distinct evolutions are in fact all *empirically equivalent*. If this is true for each set of initial kinematical quantities that has multiple solutions, this would not only save the reductionist project, but also prove (as was suggested once before, but

of adding primitive accelerations to the initial conditions) in the analogous substantivalism–relationalism debate about space. See also Arntzenius [28, Ch.5.7] on piggy-back relationalism.

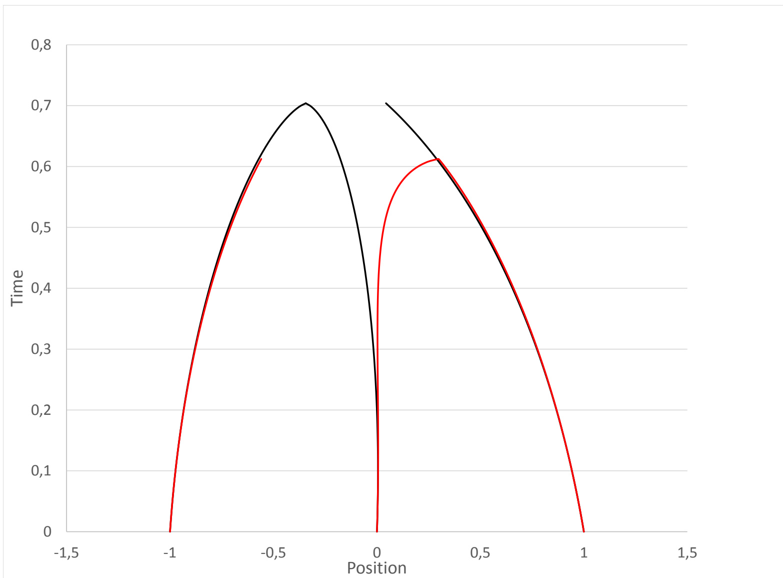

Figure 3: **The Smoking Gun:** A numerical solution of the three-body problem in one dimension (black trajectories), superimposed on an alternatve solution (red trajectories). Each three-body problem has only been solved until the first collision, as the theory breaks down at that point. The initial states of each set of three particles are identical with respect to the kinematical quantities ($G = 1$, $d_{12} = d_{23} = 1$, $v_1 = 0.2$, $v_2 = 0.1$, $v_3 = -0.5$, $a_1 = 1.25$, $a_2 = -1$, $a_3 = -1.5$), but they differ in terms of their masses ($m_1^r = 5.5$, $m_2^r = 0.125$, $m_3^r = 4.5$; $m_1^b = m_2^b = 1.2$, $m_3^b = 0.2$). Note that they do not only differ in their intrinsic masses but also their mass ratios! These different sets of masses generate empirically distinct evolutions! Particle 2 collides first with particle 3 within the red solution, but first with particle 1 within the black solution.

incorrectly, in Section 2) that the mass theory recognises distinct metaphysically possible models that are empirically indistinguishable, which violates the Principle of the Identity of Indiscernibles and Ockham's Razor.

Variation 2: Perhaps each set of infinite solutions consists of solutions that differ only with respect to the intrinsic masses but not with respect to the mass ratios. If so, the reductionist has at least partially succeeded by reducing the mass ratios, if not the absolute masses.

The easiest, most conclusive way to kill both variations with one stone is by providing a single counter-example to both. Figure 3 shows two superimposed numerical solutions of the three-body problem in one dimension. The solutions are generated from initial conditions that agree with respect to the kinematical quantities, but disagree with respect to their initial masses (which in both cases are compatible with the kinematical initial state) and *moreover their mass ratios* (against variation 2). Both solutions clearly generate *empirically distinct* evolutions (against variation 1), since in one case the middle particle collides first with the particle on the left, and in the other case its first collision is with the particle on the right[27]. Moreover, even if the mass ratios had been the same in this example, it would have served to reiterate the point made in Section 3.1 that intrinsic masses make an empirical difference. Thus, it would make salient that under variation 2 the need for fundamental intrinsic masses would remain, which anyway provide the mass ratios for free, thereby making such a partial reduction of the mass ratios good for nothing.

Secondly, the reductionist might suggest that including the 'y' and 'z' components of the acceleration might serve to remove the underdetermination and provide unique mass solutions. We should immediately feel uneasy about this suggestion: when attempting to fix n mass degrees of freedom one would expect to need n acceleration degrees of freedom, not an additional $2n$ more! More on this below (Sections 5 & 6). But even when we do allow ourselves these extra degrees of freedom, this move will not work. The one-dimensional case is still a specific instance of the three-dimensional case. In scenarios were the 'y' and 'z' components of acceleration are zero, all components of acceleration together still underdetermine the masses and thereby the evolution of the system.

Thirdly, the reductionist might bite the bullet and accept indeterminism (at the initial time only). Perhaps there are alternative, reductionist laws which allow for several possible evolutions of the initial kinematical state—one for each of the evolutions that correspond to the mass soultions compatible with that initial kinematical state—but once a specific evolution has 'begun' it follows through, deterministically, until the end. In other words, the laws are indeterministic relative to the initial instantaneous state, but not relative to an initial chunk of the evolution. At this point I can only respond by pointing out that the onus is on the reductionist to provide such indeterministic laws that generate

[27] It might be argued that one cannot compare which particle is left or right of the middle between different solutions. One could avoid this by adding an extra particle suffecently far from these free particles to be dynamically isolated from them, in order to serve as a reference for, say, 'left'. However, the two solutions are clearly not each other's mirror image, so adding an extra reference particle is not really required.

the correct set of empirically possible evolutions[28]. Even if successful, it seems that such an approach would nevertheless weaken the predictive power of the theory.

5 Bits and bobs

Have we ruled out that mass can be reduced to kinematical quantities? At least two issues need to be dealt with before we can conclude so.

The main argument rests on the substantive premise that the number of particles n is odd. This may not be true of the actual world. Especially Humeans about laws of nature might jump on this loophole, and just take the statement that n is even to be part of the best reductionist system, which justifies promoting it to the status of a law, thereby avoiding my main argument. However, first of all, it just seems that such a statement is not at all the kind of statement that is a candidate for being a law—why would it be nomologically necessary that n is even? Secondly, it could well be false of the actual world that n is even. Thirdly, even if n just happens to be even in the actual world, the reductionist still has to prove that $G\mathbf{m} = \mathbf{a}$ is solvable (where G is given by Eq. 9). The attractive nature of gravity is enough to show that even in those worlds there will be initial kinematical conditions that do not correspond to any set of positive[29] masses, namely those where the particles 'on the outside' have an acceleration that points away from all the other matter. Fourthly, assuming a non-revisionary reductionist—as suggested in Section 1—who wants to reproduce all the consequences of and the work being done by the standard form of NG (i.e. with primitive masses), the reductionist theory needs to generate all the empirically possible models of standard NG. This includes models with an odd number of particles, even if none of those represents the actual world. Finally and most importantly, the main argument still goes through for quasi-isolated subsystems of an odd number of particles. Thus, even if our universe consisted of an even number of particles, there will (probably) still be solar systems with an odd number of celestial objects. (It would have been nice for my purposes if that were true of our own solar system—ignoring asteroids etc.—but alas!)

Let us now turn to the last cluster of related issues. The focus in this paper has mainly been on accelerations. Have we ruled out a reduction of mass to any type or combination of types of kinematical quantities, or only a reduction to accelerations (and distances)? We gain some insight into this question when we return to the issue of operationally defining the absolute mass scale (Section 3.1; assuming we would have been able to fix the mass ratios). Earlier I suggested

[28] Dasgputa [29] is developing an analogue of this project in response to the accusation that relationalism about handedness, space and mass are all indeterministic. That case seems much simpler though than the case considered in this paper.

[29] Although, for e.g. a system with four masses on the vertices of a square, all with accelerations of equal magnitude pointing outwards along the diagonals, there is one unique solution if we were to allow negative masses. (It consists of masses of equal magnitude (the exact value depending on the acceleration magnitude), but the masses on one diagonal have a negative sign, whereas the masses on the other diagonal have a positive sign.)

that we could perhaps use the escape velocity scenario for this purpose. The escape velocity inequality obeyed by that scenario can be rewritten in terms of r, v and a only, suggesting that we define the absolute mass scale via some ratio of r, v and a. However, Figure 3 has not only proven that mass ratios cannot be reduced to accelerations (and distances), it also proves that the absolute mass scale cannot be defined in terms of r, v and a once we have more than two particles. For in that figure not only the initial distances and accelerations of the two superimposed solutions agreed, but also the initial velocities. Thus, an initial kinematical state containing kinematical quantities up till second order fails to solve the reductionist project.

Could we include higher-order kinematical quantities? Since these cannot be analytically determined from NG with primitive masses, it is difficult to answer this question[30], but since this would mean adding even more 'degrees of freedom' this does not seem to be a viable option (see also below). We are trying to reduce n mass degrees of freedom to more than n kinematical quantities. These extra quantities cannot be truly degrees of freedom; they cannot be *independent* of the n degrees of freedom. Either they 1) will lead to inconsistencies in the determination of the masses, or 2) they will always conspire to take on exactly the right values as to avoid inconsistencies. Such a mysterious, conspiratorial constraint—which presumably cannot even be formulated without referring to mass[31]—would be totally *unexplained*, even if imposing this constraint on the initial kinematical state would uniquely fix the evolution.

6 An additional argument against reductionism

As a little bonus, let us bring two earlier strands together, which inspire an additional argument against the reductionist project. Strand 1: Pendse approached the reductionist project as a matter of *counting degrees of freedom*. This aspect returned when we considered using the additional $2n$ degrees of freedom of the 'y' and 'z' components of the accelerations to remove the underdetermination of the masses by the 'x' components of the accelerations (and the distances) (Section 4.4). Strand 2: when considering ruling out deviant sets of initial conditions that did not correspond to any mass solutions, we realised the

[30] Perhaps the following serves as a plausibility argument for an upper bound on the order k of initial kinematical data that would guarantee removing the underdetermination of mass (although the overdetermination problem, resulting in conspiratorial (i.e. *unexplained!*) constraints, still remains). On one popular view, the "at-at" theory of motion, (initial) velocities are not in fact properties of an (initial) instant, but of an infinitesimal (initial) period of time. After all, velocity is usually defined as $\lim_{dt\to 0} \frac{r(t+dt)-r(t)}{dt}$, which is a property intrinsic to $[t, t+dt]$. In general, the initial k^{th}-order time derivative of r is a property of $[t_0, t_0+kdt]$. In a slogan: 'God was not done when he created the initial configuration and the laws, but he had to also specify the subsequent $k-1$ configurations (depending on the order of initial kinematical data that we are considering)'. Now, if $k = n+1$, this initial period (of $n+1$ instants) effectively contains $n-1$ independent sets of accelerations (and even more sets of distances). Narlikar's method then guarantees that this initial data fixes the masses.

[31] See fn. 26.

holistic and *conspiratorial* nature of the constraint on the allowed sets of initial kinematical quantities (Section 4.4; see also the end of Section 5).

On reflection, it is quite strange that we were trying to reduce the n degrees of freedom of mass, a scalar, to acceleration, which—as a vector—has $3n$ degrees of freedom, in the first place. We implicitly tried to avoid this awkwardness by only using one part of the acceleration degrees of freedom, say the 'x' components—cf. the *one-dimensional* solutions in Figure 3. However, especially in the homogeneous Euclidean space in which Newtonian Gravity lives (*pace* Knox [30]), it is arbitrary to use only one component of this vector quantity. Even if we were to do so it would be even more arbitrary to determine exactly which component we should use. We seem to have implicitly chosen some preferred axis, in a homogeneous space which has no structure to ground such a notion.

Should we then have used all components of acceleration instead? We already mentioned that this, despite *prima facie* seeming to actually make the reductionist project easier—surely more initial kinematical data will help to further pinpoint the corresponding initial masses and remove the underdetermination—it actually is of no use: one-dimensional examples of underdetermination are just specific cases of three-dimensional examples of underdetermination. In fact, adding these $2n$ degrees of freedom makes things worse. We expected that, if the reductionist project had worked at all (contrary to the conclusion of this paper), it would have fixed the n mass degrees of freedom via some set of n kinematical degrees of freedom (plus the distances and velocities which were needed additionally in the mass theory as well). If that had worked, the additional $2n$ overdetermining degrees of freedom would either 1) have lead to inconsistencies, or they would 2) have to always take on exactly the right values to not lead to any inconsistencies. But this latter situation would be extremely conspiratorial and unjustified[32]—as before in Sections 4.4 and 5. Except of course for the mass theorist, who can trivially explain why the 'y' and 'z' component of the accelerations always line up in a specific way 'depending' on the 'x' components.

Summarising, we should have been worried about reducing mass—a scalar—to acceleration—a vector, from the start!

7 Conclusion

It has been argued that Newtonian mass cannot be reduced to kinematical quantities—distance, velocity and acceleration—without losing the explanatory and predictive power of Newtonian Gravity.

[32]See again fn. 26.

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