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# Classical Logic

Preliminary Edition

Maria L. Seykora Immaculata University



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# Chapter 1

# Arguments

# Introduction

Think arguments are just a bunch of hot-headed quarrels? Not in Logic! Whether we realize it or not, we encounter Logic arguments in real life. In this chapter, section 1.1, you'll learn how Logic manifests itself in day-to-day life through everyday arguments, and what some types of everyday arguments are. Then, in section 1.2 you'll see how arguments are broken down into their components, including the main (and probably familiar) components of premise(s) and conclusion, and how these components relate to each other and individual arguments. Following this section, we'll discuss in section 1.3 the difference between arguments and another form of communication or word-grouping seen in day-to-day life: non arguments. Intrigued by the appearance of sufficient and necessary conditions in some non-arguments, we'll delve deeper into what such conditions are, and where they may appear elsewhere in Logic, in section 1.4.

# **Objectives**

- Section 1.1: Increase awareness of everyday arguments, what they are, and their characteristics.
- Section 1.2: Identify the components of arguments.
- Section 1.2: Analyze how the components of arguments relate to each other and to the argument as a whole.
- Section 1.3: Distinguish between arguments and non-arguments in real life.
- Section 1.4: Understand what sufficient and necessary conditions are.

# **Section 1.1: Everyday Arguments**

Section objective: Increase your awareness of everyday arguments, what they are, and what characterizes them.

#### **Key Terms**

The following key terms will be introduced in this chapter.

Logic: the study of arguments

Argument: a group of statements in which one or more of these statements claims to prove that another one of these is true

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**Logic**, broadly defined, is the study of arguments. In general, an **argument** is a group of statements, or sentences, in which one or more of these statements claims to prove that another one of these is true. There must be some claim somewhere *that* something is true, as opposed to explaining why something is true. An argument claims that something is true, because the statement being claimed as true is not necessarily universally established. If some fact is universally established, then it would not need to be proven true, but it may need to be explained. Notice the term "claim." A claim may be successful or not, implying that an argument may be successful in claiming that something is true, or it may not be successful in claiming that something is true. What follows is a point to be discussed furthermore in section 1.2: if an argument's claim is successful, then it is a good argument, but if its claim is not successful, then it is a bad argument. Arguments in real life may be found, though not exclusively found, in newspapers, magazines, journals, on advertisements and billboards, websites, in conversations, the classroom, the workplace, and on television.

You probably have seen arguments in some form already in your life. Here are some simple examples of arguments:

You should get a Twitter! Everyone in the sorority has one.

Chipotle is the best fast food restaurant because they use real food ingredients.

According to the weather report, it is going to rain this afternoon, so we should probably reschedule the party.

In these arguments above, the speaker is trying to convince you to accept the main point in bold. The speaker is using the information contained in the rest of the argument to try to prove the point in bold to be true.

If you are someone who is interested in sports science and medicine, or science in general, you may have come across scientific abstracts similar to the following excerpt of an abstract:

Sports nutrition supplements have previously been reported to contain undeclared doping substances. The use of such supplements can lead to general health risks and may give rise to unintentional doping violations in elite sports. To assess the prevalence of doping substances in a range of high-risk sports nutrition supplements available from Dutch web [shops, a] total of 66 sports nutrition supplements – identified as potentially high-risk products claiming to modulate hormone regulation, stimulate muscle mass gain, increase fat loss, and/or boost energy – were selected from 21 different brands and purchased from 17 web shops. All products were analyzed for doping substances by the UK life sciences testing company LGC, formerly known as the Laboratory of the Government Chemist, using an extended version of their ISO17025 accredited nutritional supplement screen. A total of 25 out of the 66 products (38%) contained undeclared doping substances...Based upon the recommended dose and the potential variability of analyte concentration, the ingestion of some products identified within this study could pose a significant risk of unintentional doping violations. In addition to inadvertent doping risks, the prescribed use of 3 products (4.5%) could likely impose general health risks.1 – Journal of Sports Science and Medicine (2021) 20, 328 – 338

Intuitively, we can tell that an argument is being made here. The authors are trying to prove a point, namely the fragments highlighted in bold above: "the ingestion of some products identified within this study could pose a significant risk of unintentional doping violations...the prescribed use of 3 products (4.5%) could likely impose general health risks." You can tell that they are trying to convince you that some high-risk supplements likely contain undeclared doping substances and could be a health hazard, because the other statements support what is in bold. The first statement, "sports nutrition supplements have previously

<sup>&</sup>lt;sup>1</sup> Erik Duiven, Luc J.C. van Loon, Laila Spruijt, Willem Koert, Olivier M. de Hon. (2021) Undeclared Doping Substances are Highly Prevalent in Commercial Sports Nutrition Supplements. *Journal of Sports Science and Medicine (20)*, 328 - 338. https://doi.org/10.52082/jssm.2021.328

been reported to contain undeclared doping substances," indicates a problem that motivates the study. The statements in bold address the problem of sports nutritional supplements containing doping substances in this first statement, and say something concerning the risks of taking them. The second statement, "the use of such supplements can lead to general health risks and may give rise to unintentional doping violations in elite sports," presents the bipartite hypothesis that the statements in bold confirm. The third and fourth statements describe the method used to come to the interpretations of the statements in bold. The fifth statement gives the results of the experiment relevant to supporting the same bolded statements. The phrase "based upon the recommended dose and the potential variability of analyte concentration" segues from these four statements to the bolded ones, in light of further background information, and the phrase "in addition to inadvertent doping risks," transitions between the two parts of the main point, or the confirmation of the bipartite hypothesis presented earlier.

Another example of an argument, posted on the American Dental Association's website, is as follows:

Every dental practice has opportunities to increase production. **COVID-19 has worked against practice production** for several reasons, including the reduced volume of patients that can be seen, a percentage of patients that still haven't returned to the practice and an increased number of no-shows that have a directly proportional effect on the use of available chair time.<sup>2</sup> – "Beyond COVID-19: 3 Ways to Boost Practice Production" by Roger P. Levin, D.D.S., *success.ada.org* 

Here, the author is trying to prove the bolded statement, namely that "COVID-19 has worked against practice production." The reasons in the second statement are centered on the bolded one, which is also the main point of the above passage. The author claims to show that COVID-19 has worked against dental practice production by citing these reasons. There has been a reduced volume of patients that can be seen, probably mostly due to at-home restrictions. A reduced volume would limit the number of patients a dental practice could work with. There has also been a percentage of patients that still haven't returned to the practice, perhaps likewise due to restrictions or maybe due to fears of COVID-19. This factor would similarly limit the number of patients the practice could work with. The practice could work with. The third reason mentioned is the increased number of no-shows who have scheduled appointments but do not show up, probably also due to complications associated with COVID-19. No-shows directly limit available chair time, because the appointments aren't canceled and the chair time remains reserved. Such unused chair time is a waste when it comes to dental practice productivity.

The advertisement industry frequently presents arguments to us in our daily lives. Here is an example:

5G speed. A14 Bionic, the fastest chip in a smartphone. An edge-to-edge OLED display. Ceramic shield with four times better drop performance. And Night mode on every camera. **iPhone has it all --- in two perfect sizes**.<sup>3</sup> —iPhone 12 and iPhone 12 mini advertisement, apple.com

Against a background of competing products, the main point being proven true here is that "iPhone has it all—in two perfect sizes." The statements claiming to support this point are that the iPhone 12 and iPhone 12 mini have 5G speed, that they have A14 Bionic, the fastest chip in a smartphone, that they have an edge-to-edge OLED display, that they have ceramic shield with four times better drop performance, and that they have Night mode on every camera. The idea is that these facts taken together are claiming to show

 $<sup>^2\</sup> https://success.ada.org/en/practice-management/dental-practice-success/dps-spring-2021/beyond-covid19-ways-to-boost-practice-production$ 

<sup>&</sup>lt;sup>3</sup> https://www.apple.com/iphone-12/

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that the iPhone 12 has everything you could ever want in a smartphone, and so are trying to convince you to buy it.

Those were some examples of everyday arguments. Now, let's see if you can identify the main point and supporting statements of the following arguments, and describe them.

[This is where the bullet point summary and section 1.1 exercises will be inserted.]

# Section 1.2: Arguments and Their Components

Section objectives:

- Identify the components of arguments.
- Analyze how the components of arguments relate to each other and to the argument as a whole.

#### **Key Terms**

The following key terms will be introduced in this chapter.

Premise: a statement in an argument that claims to give evidence or a reason

**Conclusion:** a statement in an argument that is claimed to be proven true by the evidence or reasons in the premises

**Good argument:** a group of statements in which the premises objectively succeed in proving the conclusion to be true.

**Bad argument:** a group of statements in which the premises do not objectively succeed in proving the conclusion to be true

Simple argument: an argument with one conclusion

**Complex argument:** an argument with more than one conclusion, usually divided into sub-conclusions and the final conclusion

Inference: the observed reasoning process of an argument

**Statement:** a declarative sentence, or a sentence component that can stand alone as a declarative sentence, that can be either true or false.

Declarative sentence: a type of sentence that claims to say something about the actual world

Fact: a state of affairs or event that obtains in the actual world

Proposition: the meaning of a statement that has a truth value

Meaning: the things that a statement refers to and/or the sense of all the word in a statement taken together

Truth value: an attribute assigned to a proposition or statement that is either true or false

Factual claim: a stipulation in an argument that at least one of the statements must claim to give evidence or a reason

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**Inferential claim:** a stipulation in an argument that there must be some claim that some statement follows from alleged evidence, or a reason or reasons.

Premise indicator: a word that provides a clue in identifying a premise

Conclusion indicator: a word that provides a clue in identifying a conclusion

In section 1.1, we discussed what an argument was, namely that it is a group of statements in which one or more of these statements claims to prove that another one of these is true. However, what exactly are statements? In the previous section, they were similar to sentences. Specifically, a **statement** is a **declarative sentence**, or a sentence component that can stand alone as a declarative sentence, that can be either true or false. Declarative sentences differ from interrogative, imperative, and exclamatory ones, as also from proposals and suggestions. For example, consider the following:

The clock is two minutes fast. (Declarative sentence)

If it starts snowing, then where can we buy a snow blower? (Interrogative sentence)

Feed the cat at 6 A.M. (Imperative sentence)

How beautiful is the view of the mountains! (Exclamatory sentence)

Let's wash the car. (Proposal)

I suggest that you eat more vegetables. (Suggestion)

Declarative sentences, such as the ones above, differ from the rest in that they claim to say something about the actual world, which in general is the reality we experience with the five senses. The declarative sentence "the clock is two minutes fast" claims to say something about the actual world related to the time on the relevant clock compared with the actual time. The other sentences don't say something about the actual world, at least directly. The interrogative sentence above, and others like it, may reflect a lack of knowledge regarding the actual world. In the sentence above, the place where we can buy a snow blower is unknown. The imperative sentence above, and others like it, may be addressed only to someone in the actual world, and not saying something about it. Similarly, the exclamatory sentence above, and others like it, expresses some form of emotion and doesn't necessarily give a description of the actual world. The proposal and suggestion above, and others like them, are expressions that hold between people, and do not necessarily say something about the actual world.

The following is also a declarative sentence:

Because June washed the car, in case the father would be upset, Pete instead walked the dog, and Cynthia watered the flowers.

However, what makes this declarative sentence unique is that it also consists of four sentence components that could stand alone as declarative sentences, namely:

- (1) June washed the car,
- (2) The father would be upset,
- (3) Pete walked the dog,
- (4) Cynthia watered the flowers.

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Thus, (1) - (4) above are also statements, as is the entire statement not broken down into those components. Notice that the words "because," "in case," and "and" are excluded. When identifying standalone declarative sentence components, the conjunctions, including, but not limited to, "because," "in case," "and," "therefore," "but," "yet," etc., are excluded.

Statements are also syntactical objects or objects of languages. Propositions are slightly different from statements, although the term "proposition" can be used interchangeably with the term "statement." Propositions are somewhat more abstract than statements, as we will see. In general, propositions are things that incorporate meaning. Propositions ride the middle ground between our own subjective mental states and objective reality. We can think of propositions and they can be about things.

Specifically, **propositions** are the meanings of statements that have a truth-value. Keep in mind that a statement's **meaning** includes both the things its words refer to and the sense of all the words taken together. For example, the meaning of the statement

Andrea M. Ghex, Roger Penrose, and Reinhard Genzel won the 2020 Nobel Prize in Physics.

can include the three persons themselves, the action of winning in the past, the 2020 Nobel Prize in Physics, and also the general sense of these things taken together. The truth-values are either TRUE (T) or FALSE (F). A proposition has the truth-value TRUE (T) if it is a fact. A proposition has the truth-value FALSE (F) if it is not a fact. Today, generally what facts are is common knowledge, but a formalized definition of **fact** could be "a state of affairs or event that obtains in the actual world." Some facts include:

Fleas can jump a horizontal distance of 20 cm.

There is only one English letter (Q) that does not appear in any U.S. state's name.

No number before 1,000 contains the letter "A."

Statements express propositions, and therefore they express either their truth or falsity, but not both. Statements can be true or false, and not both, precisely because they contain propositions, which are the bearers of truth-values. If a proposition has the truth-value TRUE (T), meaning it is true, then the statement which expresses it is likewise true. If a proposition has the truth-value FALSE (F), meaning it is false, then the statement which expresses it is likewise false.

Here is a way of seeing how statements are distinguished from propositions:

- (a) Dromedary camels have one hump. (T)
- (b) Los camellos dromedarios tienen una joroba. (T)
- (c) Les chameaux dromadaires ont une bosse. (T)

Statement b, "Los camellos dromedarios tienen una joroba.," is Spanish for statement a, "Dromedary camels have one hump.," and statement c, "Les chameaux dromadaires ont une bosse.," is French for statement a. Each of these statements mean the same thing, namely that Dromedary camels have one hump, and each is true (T) it is a fact that Dromedary camels have one hump, yet they each have a different grammatical syntax. Statement a operates according to English grammar rules, statement b according to Spanish grammar rules, and statement c according to French grammar rules. Because statements are syntactical objects, and a, b, and c operate according to different grammar rules, a, b, and c are different statements, yet, in a more abstract sense, they are the same proposition, because each means the same thing and propositions are the meanings of statements. When we look at a-c, we see three distinct statements, but in our minds we see the single meaning or proposition that they each refer to.

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An argument must have a claim that some statement is being proved to be true, but what exactly is meant here by claiming to prove something true? In order for there to be a claim to prove something, there must be these two claims:

- (1) Factual claim
- (2) Inferential claim.

The **factual claim** stipulates that at least one of the statements must claim to give evidence or a reason. A statement that claims to give evidence or a reason is called a **premise**. So, put another way, the factual claim stipulates that there must be at least one premise. Note that it is not necessary for the premise or premises to actually be factual or true themselves, because deciding whether or not a premise is true is, strictly speaking, outside the domain of Logic. All that is necessary is that the statement claim to give evidence or a reason. Here are two ways to think of the factual claim:

FACTUAL CLAIM = THE PREMISE(S),

and equivalently,

FACTUAL CLAIM = STATEMENT(S) CLAIMING TO GIVE EVIDENCE OR REASON(S).

The **inferential claim** stipulates that there must be some claim that some statement follows from alleged evidence, or a reason or reasons. The statement that follows from this alleged evidence or reasons is called the **conclusion**. It is the statement in an argument that is claimed to be proven true by the evidence or reasons in the premises. Note that the inferential claim is not equivalent to the conclusion. Rather, it is the objective reasoning process from premise(s) to conclusion. It has nothing to do with the arguer's subjective intentions, but is instead derived from the argument's structure. In other words, the inferential claim expresses the inference. The **inference** is the reasoning process of an argument observed by an outsider, or the claim that the conclusion follows from the premise(s). You can think of a person taking a bird's eye view of how the argument proceeds. Sometimes, the term "inference" can be used interchangeably with the term "argument." Additionally, an inference can be distinguished from an implication. An implication has to do only with the statements themselves, whereas an inference has to do with what people reason from the statements themselves. *Statements imply, but people infer*. Here are some other ways to formulate the inferential claim, each one expanding upon the previous:

INFERENTIAL CLAIM = THE INFERENCE,

equivalently,

#### INFERENTIAL CLAIM = THE ARGUMENT'S REASONING PROCESS,

equivalently,

# INFERENTIAL CLAIM = THE CLAIM THAT THE CONCLUSION FOLLOWS FROM THE FACTUAL CLAIM

and equivalently,

INFERENTIAL CLAIM = THE CLAIM THAT THE CONCLUSION FOLLOWS FROM THE PREMISE(S) (OR THE STATEMENTS CLAIMING TO GIVE EVIDENCE OR REASONS)

In order to analyze an argument in context, we have to be able to distinguish between the argument's premises and conclusions. Premise indicators and conclusion indicators help us do this. A **premise indicator** 

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is a word that provides a clue in identifying a premise. A **conclusion indicator** is a word that provides a clue in identifying a conclusion. Premise indicators include, but are not limited to,

since; as indicated by; because; for; in that; may be inferred from; as; given that; seeing that; for the reason that; in as much as; owing to .....

Note that "since" does not have the temporal meaning of "from the time that." Conclusion indicators include, but are not limited to,

therefore; wherefore; thus; consequently; we may infer; accordingly; we may conclude; it must be that; for this reason; so; entails that; hence; it follows that; implies that; as a result .....

The main point here is that premise indicators and conclusion indicators are each types of conjunctions, but they play different roles. Premise indicators are conjunctions that lead us to reason or evidence. Conclusion indicators are conjunctions that lead us from reasons or evidence to a further point. They are distinct from and not included in the premise statements and the conclusions statements themselves.

If there are no indicators in the argument passage that you are analyzing, then you may have to use context clues to figure out which statements are the premises and which is the conclusion. Ask yourself at least one of the following questions to determine the conclusion:

- (1) What single statement is claimed to follow from the others?
- (2) What is the arguer trying to prove?
- (3) What is the main point in the passage?

After identifying the conclusion, use process of elimination to determine what the premises are. The premises will normally be the remaining statements, as long as those statements claim to support the conclusion. If a statement does not claim this, then it is neither a premise nor a conclusion.

Here is a sample argument, with the statements labeled (i) - (vi):

As I stated yesterday morning, in my opening statement, (i) this is really a very simple case. (ii) The defendant burglarized Ms. Jackson's house, (iii) she saw him do it, and (iv) he confessed to the crime. (v) It is really that simple. (vi) We would ask for a verdict of guilty. – Sample Mock Trial Closing Argument For a Burglary Case, *grahamdefense.com* 

There are no premise or conclusion indicators here, so we have to look at context clues and ask ourselves the three questions above. The main point of this passage is asking for a guilty verdict at (vi). The obvious points of contention were covered: the defendant burglarized Ms. Jackson's house, she saw him do it, and he confessed that he did it. Furthermore, in context, the point of a lawyer's argument is a verdict of either guilty or not guilty. The other statements claim to support the conclusion, so by process of elimination (i) - (v) are each the premises. The factual claim just is the premises, so we label (i) - (v) as the factual claim(s) as well, either collectively or individually. The inferential claim is implicit, because there are no indicators. We have to be careful in identifying the inferential claim because it is not exactly the same as the conclusion. In order to capture the reasoning process, we could insert a conclusion indicator such as "therefore" in front of (vi), giving us "Therefore, we would ask for a verdict of guilty" for the implicit inferential claim. There may be other ways to identify the inferential claim, but the key point to remember is that it is the argument's reasoning process from premises to conclusion, and, strictly speaking, not identical with the conclusion itself. Keep in mind that the inferential claim can be explicitly introduced by a conclusion indicator, or it can be implicitly there without any conclusion indicator.

Arguments can be either simple or complex. A **simple argument** is an argument with one conclusion. A **complex argument** is an argument with more than one conclusion, usually divided into subconclusions and a final conclusion. The previous argument is a simple argument with only one conclusion (vi) and premises (i) – (v). Here is an example of a complex argument:

Consider the following standard anti-abortion argument: (i) Fetuses are both human and alive. (ii) Humans have the right to life. Therefore, (iii) fetuses have the right to life. Of course, (iv) women have the right to control their own bodies, but (v) the right to life overrides the right of a woman to control her own body. Therefore, (vi) abortion is wrong. - *Ethics: History, Theory, and Contemporary Issues* by S. Cahn and P. Markie (2012)

(iii) and (vi) are both statements that are conclusions. They are both preceded by the conclusion indicator "therefore," and they are both points that the author is trying to prove and are claimed to follow from other statements. (iii) is claimed to follow from (i) and (ii), and (vi) is claimed to follow from (iii), (iv) and (v). Answering questions (1) - (3) is also a way to distinguish between sub-conclusions and a final conclusion. Ask yourself which conclusion follows from the other(s), which one(s) prove the other, and which one is the main point in comparison with the other(s). In the case above, (vi) follows from (iii). (iii), (iv), and (v) combine to claim to show (vi), because these premises claim that a fetus' right to life overrides a woman's right to control her body in an abortion. The factual claim for a complex argument, as opposed to a simple argument as described previously in this section, can be thought of as follows:

FACTUAL CLAIM (COMPLEX ARGUMENT) = THE PREMISE(S) + SUB-CONCLUSION(S),

meaning that in the anti-abortion argument above, the factual claim is (i) - (v). The inferential claim could be "therefore, abortion is wrong," because the conclusion indicator word "therefore" helps to capture the reasoning process transitioning from the premises to the conclusion. Here, the inferential claim could be explicitly introduced by the conclusion indicator, and does not have to be implicit.

A central goal in Logic is to determine whether an argument is good or bad. What follows concerning good and bad arguments are points that will be discussed in more depth in chapters 2, 3, and 4, but here is what good and bad arguments are in general.

Arguments may be good or bad ones, depending on whether or not they actually succeed in what they claim to do. A **good argument** succeeds in what it claims to do. What does it mean for an argument to succeed in what it claims to do? If the one or more statements (premises) in the argument objectively succeed in proving that another one (conclusion) of these statements is true, then the argument succeeds in what it claims to do. These statements succeed in proving that the conclusion is true if they both fit together and each of the statement's propositions is true, or a fact. What the statements "fitting together" mean relates to validity, strength, and the inference to the best explanation, to be discussed in 2.1, 2.2, 3.1, and 4.2. Intuitively, we can tell that the following is a simple, good argument:

(i) Colorado is a state. (ii) All states are areas with boundaries. Therefore, (iii) Colorado is an area with a boundary.

The premises (i) and (ii) intuitively fit together to give the conclusion (iii), and the propositions (i) and (ii) are facts.

On the contrary, a **bad argument** does not succeed in what it claims to do. If no statements in the argument (premises) objectively succeed in proving that another (conclusion) is true, then the argument does not succeed in what it claims to do. In other words, either the statements that are premises do not fit together, or at least one of the statement's propositions is false (not a fact), or both. Here is an example of a bad argument:

A rose is a flower. (ii) A cactus is a flower. That means that (iii) a rose is a cactus.

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The premises (i) and (ii) don't fit together; just because two things are the same type of things doesn't mean that they themselves are the same thing or identical. Furthermore, the proposition (ii) is not a fact. Cacti are plants that may have flowers, but they are not flowers.

The final point for this section is putting arguments into their proper premise-conclusion format. Doing so is important because it can make it easier to analyze the arguments, and it shows that you know on a basic level how the argument is constructed. To do so, follow these steps.

- (1) Identify the conclusion if it is a simple argument, or the sub-conclusion(s) and final conclusion if it is a complex argument.
- (2) Identify the premise(s) by process of elimination.
- (3) List the premises in the order that they occur in the argument, and label the premises P1, P2, P3,...etc.
- (4) If a simple argument, list the conclusion last and label it C. If a complex argument, fill in the sub-conclusion(s) where they fit in between the premises and label them SC1, SC2, SC3,...etc., and then list the final conclusion last and label it FC.

Keep in mind that the premises always come before the conclusion or final conclusion, and that the premises, sub-conclusions, conclusion, and final conclusion must always be statements. You may have to rephrase some statements or sentences depending upon the context. You also must remove any premise indicators or conclusion indicators, because they are not included in the premises or conclusions themselves. Finally, if any statements or phrases do not claim to support the conclusion, then they must be excluded as well, because they are neither premises nor conclusions.

Here is the proper premise-conclusion reconstruction of the simple mock trial argument previously mentioned in this section.

P1: This is really a very simple case.

- P2: The defendant burglarized Ms. Jackson's house.
- P3: She saw him do it.
- P4: He confessed to the crime.
- P5: It is really that simple.
- C: We would ask for a verdict of guilty.

Notice that the phrase "as I stated yesterday morning, in my opening statement," although it is not an indicator, does not claim to support the conclusion because it is a reference to a certain time, so it must be excluded. Here is the proper premise-conclusion reconstruction of the previously discussed complex argument.

P1: Fetuses are both human and alive.P2: Humans have the right to life.SC1: Fetuses have the right to life.P4: Women have the right to control their own bodies.P5: The right to life overrides the right of a woman to control her own body.FC: Abortion is wrong.

Note that "of course" and "but" are premise indicators, and the two "therefores" are conclusion indicators, so they must be removed. The statement "consider the following standard anti-abortion argument" is a form of introduction, so it does not claim to support the conclusion and must be excluded.

[This is where the bullet point summary and section 1.2 exercises will be inserted.]

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# Section 1.3: Non-arguments

Section objective: Distinguish between arguments and non-arguments in real life.

#### Key Terms

The following key terms will be introduced in this chapter.

Non-argument: a passage in which there is no claim that anything is being proven true

Warning: a type of non-argument that alerts someone to some danger

Advice: a type of non-argument that makes a recommendation for the future

Beliefs or opinions: types of non-arguments that express what someone believes or thinks

Report: a type of non-argument that gives information about a topic or event

**Loosely associated statements:** a type of non-argument in which statements are about the same general subject without a claim to prove something

**Expository passage:** a type of non-argument that starts with a topic sentence developed by the subsequent sentence or sentences

Illustration: a type of non-argument that is an expository passage with one or more examples

Argument from example: an argument that looks like an illustration but isn't

**Explanation:** a type of non-argument that explains or sheds light on why an accepted fact is the case

Conditional statement: a type of non-argument that is an "if...then..." statement

Accepted fact: a claim that everyone, at least in the intended audience, agrees with

Explanans: the statement or statements that explains why the accepted fact is true in an explanation

Explanandum: the accepted fact to be explained by the explanans in an explanation

In section 1.1, we defined an argument as a group of statements in which one or more of these statements claims to prove that another one of these is true. In section 1.2, an argument, in addition to the factual claim, must have an inferential claim somewhere claiming to prove some conclusion true. **Non-arguments**, by contrast, are passages in which there is no claim that anything is being proven true. The different types of non-arguments consist of various expressions and groups of statements: warnings, advice, beliefs, opinions, loosely associated statements, reports, expository passages, illustrations, explanations, and conditional statements.

**Warnings** are a type of non-argument that alerts someone to some danger. They do not try to prove that there is some danger, but instead make the reader or listener aware of a danger. Here are some examples:

Don't put your hands too close to the stove.

If you walk across the street, beware of the dog.

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Watch out for COVID-19 symptoms, such as fever or chills, shortness of breath, nausea, body aches, congestion, diarrhea, and new loss of taste or smell. – Symptoms of COVID-19, *cdc.gov* 

Advice is a type of non-argument that makes some recommendation for the future or in general, such as the following:

I recommend that you take care of your mental health.

Use adversity as an opportunity. - inc.com

Use these medications according to the label instructions or as recommended by your doctor. Be careful to avoid taking too much. High doses or long-term use of acetaminophen or ibuprofen may cause liver or kidney damage, and acute overdoses can be fatal. If your child's fever remains high after a dose, don't give more medication; call your doctor instead. – *mayoclinic.org* 

**Beliefs or opinions** are types of non-arguments that express what someone believes or thinks. They may involve the words "think," "believe," and "in my opinion". Consider, for example:

I think that abortion should be banned in this state.

We believe that animal rights should be protected in the realm of scientific research.

I just can't understand why someone would listen to that loud music. It is tasteless, in my opinion.

**Loosely associated statements** are a type of non-argument in which statements are about the same general subject without a claim to prove something. They tend to resemble trains of thought having no clear point, as in the following literary excerpt:

Suppose that communal kitchen years to come perhaps. All trotting down with porringers and tommycans to be filled. Devour contents in the street. John Howard Parnell example the provost of Trinity every mother's son don't talk of your provosts and provost of Trinity women and children cabmen priests parsons fieldmarshals archbishops. From Ailesbury road, Clyde road, artisans' dwellings, north Dublin union, lord mayor in his gingerbread coach, old queen in a bathchair. My plate's empty. After you with our incorporated drinkingcup. Like sir Philip Crampton's fountain. – Excerpt from James Joyce's *Ulysses*, *berfrois.com* 

**Reports** are a type of non-argument that gives information about a topic or event, frequently found in newspapers:

Firefighters are working in extreme heat to contain a number of wildfires raging across the US west, with the largest burning in California and Oregon, as another heatwave bakes the region and puts strain on power grids. The Beckwourth Complex, which is the largest wildfire of the year in California, was raging along the Nevada state line and has burned about 140 square miles (362 sq km) as of Monday morning. State regulators have asked consumers to voluntarily "conserve as much electricity as possible" to avoid any outages starting in the afternoon. – "Wildfires blaze across western states as heatwave shatters records," *The Guardian* online

An **expository passage** is a type of non-argument that starts with a topic sentence developed by the subsequent sentence or sentences. Although these types of non-arguments can be objective, they aim to expand upon, elaborate, and develop the topic sentence, not to prove that it is true. Consider the following:

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(i)Humans have brought a wide range of animals into domestic partnerships over the past 11,000 years---as livestock, working animals, household pets, and companions. (ii) The pathways that different animal species followed into domestication are remarkably varied, shaped by the different biological constraints and opportunities of the animals brought into domestication, as well as by the different cultural contexts of their human partners. (iii) It is a journey that continues today as humans, with enhanced understanding of the process of domestication and increasingly sophisticated technology for breeding and rearing captive animals, bring an ever-expanding array of animal species, on land and sea, into domestication... - "The Domestication of Animals" (2012) by Melinda A. Zeder, *jstor.org* 

The topic sentence (i) is about humans domesticating different types of animals over the past 11,000 years, and this is expanded upon by (ii), which describes the different paths of domestication that animals took and how they were affected by various factors. (iii) expands upon (ii) further which discusses the paths of domestication that modern humans use.

Some passages look like expository passages because they have a topic sentence with subsequent sentences, but are really arguments because the subsequent sentences try to prove the topic sentence. For example:

To begin with, (i) I recognize that it is impossible that God should ever deceive me. For (ii) in every case of trickery or deception some imperfection is to be found; and (iii) although the ability to deceive appears to be an indication of cleverness or power, the will to deceive is undoubtedly evidence of malice or weakness, and so (iv) cannot apply to God. —Excerpt from Descartes' *Fourth Meditation*, English translation

The topic sentence (i) is the conclusion and main point of this passage. Just as with an expository passage it is listed first among the sentences, but (ii), (iii), and (iv) try to prove it instead merely developing or describing it. Because trickery and deception involves imperfection [(ii)], the will to deceive is evidence of malice or weakness [(iii)], and these things can't apply to God [(iv)], God cannot deceive him. How can we tell that what looks like an expository passage is really an expository passage, or an argument? The key difference between this example and the previous one is that the topic sentence for the previous one is an **accepted fact**, also known as a **claim** that everyone, at least in the intended audience, agrees with, but the topic sentence for the example above is not. What counts as an accepted fact can vary depending upon the intended audience. For example, the statement,

Time and space are relative and dependent, so they must be treated together.<sup>4</sup>

is an accepted fact for an audience that adopts modern physics, but is not an accepted fact for one that adopts classical physics. (In classical physics, time and space can be treated separately because they are considered to be independent of each other and absolute in their own right.) Because it is rarely the case that there is some statement that absolutely everyone agrees is true and, additionally, is objectively true in itself, the intended audience needs to be taken into account. If a passage is taken out of the context of its intended audience, then one may have to make a hypothetical statement saying "if we assume that the intended audience is X, then this statement is an accepted fact," or maybe even "if we assume that the intended audience is Y, then this statement is not an accepted fact," for example. If you are unsure if the statement is an accepted fact or not, you could make another type of hypothetical statement, stating "if this is an accepted fact, then this passage is not an argument," or "if this is not an accepted fact, then this passage is an argument."

Returning to the previous two examples, the claim "humans have brought a wide range of animals into domestic partnerships over the past 11,000 years---as livestock, working animals, household pets, and

<sup>&</sup>lt;sup>4</sup> Chen, T. and Chen, Z. (2016) A Bridge Connecting Classical Physics and Modern Physics. *Journal of Modern Physics*, *7*, 1378-1387. doi: 10.4236/jmp.2016.711125.

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companions" would be an accepted fact for the public in general, assuming that the dating of 11,000 years is agreed upon in general. Pets and farm animals have probably existed throughout recorded history. This accepted fact does not need to be proven true, but can be elaborated. On the contrary, the claim "I recognize that it is impossible that God should ever deceive me" is not an accepted fact, in particular for atheists and for persons who hold different views concerning God's ability to deceive. Descartes tries to prove this statement true with premises (ii) – (iii) and sub-conclusion (iv).

An **illustration** is a type of non-argument that is an expository passage with one or more examples. It is similar to an expository passage in that it is an expression providing meaning, exposition, and/or explanation, but, unlike in an expository passage, it always includes example(s). Its topic sentence may or may not be the first sentence, and this sentence will normally occur before the listed example(s). Consider:

(a) A landlocked country is one that does not touch any of the oceans. For example, Nepal, Mongolia, Andorra, Austria, Switzerland, Kazakhstan, Botswana, and Zimbabwe are landlocked countries.

These countries are listed as concrete examples to give the reader or listener an idea of what it means to be a landlocked country. An illustration may also include premise and/or conclusion indicators, even though it is a non-argument. Example:

(b) A recursive acronym is one that refers back to itself. Thus, MOM's refers to "Mom's Organic Market," VISA refers to "Visa International Service Association," ATI refers to "ATI Technologies Inc.," and CAVE refers to "Cave Automatic Virtual Environment."

Here, the conclusion indicator "thus" leads into the examples of recursive acronyms. The presence of this indicator does not imply that this passage is an argument.

Just as we saw was the case with expository passages, illustrations can be confused with arguments. An argument that looks like an illustration, but isn't, is called an **argument from example**. Example:

(c) Not all of philosophy is imprecise and lacking in rigor, in comparison with mathematics, physics, and other sciences. For example, philosophy of mathematics and advanced logic employs rigorous proofs, and some thought experiments in ethics can be tested for verification or falsification in real life.

The difference between this argument from example and the previous illustrations is that the statement (c) is not an accepted fact, whereas the statements (a) and (b) are. (c) is debated but (a) and (b) are each true by definition. A landlocked country is defined as one that does not touch any of the oceans, and a recursive acronym is defined as one that refers back to itself. Because they are true by definition, they are a claim that everyone agrees with. Arguments from example do not contain accepted facts, whereas illustrations do.

An **explanation** is a type of non-argument that explains or sheds light on why an accepted fact is true. Unlike arguments, they do not claim to prove or show *that* something is true. The goal in an argument is to establish something to be true, whereas in an explanation the goal is to elucidate something that has already been accepted as true. An explanation is composed of an explanans and explanandum, whereas an argument is composed of premises and conclusion. The **explanans** is the statement(s) that explains why the accepted fact is true in an explanation. The **explanandum** is the accepted fact to be explained by the explanans in an explanation. In an explanation, the accepted fact is explained by the explanans, but in an argument, if any of the premises are also accepted facts, then these accepted facts claim to prove the conclusion. Just as with expository passages and illustrations, the way to distinguish between arguments and explanations is to look and see if an accepted fact is being explained, or something is claiming to be proven true. If the former situation, it is an explanation; the latter, it is an argument. Here is another way of thinking about explanations:

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Explanation = explanans + explanandum

Explanans = the statement(s) that does the explaining of the accepted fact

Explanandum = accepted fact

The accepted facts in an explanation are not statements that are true by definition, but instead tend to be statements about events or scientific phenomena. Examples:

(i) On July 4, 1776, the Declaration of Independence was signed, because (ii) the Thirteen Colonies wanted to become thirteen independent states instead of being ruled by Great Britain.

(iii)The Earth's atmosphere is composed of nitrogen because, (iv) according to some scientists, the air in the Earth's atmosphere could have reacted with volcanic gases.

(i) and (iii) above are the accepted facts, or explanandums. (i) is about an event, and is explained by the explanans (ii). (iii) is a scientific phenomenon, and explained by the explanans (iv). (i) is a historical fact and (iii) is a scientific fact, but neither involves a definition. Notice the premise indicators "because" are there. Just as with illustrations, the presence of premise or conclusion indicators does not imply that it is an argument.



Figure 1.1. Expository Passages vs. Illustrations vs. Explanations.

A conditional statement is a type of non-argument that is an "if...then..." statement, such as:

If (a) the party across the street is too loud, then (b) we may have to call the police.

If (a) Nina is a bachelorette, then (b) Nina is unmarried.

If (a) the Miami Dolphins win the Super Bowl, then (b) we will be ecstatic.

For each statement, the component statement following the "if" (what's labeled "a") or before an "only if" is the antecedent. The component statement following the implicit or explicit "then" (what's

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labeled "b"), an "only if," or occurring before an "if" is the consequent. Each conditional statement consists of an antecedent and a consequent. The antecedent may occur before the consequent, as in these examples:

If Fifi is a dog, then Fifi is an animal.

If Fifi is a dog, Fifi is an animal.

Fifi is a dog only if Fifi is an animal.

The consequent may also occur before the antecedent as in the following example:

Fifi is an animal if Fifi is a dog.

Conditional statements, such as the ones above, may look like arguments, but they are in fact not arguments. A conditional statement could be the premise or conclusion of an argument, and it could be reformed as an argument. For instance, we could reform the first conditional statement above as:

Because the party across the street is too loud, we may have to call the police.

In this case, it would be an argument and no longer a conditional statement. Despite these similarities, conditional statements are not arguments.

[This is where the bullet point summary and section 1.3 exercises will be inserted.]

# Section 1.4: Sufficient and Necessary Conditions

Section objective: Understand what sufficient and necessary conditions are.

#### **Key Terms**

The following key terms will be introduced in this chapter.

Sufficient condition: a member of a set of a necessary condition

Necessary condition: a set of which a sufficient condition is a member

**Set:** a thing or a group of things

Member: a thing or group of things in a set

In the previous section, we discussed conditionals statements, which are non-arguments that are "if...then..." statements. Conditional statements contain both a sufficient condition and a necessary condition. A sufficient condition is a member of a set of a necessary condition. A necessary condition is a set of which a sufficient condition is a member. A set is a thing or group of things, whereas a member is a thing or group of things in a set. There are more ways to conceive of necessary and sufficient conditions than in terms of sets, however. A sufficient condition gives enough information for the matter at hand, whereas a necessary condition gives the required information. The occurrence of the sufficient condition is all that is needed for the occurrence of the necessary condition, and the sufficient condition cannot occur without the occurrence of the necessary condition. The sufficient condition cannot exist without the

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necessary condition, and we can infer the existence of the necessary condition from the existence of the sufficient one. The truth of the sufficient condition is guaranteed by the truth of the necessary one, and we can infer the truth of the necessary one from the truth of the sufficient one. However, we cannot infer the existence or truth of the sufficient one from the existence or truth of the necessary one, and the absence of the sufficient condition does not imply the absence of the necessary one.



Figure 1.2. Sufficient and Necessary Conditions Diagram.

The sets above are those of animals and platypuses. They each are a group of things. Platypuses are a member of the set of animals, and Birrarung and Eve are each a member of the set of platypuses. Being a platypus is a sufficient condition for being an animal because platypuses are members of the set of animals. Every platypus is necessarily an animal. Being an animal is a necessary condition for being a platypus, because animals are the set of which platypuses are members. Platypuses are a kind of animal. Additionally, being a platypus is a necessary condition for being either Birrarung or Eve. Birrarung and Eve just are particular platypuses in the San Diego Zoo. Being Birrarung or being Eve is also a sufficient condition for being a platypus. They are real-life platypuses and thus members of the set of platypuses.

There are four ways in which conditional statements can express sufficient and necessary conditions:

If A, then B. If A, B. A only if B. B, if A.

Here, A is the sufficient condition, and B is the necessary condition. This means that the four ways in which conditional statements can express the sufficient and necessary conditions in the diagram are:

If something is a platypus, then that thing is an animal.

If something is a platypus, that thing is an animal.

Something is a platypus only if that thing is an animal.

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Something is an animal, if that thing is a platypus.

Similar expressions follow for the cases of Eve, Birrarung, and platypuses. Sufficient and necessary conditions can also be expressed in other ways.

Given that Roberta is in Alaska, it follows that she is in North America.

Jared being a member of a crew team is a necessary condition for him earning a collegiate crew scholarship.

The sufficient condition "Roberta is in Alaska" follows the words "given that" and the necessary condition "she is in North America" follows the words "it follows that." Likewise, in the second statement Jared being a member of a crew team is the necessary condition for him earning a collegiate crew scholarship, which is the sufficient condition following the words "is a necessary condition for." You may also encounter in real life the following phrases indicating the presence of a sufficient or necessary condition.

Additional phrases that indicate that a sufficient condition follows are: in case, in case that, provided that, on condition that, is a necessary condition for.

Additional phrases that indicate that a necessary condition follows are: implies, implies that, is a sufficient condition for.

You may also encounter faulty examples of necessary and sufficient conditions, such as the following:

If Sam is a human being, then Sam is a boy.

Provided that the bank closes at 4pm, the sun will set.

Sam being a boy cannot be a necessary condition for Sam being a human being, and Sam being a human being is not a sufficient condition for Sam being a boy, because Sam could be a grown woman, man, a girl, baby, pre-teen, teen, etc. Similarly, the bank closing at 4pm cannot be a sufficient condition for the sun setting in the future, nor can the sun setting in the future be a necessary condition for the bank closing at 4pm, because the sun's setting is independent of the bank's schedule and is not caused by it. Barring an extraordinary event, the sun will set anyways, no matter what time the bank closes.

[This is where the bullet point summary and section 1.4 exercises will be inserted.]

[This is where a chapter 1 cumulative practice test will be inserted.]

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# Chapter 2

# Deduction

# Introduction

Have you ever heard of deduction or deductive reasoning? Perhaps you have heard of deduction or deductive reasoning in science....the type of reasoning in which you start out with a general hypothesis, examine the possibilities, and then reach a specific relevant conclusion. Well, deduction in logic is somewhat different. In section 2.1, you'll learn what deduction in logic is, and what the related concepts of validity and invalidity are. In section 2.2, you'll get into what the layout of possible and impossible worlds are, a key feature of validity and invalidity. In section 2.3, after going over the theoretical concepts behind deduction, you'll learn what some concrete and real-life deductive argument forms are. In real life, we care about whether the premises are true or not. To top it off, in section 2.4, you'll learn about factoring in the truth or falsity of the premises into deductive arguments for sound and unsound arguments, and will learn how to distinguish between them.

# **Objectives**

- Section 2.1: Understand the concepts of validity and invalidity.
- Section 2.1: Know what deductive arguments are, how arguments are defined in terms of sets, and how validity and invalidity relate to deductive arguments.
- Section 2.2: Understand the layout of the sets of all possible worlds and of all impossible worlds.
- Section 2.3: Know what some deductive argument forms are, and what real life examples of each are.
- Section 2.4: Distinguish between sound and unsound arguments.

# Section 2.1: Validity and Invalidity

Section Objectives:

- Understand the concepts of validity and invalidity.
- Know what deductive arguments are, how arguments are defined in terms of sets, and how validity and invalidity relate to deductive arguments.

#### Key Terms

Deduction: the set of all deductive arguments as a whole

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**Deductive argument:** each and every thing or member of the set of deduction which involves the claim that the conclusion follows from the premises by necessity

Argument (Sets): a set of at least two statements, where at least one member is a premise and at least one member is a conclusion

Factual Claim (Sets): for a simple argument, the subset consisting of all the statements that are premises, and for a complex argument, the subset consisting of all the statements that are either premises or sub-conclusions.

 $\subseteq$ : the symbol meaning "is a subset of"

 $\subset$ : the symbol that means that the set on the left is a proper subset of the one on the right, and included within but not identical with it

 $\supseteq$ : the symbol that means "is a superset of"

 $\supset$ : the symbol that means that the set on the left is a proper superset of the one on the right, and encompassing but not identical with it

**Inferential Claim (Sets):** the claim that the conclusion follows from the subset consisting of all the statements that are either premises or sub-conclusions, assuming these are true in all possible worlds

Validity (General): the property of an argument whereby the conclusion of that argument is true in all possible worlds, on the assumption that the premises/sub-conclusions are true in all possible worlds

Possible World (General): a way the actual world is, could have been, or could be

**Deductive Argument Indicator Words:** words such as "necessarily, absolutely, certainly, definitely, it must be the case that" that are often present in deductive arguments and show their claim of validity

Validity (For a Successful Deductive Argument): the property such that the inferential claim ( $\beta$ ) is true

**Informal Circles and Dots Test:** a type of test where you draw circles and dots within and outside of each other in order to determine if an argument is valid or invalid

**Invalidity (For an Unsuccessful Deductive Argument:** the property such that the inferential claim ( $\beta$ ) is false

Deduction is one class of arguments. Namely, **deduction** is the set of all deductive arguments as a whole. It is the class of all deductive arguments in general. The members of this set consist in deductive arguments. Each and every thing or member of the set of deduction is a **deductive argument**.

Now that we've defined sets in 1.4, we can define arguments in terms of sets. In terms of sets, an **argument** is a set of at least two statements, where at least one member is a premise and at least one member is a conclusion. More members may be premises, and if the argument is a complex argument, then more members may be sub-conclusions. More formally, an argument is:

A set  $\Gamma$  of at least two statements, call  $\varphi 1$  and  $\psi$ , where at least one member  $\varphi 1$  is a premise and at least one member  $\psi$  is a conclusion. There may be more members  $\varphi 2$ ,  $\varphi 3$ ,  $\varphi 4$ , . . .

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etc., that are premises, and if the argument is a complex argument, then more members  $\alpha 1$ ,  $\alpha 2$ ,  $\alpha 3$ , . . . etc., may be sub-conclusions.<sup>5</sup>

Returning to 1.2, an argument must include both a factual claim and an inferential claim. In terms of sets, for a simple argument, then, the **factual claim** is the subset consisting of all the statements that are premises. For a complex argument, the factual claim is the subset consisting of all the statements that are either premises or sub-conclusions. More formally, a factual claim in a simple argument is:

The subset  $\Delta \subseteq \Gamma$  such that all premises { $\varphi$ 1,  $\varphi$ 2,  $\varphi$ 3,  $\varphi$ 4, ...,  $\varphi$ n}  $\subseteq \Delta$ .

More formally, a factual claim in a complex argument is:

The subset  $\Delta' \subseteq \Gamma'$  such that all premises { $\varphi 1, \varphi 2, \varphi 3, \varphi 4, \ldots, \varphi n$ }  $\subseteq \Delta'$  and all subconclusions { $\alpha 1, \alpha 2, \alpha 3, \ldots, \alpha n$ }  $\subseteq \Delta'$ .

Notice that the symbol " $\subseteq$ " has been introduced. It is the symbol for "is a subset of" which corresponds to the definition of "subset" in section 1.4. Returning to 1.4, a subset is a type of set which is either identical to or included within another set. More formally,

 $\Delta \subseteq \Gamma$  if every member of  $\Delta$  is also a member of  $\Gamma$ .<sup>6</sup>

If every member of  $\Delta$  is also a member of  $\Gamma$ , then  $\Delta$  is either identical to or included within  $\Gamma$ , depending upon how many total members there are in  $\Gamma$ . If there is at least one member of  $\Gamma$  not in  $\Delta$ , then we could use a different symbol " $\subset$ " for " $\Delta \subset \Gamma$ " to show that  $\Delta$  is a proper subset of  $\Gamma$ , and included within  $\Gamma$  but not identical with it.7

There is also a symbol that corresponds to the definition of "superset" in 1.4, and means "is a superset of." It is the same as the symbol for "subset," but pointing in the reverse direction. It is the symbol " $\supseteq$ ." Returning to 1.4, a superset is a type of set which is either identical to or encompassing another set. More formally,

 $\supseteq \Gamma$  if every member of  $\Gamma$  is also a member of  $\Delta$ .

If every member of  $\Gamma$  is also a member of  $\Delta$ , then  $\Delta$  is either identical to it, or larger than it and encompassing it, depending upon how many total members there are in  $\Delta$ . If there is at least one member of  $\Delta$  that is not in  $\Gamma$ , then we could use a different symbol " $\supset$ " for " $\Delta \supset \Gamma$ " to show that  $\Delta$  is a proper superset of  $\Gamma$ , and encompassing  $\Gamma$  but not identical with it.

Notice, additionally, that the brackets "{}" have been introduced in the more formal definition of a factual claim previously described. In addition to being represented by the capitalized ancient Greek letters  $\Gamma$ ,  $\Delta$ , etc. as previously described, sets can be represented with their members enumerated between the curly brackets "{}," as the set of all premises and sub-conclusions in the different more formalized definitions of factual claims are.<sup>8</sup>

Returning again to 1.2, an argument must include an inferential claim. To repeat, the **inferential claim** is the argument's reasoning process. Equivalently, it is the claim that the conclusion follows from the factual claim. Equivalently, it is the claim that the conclusion follows from the subset consisting of all

<sup>&</sup>lt;sup>5</sup> This definition of arguments is adapted from the draft of Sara L. Uckelman's textbook *What is Logic?* cf.

https://sluckelman.webspace.durham.ac.uk/whatislogic/ .

 $<sup>^{6}</sup>$  Cf. https://sluckelman.webspace.durham.ac.uk/whatislogic/ .

<sup>&</sup>lt;sup>7</sup> See footnote 2.

<sup>&</sup>lt;sup>8</sup> This notation is adapted from the draft of Sara L. Uckelman's textbook *What is Logic?* cf. https://sluckelman.webspace.durham.ac.uk/whatislogic/ .

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the statements that are either premises or sub-conclusions, assuming these are true in all **possible worlds**, or from either  $\Delta$  or  $\Delta$ ', assuming that each of their members is true in all possible worlds, in the more formalized definitions of factual claims:

 $\Delta$  (or  $\Delta$ ')  $\longrightarrow \psi$ , assuming { $\varphi$ 1,  $\varphi$ 2,  $\varphi$ 3,  $\varphi$ 4, . . .,  $\varphi$ n}  $\subseteq \Delta$ ,  $\Delta$ ' is true.

With this definition of the inferential claim in mind, in general, the arguer can claim that the conclusion "follows from" the factual claim in one of two different ways:

- (1) in all possible worlds,
- (2) NOT in all possible worlds.

If the arguer claims that the conclusion follows from the factual claim (1) in all possible worlds, then the arguer is claiming that the conclusion follows by necessity. All arguments involving this type of claim are deductive arguments, and vice versa, whether or not they are good or bad ones. Good versus bad deductive arguments are discussed in section 2.3. In the other case, the arguer may claim that the conclusion follows from the premises by probability or by an inference to the best explanation. Arguments with these types of inferential claims may be inductive arguments or abductive arguments, which are discussed in chapters 3 and 4.

If the arguer claims that the conclusion follows from the premises by necessity, then the conclusion follows necessarily in all possible worlds. The idea of the conclusion following from the premises in all possible worlds leads into the definition of **validity**:

The property  $\beta$  of an argument whereby the conclusion of that argument is true in all possible worlds, on the assumption that both  $\Delta$  and  $\Delta$ ' are true in all possible worlds.

- $\beta$  (validity) thus consists in the conjunction of two components:
- (a) The assumption that the premises/sub-conclusions of argument  $\Gamma$  are true in all possible worlds,
- (b) The conclusion of  $\Gamma$  being true in all possible worlds as a result of (a).

In general, a possible world is a way the actual world is, could have been, or could be, but the definition of a possible world and possible worlds theory basics will be discussed more in section 2.2.

Here are more ways to conceptualize, in other words, the property of validity ( $\beta$ ):

- There cannot be a possible world, nor can you think of one, where the premises/subconclusions are true and the conclusion is false.
- It is impossible for the conclusion to be false given that the premises are true.
- The truth of the conclusion is completely guaranteed.
- The conclusion follows absolutely given that the premises are true.
- The conclusion follows with strict necessity given that the premises are true.
- The conclusion follows certainly given that the premises are true.
- The conclusion follows definitely given that the premises are true.
- It must be the case that the conclusion follows given that the premises are true.

Note the presence of the **deductive indicator words** above (necessarily, absolutely, certainly, definitely, it must be the case that) show the claim of validity, and are often present in deductive arguments.

Note that, because of the component (a), it does not matter for  $\beta$  whether or not the premises are true or false. Whether or not the premises are true or false matters for soundness, instead, which is discussed in section 2.3. Thus, for deductive arguments, no matter if they are good or bad ones, or whether or not each of the premises or sub-conclusions is true or not, all claim to have  $\beta$ , the property defined above.

Here are some examples of deductive arguments claiming to have  $\beta$  (validity).

 $\phi1\text{:}$  Zhi Ruo's birthday is in one of the months in English that begins with the letter "J."

 $\psi$ : Zhi Ruo's birthday is in either January, June, or July.

In this example, the imagined arguer claims this argument has validity because it is claimed that (a) assuming that  $\varphi 1$  is true in all possible worlds, (b)  $\psi$  is true in all possible worlds as a result. The options for Zhi Ruo's birthday being in only one of those three months are claimed to be the only options, given the condition that it has to be in one of the months in English that begins with the letter "J." In order for the argument to work, the arguer claims that these options are jointly exhaustive as a result of assuming that the premise is. (The claim of validity here happens to be a successful claim).

φ1: All logicians are scholars.

φ2: All biologists are scholars.

 $\psi$ : All logicians are biologists.

In this example, validity is claimed in order for this argument to work. Namely, it is claimed that (a) assuming that  $\varphi 1$  and  $\varphi 2$  are true in all possible worlds, (b)  $\psi$  is true in all possible worlds as a result. It is claimed that assuming that all logicians and all biologists are scholars, all logicians are biologists. (The claim of validity here happens to be an unsuccessful claim).

It is one thing for the arguer to claim that an argument has validity, but whether or not the argument itself has objective validity is another issue. In other words, an arguer may claim that the conclusion of an argument follows from the premises by necessity, but the arguer's claim does not imply that this same claim is successful. This claim may be either successful or not. If this claim is successful, then there is validity. A deductive argument is valid if and only if:

 $\beta$  (validity) is true9

OR

its inferential claim is true (this is another way of putting the former).

Here are some examples of valid deductive arguments. A deductive argument that has validity can have true premises/sub-conclusions and a true conclusion, such as the following:

•  $\varphi$ 1: If Garfield is a cat, then Garfield is an animal. (True)

 $\varphi$ 2: Garfield is a cat. (True)

<sup>&</sup>lt;sup>9</sup> Cf. section 1.2: a proposition has the truth-value TRUE (T) [or is true] if it is a fact. A proposition has the truth-value FALSE (F) [or is false] if it is not a fact. This same definition applies here.  $\beta$  is true if it is a fact.

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α1: Garfield is an animal. (True)

 $\varphi$ 3: If Garfield is an animal, then Garfield is a living thing. (True)

 $\psi$ : Garfield is a living thing. (True)

false premises/sub-conclusions and a false conclusion, such as the following,

φ1: All turtles are cheetahs. (False)

φ2: Some shark is a turtle. (False)

 $\psi$ : Some shark is a cheetah. (False)

and false premises/sub-conclusions and a true conclusion, such as the following:

 $\varphi$ 1: If the grass is green, then pigs can fly. (False)

φ2: The grass is green. (True)

α1: Pigs can fly. (False)

 $\varphi$ 3: If pigs have wings, then pigs cannot fly. (False)

 $\psi$ : Pigs do not have wings. (True)

BUT not true premises and a false conclusion, by definition. The first example would also be a good deductive argument, or a sound argument, and the others would not, but soundness and unsoundness is discussed more in section 2.4.  $\beta$  is true in these three examples because assuming that the premises/sub-conclusions for each argument are true in all possible worlds, you cannot think of a possible world, nor is there a possible world, where their respective conclusions are false. Try to think of a possible world where the conclusion is false for yourself. This impossibility can be confirmed by drawing **informal circles and dots** to test for validity. This informal circles and dots test is one where you draw circles and dots within and outside of each other in order to determine if an argument is valid or invalid. Let's look at the second example.  $\varphi 1$  and  $\varphi 2$  are assumed true in all possible worlds, even though they are each false. If we draw informal circles for  $\varphi 1$  we get:



φ1: All turtles are cheetahs.Figure 2.1 Informal Circles Diagram for φ1

Again, assuming  $\varphi 1$  is true in all possible worlds, we know that the circle of turtles must at least be within the circle of cheetahs, even if they are not equivalent. If we draw in  $\varphi 2$  on top of  $\varphi 1$ , we get:



 $\phi$ 1: All turtles are cheetahs. plus  $\phi$ 2: Some shark is a turtle.

#### Figure 2.2 Informal Circles Diagram for $\phi 1$ plus $\phi 2$

Assuming it is true that some shark is a turtle, we represent this premise and that one shark at least by putting a dot labeled "Shark" within the "Turtles" circle somewhere. Now, we look at the conclusion  $\psi$ : Some shark is a cheetah. If we were to represent this with an informal diagram, we would place a dot labeled "Shark" within the "Cheetahs" circle. There already is a dot labeled "Shark" within the "Cheetahs" circle in the diagram for  $\varphi 1$  and  $\varphi 2$  without us having to do anything in addition. So, the conclusion  $\psi$  will always be represented in the diagram of  $\varphi 1$  and  $\varphi 2$  no matter what, telling us that  $\psi$  will be true in all possible worlds assuming that  $\varphi 1$  and  $\varphi 2$  are. Thus, there is validity and this argument is valid. A similar method can be used to show that the other two examples are valid arguments, and I leave doing so as an exercise for the student and reader. The "if...then..." statements in these examples can be converted to diagrams of sufficient and necessary conditions, which are discussed and exemplified in section 1.4.

Returning to the arguer's claim, if this claim is unsuccessful, then there is **invalidity**. A deductive argument is invalid if and only if:

 $\beta$  (validity) is false 10

OR

its inferential claim is false (this is another way of putting the former).

As you can see, invalidity is the opposite of validity. Regarding deductive arguments, there is no middle ground between valid and invalid. Each and every deductive argument is either valid or invalid. For the invalid ones, assuming the premises are true in all possible worlds, the conclusion is not true in all possible worlds. There could be a possible world, or you could think of one, in which the premises are true and the conclusion is false. A deductive argument is invalid independently of all of its premises/sub-conclusions and conclusion being true or false. Here is an example of an invalid deductive argument with all true premises and a true conclusion:

<sup>&</sup>lt;sup>10</sup> See footnote 5.

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 $\phi1$ : If Naomi Osaka is a professional snowboarder, then Naomi Osaka is a professional extreme sports athlete. (True)

φ2: Naomi Osaka is not a professional snowboarder. (True)

ψ: Naomi Osaka is not a professional extreme sports athlete. (True)

Naomi Osaka is a professional tennis player instead, and tennis is not classified as an extreme sport, although it is true that if she were a professional snowboarder, then she would be a professional extreme sports athlete, because snowboarding is defined as an extreme sport. Each premise is true. However, the argument is invalid because  $\beta$  is false. It is false that the conclusion  $\psi$  is true in all possible worlds, on the assumption that both  $\varphi 1$  and  $\varphi 2$  are. As before, we can draw informal circles and dots to check for validity and show that this argument is invalid. If we draw informal circles for  $\varphi 1$ , then we get:



φ1: If Naomi Osaka is a professional snowboarder, then Naomi Osaka is a professional extreme sports athlete.

#### Figure 2.3 Diagram of $\varphi 1$ for Invalid Argument

Returning to section 1.4, we know that sufficient and necessary conditions are represented within conditional statements. For  $\varphi l$ , the sufficient condition is being a professional snowboarder, and the necessary condition is being a professional extreme sports athlete. The set of all things that could be classified as being a professional snowboarder is at least within the set of all things that could be classified as being a professional extreme sports athlete. To represent how these sets relate to each other, we draw the smaller circle of the sufficient condition labelled "Professional Snowboarders" within the larger circle of the necessary condition labelled "Professional Extreme Sports Athletes." If we draw in  $\varphi 2$  on top of  $\varphi l$ , we get two options.

Option 1:
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φ1: If Naomi Osaka is a professional snowboarder, then Naomi Osaka is a professional extreme sports athlete. Plus φ2: Naomi Osaka is not a professional snowboarder.







Figure 2.5 Diagram of  $\varphi 1$  plus  $\varphi 2$  for Invalid Argument, Option 2

If we add on  $\varphi 2$ , then the dot labelled "Naomi Osaka" has to be outside of the circle labelled "Snowboarders," but it could either be outside somewhere of that circle and within the circle somewhere labelled "Professional Extreme Sports Athletes," as shown in Option 1 above, or outside somewhere of that circle and also outside somewhere of the circle labelled "Professional Extreme Sports Athletes," as shown in Option 2 above. Option 2 is a possible world where assuming  $\varphi 1$  and  $\varphi 2$  are each true, the conclusion  $\psi$  is false, because the dot labelled "Naomi Osaka" within the circle labelled "Professional Extreme Sports Athletes" represents the

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possibility that Naomi Osaka could still be a professional extreme sports athlete even with  $\varphi 1$  and  $\varphi 2$  being true. Thus,  $\beta$  is false, and this deductive argument is invalid.

Now that we've learned about deductive arguments, and what validity and invalidity is for deductive arguments, let's transition into some exercises testing our understanding and knowledge of these concepts.

[This is where the bullet point summary and section 2.1 exercises will be inserted.]

# Section 2.2: Validity and Possible Worlds Theory Basics

Section Objective: Understand the layout of the sets of all possible worlds and of all impossible worlds.

#### **Key Terms**

Possible World (Specific): a member of or case in the set of all possible worlds, which is the set of all cases

Actual World: the physical world and universe we experience everyday, or could experience, with our five senses and with common sense, and which includes all present cases

Merely Possible World: a non-actual possible world that is not accessible from the actual world

**Impossible World:** a world outside of the set of all possible worlds where at least some logical contradiction is true

Law of Non-Contradiction: a principle in Logic which states that contradictory propositions, which are those of the form "A and not-A," cannot be true in the same sense at the same time

**Many-valued Logic:** a non-classical system of Logic outside the domain of Classical Logic which allows for valuations of statements other than "true" or "false"

Law of the Excluded Middle: a principle in Classical Logic which states that either a proposition or its negation is true

Intuitionistic Logic: a non-classical system of Logic that rejects the Law of the Excluded Middle

To recapitulate from section 2.1, validity is the property of an argument in which, assuming the premises are true in all possible worlds, the conclusion is true in all possible worlds. There cannot be a possible world, nor can you think of one, where the premises are true and the conclusion false. You may then ask, what is a possible world?

A **possible world** is a member of the set of all possible worlds. The set of all possible worlds is all cases. It is the set of all cases no matter the time, place, universe, dimension, condition, set, event, perspective, thought or conception, person, or being. So, a possible world would equally be a case. A convenient synonym for a possible world could be a "scenario." Possible worlds are individual and particular things, and not universal things or sets.

When we think of all cases, we should have open mindedness to the extreme. It does not matter in Logic if the possible worlds have some concrete existence or not. When thinking of or considering all the

possible worlds, the existence of perfect freedom is presupposed, no matter the being thinking or even the being itself.

When it comes to possible worlds theory basics, the set of all possible worlds, or all cases, may be infinite in number or not. Currently, the extent of the number of this set is unknown. In theory, this number seems to be infinite, but in practice, this number seems to be finite. The number of things, individuals, or substances within each possible world can be either finite or infinite. For instance, the set of all possible worlds includes the actual world. The **actual world** is the physical world and universe we experience every day, or could experience, with our five senses and with common sense. The actual world includes all present cases occurring within this physical and universe we experience everyday with the five senses. Returning to section 1.2, the states of affairs or events that obtain in the actual world could be the facts, so the actual world and the facts are related to each other in this way.

All the other non-actual worlds are those that are capable of being thought of or of even having an infinitesimal amount of being. These other non-actual worlds can be further categorized into those not accessible from the actual world, and those accessible from the actual world. Those not accessible from the actual world are the **merely possible worlds**. They are possible, but that's it. They are not accessible from the actual world, and are not capable of being a part of the actual world. They are the only possible worlds not accessible from the actual world. Merely possible worlds may include such cases as

w: The case where you go back in time to kill your grandfather.

This merely possible world is not accessible from the actual world because if it were then you would not exist in the actual world to go back in time to kill your grandfather, so that world does not exist from the standpoint of the actual world. The other non-actual possible worlds that are accessible from the actual world include all possible future cases, such as

w1: The case in which someone lives in New York City in 2088.

This case is accessible from the actual world because it could eventually be true in the current actual world that someone lives in New York City in 2088, although this is by no means necessary that it would come true in the actual world. Some other accessible worlds are all past cases, such as

w2: The case where Martin Luther King Jr. gives the "I Have a Dream" speech on August 28, 1963.

This past case is accessible from the actual world, because the actual world and the facts are related to each other, and w2 is a historical fact. Cases in other universes, dimensions, fictional worlds, and other worlds potentially in our universe are likewise accessible from the actual world. Consider:

w3: The case in which you are in a parallel universe where there is no law of gravity.

w4: The case in which Sarah Jessica Parker arrives at a dinner party early in the potential second time dimension according to string theory.

w5: The case in which the Lord of the Rings Universe exists.

w6: The case in which you travel via warp drive to another planet in the current universe where intelligent alien life forms exist.

w3 and w4 are each accessible from the actual world, because modern-day physicists have theorized that parallel universes and other time dimensions may exist within the multiverse theory and string theory. w5 is accessible from the actual world because the Lord of the Rings Universe is derived from the mind of J. R. R. Tolkein, whose legacy after his death is well-known in the actual world today. w6 is accessible from

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the actual world because war drive is an idea from Star Trek, and some contemporary scientists have hypothesized that other planets in our universe could contain intelligent life.

Made-up, hypothetical, and conditional cases are also accessible from the actual world. Consider:

w7: The case where money grows on trees throughout the world.

w8: The case where you turn right at the stop sign at an intersection instead of the way you actually turned, namely left.

w9: The case where the pool party that is originally scheduled for Wednesday is moved to Friday on the condition that it rains.

w7 is accessible from the actual world because the idea for this case derives from the well-known saying in the actual world "money doesn't grow on trees!" w8 and w9 are each accessible from the actual world, because they each involve a commonplace possibility one could think of in the actual world. w1 through w9 above are only some of many of the non-actual non-merely possible worlds, in the set of all possible worlds, accessible from the actual world.

Outside of the set of all possible worlds are the **impossible worlds**. These are the worlds where at least some logical contradiction is true. Consider:

w10: The case where it is true that the grass is green and it is true that it is not the case that the grass is green.

w11: The case of the barber paradox, where the barber is such that he shaves all and only those who do not shave themselves.

w12: The case within Zeno's paradox of motion, in which the arrow can move and the arrow cannot move.

The set of all impossible worlds includes those worlds where sentences of the form "A and not-A" hold. Sentences of this form state that contradictory propositions can be true in the same sense at the same time. Thus, sentences of this form violate the **law of non-contradiction**, which states that contradictory propositions, which are those of the form "A and not-A," CANNOT be true in the same sense at the same time. w10 is such a world in which a violation of the law of non-contradiction, a logical contradiction, is true. "A" from "A and not-A" here happens to be "the grass is green." w11 is such a world, although the violation of the law of non-contradiction is hidden. If the barber does shave himself, then he has to be one of those who do not shave themselves, because the barber only shaves those who do not shave themselves. If the barber does not shave himself, then he shaves himself and does not shave himself, which is a violation of the law of non-contradiction. w12 is likewise such a world where there is a violation of the law of non-contradiction because it cannot be the case that the arrow can both move and not move in the same sense at the same time.

There are impossible worlds where a logical contradiction would be true within the domain of Classical Logic. Consider:

w13: The case where it is neither true nor false that it will snow in March.

w14: The case where it is false that an animal is either a cat or not a cat.

The only valuations a statement within Classical Logic can have are true or false. Because, in w13, it is neither true nor false that it will snow in March, a logical contradiction is true at w13 in Classical Logic, so w13 would be an impossible world within Classical Logic. If we were to move outside of Classical Logic and into Non-classical **Many-valued Logic**, however, w13 would not be an impossible world, because

Many-valued Logic allows for valuations of statements other than "true" or "false," such as "unknown." Perhaps it is unknown in w13 that it will snow in March.

w14 would also be an impossible world within Classical Logic. Classical Logic recognizes the **Law** of the Excluded Middle, which states that either a proposition or its negation is true ( $p v \sim p$ ), but at w14 this is false. It is false that either an animal is a cat or an animal is not a cat. Within Classical Logic, a logical contradiction is true at w14. Thus, within Classical Logic, w14 is an impossible world. However, if we were to move outside the domain of Classical Logic and into Non-classical Intuitionistic Logic, for example, then w14 would not be an impossible world. Intuitionistic logic rejects the Law of the Excluded Middle, so the statement "it is false that an animal is either a cat or not a cat" would not be a logical contradiction within Intuitionistic Logic. Because it would not be a logical contradiction here, a logical contradiction would not be true at w14, and w14 would be a possible world instead of an impossible one.

Strictly speaking, then, w13 and w14 would be possible and not impossible worlds, because the set of all possible worlds includes all cases without exception, but, if we restrict the domain of the set of all possible worlds to Classical Logic, then w13 and w14 are impossible worlds. By contrast, w10, w11, and w12 would be impossible worlds, no matter if we restrict the domain of the set of all possible worlds to Classical Logic alone or not, and no matter if we do not restrict the domain at all.

What does possible worlds theory mean for validity and invalidity when it comes to deductive arguments? Again, validity is the property of an argument whereby the conclusion of that argument is true in all possible worlds, on the assumption that the premises/sub-conclusions are true in all possible worlds. Invalidity is the property of an argument whereby the conclusion of that argument is NOT true in all possible worlds, on the assumption that the premises/sub-conclusions are true in all possible worlds. Invalidity is the property of an argument whereby the conclusion of that argument is NOT true in all possible worlds, on the assumption that the premises/sub-conclusions are true in all possible worlds. Possible worlds theory helps us to think through all the possible worlds in order to either verify the truth of some conclusion assuming the premises are true, and thus show the validity of an argument, or come across a counterexample where the conclusion in question is not true, and thus show the invalidity of an argument. Thinking of all the possible worlds, such as the merely possible worlds, future worlds, past worlds, alternate universes, alternate dimensional worlds, fictional worlds, potential other worlds in currrent universe, worlds of made-up, hypothetical, or conditional cases, and lastly the actual world can help us ultimately distinguish between good and bad arguments, and lare bare any unexpected counterexamples contradictory to validity. Thinking through all of the possibilities really matters! Possible worlds theory helps facilitate doing this by providing the basics for thought experiments.

Now that we've discussed possible worlds theory basics, we are prepared to do exercises that involve thought experiments and thinking of possibile worlds in relation to validity and invalidity.

[This is where the bullet point summary and section 2.2 exercises will be inserted.]

### **Section 2.3: Deductive Argument Forms**

Section objective: Know what some deductive argument forms are, and what real life examples of each are.

#### Key Terms

Argument Based Upon Mathematics: a type of argument in which at least either the factual or inferential claim includes a mathematical derivation

**Mathematical Induction:** more complicated deductive argument based upon mathematics, in which one counts down rather than up, namely some conjecture is both true for n=1 or n=0 and true for if n=k then n=k+1

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**Argument From Definition:** a type of argument in which at least either the factual claim or inferential claim includes some definition of a word or phrase

**Categorical Syllogism:** a type of argument that consists in exactly two premises and one conclusion, each of which begin with either the words "All," "No," or "Some"

**Hypothetical Syllogism**: a type of argument that consists in exactly two premises and one conclusion, and in which the factual claim includes a statement containing both a sufficient and necessary condition

**Mixed Hypothetical Syllogism**: a type of hypothetical syllogism in which only one of the premises is a conditional statement

**Pure Hypothetical Syllogism**: a type of hypothetical syllogism in which each of the two premises and the conclusion are conditional statements

Affirming the Consequent: a fallacious argument of the form "If X, then Y. Y. Therefore, X."

Denying the Antecedent: a fallacious argument of the form "If X, then Y. Not-X. Therefore, not-Y."

**Disjunctive Syllogism:** a type of argument that consists in exactly two premises and one conclusion, in which the factual claim contains a statement of the form "either A or B" and the negation of one of the disjuncts, and the conclusion contains the other disjunct

**DeMorgan's Rules:** a type of argument with exactly one premise and one conclusion, in which either the premise is the denial of a disjunctive statement of the form "either A or B" and the conclusion is a conjunctive statement of the form "both not-A and not-B," or vice versa, or the premise is a statement that is the denial of the conjunction of A and B and the conclusion is a statement of the form "either not-A or not-B," or vice versa.

Disjunctive Statement: a type of statement of the form "either X or Y," also known as a disjunction

**Disjunct:** "X" or "Y" in a disjunctive statement of the form "either X or Y"

Conjunctive Statement: a type of statement of the form "both X and Y," also known as a conjunction

Conjunct: "X" or "Y" in a conjunctive statement of the form "both X and Y"

"---" (Set Theory): the symbol on top that is a denial or negation of what's underneath, and also refers to the complement of what's underneath

**Triple Bar (Set Theory):** a symbol that indicates equivalence, where equivalence indicates that what is on the left is a sufficient and necessary condition for what is on the right

"not" (Set Theory Lingo): a word and hyphen that means "it is not the case that"

**Upside-down "U" (Set Theory Lingo):** a symbol that mean "the intersection of" what is on the left and the right

**Rightside-up "U" (Set Theory Lingo):** a symbol that means "the union of" what is on the left and right

**Complement (Set Theory):** everything strictly speaking outside of a set in question, whether that is another set or sets or not

Union (Set Theory): the area inclusive of everything in the sets in question and in their intersections.

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**Intersection (Set Theory):** the area inclusive of where two or more sets overlap

**xNANDy Gate:** a real-life example of DeMorgan's Rules in electrical circuitry, that is equivalently broken into a negative x OR a negative y gate, and vice versa

**xNORy Gate:** a real-life example of DeMorgan's Rules in electrical circuitry, that is equivalently broken into a negative x AND negative y gate, and vice versa

**Conjunction (Argument):** a type of argument where, from the occurrence of each of a number of things, all (or at least more than one) of those same things are implied in the conclusion

**Simplification:** a type of argument where, from all of a certain number of things, at least one these things or a certain number of each of these things is implied in the conclusion

*Non sequitur*: a conclusion that does not follow from the premise(s), literally translated from the Latin as "it does not follow"

**Dilemma:** a type of argument, where the factual claim includes a disjunctive statement and two conditional statements, in which the antecedent of each is one of the disjuncts of the disjunctive statement, and the conclusion is a disjunctive statement consisting of the consequents of the two conditional statements in the factual claim

Trolley Problem: a thought experiment in ethics

In 2.1, we talked about how the conclusion in a deductive argument is claimed to follow by necessity from the factual claim, or premises, independently of whether or not it is a good or bad argument.

Here are some forms of such deductive arguments in real life that claim to have conclusions that follow by necessity<sup>11</sup>:

- (1) Argument Based Upon Mathematics
- (2) Argument from Definition
- (3) Categorical Syllogism
- (4) Hypothetical Syllogism
- (5) Disjunctive Syllogism
- (6) DeMorgan's Rules
- (7) Conjunction and Simplification
- (8) Dilemma

Regarding (1) above, an **argument based upon mathematics** is a type of argument in which at least either the factual or inferential claim includes a mathematical derivation, such as:

<sup>&</sup>lt;sup>11</sup> Some of these deductive argument forms come from Patrick J. Hurley and Lori Watson's book *A Concise Introduction to Logic* (cf. *A Concise Introduction to Logic*, 13th Edition, Patrick J. Hurley, Lori Watson, Cengage Learning, 2018. ISBN: 978-1-305-95809-8).

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In a group of 50 people, each member in the group has a birthday in December. Therefore, 19 people in that group share a birthday.

Here, the first statement is the single premise and "19 people in that group share a birthday" is the conclusion. The inferential claim is implicit. It consists in the following which involves a mathematical derivation: "There are 50 people total and 31 possible birthdays to choose from. Each person has one birthday. Therefore, 50-31 = 19 people in that group share a birthday." This argument is a good one, because the premises, including the implicit ones in the mathematical derivation in the inferential claim, objectively succeed in proving the conclusion to be true. Note that the premise is a description of the group, and does not have to include a mathematical derivation. **Mathematical induction**, despite the name, is a more complicated deductive argument based upon mathematics, in which one counts down rather than up:

Let S be a conjecture involving some natural number n.

S is true for all n if:

S is true for n=1 or n=0, and,

Assuming S is true for n=k, then S is true for n=k+1.

Let S be F(n): 0 + 1 + 2 + ... + n = [n(n+1)]/2. Let n = 1. Then,

F(1): 0 + 1 = 1 = [1(1+1)]/2 = 1, so S is true for n = 1 and condition (i) is fulfilled.

Assume S is true for n=k.

Then, F(k): 0 + 1 + 2 + ... + k = [k(k+1)]/2.

Then, F(k): 0 + 1 + 2 + ... + k + (k + 1) = [k(k+1)]/2 + (k + 1) = [k(k+1) + 2(k + 1)]/2 = [(k+2)(k+1)]/2 = [(k+1)((k+1)+1)]/2. Thus, S is true for n=k+1 and condition (ii) is fulfilled. Because (i) and (ii) are met, F(n): 0 + 1 + 2 + ... + n = [n(n+1)]/2 is true for all natural numbers n.

Here, the words in blue are the factual claim and the words in red are the inferential claim. The factual claim consists in the axiom of mathematical induction, also known as the induction axiom, and the starting assumption that S is F(n): 0 + 1 + 2 + ... + n = [n(n+1)]/2. The inferential claim consists in the mathematical derivation or proof that the conclusion, F(n): 0 + 1 + 2 + ... + n = [n(n+1)]/2 is true for all natural numbers *n*, is true. This is a well-known argument and proof, and it is a good argument because the premises, including those in the inferential claim, objectively succeed in proving that the conclusion is true.

Here is an example of a bad argument based upon mathematics:

- (1) -2 = -2(2) 4 - 6 = 1 - 3(3) 4 - 6 + 9/4 = 1 - 3 + 9/4(4)  $(2 - 3/2)^2 = (1 - 3/2)^2$ (5) 2 - 3/2 = 1 - 3/2
- (6) 2 = 1

Again, the words in blue are the factual claim and the words in red are the inferential claim. This is a bad argument because the premise and the reasoning within the inferential claim does not objectively

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support the conclusion. The error occurs in the step from line 4 to line 5.  $(1/2)^2 = (-1/2)^2$  does not imply that 1/2 = -1/2.

Regarding (2) above, an **argument from definition** is a type of argument in which at least either the factual claim or inferential claim includes some definition of a word or phrase, such as:

Ashante has pathophobia. Therefore, Ashante has a fear of phobias.

The factual claim consists in the explicit premise, "Ashante has pathophobia," and in the implicit premise "pathophobia is a fear of phobias," which is the definition of "pathophobia." This argument is a good one because the explicit and implicit premise objectively support the conclusion. The definition of "pathophobia" used is correct; "pathophobia" just is defined in that way. So, it follows by necessity that Ashante has a fear of phobias.

Here is a little bit more complicated example of an argument by definition:

Neil is a logician and a number theorist. Candace is a set theorist and a semanticist. Justice is a biologist and a statistician. So, both Neil, Candace, and Justice are mathematicians.

Number theorists, set theorists, and statisticians are defined as types of mathematicians, which definitions are the implicit premises of the factual claim. Although not all of the persons in question are logicians, semanticists, or biologists, by definition and by necessity each is a mathematician, because each is a type of mathematician by definition. This argument is a good one because the explicit and implicit premises objectively support the conclusion.

Here is an example of a bad argument by definition:

Honolulu is a city in Pennsylvania. Pennsylvania is a State of the United States. Therefore, Honolulu is in a State of the United States.

Although the conclusion here, that Honolulu is in a State of the United States, happens to be true in fact, and the premises as stated combine together correctly to give the conclusion, this argument is based upon a false definition. Honolulu by definition is not a city in Pennsylvania. It is a city in Hawaii. This argument is a bad one because the premises do not objectively support the conclusion, due to including a false definition of Honolulu.

Regarding (3) above, a **categorical syllogism** is a type of argument that consists in exactly two premises and one conclusion, each of which begin with either the words "All," "No," or "Some." For example,

P1: All athletes are persons who move their bodies.

P2: All competitive figure skaters are athletes.

C: All competitive figure skaters are persons who move their bodies.

The form of this categorical syllogism is AAA-1 and its name is "Barbara," which is discussed further in Chapter 7. Each of the premises and the conclusion begin with the word "all," and there are exactly two premises and exactly one conclusion. This is a good argument where the premises objectively support the conclusion. This well-known form is valid (do the informal circles and dots test from section 2.1 to check), and each premise is true by definition. Competitive figure skaters by definition are athletes, because athletes are persons who compete in or are proficient in a sport. For the second premise, being a person who moves their body is a necessary condition for being an athlete. Feel free to fact check the premises. Validity along with true premises gives a true conclusion in a good argument, discussed further in section 2.4.

The following example is also a categorical syllogism:

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P1: No countries bordering Swaziland are landlocked countries.

P2: Some countries bordering Zimbabwe are landlocked countries.

C: Some countries bordering Zimbabwe are not countries bordering Swaziland.

The two premises begin with either the word "no" or "some," and the conclusion begins with the word "some" as well. There are exactly two premises and one conclusion here. This categorical syllogism is a good argument because the premises objectively support the conclusion. It has a valid form, to be discussed further in Chapter 7 (again, try the informal circles and dots test from section 2.1 for yourself). Additionally, each premise is true. The countries bordering Swaziland are Mozambique and South Africa, both of which touch the Indian Ocean and so cannot be landlocked. For the second premise, the countries Zambia, Malawi, and Botswana border Zimbabwe and are each classified as being landlocked countries. Because this argument is valid and has all true premises, the conclusion must be true, independently of the fact that Botswana, Zambia, and Malawi are such countries that border Zimbabwe but do not border Swaziland.

Here is an example of a bad categorical syllogism:

P1: All carrots are vegetables. P2: All radishes are vegetables.

C: All carrots are radishes.

This argument is a categorical syllogism, because there are exactly two premises and one conclusion, and each of the premises and conclusion begin with the word "all." However, this argument is a bad one because the premises do not objectively support the conclusion. Although it is true that all carrots and all radishes are vegetables by biological definition, it is false that all carrots are radishes. They are two distinct types of vegetables. Intuitively, there is something wrong with this argument, and it is invalid, because in the actual world the premises are true and the conclusion is false.

Regarding (4) above, a **hypothetical syllogism** is a type of argument that consists in exactly two premises and one conclusion, and in which the factual claim includes a statement containing both a sufficient and necessary condition, such as:

P1: If Frida Kahlo painted *The Two Fridas*, then Frida Kahlo painted a self-portrait. P2: Frida Kahlo painted The *Two Fridas*.

C: Frida Kahlo painted a self-portrait.

The first two statements are the two premises, and the third statement below the dividing line is the conclusion. The factual claim or set of premises includes the conditional "if...then..." statement "If Frida Kahlo painted The Two Fridas, then Frida Kahlo painted a self-portrait," where painting The Two Fridas is the sufficient condition for painting a self-portrait, and painting a self-portrait is the necessary condition for painting The Two Fridas. This argument is a good one because the premises objectively support the conclusion. You can use the informal circles and dots test to test for validity, and see how it is valid. Additionally, the premises are both true: Frida Kahlo did in fact paint The Two Fridas in 1939, and if she had painted this painting, which depicts two women who are the image of Frida Kahlo, then that painting would have to be a self-portrait for her, because it would be a representation of her by her.

The previous example is what is known as a **mixed hypothetical syllogism**, in which only one of the premises is a conditional statement. In what follows, we have another example of a hypothetical syllogism, namely a **pure hypothetical syllogism**, in which each of the two premises and the conclusion are conditional statements:

P1: If Lucy lives in Virginia, then Lucy lives in the United States.

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P2: If Lucy lives in the United States, then Lucy lives in North America.

C: If Lucy lives in Virginia, then Lucy lives in North America.

This is a good argument because the premises objectively support the conclusion. You can use the informal dots and circles test to show that it is valid. Likewise, it is true that Virginia is part of the United States, and that the United States is part of North America, so each of the premises is true. Intuitively, this argument makes sense.

Here is an example of a bad hypothetical syllogism:

P1: If Lang Lang is a violinist, then Lang Lang is a musician. P2: Lang Lang is a musician.

C: Lang Lang is a violinist.

This argument is a bad one because the premises do not objectively support the conclusion. (This fallacious argument is also known as **affirming the consequent**. This is a fallacious argument of the form "If X, then Y. Y. Therefore, X." Formal fallacies such as this is discussed in Chapter 11). Do the informal circles and dots test to check. Although it is true that whoever is a violinist is also a musician by definition, and Lang Lang is in fact a musician, Lang Lang is a concert pianist and not a violinist. Here is another similarly bad hypothetical syllogism:

P1: If Lang Lang is a violinist, then Lang Lang is a musician. P2: Lang Lang is not a violinist.

C: Lang Lang is not a musician.

This argument is a bad one like the previous because the premises do not objectively support the conclusion. (This fallacious argument is also known as **denying the antecedent**. This is a fallacious argument of the form "If X, then Y. Not-X. Not-Y," discussed further in Chapter 11.) Do the informal circles and dots test to see how it's invalid. Again, although it is true that whoever is a violinist is also a musician, and it is a fact that Lang Lang is not a violinist, it is false that Lang Lang is not a musician, because he is a concert pianist.

Regarding (5) above, a **disjunctive syllogism** is a type of argument that consists in exactly two premises and one conclusion, in which the factual claim contains a statement of the form "either A or B"12 and the negation of one of the disjuncts, and the conclusion contains the other disjunct. For example, consider:

P1: Either Monique tested positive for COVID-19 when she completed the non-faulty test, or Monique tested negative for COVID-19 when she completed the non-faulty test. P2: Monique did not test positive for COVID-19 when she completed the non-faulty test.

C: Monique tested negative for COVID-19 when she completed the non-faulty test.

This argument is a disjunctive syllogism because it contains exactly two premises and one conclusion. Further, the factual claim consisting of P1 and P2 above includes both a disjunctive "either...or..." statement, namely at P1, and the negation of one of the disjuncts of the same disjunctive statement, namely at P2. P2 is the negation of the left disjunct, "Monique tested positive for COVID-19 when she completed the non-faulty test." Assuming that there is such a person named Monique who did not

<sup>&</sup>lt;sup>12</sup> Note that the disjunctive statements in this section are weak or inclusive disjunctions and not strong or exclusive disjunctions, which are of the form "either A or B and not-both A and B."

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test positive for COVID-19 when completing such a test, this argument is a good one because the premises objectively support the conclusion. Doing an informal circles and dots test shows that it's valid: no matter if the two circles representing the two disjuncts overlap somewhat or not, all that's left after the exclusion of the left disjunct is the right disjunct. Additionally, assuming P2 is true or a fact, P1 is true as well because the only two options upon completion of a non-faulty COVID-19 test are either testing positive or negative for COVID-19. Here is a more complicated example of a hypothetical disjunctive syllogism:

P1: It is not the case that Billy does not like flowers.P2: Either Billy does not like flowers, or both if health is a priority then malignant cancer may go away and if Selenium is an element then Tin is an element.

C: Both if health is a priority then malignant cancer may go away and if Selenium is an element then Tin is an element.

Again, this is a disjunctive syllogism because it consists in exactly two premises and one conclusion. The factual claim of P1 and P2 contains the "either...or..." disjunctive statement at P2, and the negation of the left disjunct "Billy does not like flowers" of the same disjunctive statement at P1 (it is possible for the right disjunct in other cases to be negated as well). Although the disjunctive statement in this argument is more complex, the form "either A or B" can be extracted.

Here is an example of a bad disjunctive syllogism:

P1: Either rainbows are colorful or atoms are bigger than quarks. P2: Rainbows are colorful.

C: It is not the case that atoms are bigger than quarks.

This argument may look like a disjunctive syllogism because one of the premises contains an "either...or..." disjunctive statement. However, the other premise does not contain the negation of one of the disjuncts in P1. This argument is a bad one because the premises do not objectively support the conclusion. The negation of the other disjunct cannot be validly derived from a positive occurrence of the one disjunct. Try the informal circles and dots test to see this for yourself. Furthermore, although P1 is true, because the first disjunct is true by definition and the other disjunct is factually false (for an entire disjunctive statement to be true only one of the disjuncts needs to be), and P2 is false, because again atoms are in fact bigger than quarks, the conclusion is false. It is the case that atoms are bigger than quarks, according to the results of numerous investigations in modern physics. This argument must be invalid for this reason as well. Intuitively, something is off with this argument if it starts with true premises yet ends up with a false conclusion.

Regarding (6) above, **DeMorgan's Rules**, or Laws, is a type of argument with exactly one premise and one conclusion, in which either the premise is the denial of a disjunctive statement of the form "either A or B" and the conclusion is a conjunctive statement of the form "both not-A and not-B," or vice versa, or the premise is a statement that is the denial of the conjunction of A and B and the conclusion is a statement of the form "either not-A or not-B," or vice versa. Here, a **disjunctive statement** or **disjunction** is a type of statement of the form "either X or Y." The disjuncts are either "X" or "Y." A **conjunctive statement** or **conjunction** is a type of statement of the form "both X and Y." The conjuncts are either "X" or "Y." Here is what this law looks like in terms of sets<sup>13</sup>:

<sup>&</sup>lt;sup>13</sup> For a proof in propositional logic of DeMorgan's Law, see the Appendix.

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Figure 2.6 Diagram of DeMorgan's Laws in Terms of Sets

Here are some notes on the symbols in the diagram. "A" and "B" represent the circles as sets. The "——" on top is a denial or negation of what's underneath. It also refers to the **complement** of what's underneath, see the definition of "complement" below. The **triple bar** is equivalence, where equivalence indicates that what is on the left is a sufficient and necessary condition for what is on the right. Departing from naive set theory lingo, DeMorgan's two laws above can be reformulated as:

- (1) Not-either A or B is equivalent to both not-A and not-B,
- (2) Not-both A and B is equivalent to either not-A or not-B.

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Again, equivalence here indicates that what is on the left is a sufficient and necessary condition for what is on the right. Additionally, **"not"** can be replaced with "it is not the case that." The **upside-down "U"** can be replaced with "the intersection of." The **rightside-up "U"** can be replaced with "the union of."

Here is where this rule comes from as shown in this image. Let's start with the image on top. For any member, assuming there is a member of a set such that it is in the complement of the union of set A and set B, then that member is outside of the union of set A and set B. Here, the complement of a set is everything strictly speaking outside of that set in question, whether that is another set or sets or not. Here, the **union** of sets is the area inclusive of everything in the sets in question and in their intersections. Later on, the intersection of sets is the area inclusive of where two or more sets overlap. If that member is outside of the union of set A and set B, then it is outside of the area inclusive of everything in A and B including their intersection. So, it is outside of A and B altogether. Whatever is outside of A and B altogether is both in the area outside of A and in the area outside of B. The complement of A is everything outside of A. So, this member, is in the complement of A. The complement of B is everything outside of B. So, this member is in the complement of B. Thus, this member is both in the complement of A and in the complement of B. So, this member is in the area inclusive of where the complement of A and the complement of B overlap. Thus, it is in the intersection of the complement of A and the complement of B. Putting it altogether, then, for any member, assuming that there is a member of a set such that it is in the complement of the union of set A and set B, then it is in the intersection of the complement of A and the complement of B. This result is equivalent to the other part of our definition of DeMorgan's Laws: the premise is the denial of the disjunction of A and B, and the conclusion is the conjunction of the denial of A and the denial of B. This is also equivalent to the left-to-right direction of (1) above.

Regarding this same image on top, we can give a proof in the opposite direction. For any member of a set, assuming there is a member of a set such that it is in the intersection of the complement of A and the complement of B, then it is in the area inclusive of where the complement of A and the complement of B overlap. If it is in this area, then it is in the area inclusive of where everything outside of A and everything outside of B overlap. Then, it is in the area outside of both A and B altogether. Then, it is in the area outside of the area inclusive of both A and B and their intersection. Then, it is in the area outside of the union of A and B, so it is in the complement of the union of A and B. Putting this all together, for any member of a set, assuming there is a member of a set such that it is in the intersection of the complement of A and the complement of B, then it is in the complement of the union of A and B. Putting this all together, for any member of a set, assuming there is a member of a set such that it is in the intersection of the complement of A and the complement of B, then it is in the complement of the union of A and B. This result is equivalent to the opposite of the other part of our definition of DeMorgan's Laws: the premise is the conjunction of the denial of A and the denial of B, and the conclusion is the denial of the disjunction of A and B. This is also equivalent to the right-to-left direction of (1) above.

Now let's turn to the image on the bottom. For any and all members, assume there is some member of a set such that it is a member of the complement of the intersection of sets A and B. So, the member that we are talking about here is outside of where sets A or B overlap. If this member is outside of where sets A and B overlap, then it could be in one of three areas: set A outside of the intersection, set B outside of the intersection, or outside of both sets altogether. If it is in set A outside of the intersection, then it is outside of set B, thus in the complement of set B, and thus in the union of the complement of set A and the complement of set B. If it is in set B outside of the intersection, then it is outside of set A, thus in the complement of set A and in the union of the complement of set A and the complement of set B. If it is outside of both sets altogether, then it is outside of both set A and set B. If it is outside of set A, it is in the complement of A, and if it is outside of set B it is in the complement of B. Thus it is in both the complement of set A and the complement of set B, so in the union of the complement of set A and the complement of set B. No matter what one of the three areas it is in, it will be in the union of the complement of set A and the complement of set B. Putting all this together, for any and all members, assuming there is some member in the complement of the intersection of sets A and B, then it is also a member of the union of the complement of set A and the complement of set B. This result is equivalent to part of our definition of DeMorgan's Laws above: the premise is a denial of the conjunctive statement of A and B and the conclusion is a statement that is a disjunction of both not-A and not-B. This is equivalent to the left-to-right direction of (2) above.

Regarding the image on the bottom, we can also give a proof in the opposite direction. For any and all members, assuming there is a member of a set such that it is a member of the union of the complement of set A and the complement of set B, then it is either in A outside of B, in B outside of A, or outside of both A and B. If it is in A outside of B, then it is outside of the intersection of A and B, so in the complement of the intersection of A and B. If it is in B outside A, then it is outside of both A and B, so in the complement of the intersection of A and B. If it is outside A, then it is outside of both A and B, so in the complement of the intersection of A and B. If it is outside A, then it is outside the intersection of A and B, so in the complement of the intersection of A and B. If it is outside of both A and B, then it is outside the intersection of both A and B, and thus in the complement of the intersection of A and B. No matter what one of the three areas it is in, it is in the complement of the intersection of A and B. Putting this together, for any and all members, assuming there is a member of a set such that it is a member of the union of the complement of set A and the complement of set B, then it is in the complement of the intersection of A and B. This result is equivalent to part of our definition of DeMorgan's Laws above in the other direction: the premise is a statement that is a disjunction of both not-A and not-B and the conclusion is a denial of the conjunctive statement of A and B. This is equivalent to the right-to-left direction of (2) above.

Here are some examples of DeMorgan's Laws in real life:

It is not the case that either a Republican or a Libertarian is President of the United States in 2022. Therefore, a Republican is not President of the United States in 2022 and a Libertarian is not President of the United States in 2022.

This is a real-life application of (1) left-to-right above. This argument is a good one and makes sense. Its premise objectively supports the conclusion, because the current President is Joe Biden who is a Democrat, meaning that neither a Republican nor a Libertarian is in 2022.

Beatrice did not wrong Billy and Beatrice did not make Billy worse off than he would have been. Therefore, it is not the case that either Beatrice did wrong Billy or Beatrice did make Billy worse off than he would have been.

What is above is a hypothetical example of (1) right-to-left above. Below, here is a hypothetical example of (2) left-to-right above:

It is not the case both that I consented to let him eat my cookies and that I desired my frenemy to come to the party. Thus, either I did not consent to let him eat my cookies or I did not desire my frenemy to come to the party.

Here is a real-life example of (2) right-to-left above:

Either it's not true that Natalie Portman is Natalie Portman or it could not have been false that Natalie Portman is Natalie Portman. Thus, it is not the case that both it's not true that Natalie Portman is Natalie Portman and it could have been false that Natalie Portman is Natalie Portman.

This argument is a good one because the premises objectively support the conclusion. Either Natalie Portman is not in fact herself, or it must be true that she is herself. This is the same thing as saying that both scenarios cannot hold true.

A more complicated real-life example of DeMorgan's laws is with electrical circuits. A **xNANDy** gate is equivalently broken into a negative x OR a negative y gate, and vice versa, replicating (2) above and illustrated in the image below.

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Figure 2.7 xNANDy gate = negative x OR negative y gate

The negative x gate is at switch 1 (S1) and the negative y gate is at switch 2 (S2). They are on the same line as the negative current, indicating either negative gate x OR negative gate y. The negative x OR negative y gate (either not-A or not-B) is converted into Q = xNANDy (not-both A and B) through an attachment, and vice versa. Similarly, an **xNORy gate** is equivalently broken into a negative x AND negative y gate, and vice versa, replicating (1) above and illustrated in the image below.



Figure 2.8 xNORy gate = negative x AND negative y gate

Again, the negative x gate is at switch 1 (S1) and the negative y gate is at switch 2 (S2). They are on parallel lines connecting the bottom negative current line to the parallel top Q = xNORy line with the negative current. The negative x and negative y parallel lines indicate the negative x AND negative y gate, because they both feed into the Q line. The negative x AND negative y gate (both not-A and not-B) is converted to the Q = xNORy gate (not-either A or B) at the two attachment sites, and vice versa. For discussion of violations of DeMorgan's Laws, or bad arguments in this regard, see chapter 8.

Regarding (7) above, **conjunction** (not to be confused with the other name for a conjunctive statement) is a type of argument where, from the occurrence of each of a number of things, all (or at least more than one) of those same things are implied in the conclusion. It simply is arguments such as the following:

Socrates taught Plato. Plato taught Aristotle. Aristotle taught Alexander the Great. So, Socrates taught Plato, and Plato taught Aristotle, and Aristotle taught Alexander the Great.

This argument is a good one because the premises objectively support the conclusion; the conclusion just is a combining of the premises with the word "and." Going in the opposite direction, **simplification** is a type of argument where, from all of a certain number of things, at least one these things or a certain number of each of these things is implied in the conclusion, such as:

Socrates taught Plato, and Plato taught Aristotle, and Aristotle taught Alexander the Great. So, Plato taught Aristotle. Socrates taught Plato.

Likewise, this argument is a good one because the premises objectively support the conclusion; the conclusion just consists in parts of the first statement. An obvious violation of conjunction or bad argument could be:

Socrates taught Plato. Plato taught Aristotle. Aristotle taught Alexander the Great. So, Confucius taught Zengzi.

Even though the conclusion is factually correct, it does not follow (by the argument form conjunction). It is a *non sequitur*.

An obvious violation of simplification could be:

Socrates taught Plato, and Plato taught Aristotle, and Aristotle taught Alexander the Great. So, Confucius taught Zengzi.

Again, the randomness of the conclusion in relation to the first statement makes it a non sequitur.

Regarding (8) above, **dilemma** is a type of argument, where the factual claim includes a disjunctive statement and two conditional statements, in which the antecedent of each is one of the disjuncts of the disjunctive statement, and the conclusion is a disjunctive statement consisting of the consequents of the two conditional statements in the factual claim.<sub>14</sub> For example,

Either the trolley driver does nothing and continues on the same track, or the trolley driver pulls the lever and diverts the trolley. If the trolley driver does nothing and continues on the same track, then the trolley will kill five people. If the trolley driver pulls the lever and diverts the trolley, then the trolley will kill one person. So, either the trolley will kill five people, or the trolley will kill one person.

This thought experiment is known as **the trolley problem** in ethics. The factual claim is the first three statements: the disjunctive statement "either the trolley rider does nothing and continues on the same track, or the trolley driver pulls the lever and diverts the trolley," and the two conditional statements "if the trolley driver does nothing and continues on the same track, then the trolley will kill five people" and "if the trolley driver pulls the lever and diverts the trolley will kill one person." The conclusion is "either the trolley will kill five people, or the trolley will kill one person."

Intuitively, these premises objectively support the conclusion, making this a good argument. For any disjunctive statement, if each of the disjuncts implies a distinct statement, then those distinct statements are the disjuncts in a new disjunctive statement that is the conclusion.

Here is a more complicated example of dilemma:

<sup>&</sup>lt;sup>14</sup> For a proof of dilemma in propositional logic, see Appendix.

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Regarding the physical substance "matter" and the mental substance "mind," either Cartesian duality, or physicalism, or idealism, or neutral monism holds.

If Cartesian duality holds, then both matter and mind are fundamental substances.

If physicalism holds, then mind is derivative from the fundamental substance matter.

If idealism holds, then matter is derivative from the fundamental substance mind.

If neutral monism holds, then both matter and mind are derivative from some other third substance.

So, regarding the physical substance "matter" and the mental substance "mind," either both matter and mind are fundamental substances, or mind is derivative from the fundamental substance matter, or matter is derivative from the fundamental substance mind, or both matter and mind are derivative from some other third substance.

What makes this example of dilemma more complex is that the disjunctive statements involve four disjuncts each instead of only two, and there are four conditional statements to cognize. This is a good argument because the premises objectively support the conclusion. Again, for any disjunctive statement, if each of the disjuncts implies a distinct statement, then those distinct statements are the disjuncts in a new disjunctive statement that is the conclusion. Here is an example of a dilemma gone wrong:

Either we turn our clocks 1 hour ahead, or we turn our clocks 1 hour behind. If we turn our clocks one hour ahead, then we lose one hour of sleep (!!). If it is fall, then we turn our clocks 1 hour behind. Thus, either we lose 1 hour of sleep, or it is fall.

This is a bad argument because the premises do not objectively support the conclusion. Combining the right disjunct of the disjunctive statement in the factual claim with the consequent of the second conditional statement to get the right disjunct in the disjunctive statement for the conclusion committs the fallacy of affirming the consequent (see 11.2).

Now that we've discussed what some deductive argument forms are and have given examples of each, of either good or bad, simple or complex arguments with or resembling those forms, let's do some exercises on these deductive argument forms.

[This is where the bullet point summary and section 2.3 exercises will be inserted.]

#### Section 2.4: Sound and Unsound Arguments

Section Objective: Distinguish between sound and unsound arguments.

#### **Key Terms**

Good Argument (Alternate): a type of argument in which both its factual and inferential claims are true

**Sound Argument**: a good deductive argument, that is both valid, or its inferential claim is true, and all of its premises are true, or its factual claim is true

Bad Argument (Alternate): a type of argument in which either the factual claim or inferential claim is false

Unsound Argument: a bad deductive argument, that is either invalid, has one or more false premises, or both

Returning to section 1.2, an argument consists of both a factual claim and an inferential claim. Now, a claim can be either true or false. The argument is a good argument if and only if both the factual claim and the inferential claim are true.

For deductive arguments, a **good argument** is called a sound argument. Only deductive arguments can be sound arguments. From (1) above, for a **sound argument** then, both the factual claim and inferential claims are true. Another way of saying that its factual claim is true is that all of the premises are true. Another way of saying that its inferential claim is true is that the argument is a valid argument, or has validity. Soundness builds upon validity, which was discussed in section 2.1. For a sound argument, all of the premises are true, in addition to the argument being valid. In other words, A deductive argument is a good argument if and only if it is a sound argument. A deductive argument is a sound argument if and only if it is both a valid argument, or its inferential claim is true, and all of its premises are true, or its factual claim is true.

Here is another way of thinking about a sound argument:

Sound argument = Valid argument (Inferential claim is true) + All premises true (Factual claim is true)

Again, a sound argument has to be both a valid argument and have all true premises, or, in other words, both its factual and inferential claims have to be true. If both of these conditions are fulfilled, then the conclusion of the sound argument must be true: returning to section 2.1, if the argument is a valid argument, then assuming the premises are true in all possible worlds, the conclusion is true in all possible worlds. If each and all of the premises are true in all possible worlds, this former assumption of validity is a true one, so the conclusion is true in all possible worlds simpliciter. So, the conclusion must be true, in the case of both a valid argument with all true premises.

Here are some examples of good deductive arguments, each of which are valid, have all true premises, and thus a true conclusion as well.

P1: All iPhones are smartphones. (T)

P2: All smartphones are devices that have functions of a computer. (T)

C: All iPhones are devices that have functions of a computer. (T)

As discussed in section 2.1, this deductive argument is valid because  $\beta$  is true, meaning that, assuming P1 and P2 are true, C is true in all possible worlds. There is no possible world where both P1 and P2 are true and C is false. Try drawing informal circles to check, as was discussed in section 2.1 (this is the unconditionally valid syllogism form of AAA-1, to be discussed in Chapter 7). Furthermore, P1 and P2 are each true by definition. Because both conditions are met, C must be true as well.

P1: Twitter is a form of social media. (T)P2: All forms of social media are things that involve interaction. (T)

C: Twitter is a thing that involves interaction. (T)

This deductive argument is also valid, and you can show this by drawing informal circles and a dot. P1 and P2 are true by definition, so C must be true as well, because both conditions are met.

P1: Three, five, and seven are prime numbers. (T) P2: If three, five, and seven are prime numbers, then all the odd numbers between two and eight are prime numbers. (T)

C: All the odd numbers between two and eight are prime numbers. (T)

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This deductive argument is also valid. It is a Modus Ponens (If A, then B; A; B), which was discussed in section 2.3 and which is always valid. P1 is true because the numbers three, five, and seven are divisible by only themselves and the number one. P2 is true because its antecedent is P1, and its consequent is true as well. When a true antecedent precedes a true consequent in a conditional statement, then entire conditional statement, P2 in this case, is true. The consequent of P2 is true, because the only odd numbers between two and eight are three, five, and seven, and these odd numbers are prime numbers. Because both conditions are met, then, C2 must be true.

On the opposite side, an argument is a **bad argument** if and only if one of the claims, either factual or inferential, is false. For deductive arguments, a bad argument is called an **unsound argument**. Only deductive arguments can be unsound arguments. This means that, for an unsound argument, at least one of the claims is false, so at least one premise is false, or the argument is invalid. In other words, a deductive argument is a bad argument if and only if it is an unsound argument, and an unsound argument is either invalid, has one or more false premises, or both. There is no middle ground between sound and unsound. Each and every deductive argument is either sound or unsound.

Here are some examples of bad deductive arguments. For all of the bad arguments that are also valid, you can draw informal circles and dots to prove validity. The informal circles and dots test can also reveal invalidity. For each of the following arguments, one of the conditions, either validity or having all true premises, is not met, rendering them unsound arguments. The following argument is valid and has a true conclusion, but has obviously false premises.

P1: Philosophy is a type of food. (F)P2: All types of food are academic subject areas studied at universities. (F)

C: Philosophy is an academic subject area studied at universities. (T)

The following argument is valid, but has a false premise and a false conclusion.

P1: Either all bachelorettes are married persons or all rocks are hard. (T) P2: Not all rocks are hard. (F)

C: All bachelorettes are married persons. (F)

P2 is false, because based upon the geological definition of rocks, all rocks have to be solid, so all rocks are hard. C is false, because bachelorettes are unmarried by definition. The following argument, although it has all true premises and a true conclusion, is invalid.

P1: All roses are plants. (T) P2: All flowers are plants. (T)

C: All roses are flowers. (T)

This looks like a sound argument, because the premises and conclusion are true: P1, P2, and C are true by biological definition. However, the informal circles test shows that it is invalid. The following argument has all true premises, but is invalid and has a false conclusion.

P1: A banana is a fruit. (T) P2: A watermelon is a fruit. (T)

C: A banana is a watermelon. (F)

Again, the informal circle and dots test reveals invalidity, and C is false because a banana is a different type of fruit from a watermelon. The following is an argument that is invalid, has false premises, and a true conclusion.

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P1: Three plus one is a number equivalent to ten. (F)

P2: Sixteen is a number equivalent to ten. (F)

C: Three plus one is a number equivalent to four. (T)

The informal dots and circle test shows invalidity, and P1 and P2 are false because they are obvious mathematical errors, whereas C is not. Finally, here is an argument that is invalid, has one false premise, and a false conclusion.

P1: Thirteen is a prime number. (T) P2: All types of clothing are prime numbers. (F)

C: All types of clothing are thirteen. (F)

The informal circles and dot test again shows invalidity, and P2 and C are obviously false, although P1 is true, because thirteen is divisible by only itself and the number one.

In sum, if we want to see whether or not a deductive argument is a sound or an unsound argument, we first look to see if it is valid or invalid with the informal circles and dots test used in section 2.1. If it is invalid, then it is an unsound argument. If it is valid, then either all premises are true or at least one premise is false. If just one premise is false, then it is an unsound argument. However, if all of the premises are true, then it is a sound argument.

Now that we've seen some simple examples of sound and unsound arguments, let's look at and evaluate more complex arguments and examples in the following exercises.

[This is where the bullet point summary and section 2.4 exercises will be inserted.]

[This is where a chapter 2 cumulative practice test will be inserted.]

### **Chapter 2 Bibliography/References**

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# Chapter 3

# Induction

### Introduction

Deduction, as discussed in chapter 2, is reasoning where it is claimed that the truth of each of the premises implies the truth of the conclusion in all possible worlds, that is, the necessary truth of the conclusion. A good deductive, or sound, argument exists if this claim is successful. In some cases in real life, such sound arguments may be hard to come by and this is where induction comes in, in which the probable as opposed to the necessary truth of the conclusion comes into play. In section 3.1, you'll learn what induction in logic is, and what the related concepts of strength and weakness are. In section 3.2, you'll build upon section 3.1 and think and learn about factoring in the truth and falsity of the premises, both individually and as a whole, into inductive arguments for cogent and uncogent arguments, and will understand how to distinguish between them. Moving from theory to practice, in section 3.3, you'll learn and understand what some concrete and real-life inductive argument forms are. Finally, in sections 3.4, 3.5, and 3.6 respectively, you'll learn about some more specific topics within induction and inductive arguments, namely induction and scientific analysis, induction and statistical analysis, and Bayesian epistemology basics.

### **Objectives**

- Section 3.1: Understand the concepts of strength and weakness.
- Section 3.1: Know what inductive arguments are, and how strength and weakness relate to inductive arguments.
- Section 3.2: Understand what cogent and uncogent arguments are, and how they can be defined in terms of set theoretic notation.
- Section 3.3: Know what some inductive argument forms are, and what real life examples of each are.

# Section 3.1: Strength and Weakness

Section objectives:

- Understand the concepts of strength and weakness.
- Know what inductive arguments are, and how strength and weakness relate to inductive arguments.

#### **Key Terms**

Induction: the set of all inductive arguments as a whole

**Strength**: the property of an inductive argument whereby the conclusion of that argument is probably true, on the assumption that the factual claim is true in all possible worlds

**"Probably true:"** words in the definition of strength that can mean either "true in 61.80% of all possible worlds" or "true in the actual world

**Golden Ratio**:  $\varphi = (1 + \sqrt{5})/2 = 1.618033988...$ 

**Principle of the Uniformity of Nature**: things or events that are or occur in one spatiotemporal region tend to be similar to or also occur in others, assuming such things or events are true and/or factual

**Weakness**: the property of an inductive argument whereby the conclusion of that argument is probably false, on the assumption that the factual claim is true in all possible worlds

**Empty Set**: represented by " $\emptyset$ ," "{}," and "{ $\emptyset$ }," the one and only set or class that cannot have any elements, is a subset of itself without being a member of itself, and is also a subset of every other set

Vacuous Truth: a statement that is true only because its subject is empty

"Vacuously True:" words meaning to be true of something that is empty, or to be true by default

In parallel with the introduction to deduction, induction is another class of arguments. **Induction** is the set of all inductive arguments as a whole. It is the class of all inductive arguments in general. The members of this set consist in inductive arguments. Each and every thing or member of the set of induction is an inductive argument.

In section 2.1, arguments, as well as the inferential claim of an argument, were defined in terms of sets. It was discussed how the arguer can claim that the conclusion "follows from" the factual claim in one of two different ways:

- (1) in all possible worlds,
- (2) NOT in all possible worlds.

Those that are claimed to follow (1) in all possible worlds are deductive arguments where the conclusion follows by necessity, as discussed in 2.1. Inductive arguments are of type (2), instead. The conclusions of inductive arguments are claimed to follow from the premises NOT in all possible worlds, because they follow by probability. Arguments that make this claim may be either good or bad inductive arguments. Good versus bad inductive arguments are discussed in section 3.3.

The idea of the conclusion following from the premises not in all possible worlds, and furthermore by probability, leads into the definition of **strength (E)**. All inductive arguments claim or are claimed by some arguer to have strength:

The property E of an argument whereby the conclusion of that argument is probably true, on the assumption that all  $\varphi_n$ ,  $\alpha_n \subseteq \Delta$  or  $\Delta'$ <sup>1</sup> are true<sup>2</sup>.

<sup>&</sup>lt;sup>1</sup> These are the delta symbols for the factual claims of simple and complex arguments, see section 2.1.

 $<sup>^2</sup>$  Note that all premises and sub-conclusions are assumed to be true in the actual world, because the definition of truth from chapter 1 employed here is: a proposition (or premise/sub-conclusion in this case) has the truth-value TRUE (T) if and only if it is a fact. Generally, a fact is something that obtains in the actual world, so what is meant by a premise/sub-conclusion being true is that it is true in the actual world.

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To recapitulate from chapter 2,  $\Delta$  is the factual claim of a simple argument and  $\Delta'$  is the factual claim of a complex argument. E (strength) consists in the conjunction of two components (E = (a) + (b)):

(a) The assumption that the premises/sub-conclusions  $(\{\varphi_1, \varphi_2, \varphi_3, \varphi_4, \ldots, \varphi_n\} \subseteq \Delta$  or  $\{\varphi_1, \varphi_2, \varphi_3, \varphi_4, \ldots, \varphi_n, \alpha_1, \alpha_2, \alpha_3, \ldots, \alpha_n\} \subseteq \Delta')$  of argument  $\Gamma$  are true,

(b) The conclusion of  $\Gamma$  being probably true as a result of (a).

(b) follows upon (a) to give us "strength" for an inductive argument. What is meant here by "probably true"? This notion may seem vague. Returning to chapter 2, for a deductive argument, the conclusion is at least claimed to be true in all possible worlds. In contrast, for inductive arguments, by "probably true" means: true in at least 61.80% of all possible worlds, thus not necessarily in the actual world.

But where does this number, 61.80%,<sup>3</sup> come from? It comes from 1 divided by the number known as the **golden ratio** as a percentage of the area under the normal distribution curve:

Golden Ratio =  $\varphi = (1 + \sqrt{5})/2 = 1.618033988...$ 

 $1/\phi = .618033989... \approx 61.80\%.$ 

By the area under the normal distribution curve, I am referring to shading in images such as the following:



Figure 3.1: Normal Distribution Curve

For the reasoning as to why 61.80% is the specific numerical percentage employed as the percentage of all possible worlds for what counts as being **probably true**, see the Appendix. In general, this specific numerical percentage seems to be the best when compared to others because it is comfortably over the crest of the normal distribution curve, and is known to occur frequently in certain proportions observed in the natural world. It is related to "nature" in the **Principle of the Uniformity of Nature (PUN)**, which is discussed in connection with an inductive argument's strength below. This number comes from the facts about nature, so it makes sense that it would be related to PUN. It can be used as a sort of heuristic device when thinking about all of the possible worlds. Something being true in 61.80% of all possible worlds intuitively correlates with that thing being probably true. Perhaps the reader will see this for themselves.

Here are some further ways of thinking about what strength (E) is for an inductive argument in general:

- Where, assuming the premises are true, the conclusion probably follows.
- Where, it is improbable that the conclusion be false given that the premises are true.
- Where, there is at least a 61.80% chance that there is a possible world where the premises/subconclusions are true and the conclusion is true as well.

<sup>&</sup>lt;sup>3</sup> This percentage may also be rooted in Bayesian epistemology, see section 3.6.

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- -Where, there is 38.20% or smaller chance that there is a possible world where the premises/sub-conclusions are true and the conclusion is false.
- Where, the truth of the conclusion is probable, and not completely guaranteed.
- Where, the conclusion likely follows given that the premises/sub-conclusions are true.
- Where, it is plausible that the conclusion follows given that the premises/sub-conclusions are true
- Where, it is reasonable to conclude that the conclusion follows given that the premises/subconclusions are true.

And there may be others.

Here are some examples of inductive arguments claiming to have strength (E).

 $\varphi_1$ : Since 2003, a tropical cyclone has hit New Jersey every year.

 $\psi$ : A tropical cyclone will hit New Jersey this year.

In this example, the imagined arguer claims that this argument has strength because (a) assuming that the premise  $\{\varphi_1\} \subseteq \Delta$  is true, (b) the conclusion  $\psi$  is probably true as a result of (a). Tropical cyclones hitting New Jersey every year since 2003 are claimed to give enough support so that  $\psi$  is probably true. Returning to the meaning of "probably true" above,  $\psi$  is claimed to be true in at least 61.80% of possible worlds, because real-life people experience tropical cyclones in New Jersey frequently. (The claim of strength here happens to be a successful claim).

 $\varphi_1$ : The S & P 500 stock market index reached a peak today.

 $\varphi_2$ : Stock market index peaks are like the peaks and valleys on sine waves.

 $\psi$ : The S & P 500 stock market index will reach a valley equivalent to an all-time low soon.

In this example, it is claimed that this argument has strength because (a) assuming that the premises  $\{\varphi_1, \varphi_2\} \subseteq \Delta$  are true, (b) the conclusion  $\psi$  is probably true as a result of (a). The S & P 500 stock market index peaks and valleys are claimed to be like the peaks and valleys on sine waves in mathematics, and thus the peak will reach a valley equivalent to an all-time low soon. Returning to the meaning of "probably true" above,  $\psi$  is claimed to be true in at least 61.80% of all possible worlds, because the analogy being drawn between the stock market index peaks and valleys and the peaks and valleys of sine waves may seem to involve about a 60% similarity when it comes to mental imagery (see below). (It does not; there are too many details left out in this analogy - the claim of strength here happens to be an unsuccessful claim).

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Figure 3.2: Stock Market Index and Sine Wave Peaks and Valleys

It is one thing for the arguer to claim that an argument has strength, but whether or not the argument itself has objective strength is another issue. In other words, an arguer may claim that the conclusion of an argument follows as probably true from the premises, but the arguer's claim does not imply that this same claim is successful. This claim may be either successful or not. If this claim is successful, then there is strength. An inductive argument is strong if and only if:

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(i) E (strength) is true<sup>4</sup> of  $\Gamma$  (inductive argument)

OR

(ii) its inferential claim is true<sup>5</sup>

OR

(iii)  $E \subseteq \Gamma^6$ .

Regarding the second option above, an inductive argument's inferential claim is true if and only if the inferential claim is consistent with the principle of the uniformity of nature (PUN)<sup>7</sup> (Hurley and Watson, 2018). The principle of the uniformity of nature states that things or events that are or occur in one spatiotemporal region tend to be similar to or also occur in others, assuming such things or events are true and/or factual. In other words, the future tends to replicate the past, and regularities in one spatiotemporal region tend to be regularities in others, assuming such things or events are true and/or factual. Furthermore, the inferential claim is consistent with PUN if it is based upon the following to a high degree:

- Past observations
- One's experiences
- What one would naturally expect to occur
- What would be the least surprising
- Similarities
- Other probabilistic support
- and conditioned prior beliefs may be a factor...<sup>8</sup>

again, assuming the above factors are true and/or factual. PUN carries with itself the assumption that such things, events, and factors in question are true and/or factual. Regarding PUN itself, this assumption is not questioned.

Past observations could include: "all rocks have been observed to be gray," "most dinosaur fossils have been found in the desert." One's experiences could include: "some places in India are very rainy," "all sweaters of this type I've observed are pink with white polka dots." What one would naturally expect to occur could include: "thunder occurs shortly after the presence of lightning," "based on Shantaya's past personality, she would be friendly in this situation." What would be the least surprising could include: "I expect the sidewalk to be wet when it rains," "environmental scientists expect global atmospheric temperatures to increase in the presence of fossil fuels." Similarities could include: "this blue Jeep resembles that black Jeep," "these Australian aboriginals resemble these Native Americans." Other probabilistic support could include: "in a group of exactly 23 people, there is at least a 50% chance of two people by comparison having the same birthday," and similar statements.

<sup>&</sup>lt;sup>4</sup> Cf. section 1.2: a proposition has the truth-value TRUE (T) [or is true] if it is a fact. A proposition has the truth-value FALSE (F) [or is false] if it is not a fact. This same definition applies here. E is true if it is a fact.

<sup>&</sup>lt;sup>5</sup> This is another way of putting the former "E is true of  $\Gamma$ ."

<sup>&</sup>lt;sup>6</sup> This is another way of putting the former two, "E is true of  $\Gamma$ " and "its inferential claim is true." This way may be relevant for set theory connections later on in this book.

<sup>&</sup>lt;sup>7</sup> David Hume, in his work *A Treatise on Human Nature*, argues that PUN cannot be based upon any reasoning. There is only either demonstrative/necessary or moral/"probable" reasoning, and PUN is either based upon this demonstrative or "probable" reasoning. It cannot be based upon demonstrative reasoning, because demonstrative reasoning establishes conclusions that hold with necessity, whereas PUN does not, and it cannot be based upon "probable" reasoning, because in that case it would beg the question (PUN, which is probable reasoning, is based upon probable reasoning). See (Henderson, 2020). For reasoning as to why PUN in this book does not beg the question (or involve an infinite regress), I argue in response to Hume's objection and the scholarly debate surrounding Hume's view, please see the Appendix.

<sup>&</sup>lt;sup>8</sup> See section 3.6 for Bayesian epistemology basics.

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If another way of putting (i) above is (ii), then E (strength) is the same as the inferential claim and vice versa. Here's how. Consistency with PUN is basically the same as being true in at least 61.80% possible worlds. Both PUN and 61.80% come from the golden ratio ( $\varphi$  with no subscript). Consistency with past observations, one's experiences, what one would naturally expect to occur, what would be the least surprising, similarities, and other probabilistic support etc. intuitively would support the conclusion being probably true, or true in at least 61.80% of all possible worlds.

Here are some examples of strong inductive arguments. An inductive argument that has strength can in fact have:

true premises/sub-conclusions and a probably true conclusion, such as the following,

 $\varphi_1$ : Every map of the United States that I (the author) have seen shows California on the Pacific Coast.

 $\psi$ : California is a Western state.

The premise  $\varphi_1$  and the conclusion  $\psi$  are each true because they are facts.<sup>9</sup> This inductive argument is a strong argument, because E is true of it. Assuming that  $\varphi_1$  is true (aside from the fact that it is),  $\psi$  is probably true.  $\psi$  is probably true, or true in at least 61.80% of all possible worlds, because I have seen over 400 maps of the U.S. showing California on the Pacific Coast, and 400 is the number associated with a 95%  $\geq$  61.80% confidence interval for generalizations such as in this example.<sup>10</sup>

It can have false premise(s)/sub-conclusion(s) and a probably false conclusion, such as the following,

 $\varphi_1$ : In my experience, all cereal boxes have flowers on them.

 $\varphi_2$ : Cereal boxes are similar to frogs.

 $\alpha_1$ : The next frog I see will have flowers on it.

 $\varphi_3$ : Flowers are similar to dumptrucks.

 $\psi$ : The next frog I see will have dumptrucks on it.

Premises  $\varphi_1$ ,  $\varphi_2$ ,  $\varphi_3$  and sub-conclusion  $\alpha_1$  each are false because each is not a fact.<sup>11</sup> However, this inductive argument is still a strong argument, because its inferential claim is true. Assuming that  $\varphi_1$ ,  $\varphi_2$ ,  $\varphi_3$  and  $\alpha_1$  each are true and/or fact (even though they aren't), the inferential claim ({ $\varphi_1, \varphi_2, \varphi_3, \alpha_1$ }  $\subseteq$  $\Delta$ ') is consistent with PUN.  $\varphi_1$  is based upon past observations.  $\varphi_2$  and  $\varphi_3$  are based upon similarities.  $\alpha_1$ is based upon what one would naturally expect to occur. If you have observed that all cereal boxes have flowers on them, and if cereal boxes are similar to frogs, then you would naturally expect that frogs would have flowers on them as well, because they are similar. Likewise, going from  $\alpha_1$  and  $\varphi_3$  to  $\psi$ , if the next frog seen will have flowers on it, and if flowers are similar to dumptrucks, then you would naturally expect that the next frog seen will also have dumptrucks on it, in virtue of that similarity.

It can have false premise(s)/sub-conclusion(s) and a probably true conclusion, such as the following,

 $\varphi_1$ : All the past U.S. presidents have been women.

<sup>&</sup>lt;sup>9</sup> See footnote 2.

<sup>&</sup>lt;sup>10</sup> For more information on confidence intervals, see section 3.5.

<sup>&</sup>lt;sup>11</sup> See footnote 2.

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 $\psi$ : A future U.S. president will eventually be a woman (with all other things being equal).

Premise  $\varphi_1$  is false, because no past U.S. presidents from 2022 and before have been women, but conclusion  $\psi$  is probably true because in the long run that seems to be where the presidency is heading. However, this inductive argument is still a strong argument, because E is true of it. Assuming that  $\{\varphi_1\} \subseteq$  $\Delta$  is true (even though it isn't;  $\varphi_1$  is false),  $\psi$  is probably true, or true in at least 61.80% of all possible worlds. A skilled position being held by women 100% in the past would have at least a 61.80% chance of being held by a woman again in the future, with all other things being equal.

It CANNOT in fact have all true premises and a probably false conclusion, however. Strength, by definition, is a probably true conclusion following from all (assumed) true premise(s)/subconclusion(s); a probably false conclusion following from all true premises is inconsistent with this definition. For the other options above, starting with at least one false premise is irrelevant to the notion of strength, and a probably true conclusion following from all true premises is consistent with the definition of strength.

Returning to the arguer's claim for an inductive argument in general, if this claim is unsuccessful, then there is weakness. An inductive argument is weak if and only if:

E (strength) is false<sup>12</sup> of  $\Gamma$  (argument)

OR

its inferential claim is false<sup>13</sup>

OR

 $E = \emptyset \subseteq \Gamma^{14}.$ 

" $\emptyset$ " here is **the empty set**, or class.<sup>15</sup> The empty set can also be represented by "{}." The empty set is the one and only set that cannot have any elements. It is the unique collection of no objects. There are no elements of the empty set. However, it is a subset of itself (  $\{\emptyset\}$  )<sup>16</sup>, without being a member of itself, and it is also a subset of every other set17, each of which are necessarily non-empty and have at least one member.<sup>18</sup> For " $E = \emptyset \subseteq \Gamma$ ," E (strength) is vacuously true of  $\Gamma$ . A vacuous truth is a statement that is true only because its subject is empty, and to be "vacuously true" means to be true of something that is empty, or by default. E is true of  $\Gamma$  because it is the empty set, which is a subset of every set, including  $\Gamma$ . It is true of  $\Gamma$  by default, and not because any substantive claim about  $\Gamma$  is being made. The claim is meaningless.

If E is false of  $\Gamma$ , then:

<sup>&</sup>lt;sup>12</sup> See footnote 3.

<sup>&</sup>lt;sup>13</sup> This is another way of putting the former.

<sup>&</sup>lt;sup>14</sup> This is another way of putting the former two, and may be relevant for set theory connections later on in this book.

<sup>&</sup>lt;sup>15</sup> For a proof that the empty or null class is also a set, see Appendix.

<sup>&</sup>lt;sup>16</sup> The empty set is a subset of itself because no subclass that is it has any elements to begin with. This property is also known as "vacuous transitivity." The empty set is also "vacuously swelled," because, if it is both the antecedent and consequent in a conditional statement, the entire conditional statement is true (because both are false because it registers as false). <sup>17</sup> For a proof that the empty set is a subset of every other set, see the Appendix.

<sup>&</sup>lt;sup>18</sup> Additionally, George Boole 1847 thought of the empty set as nothing or an elective symbol "0," Gottlob Frege 1884 thought of zero as a logical object or extension of the concept "not identical with itself" (but not as a set or class, Georg Cantor 1880 thought of the empty set as a sort of predication for being empty, Ernest Zermelo 1908 considered the existence of an empty set to be an axiom in set theory, and Felix Hausdorff 1914 thought that the empty set A = 0 "vanishes" and exists with no elements (Kanamori 2003, 275-276).

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the conclusion of the inductive argument is probably false, on the assumption that  $\Delta$  or  $\Delta'$  are true.

In other words, the argument is inconsistent with the definition of strength. If its inferential claim is false, then the inferential claim will not be consistent with PUN, and will not be based upon the abovementioned factors to a high degree. "Probably false" means:

false in at least 61.80%, OR true in 38.20% or fewer of all possible worlds

As you can see, **weakness** is the opposite of strength. Regarding inductive arguments, there is no middle ground between strong and weak. Each and every inductive argument is either strong or weak, because either the conclusion is probably true or not probably true (probably false), assuming that all the premises are. An inductive argument is weak independently of all of its premises/sub-conclusions and conclusion being true or false. Here is an example of a weak inductive argument with all true premises and a true conclusion:

 $\varphi_1$ : Princeton University admitted 4.38% of undergraduate applicants in 2021.

 $\psi$ : Hurricane Ida hit the United States in 2021.

Both premise  $\varphi_1$  and conclusion  $\psi$  are true because they are facts.<sup>19</sup> However, this inductive argument is weak because E is false of it. Assuming that  $\varphi_1$  is true in all possible worlds (and it is),  $\psi$  is probably false. It is false in at least 61.80% of all possible worlds, because there is no known significant or direct correlation between Princeton University's admission rate and tropical cyclones hitting the United States. It seems there is a less than 1% correlation between a university's admission rate and a certain hurricane hitting the United States in a given year. The association between the two is random. Additionally, it is a weak argument because its inferential claim is inconsistent with PUN. One would not expect a hurricane to occur due to Princeton's admissions rate being a certain percentage, all in the same year.

There can also be a weak argument with all true premise(s)/sub-conclusion(s) and a probably false conclusion, false premise(s)/sub-conclusion(s) and a probably true conclusion, false premise(s)/sub-conclusion(s) and a probably false conclusion.

Now that we've learned about inductive arguments, and what strength and weakness is for inductive arguments, let's transition into some exercises testing our understanding and knowledge of these concepts.

[This is where the bullet point summary and section 3.1 exercises will be inserted.]

# Section 3.2: Cogency and Uncogency

Section objective: Understand what cogent and uncogent arguments are, and how they can be defined in terms of set theoretic notation.

#### **Key Terms**

 $\rightarrow$ : the symbol that means "if A then B," or "B follows from A," or "assuming A, then B," where A is whatever is to the left side of the arrow and B is whatever is to the right side of the arrow

**Good argument (Set theory)**: a type of argument in which both  $\Delta$  (or  $\Delta'$ ) and  $\Delta$  (or  $\Delta'$ )  $\rightarrow \psi$  are true

<sup>&</sup>lt;sup>19</sup> See footnote 3.

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**Bad argument (Set theory)**: a type of argument in which it is not the case that both  $\Delta$  (or  $\Delta'$ ) and  $\Delta$  (or  $\Delta'$ )  $\rightarrow \psi$  are true

**Cogent argument**: a good inductive argument that is both a strong argument, or has its inferential claim true, and has all true premises and the total evidence requirement met, or has its factual claim true

**Cogent argument (Set theory)**: and inductive argument in which **both**  $E \subseteq \Gamma$ ; **and** for all  $\phi_n$ ,  $a_n$  in  $\{\phi_1, \phi_2, \phi_3, \phi_4, \ldots, \phi_n, \alpha_1, \alpha_2, \alpha_3, \ldots, \alpha_n\} \subseteq \Delta'$  (or  $\Delta$ ),  $\phi_n$ ,  $a_n$  is true; **and** for each  $\phi_n$ ,  $\alpha_n$  in  $\{\phi_1, \phi_2, \phi_3, \phi_4, \ldots, \phi_n, \alpha_1, \alpha_2, \alpha_3, \ldots, \alpha_n\} \subseteq \Delta'$  (or  $\Delta$ ),  $\phi_n$  and  $\alpha_n$  are coherent with D, where D is the domain of the conclusion such that  $\psi \in D$ 

**Uncogent argument**: a bad inductive argument in which it is not the case that it is both a strong argument, or has its inferential claim true, and has all true premises and the total evidence requirement met, or has its factual claim true

**Uncogent argument (Set theory)**: an inductive argument in which either  $E = \emptyset \subseteq \Gamma$ ; or for some  $\phi_n, a_n$  in  $\{\phi_1, \phi_2, \phi_3, \phi_4, \dots, \phi_n, \alpha_1, \alpha_2, \alpha_3, \dots, \alpha_n\} \subseteq \Delta'$  (or  $\Delta$ ),  $\phi_n, a_n$  is false; or for some  $\phi_n, \alpha_n$  in  $\{\phi_1, \phi_2, \phi_3, \phi_4, \dots, \phi_n, \alpha_1, \alpha_2, \alpha_3, \dots, \alpha_n\} \subseteq \Delta'$  (or  $\Delta$ ),  $\phi_n$  or  $\alpha_n$  is not coherent with D, where D is the domain of the conclusion such that  $\psi \in D$ .

**Total evidence requirement**: all evidence relevant to an inductive argument cannot be left out by the premises as a whole

**Total evidence requirement (Set theory)**: a cogent's requirement for the premises and sub-conclusions that for each  $\phi_n$ ,  $\alpha_n$  in the set  $\{\phi_1, \phi_2, \phi_3, \phi_4, \dots, \phi_n, \alpha_1, \alpha_2, \alpha_3, \dots, \alpha_n\} \subseteq \Delta'$  (factual claim),  $\phi_n$  and  $\alpha_n$  are coherent with D, where the set D is the domain of the conclusion such that  $\psi \in D$ 

**Being true as a whole**: a property that applies to premises and sub-conclusions when they meet the total evidence requirement

"Evidence" (Total evidence requirement): anything in the actual world

**Coherence (Total evidence requirement)**: a property of the premises  $\phi_n$  and sub-conclusions  $\alpha_n$  in an inductive argument as part of the total evidence requirement, where each  $\phi_n$ ,  $\alpha_n$  fits with all of the evidence in D and no  $\phi_n$ ,  $\alpha_n$  contradicts any of the evidence in D

 $\epsilon$ : the symbol that is the memberships relation in set theory, meaning "is a member (or element) of "

**Domain of the conclusion (Total evidence requirement)**: the set of the general subject matter that the conclusion is a member of, and also where the subject and predicate of the conclusion intersect

Subject: in general, all the words that occur before the verb in a statement

Predicate: in general, all the words including the verb and those that follow after in a statement

Good and bad arguments have already been discussed, starting in chapter 1. Returning to 1.1, good and bad arguments were defined in a very general way. A **good argument** succeeds in what it claims to do. One or more of the statements, the premises, objectively succeed in proving that another one, the conclusion, is true. A **bad argument** does not succeed in what it claims to do. None of the statements that are premises objectively succeed in proving that another of the statements, the conclusion, is true.

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Then, in 1.2, arguments were defined more specifically in terms of factual and inferential claims. In order for an argument to have its characteristic claim that a statement is being proven true, it must have both a factual claim and an inferential claim.

Remember,

FACTUAL CLAIM (SIMPLE ARGUMENT) = PREMISE(S),

or FACTUAL CLAIM (COMPLEX ARGUMENT) = PREMISE(S) + SUB-CONCLUSION(S)

INFERENTIAL CLAIM = THE CLAIM THAT THE CONCLUSION FOLLOWS FROM THE FACTUAL CLAIM

Furthermore, in 2.1 simple set theoretic notation for arguments, factual claims, and inferential claims was introduced.

An argument is a set  $\Gamma$  of at least two statements, call  $\phi_1$  and  $\psi$ , where at least one member  $\phi_1$  is a premise and at least one member  $\psi$  is a conclusion. There may be more members  $\phi_2$ ,  $\phi_3$ ,  $\phi_4$ , ... etc., that are premises, and, if the argument is a complex argument, then more members  $\alpha_1$ ,  $\alpha_2$ ,  $\alpha_3$ , ... etc., may be sub-conclusions.

A factual claim for a simple argument is the strict subset of an argument consisting of all the statements that are premises:

the subset  $\Delta \subset \Gamma$  such that all premises  $\{\phi_1, \phi_2, \phi_3, \phi_4, \ldots, \phi_n\} \subseteq \Delta$ .

A factual claim for a complex argument is the strict subset of an argument consisting of all the statements that are either premises or sub-conclusions:

the subset  $\Delta' \subset \Gamma$  such that all premises of  $\Gamma \{\phi_1, \phi_2, \phi_3, \phi_4, \ldots, \phi_n\} \subseteq \Delta'$  and all subconclusions of  $\Gamma \{\alpha_1, \alpha_2, \alpha_3, \ldots, \alpha_n\} \subseteq \Delta'$ .

An inferential claim is the claim that the conclusion follows from the subset consisting of all the statements that are either premises or sub-conclusions, assuming these are true in all possible worlds:

 $\Delta$  (or  $\Delta'$ )  $\rightarrow \psi$ .

Here, " $\rightarrow$ " means "if A then B," or "B follows from A," or "assuming A, then B," where A is whatever is to the left side of the arrow and B is whatever is to the right side of the arrow. Above, A is " $\Delta$  (or  $\Delta'$ )," and B is " $\psi$ ." What is meant here is assuming { $\phi_1, \phi_2, \phi_3, \phi_4, \ldots, \phi_n, \alpha_1, \alpha_2, \alpha_3, \ldots, \alpha_n$ }  $\subseteq \Delta$ ,  $\Delta'$  is true in all possible worlds,  $\psi$  is probably true. This symbol is discussed in more depth in chapter 8 with propositional logic.

We define good and bad arguments in a more precise way. Returning to 2.4, another way of saying the premises of an argument objectively succeed in proving its conclusion to be true, or not, is:

- (1) An argument is a good argument if and only if both the factual claim and the inferential claim are true; otherwise, it is a bad argument.
- (2) Expanding upon this definition of good and bad arguments using set theoretic notation, we get:
- (3) An argument is a good argument if and only if both  $\Delta$  (or  $\Delta'$ ) and  $\Delta$  (or  $\Delta'$ )  $\rightarrow \psi$  are true; otherwise, it is a bad argument

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In chapter 2, a good deductive argument was defined as a sound argument, and a bad deductive argument was defined as an unsound argument. Transitioning to inductive arguments, a good inductive argument is a **cogent argument**, and a bad inductive argument is an **uncogent argument**. Only inductive arguments can be cogent or uncogent arguments. In other words, an inductive argument is a good argument if and only if it is a cogent argument, and an inductive argument is a bad argument if and only if it is an uncogent argument.

There is no middle ground between cogent and uncogent. Each and every inductive argument is either cogent or uncogent. What is a cogent or uncogent argument? From (i) above, a cogent argument has factual and inferential claims that are true. Another way of saying that its factual claim is true is that each of the premises or sub-conclusions are true and the **total evidence requirement** is met. Another way of saying that its inferential claim is true is that the argument is a strong argument, or has strength. Cogency builds upon strength, which was discussed in section 3.1. For a cogent argument, each of the premises or sub-conclusions are true, and the total evidence requirement is met, in addition to the argument being strong. Otherwise, if either of these claims is false, or one of the three conditions not met, the inductive argument is an uncogent argument. If either one premise is false or the total evidence requirement not met, then the entire factual claim is false. In other words,

an inductive argument is a cogent argument if and only if it is both a strong argument, or its inferential claim is true, and both each of its premises are true and the total evidence requirement is met, or its factual claim is true. Otherwise, an inductive argument is an uncogent argument.

Here is another way of thinking about a cogent argument:

Cogent argument = Strong argument (Inferential claim is true) + Each of premises or subconclusions true and total evidence requirement met (Factual claim is true)

Again, if one of these conditions is not met, or one of these claims false, then it is uncogent.

As part of the factual claim being true, the total evidence requirement must be met.<sup>20</sup> The total evidence requirement is met if and only if the premises are true as a whole, or in a complete sense. What is meant here by "true as a whole, or in a complete sense"? In the words of Patrick J. Hurley and Lori Watson, "[t]he premises must not exclude or overlook some crucial piece of evidence that undermines the stated premises and requires a different conclusion" (Hurley and Watson 2018, 51). Namely, all evidence relevant to the domain of the conclusion and the inductive argument itself must be taken into account in the premises and/or sub-conclusions. In sum, we may stipulate the following definitions.

The premises in a simple inductive argument are true as a whole if and only if they meet the total evidence requirement.

The premises and sub-conclusions in a complex inductive argument are true as a whole if and only if they meet the total evidence requirement.

The premises in a simple inductive argument meet the total evidence requirement if and only if they do not leave out any of the evidence relevant to the inductive argument that it is in.

The premises and sub-conclusions in a complex inductive argument meet the total evidence requirement if and only if they do not leave out any of the evidence relevant to the inductive argument that it is in.

<sup>&</sup>lt;sup>20</sup> The total evidence requirement has also been discussed in Hurley and Watson (2018), cf. page 51.

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What is meant here by **"evidence**"? It means "anything in either the physical or non-physical actual world." The words "physical or non-physical actual world" are emphasized in this definition in order to prevent cognitive bias, especially cognitive bias for things in the physical actual world over things in the non-physical actual world. The actual world is discussed in 2.2. We may also define the total evidence requirement using set theoretic notation, in order to maintain consistency with the previous definitions in other sections.

The premises  $\phi_1, \phi_2, \phi_3, \phi_4, \ldots, \phi_n$  in a simple inductive argument meet the total evidence requirement if and only if for each premise  $\phi_n$  in the set  $\{\phi_1, \phi_2, \phi_3, \phi_4, \ldots, \phi_n\} \subseteq \Delta$  (factual claim)  $\subset D$ ,  $\phi_n$  is coherent with D, where the set D is the domain of the conclusion such that  $\psi \in D$ .

The premises  $\phi_1, \phi_2, \phi_3, \phi_4, \ldots, \phi_n$  and sub-conclusions  $\alpha_1, \alpha_2, \alpha_3, \ldots, \alpha_n$  in a complex inductive argument meet the total evidence requirement if and only if for each  $\phi_n, \alpha_n$  in the set  $\{\phi_1, \phi_2, \phi_3, \phi_4, \ldots, \phi_n, \alpha_1, \alpha_2, \alpha_3, \ldots, \alpha_n\} \subseteq \Delta'$  (factual claim),  $\phi_n$  and  $\alpha_n$  are coherent with D, where the set D is the domain of the conclusion such that  $\psi \in D$ .

What is meant here by "is/are coherent with"? This phrase here means "fits with all and doesn't contradict any evidence in."  $\phi_n$  is coherent with D if and only if each  $\phi_n$  fits with all of the evidence in D and no  $\phi_n$  contradicts any of the evidence in D. Fitting with all and not contradicting any of the evidence is another way of saying that no relevant evidence that could change the conclusion is left out, as described in the non-set theoretic definition. Note that circular justifications, where some  $\phi_n$  justifies some  $\phi_{n'}$  and the same  $\phi_{n'}$  justifies the same  $\phi_n$ , may not matter here because the sense of justification of the premises is one that is more systematic and holistic. The total evidence requirement by definition focuses on the truth of the premises as a whole, and not on the premises individually. With inductive arguments, the truth of the premises as a whole comes into play, because the conclusions of such arguments are not claimed to be true in all possible worlds, contrary to deductive arguments, where the truth of the premises as a whole is irrelevant, because the conclusions of such arguments are claimed to be true in all possible worlds.

What is meant here by " $\in$ "? This symbol is the membership relation in set theory, meaning "is a member (or element) of." Here, a member and element are the same thing. They both refer to objects in sets, represented by variables such as x, y, z, a, b, c....etc., in set theory.  $x \in Y$  means "x is a member (or element) of Y," where x is an object and Y is a set. In the context above,  $\psi \in D$  means " $\psi$  is a member (or element) of D," or the conclusion  $\psi$  is a member of the set of the domain of the conclusion.

What exactly is the **domain of the conclusion**, D? This is the set of the general subject matter that the conclusion is a member of. Any statement will always have both a subject and a predicate, where the **subject** in general is all the words before the verb, and the **predicate** in general is all the words including the verb and following. The domain of the conclusion is where the subject and predicate intersect. Take, for example, the conclusion "Climate change is dependent upon the rate of fossil fuel exchange with the environment." The subject in general here is climate change, and the predicate in general here is fossil fuel environmental exchange rates. For this example conclusion, then, the domain of the conclusion would be the set where the set of climate change matters or things and the set of fossil fuel environmental exchange rates intersect or overlap.

Here is an example of an argument that meets the total evidence requirement.

 $\phi_1$ : Human energy output follows the first law of thermodynamics: the rate of change of bodily macronutrient stores is equal to the difference between energy intake and energy expenditure (E<sub>S</sub> = E<sub>I</sub> - E<sub>0</sub>).<sup>21</sup>

<sup>&</sup>lt;sup>21</sup> See Hall et. al. (2012).

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 $\phi_2$ : Numerous studies have been published indicating a link between energy expenditure and caloric requirement.<sup>22</sup>

 $\phi_3$ : Studies suggesting no link between energy expenditure and caloric requirement have been plausibly explained away.<sup>23</sup>

 $\psi$ : Energy expenditure for each biological human being requires a certain amount of calories.

The premises  $\phi_1$ ,  $\phi_2$ ,  $\phi_3$  meet the total evidence requirement because they do not leave out any of the evidence relevant to the inductive argument it is in. The argument concerns energy expenditure and caloric requirement for humans.  $\phi_2$  states this link has verification in numerous studies, and  $\phi_3$  states that the opponents, the studies contradicting this verification of this linkage, can be plausibly explained away. A more universal law of physics, the first law of thermodynamics, is stated in  $\phi_1$  to ground energy expenditure requiring a certain amount of calories in a more universal way. Because both sides are taken into account, and a more universal scientific law is employed in support, no relevant evidence is left out. Additionally, each premise  $\phi_1$ ,  $\phi_2$ ,  $\phi_3$  is coherent with D, which, in this example is "energy expenditure and caloric requirement." Each premise fits with all of the evidence in D, in this case the various statements about the science experiments and relevant scientific laws. All of the evidence in D would be either for this link or against it, and there are multiple experiments for it and plausible explanations of the contrary experiments, making  $\phi_2$  and  $\phi_3$  consistent. The first law of thermodynamics gives general support, and the experiments and the explaining away thereof give particular support, making  $\phi_1$ ,  $\phi_2$ , and  $\phi_3$  consistent. No premise contradicts any of the evidence in D, because it would take more evidence than currently available to undermine the premises' consistency.

Here is an example of a hypothetical argument that does not meet the total evidence requirement.

 $\phi_1$ : No one has observed a teddy bear that is black with white polka dots

 $a_1$ : No teddy bears are black with white polka dots.

 $\phi_2$ : No one has ever observed any stuffed animal on Pluto.

 $\psi$ : No teddy bears that are black with white polka dots are on Pluto.

(Bob has observed some teddy bears that are black with white polka dots on Pluto.)

These premises and sub-conclusions in this complex argument as a whole do not meet the total evidence requirement, because they leave out some relevant evidence, namely the statement in parentheses above, beneath the conclusion  $\psi$ . The evidence that Bob has observed some teddy bears that are black with white polka dots on Pluto is relevant to this argument, because  $\phi_1$  talks about observations of black teddy bears with white polka dots, and  $\phi_2$  talks about stuffed animals being on Pluto. However, this evidence is ignored. It could give evidence against the conclusion, because it undermines both  $\phi_1$  and  $\phi_2$ . Additionally, the factual claim is not coherent with the domain of the conclusion. The domain of the conclusion here is "black teddy bears with white polka dots and Pluto." The factual claim does not follow from the conjunction of all  $\phi_d$  in this domain, because both  $\phi_1$  and  $\phi_2$  contradict the evidence that Bob has observed some teddy bears that are black with white polka dots on Pluto, which evidence is a member of the domain. Bob is someone who has observed a teddy bear that is black with white polka dots, contradicting  $\phi_1$ , and Bob is someone who has observed a stuffed animal on Pluto, contradicting  $\phi_2$ .

<sup>&</sup>lt;sup>22</sup> See Hall, et. al. (2012), Johnstone et. al. (2005), Edholm, et. al. (1955), Levine et. al. (1999), Roberts et. al. (1996), Joosen et. al. (2005), Saltzman and Roberts (1995), Church et. al. (2011), Donnelly et. al. (2003), to name some.

<sup>&</sup>lt;sup>23</sup> See Hall et. al. (2012), Heymsfield et. al. (2007), Hall (2010), Hall (2008), Hall et. al. (2011), to name a few.

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Here is another way to think about a cogent argument. Recall, again from 3.1, the definition of strength:

The property E of an argument whereby the conclusion of that argument is probably true in all possible worlds, on the assumption that  $\Delta$  or  $\Delta'$  are true in all possible worlds.

- E (strength) thus consists in the conjunction of two components (E = (a) + (b)):
- (1) The assumption that the premises/sub-conclusions ( $\Delta$  or  $\Delta'$ ) of argument  $\Gamma$  are true in all possible worlds,
- (2) The conclusion of  $\Gamma$  being probably true in all possible worlds as a result of (a).

Cogency, here, just is strength with component (a) verified, plus the total evidence requirement being met.

Again, a cogent argument has to be both a strong argument and have both all true premises and the total evidence requirement met, or, in other words, both its factual and inferential claims have to be true. If all of these conditions are fulfilled, then the conclusion of the cogent argument is probably true.

We may now define a cogent and an uncogent argument in set theoretic notation. An inductive argument is a cogent argument if and only if

#### BOTH

E ⊆ Γ (E is true of Γ OR Γ's inferential claim [Δ (or  $\Delta'$ ) → ψ] is true ),

AND

for all  $\phi_n$ ,  $a_n$  in  $\{\phi_1, \phi_2, \phi_3, \phi_4, \dots, \phi_n, \alpha_1, \alpha_2, \alpha_3, \dots, \alpha_n\} \subseteq \Delta'$  (or  $\Delta$ ),  $\phi_n$ ,  $a_n$  is true,

AND

for each  $\phi_n$ ,  $\alpha_n$  in  $\{\phi_1, \phi_2, \phi_3, \phi_4, \dots, \phi_n, \alpha_1, \alpha_2, \alpha_3, \dots, \alpha_n\} \subseteq \Delta'$  (or  $\Delta$ ),  $\phi_n$  and  $\alpha_n$  are coherent with D, where D is the domain of the conclusion such that  $\psi \in D$ .

Otherwise, an inductive argument is an uncogent argument. For an uncogent argument, at least one of the claims is false.<sup>24</sup>

Here are some examples of good inductive arguments, each of which are strong, have all true premises and the total evidence requirement met, and thus a true conclusion as well.

 $\phi_1$ : Navy blue Jeeps have steering wheels.

 $\phi_2$ : Navy blue Jeeps are similar to yellow Jeeps.

 $^{24}$  Equivalently, an inductive argument is an uncogent argument if and only if EITHER

- $E = \emptyset \subseteq \Gamma$  (E is false of  $\Gamma / \Gamma$ 's inferential claim is false)
- OR

for some  $\phi_n$ ,  $a_n$  in  $\{\phi_1, \phi_2, \phi_3, \phi_4, \dots, \phi_n, \alpha_1, \alpha_2, \alpha_3, \dots, \alpha_n\} \subseteq \Delta'$  (or  $\Delta$ ),  $\phi_n$ ,  $a_n$  is false

OR

for some  $\phi_n$ ,  $\alpha_n$  in  $\{\phi_1, \phi_2, \phi_3, \phi_4, \dots, \phi_n, \alpha_1, \alpha_2, \alpha_3, \dots, \alpha_n\} \subseteq \Delta'$  (or  $\Delta$ ),  $\phi_n$  or  $\alpha_n$  is not coherent with D, where D is the domain of the conclusion such that  $\psi \in D$ .
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 $\varphi$ : The next yellow Jeep I observe will have a steering wheel.

This argument is strong because its inferential claim is consistent with PUN. Assuming  $\phi_1$  and  $\phi_2$  are true in all possible worlds,  $\varphi$  is probably true because  $\varphi$  is what one would naturally expect to be the case. If Jeeps of one color have steering wheels (and they do in fact), and we are comparing Jeeps of different colors, one would naturally expect the next Jeep you see of that different color to also have a steering wheel, maybe with at least a 90% probability or degree of certainty. In spacetime, the different colors of Jeeps in general do not change how each Jeep is manufactured with respect to its crucial parts, unless there is some sort of freak accident or occurrence.

Additionally, this argument has all true premises, because  $\phi_1$  is a fact and  $\phi_2$  is a fact that is somewhat of a tautology.<sup>25</sup> Navy blue Jeeps are similar to yellow Jeeps because they are both Jeeps.

Additionally, this argument meets the total evidence requirement. Both  $\phi_1$  and  $\phi_2$  as a whole are coherent with the domain of the conclusion, which is "observations of yellow Jeeps and steering wheels."  $\phi_1$  and  $\phi_2$  each fit with all of the evidence  $\phi_d$  in this domain, because they talk about Jeeps and steering wheels, and they do not contradict any of the evidence in this domain, because it is highly likely that all yellow Jeeps in the set of this domain have steering wheels, and also that all yellow Jeeps in this set can be compared with all other Jeeps of different colors. Because all three conditions are met, this argument is a cogent argument.

 $\phi_1$ : In France, from 2016 to 2017, 91% of aggressors of sexist acts were men.<sup>26</sup>

 $\alpha_1$ : In France, from 2016 to 2017, aggressors of sexist acts were men.

 $\phi_2$ : According to the Council of Europe's Human Rights Channel, sexist acts are based upon the idea that some persons are inferior because of their sex.<sup>27</sup>

 $a_2$ : Sexist acts are based upon the idea that some persons are inferior because of their sex.

 $\varphi$ : In France, from 2016 to 2017, aggressors of acts based upon the idea that some persons are inferior because of their sex were men.

This argument is a strong argument because E (strength) is true of it. Assuming  $\phi_1$ ,  $\phi_2$ ,  $\alpha_1$ ,  $\alpha_2$  are true, then  $\varphi$  is probably true. Reasoning from  $\phi_1$  to  $\alpha_1$ , if 91% of a group of people have a certain property, then it is probably true that that entire group has that same property. One would expect this to be the case because 91% is a significantly large proportion. Reasoning from  $\phi_2$  to  $\alpha_2$ , if an authority, the Council of Europe's Human Rights Channel in this case, makes a statement in their area of expertise, then that statement is probably true. This conclusion would be the least surprising to follow from  $\phi_2$  because experts by and large are trained to have accurate knowledge in their respective domains. Combining  $\alpha_1$  and  $\alpha_2$  together gives us  $\varphi$ , which one would expect to follow by definition.

Additionally, this argument has all true premises because both  $\phi_1$  and  $\phi_2$  are facts (see footnotes 24 and 25 for fact-checking and verification).

Additionally, this argument meets the total evidence requirement because  $\phi_1$ ,  $\phi_2$ ,  $\alpha_1$ , and  $\alpha_2$  as a whole are coherent with the domain of the conclusion, which is "aggressors of acts based upon the idea that some persons are inferior because of their sex and men in France from 2016-2017." This domain is specific

<sup>&</sup>lt;sup>25</sup> Cf. section 1.2: a proposition has the truth-value TRUE (T) [or is true] if it is a fact.

<sup>&</sup>lt;sup>26</sup> See Human Rights Channel (https://human-rights-channel.coe.int/stop-sexism-quiz-en.html) and HCE Republique

Francaise (https://www.haut-conseil-egalite.gouv.fr/IMG/pdf/hce\_etatdeslieux-sexisme-vf-2.pdf).

<sup>&</sup>lt;sup>27</sup> See Human Rights Channel (https://human-rights-channel.coe.int/stop-sexism-en.html).

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enough that each premise and sub-conclusion fits with it, and there are no contradictory definitions or statistical analyses. Because all three conditions are fulfilled, this argument is a cogent argument.

 $\phi_1$ : 84,335,708 people in the U.S. tested positive for COVID-19.<sup>28</sup>

p: Approximately 84,335,708 people in the U.S. have had COVID-19.29

(Although there may have been some evidence for false positive and false negative results).<sup>30 31</sup>

This argument is a strong argument because its inferential claim is consistent with PUN. Assuming 84,335,708 people in the U.S. tested positive for COVID-19, past observations of commercial testing for diseases in general in the U.S. would indicate that approximately the same number actually have COVID-19 in the U.S. In general, such tests have tended to be more accurate than not.

Additionally, this argument's premise is true because  $\phi_1$  is a fact.<sup>32</sup>

Additionally, this argument meets the total evidence requirement because  $\phi_1$  is coherent with the domain of the conclusion "number of people in the U.S. with COVID-19."  $\phi_1$  fits with the evidence because it gives the precise number in question with respect to positive tests, and it does not contradict any  $\phi_d$  in the domain. Although there has been relevant evidence of both false positive and false negative COVID-19 tests, these tests probably would not be significant enough to shift the actual number of persons with COVID-19, because the detraction from the total number of persons actually having had COVID-19 due to false positives would about equal the addition to the total due to false negatives. Because all three of the conditions are met, this argument is a cogent argument.

For an uncogent argument, again, at least one of the claims is false, so at least one premise is false, or the total evidence requirement is not met, or the argument is weak. Here are some examples of bad inductive, or uncogent, arguments.

 $\phi_1$ : A red Royal Gala apple is similar in shape to a Pink Crisp apple.

 $\phi_2$ : A Pink Crisp apple is similar in shape to a green Granny Smith apple.

 $\varphi$ : A Royal Gala apple is similar in color to a Granny Smith apple.

This argument has all true premises and meets the total evidence requirement, but it is weak. Both  $\phi_1$  and  $\phi_2$  are true because they are each facts. Furthermore, it meets the total evidence requirement, because both  $\phi_1$  and  $\phi_2$  as a whole are coherent with the domain of the conclusion, which here is "Royal Gala apple color and Granny Smith apple color." They fit with each piece of evidence  $\phi_d$  in the domain, because a Royal Gala apple's color is generally red and a Granny Smith apple's color is green, and  $\phi_1$  and  $\phi_2$  mention those colors even though they are mainly about shapes. They also do not contradict any evidence in the domain, because  $\phi_1$  states that a Royal Gala apple is red, and  $\phi_2$  states that a Granny Smith apple is green. The conclusion's domain here is small enough that other evidence is irrelevant.

However, this argument is weak because its inferential is false, or inconsistent with PUN. Assuming  $\phi_1$  and  $\phi_2$  are true (and they each are), it is unlikely that  $\varphi$  would be probably true. Similarities between

<sup>&</sup>lt;sup>28</sup> Statistic as of June 27, 2022 1:39 PM ET. See CDC COVID Data Tracker (https://covid.cdc.gov/covid-data-

tracker/#cases\_totaltests).

<sup>&</sup>lt;sup>29</sup> As of the date in footnote 24.
<sup>30</sup> See (Liu and Ruslin, 2021).

<sup>&</sup>lt;sup>31</sup> See (Boukli et. al., 2020).

 $<sup>^{32}</sup>$  See footnote 23.

See Toothole 23

apple shapes do not correlate with similarities between apple colors. Shapes and colors are distinct properties in spacetime. Because one of the conditions is not met, this argument is an uncogent argument.

 $\phi_1$ : 96% of the states in the United States of America are contiguous states of the United States.

 $\phi_2$ : Alaska is a state in the United States of America.

 $\varphi$ : Alaska is a contiguous state of the United States.

This argument has all true premises and is strong, but it does not meet the total evidence requirement.  $\phi_1$  is true because it is a fact. 48/50 = 96% of the states in the U.S. are contiguous, or grouped together uninterrupted by other land or water.  $\phi_2$  is true because it is a fact. This argument is a strong argument, because E is true of it.  $\varphi$  is probably true, on the assumption that  $\phi_1$  and  $\phi_2$  are true. If someone picks any state from the set of states in the U.S., then there is a 96% chance that that state is a contiguous state. 96% is greater than 61.80%, so it is true in at least 61.80% of the possible worlds that whatever state is chosen is a contiguous state. Because of the definition of "probably true," then, it is probably true that whatever state is picked, even if it is Alaska, is a contiguous state.

This argument does not meet the total evidence requirement, because  $\phi_1$  and  $\phi_2$  as a whole are not coherent with  $\varphi$ 's domain, which is "Alaska and contiguous states of the United States." It turns out this domain or set is the empty set ( $\varnothing$ ). This set is the set that has no members because Alaska cannot be a contiguous state of the U.S.; it is a non-contiguous state. The factual claim  $\Delta$  cannot fit with the domain because it has two members,  $\phi_1$  and  $\phi_2$ , whereas the empty set has no members. Because it has members and the domain does not, it also contradicts with the domain and the evidence in it. The fact that Alaska is a non-contiguous state is not considered. Then, because one of the conditions is not met, this argument is an uncogent argument.

 $\phi_1$ : All grizzly bears observed by scientists to date<sup>33</sup> have brown fur.

 $\phi_2$ : All brown fur observed to date is gobbledygook.

 $\phi_3$ : All gobbledygook things observed to date are fuzzy.

 $\varphi$ : The next grizzly bear observed by scientists will be fuzzy.

This argument is strong and meets the total evidence requirement, but it has false premises. It is a strong argument because its inferential claim is consistent with PUN. If all observations of certain things have had the same property in the past, and the observations of properties build upon each other, then one would naturally expect those certain things to all have some same property. In other words, if all past observations of thing X have property A, all past observations of things with property A also have property B, and all past observations of things with property B also have property C, then it is likely that the next observation of X will be that of X having C.

Additionally, this argument meets the total evidence requirement, because the premise does not leave out any evidence relevant to the argument that could end up changing the argument's conclusion.  $\varphi \in D$ , which is "grizzly bear observations and fuzziness." The only evidence relevant to this domain are the grizzly bear observations, stated in  $\phi_1$ , fuzziness, stated in  $\phi_3$ , and the connecting brown fur, where the fuzziness is on grizzly bears, stated in  $\phi_2$ .

<sup>&</sup>lt;sup>33</sup> This was last edited on June 28, 2022. This same date applies to  $\phi_2$  and  $\phi_3$ .

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However, both  $\phi_2$  and  $\phi_3$  are false, although  $\phi_1$  and the conclusion  $\varphi$  are each true in themselves. The word "gobbledygook" in the false premises is meaningless, and those premises are not facts. Because one of the required conditions is not met, then, this argument is an uncogent argument.

Now that we've seen some simple examples of cogent and uncogent arguments, let's look at and evaluate more complicated arguments and examples in the following exercises.

[This is where the bullet point summary and section 3.2 exercises will be inserted.]

## **Section 3.3: Inductive Argument Forms**

Section objective: Know what some inductive argument forms are, and what real life examples of each are.

#### **Key Terms**

**Prediction:** a type of argument in which the factual claim includes at least one statement in the present or past about members of a group having some property or attribute, and the inferential claim includes reasoning from these members of a group having some property or attribute to some future statement about some member(s) of this group having the same property or attribute

**Analogical Argument:** a type of argument in which the factual claim contains evidence that at least one entity or object has n properties and at least one entity has n+1 properties, and the inferential claim includes reasoning from this evidence to another entity having the  $n + 1^{nth}$  property

**Strong Analogical Argument:** an analogical argument in which there is either a systemic or causal relationship between all n properties in total and the  $n + 1^{nth}$  property(ies)

Weak Analogical Argument: an analogical argument in which it is not the case that there is either a systemic or causal relationship between all n properties in total and the  $n + 1^{nth}$  property(ies)

**Systemic Relationship:** a type of relationship that exists between properties in a strong analogical argument if and only if one, or a group of them, is a sufficient condition for another, or a group of others

**Causal Relationship:** a type of relationship that exists between properties in a strong analogical argument if and only if one, or a group of them, is a necessary condition for another, or a group of others

"group of them": in this textbook's definitions of "systemic relationship" and "causal relationship," what can either be the intersection or the union of the properties, more specifically the sets of the properties, in question

**Systemic Relationship (Sets):** a type of relationship that exists between properties if and only if one, or either the intersection or union of each of some properties in a group, is a subset of, or identical to or included within, another property, or either the intersection or union of each of others in a group

Natural Numbers: the whole numbers 0, 1, 2, 3, .... etc. counting to infinity

Systemic Relationship (Formal): set Z is in a systemic relationship with set Z' if and only if  $Z \subseteq Z'$ , and Set Z' is in a systemic relationship with set Z if and only if  $Z' \subseteq Z$ , where  $Z = \{z_1 \cup z_2 \cup z_3 \cup ... \cup z_n \text{ OR } z_1 \cap z_2 \cap z_3 \cap ... \cap z_n\}$ ,  $Z' = \{z_{n+1} \cup z_{n+2} \cup z_{n+3} \cup ... \cup z_{n+n'} \text{ OR } z_{n+1} \cap z_{n+2} \cap z_{n+3} \cap ... \cap z_{n+n'}\}$ , where  $z_1 ... z_n$ ,  $z_{n+1}$ ,  $... z_{n+n'}$  are properties and for all natural numbers n, n', n' > 0 **Causal Relationship** (Sets): a type of relationship that exists between properties if and only if one, or either the intersection or union of each of some attributes in a group, is a superset of, or identical to or encompassing, another property, or either the intersection or union of each of the others in a group

**Causal Relationship (Formal):** set Z is in a causal relationship with set Z' if and only if  $Z \supseteq Z'$ , and Set Z' is in a causal relationship with set Z if and only if  $Z' \supseteq Z$ , where  $Z = \{z_1 \cup z_2 \cup z_3 \cup \ldots \cup z_n \text{ OR } z_1 \cap z_1 \cup z_2 \cup z_3 \cup \ldots \cup z_n \}$  $z_2 \cap z_3 \cap \ldots \cap z_n$ ,  $Z' = \{z_{n+1} \cup z_{n+2} \cup z_{n+3} \cup \ldots \cup z_{n+n'} \text{ OR } z_{n+1} \cap z_{n+2} \cap z_{n+3} \cap \ldots \cap z_{n+n'}\}$ , where  $z_1...z_n, z_{n+1}, ...z_{n+n'}$  are properties and for all natural numbers n, n', n' > 0

Strong Analogical Argument (Formal): an analogical argument where either  $Z \subseteq Z'$  or  $Z \supseteq Z'$ 

Weak Analogical Argument (Formal): an analogical argument where it is not the case that either  $Z \subseteq Z'$  or  $Z \supseteq Z'$ 

Generalization: a type of argument in which the factual claim includes a sample of a group having a property and the inferential claim includes reasoning from this sample of a group having a property to the entire group from which the sample was taken having that same property

**Representative Sample:** a type of sample that is both large and random

Atypical Sample: a type of sample that is random

Unrepresentative Sample: a type of sample that is either small or non-random

Statistical Generalization: a type of generalization that uses a statistical sample, in which the members of the sample are chosen

Non-statistical Generalization: a type of generalization that uses a non-statistical sample, in which the members of the sample are not chosen

Large Sample: a type of sample in which the number of members *n* is at least equivalent to  $\frac{4Z^2\sigma^2}{W^2}$ , where Z represents a score for a desired confidence interval,  $\sigma$  is the variance of the members, and W is twice the margin of error of the sample

Small Sample: a type of sample in which it is not the case that the number of members n is at least equivalent

to  $\frac{4Z^2\sigma^2}{W^2}$ , where Z represents a score for a desired confidence interval,  $\sigma$  is the variance of the members,

and W is twice the margin of error of the sample

**Non-Random Sample:** a type of sample in which there are biases such as the following - non-equitable, non-balanced, or non-objective representation of all participants, improper gender, race, socioeconomic, etc. distribution, premature terminations of any sort, time-related factors, cause-effect mix-ups of any sort, cherry-picking or confirmation bias, ignorance of relevant parts or groups, or arbitrary rejections, favoritism, intentionally searching for correlations, observer selection, volunteer bias, etc.

Random Sample: a type of sample which involves selecting members or individuals each with equivalent probabilities and without bias

Strong Generalization: a generalization that has a representative sample

Weak Generalization: a generalization that has an unrepresentative sample

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**Argument From Authority:** a type of argument in which the factual claim includes a citation of an authority or authorities backing up a statement or statements, and the inferential claim includes reasoning from this authority's or authorities' support to the conclusion of the statement(s) being probably true

Strong Argument From Authority: an argument from authority that includes only qualified authorities

Weak Argument From Authority: an argument from authority that includes at least one unqualified authority

**Qualified Authority:** anyone who both has expertise in the relevant field, and lacks bias and prejudice, and lacks a motive to lie and disseminate misinformation

**Unqualified Authority:** anyone who either lacks expertise in the relevant field, or has bias or prejudice, or has a motive to lie or disseminate misinformation

**Argument From Signs:** a type of argument in which the factual claim includes the description of some sign(s), and the inferential claim includes reasoning from the description of some sign(s) to the conclusion that the description is probably true

Sign (Argument From Signs): any kind of message produced by an intelligent being

Intelligent Being (Argument From Signs): in general, any type of agent with at least some potential to create or analyze

**Generalized Causal Inference:** a type of argument in which either the factual claim includes some sort of cause, loosely defined, and the inferential claim includes reasoning from that cause to its effect, loosely defined, being probably true in the conclusion, or the factual claim includes some sort of effect, loosely defined, and the inferential claim includes reasoning from that effect to its cause, loosely defined, being probably true in the conclusion

"Correlation does not necessarily imply causation": the principle that two things being correlated in reality do not necessarily imply that one causes the other

**Proportional Syllogism:** a type of argument that consists in exactly three statements, in which the factual claim consists in exactly one statement, the major premise, about a proportion of members of a set having some property plus exactly one statement, the minor premise, about an individual or object being a member of that set, and the inferential claim consists in reasoning from those two statements to the third statement or probably true conclusion that the individual or object has the property in question

**Major Premise (Proportional Syllogism)**: the first premise in the factual claim of a proportional syllogism that is of the form "X proportion of Y are P," where X is some fraction or percentage, Y is a set containing members, and P is a property

**Minor Premise (Proportional Syllogism)**: the second premise in the factual claim of a proportional syllogism that is of the form "O is a Y," where O is an object and Y is a set containing members

**Conclusion (Proportional Syllogism)**: the third statement of a proportional syllogism that is of the form "O is P," where O is an object and P is a property

**Strong Proportional Syllogism:** a proportional syllogism in which the conclusion follows from the factual claim with at least a 61.80% chance

**Weak Proportional Syllogism:** a proportional syllogism in which it is not the case that the conclusion follows from the factual claim with at least a 61.80% chance

**General Scientific Argument:** a type of argument that is a very general summary of scientific findings and may appear in thought processes or writings in reference to a larger body of more specific scientific findings, and that can be further classified as either an argument for the discovery of a scientific law or an argument for the application of a scientific law

**Argument For the Discovery of a Scientific Law:** a type of argument in which the factual claim includes a statement or statements about observed instances of a certain effect, and the inferential claim includes reasoning from these instances to the conclusion of a named law governing these instances being probably true

**Argument For the Application of a Scientific Law:** a type of argument in which the factual claim includes references to some known scientific law and a circumstance in which this scientific law could be applied, and the inferential claim includes reasoning from applying this scientific law to this circumstance and saying that this application is probably true in the conclusion

**Argument From Compassion:** a type of argument in which the factual claim includes some evidence that someone is a victim of circumstances, and the inferential claim includes reasoning from such evidence to the probably true conclusion that the person in question is deserving of some benefit or compassion in recompense

Argument From Example (Factual and Inferential Claims): a type of argument in which the factual claim includes some example(s) and the inferential claim includes reasoning from such example(s) to some conclusion being probably true

**General Rule Argument:** a type of argument in which the factual claim includes a general rule and an individual or group that falls under the subject of the general rule, and the inferential claim includes reasoning from applying this general rule to the individual or group, concluding that it is probably true that the individual or group meets this general rule

General Rule: a type of rule that is not necessarily true, and is meant to apply in most but not all cases

**Composition:** a type of argument in which the factual claim includes some general statement(s) about the parts of something having a property, and the inferential claim includes reasoning from that general statement(s) to the whole thing or class having that same property in the probably true conclusion that is a class statement

**General Statement:** a type of statement that says something about each and every member of a class and that employs distributive predication

**Distributive Predication:** a type of predication in which an attribute or property is said of each and every member of a class

**Class Statement:** a type of statement that says something about some class as a whole and that employs collective predication

Collective Predication: a type of predication in which an attribute or property is said of a whole class

**Division:** a type of argument in which the factual claim includes some class statement about a whole thing or class having the same property, and the inferential claim includes reasoning from that class statement to the probably true conclusion that some part or parts of the whole thing in question each have the same property

Class: either a set or a group of sets whose members as a whole share some property

Now that we've talked about strong and weak arguments in 3.1, and cogent and uncogent arguments in 3.2, let's talk about what some inductive argument forms are and go through real life examples of each. Here are some, but not necessarily all, forms of inductive arguments<sup>34</sup>:

- (1) Prediction
- (2) Analogical Arguments
- (3) Generalization
- (4) Argument from Authority
- (5) Argument Based on Signs
- (6) Generalized Causal Inference
- (7) Proportional Syllogism
- (8) General Scientific Arguments
- (9) Miscellaneous Inductive Arguments

Regarding (1) above, a **prediction** is a type of argument in which the factual claim includes at least one statement in the present or past about members of a group having some property or attribute, and the inferential claim includes reasoning from these members of a group having some property or attribute to some future statement about some member(s) of this group having the same property or attribute.

Here is an example of a simple cogent prediction:

 $\phi_1$ : All past observations of giant squid are of them having eyes at about 27 cm in diameter.<sup>35</sup>

p: The next observation of a giant squid will be of it having eyes at about 27 cm in diameter.

The factual claim is  $\phi_1$ , which is a past statement about all the past members of the group of giant squid observations having the property "eyes at about 27 cm in diameter." The inferential claim is the reasoning from  $\phi_1$  to the future statement,  $\varphi$ , about some next member of the group of giant squid observations having this same property, "eyes at about 27 cm in diameter."

This argument is a cogent argument because it is strong,  $\phi_1$  is true, and the total evidence requirement is met. It is strong because its inferential claim is true, or consistent with PUN. The next observation of a giant squid having in the future having eyes at about 27 cm in diameter would be spatiotemporally consistent with all past observations of giant squid being like that.

 $\phi_1$  is true because it is a fact (see footnote 36).

<sup>&</sup>lt;sup>34</sup> Some of these inductive argument forms come from Patrick J. Hurley and Lori Watson's book *A Concise Introduction to Logic* (cf. Hurley and Watson, 2018).

<sup>&</sup>lt;sup>35</sup> See Nilsson, et. al. (2012).

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This argument meets the total evidence requirement because  $\phi_1$  is coherent with the domain of the conclusion, "future giant squid observation and squid having eyes at about 27 cm in diameter."  $\phi_1$  fits with and does not contradict any of the evidence or  $\phi_d$  in this domain. It is equally about giant squid having eyes at about 27 cm in diameter, and does not contradict any evidence in this domain, because this domain is precise and would mainly concern giant squid having eyes at about 27 cm in diameter in the present, future, and past, which is what  $\phi_1$  is about.

Here is a more complicated example of a prediction that is  $cogent^{36}$ :

 $\phi_1$ : The MLS soccer team Chicago Fire has only lost 1/20 home matches against the Columbus Crew, avoiding defeat in the last 8.

 $\phi_2$ : The Chicago Fire lost for the ninth time in their last 13 MLS matches, but recorded back-to-back wins against other teams after winning one of its first six matches also against other teams.

 $\phi_3$ : The Columbus Crew have won 2 of their last 3 away matches, and have only lost 2 of their last 11 away matches.

 $\varphi$ : The soccer match between the home team Chicago Fire and the away team Columbus Crew on July 9<sup>th</sup>, 2022 will end in a draw.

The past or present statement about some members of a group having a certain property or attribute in the factual claim is a bit harder to see here. The factual claim  $\{\phi_1, \phi_2, \phi_3\}$  makes past statements about various soccer matches and their results. The inferential claim includes reasoning from this factual claim to the future statement about a soccer match with the result of ending in a draw for  $\varphi$ .

This argument is cogent because it is strong, has all true premises, and meets the total evidence requirement. The argument is strong because, assuming the premises are true, the conclusion is probably true. If the Chicago Fire have rarely lost a home match against the Columbus Crew, and if the Chicago Fire are currently on a winning streak, and if the Columbus Crew are also on a winning streak when it comes to away matches, then, with at least a 61.80% likelihood, the future match (at the time of the example's creation)<sup>37</sup> with the Chicago Fire home and the Columbus Crew away ends in a draw, because the statistics in the factual claim seem to have them evenly matched.

The argument has all true premises because both  $\phi_1$ ,  $\phi_2$ , and  $\phi_3$  are facts, as of the creation of this example on July 7, 2022.

This argument meets the total evidence requirement because its factual claim is coherent with the domain of the conclusion, "July 9<sup>th</sup>, 2022 soccer match and Chicago Fire home and Columbus Crew away." Each of the premises fits with and is relevant to the match outcome (at the time), because they match the summary of all relevant info (see footnote 37), and they do not contradict any info in the domain of the conclusion, because they represent the stats before July 9<sup>th</sup>, 2022.

Here is an example of a prediction that is uncogent:

<sup>&</sup>lt;sup>36</sup> See *Major League Soccer* (https://www.mlssoccer.com/competitions/mls-regular-season/2022/matches/chivsclb-07-09-2022/ ; https://www.mlssoccer.com/competitions/mls-regular-season/2022/matches/chivsclb-07-09-2022/odds ). This example was created on July 7, 2022 before the predicted match on July 9, 2022.

<sup>&</sup>lt;sup>37</sup> The actual result was Chicago 2 – Columbus 3. See *Chicago Fire FC vs. Columbus Crew*: (https://www.espn.com/soccer/match/ /gameId/623425).

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 $\phi_1$ : One piece of bread has nourished me (the author) in the past.

 $\varphi$ : The next piece of bread I eat will be nourishing.

The factual claim  $\phi_1$  is a past statement about some member of the set of pieces of bread having the property "nourishing me (the author)," and the inferential claim includes reasoning from  $\phi_1$  to the future statement,  $\phi$ , that some next member of the set of pieces of bread having the same property of nourishing me.

This argument is an uncogent argument because it is weak. It is weak because its inferential claim is false, or not consistent with PUN. One would not expect the next piece of food of anything, including bread, to be nourishing off of just one previous instance of the same piece of food being nourishing; many more observations are needed.

Regarding (2) above, an **analogical argument**<sup>38</sup>, or one that proceeds by analogy, is a type of argument in which the factual claim contains evidence that at least one entity or object has n properties and at least one entity has n+1 properties, and the inferential claim includes reasoning from this evidence to another entity having the  $n + 1^{nth}$  property. An analogical argument is strong if and only if there is either a systemic or causal relationship between all n properties in total and the  $n + 1^{nth}$  property(ies). Otherwise, it is weak. A systemic or causal relationship here would be consistent with PUN, and a non-systemic or non-causal would not be. Mathematical, arithmetical, definitional, and geometrical relationships are probably systemic or causal depending on context. Sometimes, using your own judgment from your own experiences can help determine if there is a systemic or causal relationship. Additionally, if entities A and B are the same type of substance or living thing, then a systemic or causal relationship between attributes is more likely.

We can define more concretely what a systemic or causal relationship between properties is. A **systemic relationship** between properties exists if and only if one, or a group of them, is a sufficient condition for another, or a group of others. A **causal relationship** between properties exists if and only if one, or a group of them, is a necessary condition for another, or a group of others. The "**group of them**" in these two previous definitions can either be the intersection or the union of the properties, more specifically the sets of the properties, in question. Systemic and causal relationships between properties at root is thinking about sufficient and necessary conditions in a different way, this time with a more empirical flavor, because the properties in question are usually those perceived by the five senses in the actual world.

Returning to 1.4, we defined sufficient and necessary conditions in terms of sets. We can do the same for systemic and causal relationships between properties in this case:

A systemic relationship between properties exists if and only if one, or either the intersection or union of each of some properties in a group, is a subset of, or identical to or included within, another property, or either the intersection or union of each of others in a group.

Here is a more formalized and less wordy version of the above definition.

Set Z is in a systemic relationship with set Z' if and only if  $Z \subseteq Z'$ , and Set Z' is in a systemic relationship with set Z if and only if  $Z' \subseteq Z$ .

 $Z = \{z_1 \cup z_2 \cup z_3 \cup \ldots \cup z_n \text{ OR } z_1 \cap z_2 \cap z_3 \cap \ldots \cap z_n\}, Z' = \{z_{n+1} \cup z_{n+2} \cup z_{n+3} \cup \ldots \cup z_{n+n'} \text{ OR } z_{n+1} \cap z_{n+2} \cap z_{n+3} \cap \ldots \cap z_{n+n'}\}, \text{ where } z_1 \ldots z_n, z_{n+1}, \ldots z_{n+n'} \text{ are properties and for all natural numbers } n, n', n' > \text{ (is greater than) } 0.$ 

<sup>&</sup>lt;sup>38</sup> This type of argument is opposed to a weak one, which is discussed further in chapter 6.

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The natural numbers are defined as the whole numbers 0, 1, 2, 3, .... etc. counting to infinity.

A causal relationship between properties exists if and only if one, or either the intersection or union of each of some attributes in a group, is a superset of, or identical to or encompassing, another property, or either the intersection or union of each of the others in a group.

Similar to what's above, here is a more formalized and less wordy version of this definition.

Set Z is in a causal relationship with set Z' if and only if  $Z \supseteq Z'$ , and Set Z' is in a causal relationship with set Z if and only if  $Z' \supseteq Z$ .

Again,  $Z = \{z_1 \cup z_2 \cup z_3 \cup \ldots \cup z_n \text{ OR } z_1 \cap z_2 \cap z_3 \cap \ldots \cap z_n\}$ ,  $Z' = \{z_{n+1} \cup z_{n+2} \cup z_{n+3} \cup \ldots \cup z_{n+n'} \text{ OR } z_{n+1} \cap z_{n+2} \cap z_{n+3} \cap \ldots \cap z_{n+n'}\}$ , where  $z_1 \ldots z_n, z_{n+1}, \ldots z_{n+n'}$  are properties and for all natural numbers n, n', n' > 0.

Putting this altogether, then, an analogical argument is strong if and only if either  $Z \subseteq Z'$  or  $Z \supseteq Z'$ . Otherwise, it is weak.

Here is a simple example of a cogent analogical argument:

 $\phi_1$ : Jupiter's moon Europa has a major ice sheet like Antarctica does.

 $\phi_2$ : There is life somewhere beneath Antarctica's major ice sheet.

 $\varphi$ : There is life somewhere beneath Europa's major ice sheet.

The factual claim states that Europa has the property of "major ice sheet" ( $\phi_1$ ) and that Antarctica has the properties of "major ice sheet" and "existence of life somewhere beneath major ice sheet" ( $\phi_1$  and  $\phi_2$ ) The inferential claim includes reasoning from the factual claim to the other entity, Europa, having the property of "existence of life beneath major ice sheet" for the conclusion  $\varphi$ .

This argument is cogent because it is strong, has all true premises, and meets the total evidence requirement. It is strong because the set  $Z = \{\text{``major ice sheet''}\} \supseteq Z' = \{\text{``existence of life somewhere beneath major ice sheet''}\}$ . Z is in a causal relationship with Z', and Z' is in a systematic relationship with Z. The existence of life somewhere beneath a major ice sheet would require a major ice sheet, and so Z' would be a proper subset of Z.

This argument has all true premises because  $\phi_1$  and  $\phi_2$  are common-knowledge facts.

This argument meets the total evidence requirement because the factual claim is coherent with the domain of the conclusion, "the existence of life beneath Europa's major ice sheet." It fits with all the evidence in this domain, because it sums up the common argument that most scientists give for the existence of life on Europa. It does not contradict any of the evidence in this domain, because currently there are no major arguments against the probable existence of life here. Other than Earth, Europa is the most likely location in our Solar System for the existence of extraterrestrial life.

Here is a more complicated example of a cogent analogical argument<sup>39</sup>:

 $\phi_1$ : Assuming God exists, God's Knowledge is measured by eternity, just as God's Being is measured by eternity.

<sup>&</sup>lt;sup>39</sup> The inspiration for this example comes from Aquinas' Summa Theologiae, ST Ia.14.13. See Bosley and Martin (2006).

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 $\phi_2$ : Assuming God exists, God's Being as measured by eternity as a whole exists all at once and surrounds the whole of time.

 $\varphi$ : Assuming God exists, God's Knowledge as a whole exists all at once and surrounds the whole of time.

The factual claim describes God's knowledge as having the property "measured by eternity" and God's Being as having the properties "measured by eternity" and "as a whole existing all at once and surrounding the whole of time." The inferential claim includes reasoning from  $\phi_1$  and  $\phi_2$  to God's Knowledge also having the property "as a whole exists all at once and surrounds the whole of time."

This argument is a cogent argument because it is strong, has all true premises, and meets the total evidence requirement. This is a strong analogical argument because the set  $Z = \{\text{``measured by eternity''}\} \supseteq Z' = \{\text{``as a whole existing all at once and surrounding the whole of time''}\}. Z is in a causal relationship with Z', and Z' is in a systemic relationship with Z. Whatever as a whole exists all at once and surrounds the whole of time would necessarily be measured in some way in some aspect by eternity because it exists outside time, so Z' would be a subset of Z.$ 

This argument has all true premises because, assuming God exists,  $\phi_1$  and  $\phi_2$ , are true by definition in theology on some level. Because God is defined as an eternal being, and God's Knowledge would be proper to God, God's Knowledge would be proper to an eternal being, and thus measured by eternity. God's Being is defined as that of an eternal being, and thus would also be measured by eternity, and is also defined as a whole existing all at once and surrounding the whole of time, in reference to the divine attributes of eternity, aseity, transcendence, and oneness.

This argument meets the total evidence requirement because the factual claim ( $\phi_1$  and  $\phi_2$ ) is coherent with the domain of the conclusion, "assuming God exists, God's Knowledge and entire existence all at once surrounding the whole of time." It fits with  $\varphi_d$  this specific domain, because  $\phi_1$  and  $\phi_2$  are about God's Knowledge and God's entire Existence all at once surrounding the whole of time. It does not contradict any  $\varphi_d$  in this domain because it is about Divine Attributes, like the conclusion, all of which Attributes inhere in the single Divine Essence or Substance in theology.

Here is an example of an uncogent analogical argument:

 $\phi_1$ : The sun is similar to a lightbulb.

 $\phi_2$ : The sun is bright, circular, and gives off heat.

 $\phi_3$ : A light bulb is bright, circular, gives off heat, and is fragile.

 $\varphi$ : The sun is fragile.

The factual claim includes the sun having 3 properties, "brightness," "circularity," and "giving off heat" ( $\phi_2$ ) Additionally, it includes a light bulb having 4 properties: "brightness," "circularity," "giving off heat," and "fragility" ( $\phi_3$ ) The inferential claim includes reasoning from the factual claim to the conclusion  $\varphi$  that the sun has the additional property of "fragility."

This argument is uncogent because it is a weak analogical argument. It is not the case that either  $Z \subseteq Z'$  or  $Z \supseteq Z'$ , where  $Z = \{$ "brightness," "circularity," "giving off heat" $\}$  and  $Z' = \{$ "fragility" $\}$ . It is the case that there are things that have the property of fragility that do not have any of the properties of "brightness," "circularity," and "giving off heat," such as a black square glass vase, so Z' cannot be a subset of or equal to Z. It is also the case that there are some things that have the property of circularity but do not have the property of fragility, such as golf balls, so the union of all the properties in Z cannot be a subset of

Z'. It is also the case that there are things that have all the properties of brightness, circularity, and giving off heat, but do not have the property of fragility, such as the lamp in a lighthouse, so the intersection of all the properties in Z cannot be a subset of Z'.

Regarding (3) above, a **generalization** is a type of argument in which the factual claim includes a sample of a group having a property and the inferential claim includes reasoning from this sample of a group having a property to the entire group from which the sample was taken having that same property. For a generalization that is a strong one, the sample must be a representative sample. If the sample is **unrepresentative**, then the generalization is a weak one, and vice versa. A **representative sample** is a type of sample that is both large and random (also known as atypical). If a sample is either small or non-random (or typical), then it is an unrepresentative sample and not a representative one. The goal is for the sample to match the characteristics of the group in question.

What makes a sample a large one, as opposed to a small one?

A sample is large if and only if its number of members n is at least equivalent to  $\frac{4Z^2\sigma^2}{W^2}$ , where Z represents a score for a desired confidence interval,  $\sigma$  is the variance of the members, and W is twice the margin of error of the sample. Otherwise, a sample is small.

In general, this number will be at least around 400, in order to match up with the commonly-used 95% confidence interval. The reasoning behind this definition of a **large sample**, and what the confidence interval, variance, and margin of error are, is discussed in more depth in section 3.5.

What makes a sample a random one, as opposed to a non-random one? A sample is **non-random** if it incorporates any of the following biases:

- Non-equitable, non-balanced, or non-objective representation of all participants
- Improper gender, race, socioeconomic, etc. distribution
- Premature terminations of any sort
- Time-related factors
- Cause-effect mix-ups of any sort
- Cherry-picking or confirmation bias<sup>40</sup>
- Ignorance of relevant parts or groups, or arbitrary rejections
- Favoritism
- Intentionally searching for correlations
- Observer selection<sup>41</sup>
- Volunteer bias<sup>42</sup>

If the sample does not incorporate any such biases as stated above, then it is random. A **random sample** involves members or individuals each with equivalent probabilities and without bias. Intuitively, a sample, at least about 400 in number that is random, of a certain group sharing a certain common characteristic, lets us reason with at least a 61.80% likelihood that that entire group has that certain characteristic as well.43 Furthermore, an inferential claim with a representative sample in a **strong** 

<sup>&</sup>lt;sup>40</sup> Cherry-picking is defined as picking members or data of a sample to confirm a position while ignoring members or data that disconfirms that position, see Klass, (2014). Confirmation bias is defined as the tendency to favor info, data, or evidence to confirm or support one's beliefs or values in spite of contradictory info, data, or evidence, see Risen and Gilovich, (2007).

<sup>&</sup>lt;sup>41</sup> Observer selection is defined as some attribute or property of data or members of a group being correlated with the observer of the data or members itself, see *Observer selection effects* (https://www.fhi.ox.ac.uk/wp-content/uploads/W6-Observer-selection-effects.pdf).

<sup>&</sup>lt;sup>42</sup> Volunteer bias is defined as volunteers for a study's sample differing significantly in relevant characteristics compared with non-volunteers, see Tripepi, et. al. (2010).

<sup>&</sup>lt;sup>43</sup> Even if sample size is not the be-all and end-all in this case, the related confidence interval is normally 95%, which is significantly greater than 61.80%.

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**generalization** would be consistent with PUN, because one would expect a random sample at least about 400 in number to say something true about the entire group in question. It would be unsurprising for a random sample at least about 400 in number to say something true about the entire group.

If the sample used is a statistical sample, then the generalization is a **statistical generalization**. If the sample used is a non-statistical sample, then the generalization is a **non-statistical generalization**. Statistical samples are created through sampling, or choosing members of one group or groups to be part of a new group, whereas non-statistical samples are not. Statistical samples involve some sort of choosing with regards to their members, but non-statistical samples do not.

Here is a simple example of a cogent generalization.

 $\phi_1$ : Throughout all of recorded history, the sun has risen after it has set.

 $\varphi$ : The sun rises after it sets.

This generalization is a non-statistical generalization because no choosing is involved – the rising and setting of the sun throughout all of recorded history is something that has occurred. The factual claim, or  $\phi_1$  is a statement about all of the past observations in recorded history of the sun rising after it setting. The inferential claim includes reasoning from this statement to the conclusion  $\varphi$  that the entire group of observations about the sun, past, present, and future, are that it rises after it sets.

This argument is cogent because it is strong, has all true premises, and meets the total evidence requirement. It is a strong argument because it has a representative, or a large and random, sample  $(\phi_1)$ . The sample is large because it involves at least 400 instances – each instance would correspond to a day, and there have been far more than 400 days throughout all of recorded history. The sample is random because there are no biases involved and each member involves an equivalent probability. The sample is based upon consistent facts throughout recorded history, and each rising and setting of the sun has occurred one after the other consistently, so they each have the same chance of occurring and thus have equivalent probabilities.

This argument has all true premises because  $\phi_1$  is a fact.

This argument meets the total evidence requirement because  $\phi_1$  is coherent with the domain of the conclusion, "the sun and rising after setting."  $\phi_1$  fits with all of the evidence  $\phi_d$  in this domain because it is about all observations in recorded history of the sun and its rising after setting. It does not contradict any of the evidence in this domain, because the sun intersecting rising and setting includes all of the observations of the sun rising and setting throughout recorded history.

Here is an example of an uncogent generalization<sup>44</sup>.

 $\phi_1$ : In 2017 in the United States population, approximately 20.1% of women met the CDC's physical activity guidelines.

 $\phi_2$ : In 2017 in the United States population, approximately 28.8% of men met the CDC's physical activity guidelines.

 $a_1$ : In 2017 in the United States population, more men than women met the CDC's physical activity guidelines.

 $\phi_3$ : In 2017 in the United States population, approximately 25.3% of urban dwellers met the CDC's physical activity guidelines.

<sup>44</sup> cf. Whitfield, et. al. (2019).

 $\phi_4$ : In 2017 in the United States population, approximately 19.6% of rural dwellers met the CDC's physical activity guidelines.

 $\varphi$ : In 2017 in the United States population, more urban dwellers who are men met the CDC's physical activity guidelines than rural dwellers who are women.

This argument is a statistical generalization, because it has statistical samples whose members were chosen through the Center of Disease Control (CDC)'s National Health Interview Survey (NHIS)(see footnote 45). The factual claim includes 4 samples ( $\phi_1$ ,  $\phi_2$ ,  $\phi_3$ ,  $\phi_4$ ). The inferential claim includes reasoning from the factual claim to the conclusion  $\varphi$  that compares all of the 4 samples of the population with respect to the property of meeting the CDC's physical activity guidelines.

This generalization is an uncogent one because it is weak. It has unrepresentative samples. Although the samples at  $\phi_1$ ,  $\phi_2$ ,  $\phi_3$ , and  $\phi_4$  are each large, because the number of people who got the survey were at least 21,781 with at least a 53.0% response rate to surveys, giving a sample size number of at least 11,543, which is much greater than around 400, nevertheless, they are each non-random, because there is a significant bias (Whitfield et. al., 2019). There are biases in the wording of the survey itself. Firstly, the assessment is strictly related to leisure-time activity, which activity may not include the relevant occupational or domestic physical tasks that often accompany those living in rural areas (ibid.). Secondly, "light-intensity and moderate-intensity activity" are grouped together in a single question, which is misleading because the CDC's physical activity guidelines only pertain to physical activities of at least moderate intensity (ibid.).

Regarding (4) above, an **argument from authority** is a type of argument in which the factual claim includes a citation of an authority or authorities supporting a statement or statements, and the inferential claim includes reasoning from this authority's or authorities' support to the conclusion of the statement(s) being probably true. An argument from authority is strong if and only if it includes only qualified authorities. An argument from authority is weak if and only if it includes at least one unqualified authority.

A qualified authority is anyone who meets each of the following three conditions:

- (1) Has expertise in the relevant field—i.e. has experience or a certain advanced degree in a certain area, meets a performance standard, has the ability to perceive or recall relevant info, etc.
- (2) Lacks bias and prejudice—i.e. there is some form of mathematical equality, there is no favoritism or one-sidedness, etc.
- (3) Lacks a motive to lie and disseminate misinformation—i.e. there are no large amounts of money involved, no predispositions towards certain conclusions, etc.

Otherwise, the authority is an **unqualified authority**. An inferential claim that involves a qualified authority who meets each of these three conditions would also be true, or consistent with PUN, because one would expect that someone who both has expertise, and lacks bias and prejudice, and lacks a motive to lie or disseminate misinformation, would support or say something that is probably true in their area of expertise. Some things to keep in mind are that an authority could have expertise in more than one area, such as a medical doctor having expertise in both neuroscience and biology, and that some areas have little or no qualified authorities, such as politics and religion where there are numerous heated debates.

Here is an example of a simple cogent argument from authority.

 $\phi_1$ : According to famous mathematician Terence Tao, "if one were to strictly adhere to type conventions (analogous to using a strongly typed language in software engineering) then one would have to distinguish between 'the natural number 1', 'the integer 1', 'the

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real number 1', and the 'extended real number 1', and these are all technically of different types and thus not necessarily comparable with each other."<sup>45</sup>

 $\varphi$ : If one were to strictly adhere to type conventions (analogous to using a strongly typed language in software engineering) then one would have to distinguish between "the natural number 1", "the integer 1", "the real number 1", and the "extended real number 1", and these are all technically of different types and thus not necessarily comparable with each other.

The factual claim here includes a statement of mathematics about type conventions and the natural number 1, the integer 1, the real number 1, and the extended real number 1 backed up by the famous mathematician Terence Tao ( $\phi_1$ ). The inferential claim includes reasoning from Tao's support for this statement to this statement itself being probably true.

This argument is cogent because it is strong, has all true premises, and meets the total evidence requirement. It is strong because its inferential claim is consistent with PUN, or involves an authority who meets the above three conditions. Terence Tao is a prodigious, learned, and award-winning mathematician, so the authority has expertise. He lacks bias or prejudice because the statement is given on a blog post in response to a technical question requiring a precise answer, which leaves little room for prejudice or bias. He lacks a motive to lie or disseminate misinformation, because mathematics is an objective discipline.

This argument has all true premises because  $\phi_1$  is a fact (see footnote 46).

This argument meets the total evidence requirement because  $\phi_1$  is coherent with the domain of the conclusion, "conditional strict adherence to type conventions and the natural number 1, the integer 1, the real number 1, and the extended real number 1 being technically of different types and not comparable."  $\phi_1$  fits with all of the evidence  $\phi_d$  in this domain because a cited instance of someone claiming to give support for what's in this domain is pertinent to it. It does not contradict any evidence in this domain because the domain itself is narrow and precise.

Here is a more complicated example of a cogent argument from authority.

 $\phi_1$ : According to legendary fighter Ronda Rousey, "the best way to take a punch is to look at it...someone could hit you with the hardest punch that they have, but as long as you see it, it's not going to knock you out."<sup>46</sup>

 $\phi_2$ : According to the famous actor and wrestler Dwayne "The Rock" Johnson, "the ego can be the great success inhibitor."<sup>47</sup>

*q*: The best way to take a punch is to look at it; the ego can be the great success inhibitor.

The factual claim includes two statements ( $\phi_1$  and  $\phi_2$ ) that are about authorities, Ronda Rousey and Dwayne Johnson, supporting other statements. The inferential claim includes reasoning from the factual claim to the conclusion  $\varphi$  that both of the other statements are probably true.

This argument is cogent because it is strong, has all true premises, and meets the total evidence requirement. It is strong because it includes only qualified authorities. Ronda Rousey is a qualified authority to support a statement about fighting, because she has the relevant expertise - she is a winning professional fighter and wrestler. She is not biased or prejudiced in this context (online *Esquire* article – see footnote 47) because advice is being given on an objective technique after five years of experience. There are not

<sup>&</sup>lt;sup>45</sup> See *terrytao.wordpress.com* (https://terrytao.wordpress.com/books/analysis-i/comment-page-13/#comment-649225).

<sup>&</sup>lt;sup>46</sup> See *esquire.com* (https://www.esquire.com/sports/interviews/a36781/ronda-rousey-fighting-advice/)

<sup>&</sup>lt;sup>47</sup> See *Dwayne Johnson on Twitter* (https://twitter.com/therock/status/760869801573044224).

necessarily any individuals, groups, or beliefs involved, and it is not as if only the first piece of information from her experience is being considered. She does not have a motive to lie or disseminate misinformation, because the Esquire magazine interview is for her benefit, so it is in her best interest to tell the truth and be accurate. Dwayne "The Rock" Johnson is a qualified authority to support a statement about success, because he has had success as an actor, businessman, producer, professional wrestler, writer, host, activist, and philanthropist. He lacks bias and prejudice here because it is to his disadvantage to say this general statement ( $\phi_2$ ), so it is unlikely that he would be biased or prejudiced towards his own views. He is describing his personal philosophy about something that is hard for him to do (see footnote 48). It is in relation to experiences of his lack of successes in the past, which may be embarrassing to talk about (see footnote 48). This statement also does not include any direct references to individuals or groups, and is made in 2016 after years of experience with his work (see footnote 48). Although it mentions a belief, it is one to his disadvantage so there is likely no bias or prejudice (see footnote 48). He lacks a motive to lie or disseminate misinformation because he is not paid to post tweets on Twitter, and Twitter allows readers to report Tweets.

It has all true premises because  $\phi_1$  and  $\phi_2$  are both facts (see footnotes 47 and 48).

It meets the total evidence requirement because the factual claim is coherent with the domain of the conclusion "the best way to take a punch and looking at it; the ego and the ability to be the great success inhibitor." It fits with all the evidence in this domain because it consists in the citation of statements that are similar to what's in this domain. It does not contradict any of the evidence in this domain, because this domain is precise. Also, other resources from a Google search indicate that you should watch the movement of the attacker and keep eye contact when taking punches.

Here is an uncogent example of an argument from authority.<sup>48</sup>

 $\phi_1$ : According to GoodNews Roundup, the current Democratic Party is the absolute best party of all time.<sup>49</sup>

 $\varphi$ : The current Democratic Party is the absolute best party of all time.

The factual claim includes a statement about politics supported by the authority, GoodNews Roundup ( $\phi_1$ ). The inferential claim includes reasoning from the factual claim to the conclusion that this statement about politics is probably true ( $\varphi$ ).

This argument is uncogent because it is weak. It is weak because it includes at least one unqualified authority, GoodNews Roundup. GoodNews Roundup is an unqualified authority because this person or group has bias. They have bias because their support of this statement is their "completely unscientific (and yet very strongly held) opinion" (GoodNews Roundup, 2019). In addition to the field of politics in general involving mostly opinions, the authority in question gives their own opinion.

Regarding (5) above, an **argument from signs** is a type of argument in which the factual claim includes the description of some sign(s), and the inferential claim includes reasoning from the description of some sign(s) to the conclusion that the description is probably true. A **sign** is any kind of message produced by an intelligent being. Signs can range from road signs with pictures and words, to sound creations, to Braille, to secret signs with secret handshakes, to text lingo, to name just a few examples. An **intelligent being**, generally defined, is any type of agent with at least some potential to create or analyze. Keep in mind that some signs may be intentionally or accidentally misleading.

<sup>&</sup>lt;sup>48</sup> Disclaimer: this example or others like it do not necessarily reflect the author's political views.

<sup>&</sup>lt;sup>49</sup> See GoodNews Roundup (2019). In support of this premise, the article writer gives 10 reasons: "Nancy Pelosi [does a really good job]...Democrats are introducing huge amounts of cool legislation...Democrats are taking on Trump's Family where necessary...Democrats are fighting hard for the rights of all Americans...Democrats Are Hilarious...Democrats are taking oversight VERY seriously...Democrats are not afraid to take on the big guys...Even the centrists[..]are psyched up and have some cool ideas...Democrats Are Fighting to Save the Planet...Democrats Have Amazing Candidates for 2020."

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Here is a simple cogent argument from signs.

 $\phi_1$ : The speed limit sign says that the speed limit on some road is 65.

 $\varphi$ : The speed limit on some road is 65 mph.

The factual claim includes the description of some speed limit sign, where the description says that the speed limit on some road is 65 ( $\phi_1$ ). The inferential claim includes reasoning from this description to the conclusion that this description, that the speed limit on some road is 65 mph, is probably true.

This argument is a cogent argument because it is strong, has all true premises, and meets the total evidence requirement. It is strong its inferential claim is true, or consistent with PUN. One would expect that a speed limit sign saying that the speed limit on some road is 65 to indicate to be probably true that the speed limit on some road is 65 mph, because that is the purpose of numbers on speed limit signs in general.

This argument has all true premises because  $\phi_1$  is a fact; for instance, there is a speed limit sign on U.S. Road 66 in Oklahoma that says that the speed limit is 65.<sup>50</sup>

This argument meets the total evidence requirement because the factual claim  $\phi_1$  is coherent with the domain of the conclusion, "speed limit on some road and 65 mph." It fits with all of the evidence  $\phi_d$  in this domain because it says the same thing. It does not contradict any of the evidence in this domain because it gives an instance of both a speed limit on some road and 65 mph.

Here is a more complicated argument from signs that could be cogent.

 $\phi_1$ : Ximena:

 $\varphi$ : Ximena is saying that she believes me.

The factual claim includes the description of some sign, a text-message line ( $\phi_1$ ), through 4 emojis, an eye, a bee, a leaf, and a ewe, meaning Ximena is saying "I believe you." The inferential claim includes reasoning from this description in the factual claim to the conclusion  $\varphi$  that this description of Ximena saying that she believes me is probably true.

This argument could be cogent because it is strong, it could have all true premises and it meets the total evidence requirement. It is strong because its inferential claim is true, or is consistent with PUN. There are similarities between this symbolic-phonetic text-emoji lingo and other symbolic-phonetic text-emoji lingos and abbreviations in past spatiotemporal areas. These symbols are a pictoral way of conveying Ximena's message: O = "eye" + O = "bee" + O = "leaf" + O = "ewe", which is practically equivalent to "eye-bee-leaf-ewe," which is onomatopoetically equivalent to "I believe you." A similar type of communication occurs with capitalized letters with: "FR" (for real), "ICYMI" (in case you missed it), etc.

This argument could have all true premises because  $\phi_1$  is a hypothetical situation of a text message that could be a fact.

This argument meets the total evidence requirement because  $\phi_1$  is coherent with the domain of the conclusion, "Ximena and Ximena saying that she believes me." It fits with all of the evidence in this domain because it is one way to represent that Ximena is saying that she believes me. It does not contradict any of the evidence in this domain because there is no interpretation of this hypothetical text message line that contradicts Ximena and Ximena saying that she believes me.

<sup>&</sup>lt;sup>50</sup> See *Projects – Oklahoma Route 66 Association* (https://www.oklahomaroute66.com/projects).

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Here is an uncogent argument from signs.

 $\phi_1$ : Some 2012 Apple advertisement says that the iPad is 4G capable in Australia.<sup>51</sup>

 $\varphi$ : Some 2012 Apple iPad is 4G capable in Australia.

The factual claim includes the description of some 2012 Apple advertisement sign in Australia, where the description is that it says that the iPad is 4G capable. The inferential claim includes reasoning from this description to the probably true conclusion that some 2012 Apple iPad is 4G capable in Australia.

This argument is uncogent because it does not meet the total evidence requirement. The factual claim  $\phi_1$  is not coherent with the evidence in the domain of the conclusion, "some 2012 Apple iPad and 4G capability in Australia," because it contradicts the evidence  $\phi_d$  in this domain that Apple misled consumers by advertising the iPad as 4G in 2012, and then had to pay a fine of \$2.25m to the Australian Consumer and Competition Commission (see footnote 51).

Regarding (6) above, a **generalized causal inference** is a type of argument in which either the factual claim includes some sort of cause, loosely defined, and the inferential claim includes reasoning from that cause to its effect, loosely defined, being probably true in the conclusion, or the factual claim includes some sort of effect, loosely defined, and the inferential claim includes reasoning from that effect to its cause, loosely defined, being probably true in the conclusion. A causal inference can either proceed from cause to effect, or from effect to cause. When thinking about causal inferences, keep in mind the principle that "**correlation does not necessarily imply causation**." Just because two things may be correlated in reality, does not mean that one causes the other. Statistical correlations may reveal little info. Often, on top of correlation, there are deeper reasons behind causation that also need to be validated, when it comes to a thing causing another or a thing being an effect of another. Some other factors to be considered may be your own experience, cause-effect mix-ups, the complex nature of motivation, various scientific laws, etc.

Here is a simple generalized causal inference that could be cogent.

 $\phi_1$ : Rohan has dark yellow urine, decreased urine output, dry mucous membranes, a headache, and is extremely thirsty, after being out in the 100 degree Fahrenheit sunny weather all morning and afternoon.

 $\varphi$ : Rohan is dehydrated.

This argument proceeds from effect to cause. The factual claim includes the loosely defined effects or symptoms of dehydration (dark yellow urine, decreased urine output, dry mucous membranes, a headache, extreme thirst) at  $\phi_1$  and the inferential claim includes reasoning from that effect to the loosely defined cause of Rohan being dehydrated.

This argument could be cogent because it is strong, it could have all true premises, and it meets the total evidence requirement. It is strong because its inferential claim is true. One would expect that anyone who has both dark yellow urine, decreased urine output, dry mucous membranes, a headache, and is extremely thirsty after being out in 100 degree Fahrenheit weather would also be dehydrated. These are some of the commonly known symptoms of dehydration.

It could have all true premises because  $\phi_1$  is a hypothetical scenario that could be a fact.

It meets the total evidence requirement because  $\phi_1$  is coherent with the domain of the conclusion, "Rohan and being dehydrated." It fits with all of the evidence in this domain, because it describes symptoms that someone named Rohan could experience from being dehydrated. It does not contradict any of the

<sup>&</sup>lt;sup>51</sup> See Connelly and Technology Reporter (2012).

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evidence in this domain because it does not contradict any combination of symptoms that someone named Rohan or anyone could experience from dehydration in 100 degree Fahrenheit weather.

Here is a more complicated cogent generalized causal inference.52

 $\phi_1$ : There is reduced unemployment rate in the U.S. since June 2020.<sup>53</sup>

 $\phi_2$ : The value of the U.S. dollar has decreased since 2008.<sup>54</sup>

 $\phi_3$ : The U.S. Federal deficit has increased since 2002.<sup>55</sup>

 $\varphi$ : The U.S. government has increased its spending recently.<sup>56</sup>

This argument proceeds from effect to cause. The factual claim includes the effects, looselydefined, of reduced unemployment rate since 2020 ( $\phi_1$ ), the U.S. dollar value decreasing since 2006 ( $\phi_2$ ), and increased U.S. Federal deficit since 2002 ( $\phi_3$ ). The inferential claim includes reasoning from this factual claim to the cause, loosely-defined, of recent increased U.S. government spending being probably true ( $\phi$ ).

This argument is cogent because it is strong, has all true premises and sub-conclusions, and meets the total evidence requirement. The inferential claim is true because it is consistent with PUN. Assuming that  $\phi_1$ ,  $\phi_2$ , and  $\phi_3$  are true, it is likely that a combination of reduced unemployment rate, decreased U.S. dollar value, and increased U.S. Federal deficit in the United States for similar time periods would be the effect of the U.S. government increasing its spending recently. Reduced unemployment rate can only occur if the money needed to pay newly employed individuals is spent by the U.S. government's Treasury or central banks on some level. The decrease in U.S. dollar value can occur when the U.S.'s central bank the Federal Reserve lowers interest rates and thus creates more dollars out of thin air to be put into circulation. It could also be due to economic inflation. The U.S. government increases its spending through putting more dollars into circulation. The increased U.S. Federal deficit can only occur if the U.S. Federal expenses are greater than the revenue, which expenses indicate increased U.S. government spending.

This argument has all true premises because  $\phi_1$ ,  $\phi_2$ , and  $\phi_3$  are facts (see footnotes 53, 54, and 55).

This argument meets the total evidence requirement because the factual claim is coherent with the domain of the conclusion, "the U.S. government and recent increased spending." It fits with all of the evidence in this domain, because the U.S. government has spent \$195 billion more dollars on "Education, Training, Employment, and Social Services" in 2021 than in 2018, in line with the reduction in unemployment rate since 2020 (see footnote 56). Both the U.S. dollar value decrease and the increased (from 2019) federal spending to approximately \$4.8 trillion have occurred in 2021.<sup>57</sup> The U.S. Federal deficit increase is the result of increased U.S. government spending surpassing the U.S. government's revenue. It does not contradict any of the evidence in this domain because reduced unemployment rates, decreased dollar value, and increased federal deficit are each occurrences sufficient for the prior indication of the U.S. government and its recent increased spending.

Here is an uncogent generalized causal inference.

 $\phi_1$ : The President of the United States vetoed a beneficial law.

<sup>&</sup>lt;sup>52</sup> This example was last edited on July 30, 2022.

<sup>&</sup>lt;sup>53</sup> See Bureau of Labor Statistics (2022).

<sup>&</sup>lt;sup>54</sup> See Amadeo (2022).

<sup>&</sup>lt;sup>55</sup> See U.S. Treasury Data Lab (2022).

<sup>&</sup>lt;sup>56</sup> See U.S. Treasury Data Lab (2022).

<sup>&</sup>lt;sup>57</sup> See Amadeo (2022).

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#### $\varphi$ : The U.S. is falling apart.

This argument proceeds from cause to effect. The factual claim includes the loosely-defined cause of the president vetoing a beneficial law ( $\phi_1$ ). The inferential claim includes reasoning from this cause to the loosely-defined effect of the country falling apart being probably true in the conclusion ( $\phi$ ).

This argument is uncogent because it is weak. Its inferential claim is false because it is inconsistent with PUN. It would be surprising if the country were to fall apart as a result of the president vetoing a beneficial law, because Congress, as part of the U.S.'s system of checks and balances, could respond by overriding the veto. Even if the two were correlated in reality, this would not imply that one causes the other, because a country falling apart would probably be due to a systematic error instead of one presidential act, in light of many factors.

Regarding (7) above, a **proportional syllogism** is a type of argument in which the factual claim consists in exactly one statement about a proportion of members of a set having some property plus exactly one statement about an individual or object being a member of that set, and the inferential claim consists in reasoning from those two statements to the probably true conclusion that the individual or object has the property. A proportional syllogism argues from some generalization being true to it being true in a particular case. A proportional syllogism generally has the following form:

 $\phi_1$ : X proportion of Y are P.

 $\phi_2$ : O is a Y.

*φ*: O is a P.

Here, X is some fraction or percentage, Y is a set containing members, P is a property, and O is an object.  $\phi_1$  is the **major premise**,  $\phi_2$  is the **minor premise**, and  $\phi$  is the **conclusion**. Each proportional syllogism consists in exactly three statements: exactly one major premise and one minor premise for the factual claim, plus the conclusion. Proportional syllogisms are similar to categorical syllogisms in structure, but they claim that their conclusions follow with a high degree of probability and not with necessity, as is the case with categorical syllogisms. A proportional syllogism is strong if and only if the conclusion follows with at least a 61.80% chance; otherwise, it is weak.

Here is a simple cogent proportional syllogism.

 $\phi_1$ : 20 of the 26 letters in the English alphabet are consonants.

 $\phi_2$ : Z is a letter in the English alphabet.

 $\varphi$ : Z is a consonant.

The factual claim includes the major premise  $\phi_1$ , stating that the 20/26 fraction or proportion of letters in the set of the English alphabet have the property of being a consonant. It also includes the minor premise  $\phi_2$ , stating that some object "Z" is a letter that is a member of the set of the English alphabet. The inferential claim consists in reasoning from the major and minor premises to the claimed probably true conclusion that the same object "Z" has the same property of being a consonant.

This argument is a cogent argument because it is strong, has all true premises, and meets the total evidence requirement. It is strong because the conclusion follows with at least a 61.80% likelihood, specifically a 20/26 = 76.92% likelihood for the 20 out of the 26 letters in the alphabet that are consonants (all except A, E, I, O, U, Y).

It has all true premises because both  $\phi_1$  and  $\phi_2$  are facts; they are common knowledge.

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It meets the total evidence requirement because both  $\phi_1$  and  $\phi_2$  are coherent with the domain of the conclusion, "Z and being a consonant." They fit with all of the evidence  $\phi_d$  in this domain because they describe the set of letters in relation to consonants and one letter Z. They don't contradict any of the evidence in this domain because neither denies that Z is a consonant, which is a fact.

Here is a more complicated cogent proportional syllogism.

 $\phi_1$ : Approximately 6.15% of the countries in the world are in South America.

 $\phi_2$ : Bosnia and Herzegovina is a country in the world.

 $\varphi$ : Bosnia and Herzegovina is not in South America.

This argument is somewhat more complicated than the previous, because the conclusion includes the negative version of the property mentioned in  $\phi_1$ . The factual claim includes the major premise  $\phi_1$ , which states that approximately the proportion or percentage of 6.15 of the countries that are members of the set of world countries have the property of being in South America. It also includes the minor premise  $\phi_2$ , stating that the object "Bosnia and Herzegovina" is a country that is a member of the set of world countries. The inferential claim includes reasoning from this factual claim to the claimed probably true conclusion that the same object "Bosnia and Herzegovina" has the property of NOT being in South America.

This argument is cogent because it is strong, has all true premises, and meets the total evidence requirement. It is strong because the conclusion follows with at least a 61.80% likelihood, specifically an approximate 93.85% likelihood. If approximately 6.15% of the countries in the world are in South America, then it automatically follows that approximately 100-6.15 = 93.85% of the countries in the world are not in South America.

This argument has all true premises because both  $\phi_1$  and  $\phi_2$  are facts.  $\phi_2$  is common knowledge.

This argument meets the total evidence requirement because both  $\phi_1$  and  $\phi_2$  are coherent with the domain of the conclusion, "Bosnia and Herzegovina and not being in South America." They fit with all of the evidence in this domain because they describe the percentage of countries in the world that are in South America and Bosnia and Herzegovina as one of the countries in the world. They do not contradict any of the evidence in this domain because neither  $\phi_1$  nor  $\phi_2$  denies that Bosnia and Herzegovina is not in South America, which is a fact because it is in Europe and not South America.

Here is an uncogent proportional syllogism.

 $\phi_1$ : About 70 to 80 percent of the world's population has brown eyes.<sup>58</sup>

 $\phi_2$ : Amanda Seyfried is a member of the world's population.

 $\varphi$ : Amanda Seyfried has brown eyes.

The factual claim includes the major premise  $\phi_1$ , stating that the percentage or proportion of 70 to 80 of the members or people of the set of the world's population has the property of brown eyes. It also includes the minor premise  $\phi_2$ , stating that the individual Amanda Seyfried is a member of the set of the world's population. The inferential claim includes reasoning from the factual claim to the claimed probably true conclusion that Amanda Seyfried also has the property of brown eyes.

This argument is uncogent because it does not meet the total evidence requirement. The factual claim is not coherent with all of the evidence in the domain of the conclusion, "Amanda Seyfried and having brown eyes." It contradicts the fact that Amanda Seyfried has green instead of brown eyes. Although this is

<sup>&</sup>lt;sup>58</sup> See WorldAtlas (2022).

a **strong proportional syllogism**, because the conclusion follows with at least a 61.80% likelihood, specifically at least a 70% likelihood, and both premises are facts and thus true (see footnote 58), Amanda Seyfried is in fact a member of the minority of the world population in this case.

Regarding (8) above, a **general scientific argument** is a type of argument that can be further classified as either an argument for the discovery of a scientific law or an argument for the application of a scientific law. These types of arguments are very general summaries of scientific findings, and may appear in thought processes or writings in reference to a larger body of more specific scientific findings.

An **argument for the discovery of a scientific law** is a type of argument in which the factual claim includes a statement or statements about observed instances of a certain effect, and the inferential claim includes reasoning from these instances to the conclusion of a named law governing these instances being probably true.

An **argument for the application of a scientific law**<sup>59</sup> is a type of argument in which the factual claim includes references to some known scientific law and a circumstance in which this scientific law could be applied, and the inferential claim includes reasoning from applying this scientific law to this circumstance and saying that this application is probably true in the conclusion.

Here is a simple cogent general scientific argument.

 $\phi_1$ : As a result of millions of past observations of instances of floating objects in water or air, the weight of the displaced fluid or air is equivalent to the buoyant force  $(F_b)$ , the weight of objects in vacuums minus the weight of objects in fluids, and also equivalent to the density of the fluid or gas multiplied by the submerged volume times the gravity.

 $\varphi$ : Wherever there is some buoyant force ( $F_b$ ) for floating objects, Archimedes' Principle,  $F_b = \rho Vg$ , governs it.

This is an argument for the discovery of a scientific law. The factual claim includes a statement  $(\phi_1)$  about the millions of observed past instances of floating objects in water or air having the buoyant force (also the weight of the object in a vacuum minus the weight of the objects in a fluid) being equivalent to the density of the fluid or gas multiplied by the submerged volume times the gravity. The inferential claim includes reasoning from these instances to the probably true conclusion  $\varphi$  that the scientific law named Archimedes' Principle,  $F_b = \rho Vg$ , governs these instances.

This argument is cogent because it is strong, has all true premises, and meets the total evidence requirement. It is strong because its inferential claim is true, or consistent with PUN. If millions of past

<sup>&</sup>lt;sup>59</sup> Please note that Hurley and Watson (2018), classify the argument for the application of a scientific law as a deductive argument, because "scientific laws are widely considered to be generalizations that hold for all times and all places" (38-39). This argument is classified as an inductive argument because a distinction is made here between practical certainty and certainty or absolute certainty. Hurley and Watson (2018)'s classification of this type of argument as deductive is based upon its practical certainty. Such arguments are claimed to have conclusions that hold with practical certainty, but are distinguished from arguments that have conclusions that hold with certainty or absolute certainty, such as arguments in geometry (Hurley and Watson 2018, 38). My classification of an argument as a deductive argument requires that it be based upon either certainty or absolute certainty in all possible worlds, and not merely practical certainty. Practical (or moral) certainty is a high enough degree of probability for an argument with the application at hand to be carried out successfully, but it is not necessarily 100% certain. Certainty is the 100% likelihood for an argument such that you personally cannot think of any possible world in which the premises/sub-conclusions are true and the conclusion false. Absolute certainty is the 100% likelihood for an argument such that BOTH you personally cannot think of any possible world in which the premises/sub-conclusions are true and the conclusion false AND distinguished from anyone's thinking, there cannot be any possible world in which the premises/sub-conclusions are true and the conclusion false. Absolute certainty may be a problem for deduction, because in light of an article the widely-known valid deductive argument form of modus ponens may have some counterexamples. See McGee (1985), Piller (1996), and Mandelkern (2020) for more information.

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observations of a certain effect were equivalent to a buoyant force, also equivalent to  $\rho Vg$ , then it is probably true that there would be some law  $F_b = \rho Vg$  governing those instances and others like it.

This argument has all true premises, because  $\phi_1$  is a fact based upon the common knowledge of millions of observations of instances of floating objects in water or air.

This argument meets the total evidence requirement because  $\phi_1$  is coherent with the domain of the conclusion, "Buoyant force ( $F_b$ ) existence for floating objects and Archimedes' Principle ( $F_b = \rho Vg$ )." It fits with all of the evidence in this domain because it is a description of this domain. It does not contradict any of the evidence in this domain because there are no past observed instances in which the buoyant force does not equal the density of the fluid or gas multiplied by the submerged volume times the gravity.

Here is a more complicated general scientific argument that could be cogent.

 $\phi_1$ : According to the Pauli exclusion principle, no two identical fermions can simultaneously occupy the same quantum state in a quantum system.

 $\phi_2$ : Scientist X observes two electrons<sup>60</sup> simultaneously in the same orbital each have the same spin projection of  $\frac{1}{2}$ .

 $\varphi$ : Scientist X's observation is incorrect.

This argument is an argument for the application of a scientific law. This argument is somewhat more complicated than the previous because the application of the scientific law to the circumstance negates the circumstance itself. The factual claim includes a reference to the known scientific law of the Pauli exclusion principle ( $\phi_1$ ), and a circumstance of two electrons simultaneously in the same orbital and having the same spin projection of  $\frac{1}{2}(\phi_2)$ . The inferential claim includes reasoning from applying this law at  $\phi_1$  to the circumstance mentioned at  $\phi_2$  and concluding at  $\varphi$  that it is probably true that scientist X's observation of the circumstance at  $\phi_2$  is incorrect.

This argument could be cogent because it is strong, could have all true premises, and meets the total evidence requirement. It is strong because its inferential claim is true, or is consistent with PUN. It would not be surprising for scientist X's observations to be incorrect, if scientist X observed two electrons simultaneously in the same orbital with the same spin projection of  $\frac{1}{2}$ , because this is impossible according to the Pauli exclusion principle. At the very least, the two electrons would have to have different spin projections in order to not be in the same quantum state.

This argument could have all true premises, because  $\phi_1$  is a fact, and  $\phi_2$  is a hypothetical scenario that could be a fact if X is some actual scientist.

This argument meets the total evidence requirement because the factual claim is coherent with the domain of the conclusion, "scientist X's observation in this case [an observation contradictory to the Pauli exclusion principle] and incorrectness." It fits with all of the evidence in this domain because it provides the observation in this case at  $\phi_2$  and its incorrectness at  $\phi_1$ . It does not contradict any of the evidence in this domain because there is no available reason why scientist X's observation in this case would be correct.

Here is an uncogent general scientific argument.<sup>61</sup>

 $\phi_1$ : Serological semen detection is capable of determining the correct seminal fluid originator to being a member of 5% of the biological male population.

<sup>&</sup>lt;sup>60</sup> Electrons are types of fermions.

<sup>&</sup>lt;sup>61</sup> This example is inspired by Posley (2020).

 $\phi_2$ : In 1989, a serology expert correctly identified Andrew Johnson as a member of 5% of the biological male population who could be the originator of seminal fluid in a Wyoming 1989 criminal case.

 $\varphi$ . And rew Johnson is the correct seminal fluid originator in this Wyoming 1989 criminal case.

This argument is an argument for the application of a scientific law. The factual claim includes a reference to a known scientific law of serological semen detection determining the correct originator to being within a certain group of 5% of the biological male population ( $\phi_1$ ), and also a reference to a Wyoming 1989 criminal case circumstance in which some member of the biological male population, Andrew Johnson, had his semen determining him to be a member of 5% of the biological male population that could be the correct originator of the semen ( $\phi_2$ ). The inferential claim includes reasoning from applying this serological semen detection law to this circumstance to the probably true conclusion  $\varphi$  that Andrew Johnson is the correct seminal fluid originator in this case.

This argument is uncogent because it does not meet the total evidence requirement. The factual claim is not coherent with the domain of the conclusion, "Andrew Johnson and correct seminal fluid originator for the Wyoming 1989 criminal case." It does not fit with all of the evidence in this domain, because there is the evidence that in 2013 DNA tests excluded Andrew Johnson from being the correct seminal fluid originator; the DNA from the semen was instead that of the victim's then-fiance (Posley 2020). Johnson was then given an "order of actual innocence" (ibid.).

Regarding (9) above, there are a few miscellaneous inductive argument forms that you may run into in real life: argument from compassion, argument from example, general rule argument, and good composition and good division.

Briefly, an **argument from compassion** is a type of argument in which the factual claim includes some evidence that someone is a victim of circumstances, and the inferential claim includes reasoning from such evidence to the probably true conclusion that the person in question is deserving of some benefit or compassion in recompense.

Here is an example of an argument from compassion that could be cogent.

 $\phi_1$ : Modern-day Ukrainian people have been victims of unjust Russian attacks

 $\varphi$ : Modern-day Ukrainian people should not be penalized for seeking asylum in other countries.

The author leaves the explanations as to why this argument is an argument from compassion, and why it could be cogent, as an exercise for the reader.

Briefly, an **argument from example** is a type of argument in which the factual claim includes some example(s) and the inferential claim includes reasoning from such example(s) to some conclusion being probably true.

Here is an example of an argument from example that could be cogent.

 $\phi_1$ : For example, bananas have high levels of potassium.

 $\phi_2$ : For example, oranges have high levels of vitamin C.

 $\phi_3$ : For example, kiwis have antioxidants and are high in vitamin C.

 $\phi_4$ : For example, apples are high in quercetin.

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 $\varphi$ : Fruits are healthy.

The author leaves the explanations as to why this argument is an argument from example, and why it could be cogent, as an exercise for the reader.

Briefly, a **general rule argument** is a type of argument in which the factual claim includes a general rule and an individual or group that falls under the subject of the general rule, and the inferential claim includes reasoning from applying this general rule to the individual or group, concluding that it is probably true that the individual or group meets this general rule. Some general rules include, but are not limited to, "birds fly," "Christians believe in God," "planets in the Milky Way Galaxy revolve around stars," "artists are creative," "sports are fun," etc. Note that general rules are not necessarily true; they are meant to apply in most but not all cases.

Here is an example of a cogent general rule argument.

 $\phi_1$ : As a general rule, unkindness weakens relationships in society.

 $\phi_2$ : Bullying is an act of unkindness.

 $\varphi$ : Bullying weakens relationships in society.

The author leaves the explanations as to why this argument is a general rule argument, and why it is cogent, as an exercise for the reader.

Briefly, **composition** is a type of argument in which the factual claim includes some general statement(s) about the parts of something having a property, and the inferential claim includes reasoning from that general statement(s) to the whole thing or class having that same property. The conclusion would be a class statement, and would be probably true. A **general statement** is a type of statement that says something about each and every member of a class. It employs **distributive predication**, which is a type of predication in which an attribute or property is said of each and every member of a class. A **class statement** is a type of statement that says something about some class as a whole. It employs **collective predication**, which is a type of predication in which an attribute or property is said of a whole class.





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 $\phi_1$ : Each card in a deck of cards has physical weight.

 $\varphi$ : An entire deck of cards has physical weight.

The author leaves the explanations as to why this argument is composition, and why it is cogent, as an exercise for the reader.

Briefly, **division** is loosely the opposite of composition. Division is a type of argument in which the factual claim includes some class statement about a whole thing or class having the same property, and the inferential claim includes reasoning from that class statement to the probably true conclusion that some part or parts of the whole thing in question each have the same property.

What is the difference between a class and a set? We already defined a set in chapter 1 as a thing or a group of things. A class builds upon sets. A **class** is either a set or a group of sets whose members as a whole share some property. The word "class" instead of "set" is used in the definitions of composition and division because there is talk of entire things having some property.

Here is an example of cogent division.

 $\phi_1$ : The Callahan family has a last name.

 $\varphi$ : Each member of the Callahan family has a last name.

The author leaves the explanations as to why this argument is division, and why it is cogent, as an exercise for the reader.

Now that we've gone through some inductive argument forms and real-life examples, let's get into some more exercises involving these.

[This is where the bullet point summary and section 3.3 exercises will be inserted.]

[This is where a chapter 3 cumulative practice test will be inserted.]

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# **Chapter 4**

# **Inference to the Best Explanation**

# Introduction

Inferences to the best explanation are a relatively new, yet nonetheless crucial, form of reasoning. In this chapter, you will learn what an inference to the best explanation is, how it is distinguished from abduction in a sense, how it is formalized, and what good and bad inferences to the best explanation are.

# Objectives

Understand what an inference to the best explanation is, how it is distinguished from abduction, how it is formalized, and identify and explain what a good inference to the best explanation is as opposed to a bad one.

# Section 4.1: What Inference to the Best Explanation Is

Section objectives:

- Understand what an inference to the best explanation is.
- Understand how an inference to the best explanation is distinguished from abduction.
- Understand how an inference to the best explanation is formalized.
- Identify and explain what a good inference to the best explanation is as opposed to a bad one.

#### **Key Terms**

Abduction: the set of all abductive arguments

Abductive argument: explanatory reasoning in generating, adopting, or discovering worthy scientific hypotheses in order to avoid contradictions or surprising observations and evidence, put forth by Charles Sanders Pierce, that is part of the logic of science

**Inference to the Best Explanation (IBE):** the modern sense of "abductive argument;" explanatory reasoning in the justification of hypotheses etc., that is mainly part of logic and philosophy

Abduction (Formal): the set A such that A:  $a_{h1}$ ,  $a_{h2}$ ,  $a_{h3}$ ,...,  $a_{hn}$ ;  $a_{m1}$ ,  $a_{m2}$ ,  $a_{m3}$ , ...,  $a_{mn}$ , where n is any number,  $a_{hn}$  is some abductive argument, and  $a_{mn}$  is some inference to the best explanation

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**IBE Clause:** the clause in a formalized inference to the best explanation that states that, given evidence E and hypotheses  $H_1, \ldots, H_n$  of E, if  $H_i$  explains E better than any of the other hypotheses, infer that  $H_i$  is closer to the truth than any hypothesis  $H_1, \ldots, H_n$ 

**Theoretical Virtues:** also known as explanatory virtues, the qualities that a hypothesis possesses that makes it better off than it otherwise would have been, which include: simplicity, generality (or universal applicability), coherence with scientific theories, and fit

 $H_1, ..., H_n$ : the set of all the hypotheses you can think of as candidates to explain evidence E in a formalized inference to the best explanation, where each hypothesis in this group is a single statement

Candidate Hypothesis: a statement that could explain the evidence

**Good Inference to the Best Explanation (General):** an inference to the best explanation that succeeds in establishing some link between explanation of the evidence and the (non-necessary) truth of the hypothesis or statement that best explains such evidence

**Bad Inference to the Best Explanation (General):** an inference to the best explanation that does not succeed in establishing some link between explanation of the evidence and the (non-necessary) truth of the hypothesis or statement that best explains such evidence

Good Inference to the Best Explanation (Formal): an inference to the best explanation in which both all premises, or  $P_1 \dots P_n$  and  $P_{n+1}$ , are true and the IBE clause is met

**Bad Inference to the Best Explanation (Formal):** an inference to the best explanation in which it is not the case that both all premises,  $P_1 \dots P_n$  and  $P_{n+1}$ , are true and the IBE clause is not met

Factual Claim (IBE): the condition of all premises being true

Inferential Claim (IBE): the condition of the IBE clause being met

**Factual Claim (Simple IBE – Set Theory):** the subset  $\Delta \subset \Gamma$ , where  $\Gamma$  is the argument with the inference to the best explanation, such that the set of premises  $\{P_1 \dots P_n, P_{n+1}\} = \Delta$ 

**Factual Claim (Complex IBE – Set Theory):** the subset  $\Delta' \subset \Gamma$ , where  $\Gamma$  is the argument with the inference to the best explanation, such that the set of premises  $\{P_1 \dots P_n, P_{n+1}\} \subset \Delta'$  and the set of sub-conclusions  $\{a_1, a_2, a_3, \dots, a_n\} \subset \Delta'$ 

**Inferential Claim (IBE – Set Theory):** IBE clause =  $\{\Delta (\text{or } \Delta') \rightarrow \psi\}$ 

→ (Inferential Claim): a symbol which indicates the reasoning process within an inferential claim

Good Inference to the Best Explanation (Set Theory): an inference to the best explanation in which both

 $\{P_1 \dots P_n, P_{n+1}\} \subset \Delta \neq \emptyset \subset \Gamma$  (or  $\{P_1 \dots P_n, P_{n+1}; a_1, a_2, a_3, \dots a_n\} \subset \Delta' \neq \emptyset \subset \Gamma$ ), and IBE clause =  $\{\Delta (\text{or } \Delta') \rightarrow \psi\} \neq \emptyset \subset \Gamma$ 

**Bad Inference to the Best Explanation (Set Theory):** an inference to the best explanation in which it is not the case that both  $\{P_1 \dots P_n, P_{n+1}\} \subset \Delta \neq \emptyset \subset \Gamma$  (or  $\{P_1 \dots P_n, P_{n+1}; a_1, a_2, a_3, \dots a_n\} \subset \Delta' \neq \emptyset \subset \Gamma$ ), and IBE clause =  $\{\Delta \text{ (or } \Delta') \rightarrow \psi\} \neq \emptyset \subset \Gamma$ 

" $\neq \emptyset \subset \Gamma$ ": a group of symbols that are another way of saying that the set or entity in question "is true," and for the argument  $\Gamma$  it means "is not equal to the empty set, which is a proper subset of the set of the argument  $\Gamma$ "

"=  $\phi$ ": symbols which mean that something is equivalent to the empty set

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" $\neq \emptyset$ ": symbols which mean that something is not equivalent to the empty set

Falsity (Alternate): the state of being equivalent to the empty set

Truth (Alternate): the state of not being equivalent to the empty set

In chapter 2, it is discussed how, for the inferential claim of an argument, the arguer can claim, or it can be claimed, that the conclusion "follows from" the factual claim in one of two ways:

- (1) in all possible worlds
- (2) NOT in all possible worlds

Chapter 2 discusses option (1), namely deductive arguments. Option (2) includes both inductive arguments, abductive arguments, and arguments with an inference to the best explanation. Chapter 3 discusses inductive arguments, whereas this chapter, chapter 4, discusses arguments with an inference to the best explanation as distinguished from abductive arguments.

Arguments with an inference to the best explanation, or, in other words, **inferences to the best explanation (IBE)**, loosely, are distinguished from the set of all abductive arguments, or **abduction**. An **abductive argument** has two main senses: one historical and one modern. The historical sense of an abductive argument is explanatory reasoning in generating, adopting, or discovering worthy scientific hypotheses in order to avoid contradictions or surprising observations and evidence, put forth by Charles Sanders Peirce, that is part of the logic of science.<sup>76</sup> The modern sense of an abductive argument is explanatory reasoning in the justification of hypotheses etc., that is mainly part of logic and philosophy, and what is called "inference to the best explanation."

The following may be considered an example of an inference to the best explanation in logic and philosophy.

*Bank Robbery Case:* The bank was robbed yesterday. Maria bought a gun two days ago. Maria needed money to pay her bookmaker. Maria was seen near the bank earlier in the day. The bookmaker just received a check. Therefore, it was Maria who robbed the bank yesterday.

The explanatory reasoning here is in the first five statements, which are clues justifying the hypothesis "it was Maria who robbed the bank yesterday." Most likely, the best explanation of the above statements (assuming they are true), is that it was Maria who robbed the bank yesterday. Contrast this with the following, which may be considered an example of an abductive argument within the logic of science.

*Flashing Bright-Green Lights Case:* You are staying at a ski resort in Aspen, Colorado. You observe a group of flashing bright-green lights outside the window in the room you are staying at about 10:30pm. These flashing bright-green lights phenomena would not be surprising if: either, an alien UFO is nearby, there is a group of jets with green lights nearby, there is a fireworks show with bright-green fireworks nearby, or you are experiencing a schizophrenic delusion. The hypothesis "there is a fireworks show with bright-green fireworks nearby" is most likely to be the case, and therefore probably true.

This argument uses explanatory reasoning to generate worthy hypotheses in order to avoid the surprising observations of the bright-green lights. Each of the four hypotheses generated is a case that could explain the presence of the bright-green lights. The explanatory reasoning comes in with weighing the

<sup>&</sup>lt;sup>76</sup> See Douven 2021 and Carneades.org 2015 for more information.

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probability or likelihood of truth of each of the hypotheses, which the hypothesis "there is a fireworks show with bright-green fireworks nearby" being the most likely and therefore the most worthy one. Building upon these examples, here is an example of abduction, or the set of all abductive arguments, whether in the historical or modern sense.

Let A be the set such that A:  $a_{h1}$ ,  $a_{h2}$ ,  $a_{h3}$ ,...,  $a_{hn}$ ;  $a_{m1}$ ,  $a_{m2}$ ,  $a_{m3}$ , ...,  $a_{mn}$ , where n is any number.

The set A here is "Abduction," and its elements are all the abductive arguments. Any element  $a_{hn}$  is some abductive argument (in the historical sense), whereas any element  $a_{mn}$  is some inference to the best explanation (the modern sense of an abductive argument). Let the Bank Robbery Case =  $a_{m1}$  and the Flashing Bright-Green Lights Case =  $a_{h1}$ . The other  $a_{mn}$  are different inferences to the best explanation, potentially infinite in number, and the other  $a_{hn}$  are different abductive arguments, also potentially infinite in number. (Building block examples)

Because Peirce's version of abductive arguments, namely what he calls "abduction," involves insight into inventing new hypotheses and ranking untested hypotheses, it is more concerned with the discovery of viable hypotheses that one may have reason to suspect as being true (Mohammadian 2021, 8, 19; Cabrera 2022, 9). It is a more abstract notion and not as technical, and also not as concerned with the justification of hypotheses or with which explanation is the best explanation of the facts, data, or premises in an argument (Mohammadian 2021, 16, 19; Cabrera 2022, 3-4). Abduction is more abstract than inference to the best explanation, because abduction involves untested hypotheses whereas inference to the best explanation involves tested hypotheses, presumably (Mohammadian 2021, 19).

With this background in mind, here is the most recent and accepted formalization of IBE in philosophy, where n is any number.

 $P_1 \dots P_n$ : Evidence *E*.

 $P_{n+1}$ : Hypothesis  $H_i$  of  $H_1, \ldots, H_n$  explains E better than any of the other  $H_1, \ldots, H_n$ .

IBE Clause: Given evidence E and hypotheses  $H_1, ..., H_n$  of E, if  $H_i$  explains E better than any of the other hypotheses, infer that  $H_i$  is closer to the truth than any hypothesis  $H_1, ..., H_n$ .

C:  $H_i$  is closer to the truth than any hypothesis  $H_1, ..., H_n$ .<sup>77</sup>

There is a lot to unpack here. Each inference to the best explanation has to fit this form. The evidence E at  $P_1 \dots P_n$  could be any statement or group of statements claiming to say something about the actual world, whether or not the statement(s) are true or false. Furthermore, such evidence may be in the form of facts and/or data from various sources. At the next premise,  $H_1, \dots, H_n$  is the set of all the hypotheses you can think of as candidates to explain evidence E. Each hypothesis in this group is a single statement, such as "the speed of light in vacuum is the same for all observers, regardless of the motion of the light source or the observer."<sup>78</sup> In order for some  $H_i$  to explain the evidence better than any of the other hypotheses in question, it must have what's called theoretical, or explanatory, virtues. Theoretical virtues are the qualities that a hypothesis possesses that makes it better off than it otherwise would have been. They include: simplicity, generality (or universal applicability), coherence with scientific theories, and fit with what is already known (Douven 2021 and Wireless Philosophy 2015). Currently, what these virtues consist in is somewhat intuitive (Douven 2021). The more explanatory virtues a hypothesis has, and to a higher degree, the better it is. Additionally, a hypothesis that explains the evidence better than any of the others may be

<sup>&</sup>lt;sup>77</sup> See Douven 2021.

<sup>&</sup>lt;sup>78</sup> This is the second postulate of Einstein's special theory of relativity.

better at eliminating the impossible than the others (CrashCourse, 2016).<sup>79</sup> Note that a more informative hypothesis is not necessarily a better one, and not necessarily one that would possess any of the theoretical virtues; a more informative theory is also more likely to be false, and have more equivocations and more contradictions, according to van Fraassen 1989 (Douven 2021).

Ideally, the set of all the hypotheses you can think of,  $H_1, ..., H_n$ , is equivalent to the set of all possible **candidate hypotheses**. A candidate hypothesis is a statement that could explain the evidence. However, because  $H_1, ..., H_n$  may not include all of the possible candidate hypotheses, or there is a mismatch between  $H_1, ..., H_n$  and the set of all possible candidate hypotheses, the **IBE Clause** above is needed. Note that the IBE Clause is NOT a premise. Another way of stating this clause is, on the assumption of  $P_1 \ldots P_n$  and  $P_{n+1}$ , the conclusion, that  $H_i$  is closer to the truth than any hypothesis  $H_1, ..., H_n$ , only if  $H_i$  is sufficiently good or satisfactory as an explanation, follows. Note that  $H_i$  is *closer to the truth than any hypothesis*  $H_1, ..., H_n$ , and not "probably" or "approximately true." This is a qualified and relative statement concerning the truth of  $H_i$ .  $H_i$  is not probably true, because being probably true means being true in a certain percentage of possible worlds, which does not apply here. Rather,  $H_i$  is more true in relation to the other hypotheses  $H_1, ..., H_n$ .  $H_i$  is also not approximately true. Approximate truth may imply a quantitative or numerical distance from the truth, but  $H_i$  is closer to the truth here because it has a quality that is closer to the truth over the quality of any hypothesis  $H_1, ..., H_n$ . Although the truth of the conclusion does not follow probably or approximately, it nevertheless, as stated above, does not follow by necessity. It does not follow in all possible worlds.

**Inferences to the best explanation** aim to establish some link between explanation of the evidence and the (non-necessary) truth of the hypothesis or statement that best explains such evidence. A **good inference to the best explanation** succeeds in establishing this link, whereas a **bad inference to the best explanation** does not succeed in establishing this link. A good inference to the best explanation may be defined as follows.

An inference to the best explanation is a good inference to the best explanation if and only if:

(1) All premises are true

AND

(2) The IBE clause is met.

Otherwise, an inference to the best explanation is a bad inference to the best explanation.

Number 1 above is the factual claim for an IBE, and number 2 above is the inferential claim for an IBE. Turning back to a formalized IBE, an inference to the best explanation is a good one, where  $H_i$  is closer to the truth than any hypothesis  $H_1, ..., H_n$ , if and only if all premises are true – that is, both  $P_1 ..., P_n$ , and  $P_{n+1}$  are true -- and the IBE Clause is met. Otherwise, an inference to the best explanation is a bad inference to the best explanation.

The factual claim for a simple inference to the best explanation, which has premise(s) and one conclusion, can be defined in set theory as follows:

the subset  $\Delta \subset \Gamma$ , where  $\Gamma$  is the argument with the inference to the best explanation, such that the set of premises  $\{P_1 \dots P_n, P_{n+1}\} = \Delta$ .

<sup>&</sup>lt;sup>79</sup> Another way to determine which hypothesis is the better explanation of the evidence is to use Bayesian Epistemology, discussed in chapter 3 of this book and Cabrera (2022).

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The **factual claim** for a complex inference to the best explanation, which has premise(s), subconclusion(s), and one conclusion, can be defined in set theory as follows:

the subset  $\Delta' \subset \Gamma$ , where  $\Gamma$  is the argument with the inference to the best explanation, such that the set of premises  $\{P_1 \dots P_n, P_{n+1}\} \subset \Delta'$  and the set of sub-conclusions  $\{a_1, a_2, a_3, \dots, a_n\} \subset \Delta'$ .

The **inferential claim** for an inference to the best explanation, whether simple or complex, is equivalent to the IBE clause. In other words,

IBE clause = { $\Delta$  (or  $\Delta'$ )  $\rightarrow \psi$ }

is the inferential claim here, where  $\psi$  is the statement that is the conclusion, and the " $\rightarrow$ " indicates the reasoning process from the factual claim ( $\Delta$  (or  $\Delta'$ )) to the conclusion  $\psi$ .

Thus, if we are to define a good inference to the best explanation in set theory, we may define it as thus.

An inference to the best explanation is a good inference to the best explanation if and only if

BOTH

 $\{P_1 \dots . P_n, P_{n+1}\} = \Delta \neq \emptyset \subset \Gamma \; (\text{or} \; \{P_1 \dots . P_n, P_{n+1}; a_1, a_2, a_3, \dots \; a_n\} = \Delta' \neq \emptyset \subset \Gamma)$ 

AND

IBE clause =  $\{\Delta \text{ (or } \Delta') \rightarrow \psi\} \neq \emptyset \subset \Gamma$ ; otherwise, it is a bad inference to the best explanation.

Here, " $\neq \emptyset \subset \Gamma$ " is another way of saying that the set or entity in question "is true." " $\neq \emptyset \subset \Gamma$ " means "is not equal to the empty set, which is a proper subset of the set of the argument  $\Gamma$ ." All sets, whether they are subsets of another set or not, are either empty or not empty. If they are empty, then they are equivalent to the empty set (=  $\emptyset$ ). Equivalence to the empty set is also falsity<sup>80</sup>, so any set that is equivalent to the empty set is false. If they are not empty, then they are not equivalent to the empty set ( $\neq \emptyset$ ). Not being equivalent to the empty set is also truth, so any set that is not equivalent to the empty set is true. In the context of the argument  $\Gamma$  and the conditions for  $\Gamma$  being a good inference to the best explanation, then,  $\{P_1 \dots P_n, P_{n+1}\} = \Delta \neq \emptyset \subset \Gamma$  (or  $\{P_1 \dots P_n, P_{n+1}; a_1, a_2, a_3, \dots a_n\} = \Delta' \neq \emptyset \subset \Gamma$ ) means " $\{P_1 \dots P_n, P_{n+1}\} \subset \Delta$  is true (or  $\{P_1 \dots P_{n+1}; a_1, a_2, a_3, \dots a_n\} = \Delta' = \{\Delta \text{ (or } \Delta') \rightarrow \psi\}$  $\neq \emptyset \subset \Gamma$  means "IBE clause =  $\{\Delta \text{ (or } \Delta') \rightarrow \psi\}$  is true."

Here is an example of a good inference to the best explanation.

Nathan Leopold and Richard Loeb decided to plan to execute the perfect crime in the Chicago area. Specifically, they had the motive of kidnapping and murdering Bobby Franks. Leopold's glasses were found near Bobby Franks' body. The destroyed and stolen typewriter on which Loeb and Leopold had written the ransom note was found in a lagoon near Chicago. Leopold and Loeb's alibi was exposed when Leopold's chauffer told the police he was repairing the car while Leopold and Loeb had already claimed to be using it. Loeb and Leopold later confessed to participating in the murder of Bobby Franks. The most logical explanation is that both Nathan Leopold and Richard Loeb together killed Bobby Franks.

<sup>&</sup>lt;sup>80</sup> Equivalence to the empty set may also be non-applicability.
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A good inference to the best explanation may appear as above in real life, but it is also formalized as follows as stated above.

 $P_1$ : Nathan Leopold and Richard Loeb decided to plan to execute the perfect crime in the Chicago area.

 $P_2$ : Leopold and Loeb had the motive of kidnapping and murdering Bobby Franks.

 $P_3$ : Leopold's glasses were found near Bobby Franks' body.

 $P_4$ : The destroyed and stolen typewriter on which Loeb and Leopold had written the ransom note was found in a lagoon near Chicago.

 $P_5$ : Leopold and Loeb's alibi was exposed when Leopold's chauffer told the police he was repairing the car while Leopold and Loeb had already claimed to be using it.

 $P_6$ : Loeb and Leopold later confessed to participating in the murder of Bobby Franks.

 $P_7$ : Hypothesis "both Nathan Leopold and Richard Loeb together killed Bobby Franks" of other hypotheses implicating other potential candidates explains  $P_1...P_6$  better than those other hypotheses.

IBE Clause: Given evidence  $P_1 \dots P_6$  and the other hypotheses implicating other potential candidates explaining  $P_1 \dots P_6$ , if "both Nathan Leopold and Richard Loeb together killed Bobby Franks" explains  $P_1 \dots P_6$  better than any of the other hypotheses, infer that "both Nathan Leopold and Richard Loeb together killed Bobby Franks" is closer to the truth than any of the other hypotheses implicating other potential candidates.

C: It is closer to the truth than any of the other hypotheses implicating other potential candidates that both Nathan Leopold and Richard Loeb together killed Bobby Franks.

This inference to the best explanation is a good one because it both has all true premises and the IBE clause is met.  $P_1$ ,  $P_2$ ,  $P_3$ ,  $P_4$ ,  $P_5$ , and  $P_6$  are each true because each of them is a fact.  $P_7$  is true because it is a fact that no other persons besides Nathan Leopold and Richard Loeb were considered to be candidates for the murder. Each of these facts has substance and is not equivalent to the empty set, so each is also true in this sense. The IBE clause is met because the hypothesis "both Nathan Leopold and Richard Loeb together killed Bobby Franks" has the most explanatory virtues out of any other potential hypotheses involving other candidates for the crime. It is the simplest hypothesis, because Leopold and Loeb had directly confessed that they committed the crime, and it is also the hypothesis with the best fit with background knowledge, because it corroborates with the other pieces of evidence in  $P_1...P_6$ . The IBE Clause has substance here, so it is not equivalent to the empty set, and thus also true in this set-theoretical sense.

The author leaves the composition of an example of a bad inference to the best explanation, and the explanation why thereof, as an exercise for the reader.

Now that we've learned and thought more about inferences to the best explanation, let's do some exercises testing our understanding of what they are and how they operate in real life.

[This is where the bullet point summary and section 4.1 exercises will be inserted.]

[This is where a chapter 4 cumulative practice test will be inserted.]

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# **Chapter 5**

# Logic and Philosophy of Language

# Introduction

[Content in development.]

# **Objectives**

- Section 5.2: Understand what the different types of theories of truth are, and how different truthbearers each can be either true or false under some theories.
- Section 5.3: Understand what counts as a fact, what may count as a fact, and what does not count as a fact.

# Section 5.1: Meaning

[This is where the bullet point summary and section 5.1 exercises will be inserted.]

# **Section 5.2: Theories of Truth**

Section objectives:Understand what the different types of theories of truth are, and how different truthbearers each can be either true or false under some theories.

#### **Key Terms**

**Theory of Truth:** a theory in the contemporary philosophical literature that states what it is for a statement to be true and/or false, and/or what it is for a proposition to have the truth value TRUE and/or FALSE

Truthmaking Principle: for every truth, there is something that makes it true

**Correspondence Theory of Truth:** also known as the classical view of truth, a type of theory of truth, present in Aristotle's and St. Thomas Aquinas' writings at least, stating that a statement is true if it corresponds to reality, where reality is the way things are or the facts

**Correspondence Theory of Truth (General):** a type of theory of truth that states that a truthbearer is true if and only if it corresponds with a truthmaker

Truthbearer: beliefs, thoughts, ideas, judgments, statements, assertions, utterances, sentences, or propositions

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Truthmaker: facts, states of affairs, events, things, tropes, or properties

**Neo-classical Correspondence Theory of Truth:** a theory of truth that is a slight modern variation on the correspondence theory of truth

Ontological Thesis: a thesis relating to the idea of existence

**Correspondence Theory of Truth (Ontological Thesis):** a version of the correspondence theory of truth, or a neo-classical theory of truth, that states that a statement is true if and only if there *exists* a fact to which it corresponds; otherwise, it is false

**Identity Theory of Truth:** a type of theory of truth put forth by Bertrand Russell and G. E. Moore initially, stating that a statement's proposition has the truth value TRUE if and only if it is a fact; otherwise, it has the truth value FALSE

"if": a word that indicates that an antecedent follows

"only if": words that indicate that a consequent follows

"if and only if": words that indicate that what is on the left is an antecedent for the consequent on the right, and what is on the right is an antecedent for the consequent on the left

Antecedent: part of an "if...then...", "only if", or "if and only if" conditional statement that includes a sufficient condition

**Consequent:** part of an "if...then...", "only if", or "if and only if" conditional statement that includes a necessary condition

**Coherence Theory of Truth:** a type of theory of truth that states that a truthbearer is true if and only if it is part of a coherent system of truthbearers and/or truthmakers, otherwise, it is false, where coherence is either some form of entailment or mutual explanatory support, and the system of truthbearers and/or truthmakers is ideally the set of truthbearers and truthmakers each of which an omniscient being would know the truth of

**Omniscient Being (Coherence Theory of Truth):** a type of being who knows every truthbearer and/or truthmaker, who rationally orders each proposition as TRUE or FALSE, and who is capable of knowing truths that people normally wouldn't be capable of

**Logical Entailment (Coherence Theory of Truth):** what is part of the coherence theory of truth under a version of it where the truthbearer follows from combining members of the set of truthbearers and truthmakers somehow through logical rules, such as modus ponens and modus tollens

**Mutual Explanatory Support (Coherence Theory of Truth):** what is part of the coherence theory of truth under a version of it where each truthbearer or truthmaker in the set of truthbearers and truthmakers is explained by at least one other truthbearer or truthmaker in the same set

**Circular Explanation:** where one truthbearer or truthmaker explains another, and that same other truthbearer or truthmaker explains the former

**Neo-classical Coherence Theory of Truth:** a type of theory of truth that is the same as the coherence theory of truth, except that coherence involves a relation from the content of a truthbearer to the content of a truthbearer; there is no relation to the world involved

**Pragmatist Theory of Truth:** a type of theory of truth that generally states that a truthbearer is true if and only if it is useful; otherwise, it is false

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**Consensus Theory of Truth:** a type of theory of truth that states that a truthbearer is true if and only if there is a consensus, either actual or ideal, among the general population

Actual Consensus (Consensus Theory of Truth): a consensus in the actual world, physical or non-physical

**Ideal Consensus (Consensus Theory of Truth):** a hypothetical consensus that would be true if something were the case

**General Population (Consensus Theory of Truth):** at least 97.72% of the currently living population on Earth

**Constructivist Theory of Truth:** a type of theory of truth that states that a truthbearer is true if and only if it is constructed somehow by the scientific community at large through one of the various scientific methodologies; otherwise, it is false

Science (Constructivist Theory of Truth): what results from trying to prove a theory or idea wrong and failing

**Pseudoscience (Constructivist Theory of Truth):** faulty science or otherwise that results from trying to prove a theory or idea or confirm it

Verificationist Theory of Truth: a type of theory of truth that states that a truthbearer is true if and only if it is verified in the actual world in some way, either empirically or conceptually at least; otherwise it is false

**Empirical Verification:** a type of verification that is *a posteriori* at least by the five senses and common sense in the physical actual world

a posteriori: Latin words that mean "from what comes after [in the world]"

**Conceptual Verification:** a type of verification that is *a priori* at least by the mind in the non-physical actual world

a priori: Latin words that mean "from what is before [in the mind]"

**Redundancy Theory of Truth:** a type of theory of truth, equivalently the prosentential theory of truth, that uses a recursive definition of truth and states that a truthbearer is true if and only if the truthbearer itself is true; otherwise, it is false

Recursive Definition of Truth: "x" is true if and only if "x," where x is a truthbearer; otherwise, x is false

**Liar's Paradox:** the paradox upon considering the case where the truthbearer x is false, or not true, under the recursive definition of truth; then, "x is not true" is true if and only if "x is not true;" however, if "x is not true," then it cannot be the case that "x is not true" is true; also, if "x is not true" is true, then it cannot be the case that "x is not true," and not true, which is a contradiction

**Revision Theory of Truth:** a type of theory of truth which states that truth needs to be revised due to the liar's paradox

**Tarski's Semantic Theory of Truth:** a type of theory of truth that states that a truthbearer "x" is true if and only if x; otherwise, x is false

**Performative Theory of Truth:** a type of theory of truth in which ascribing truth to a truthbearer just is to license endorsement of belief in the truth of the truthbearer

**Prosentences:** placeholders for sentences, such as "is true," just as pronouns are placeholders for their antecedents called nouns

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**Pluralist Theory of Truth:** a type of theory of truth that can include some combination of at least more than one of the following - correspondence theory of truth, neo-classical correspondence theory of truth, identity theory of truth, coherence theory of truth, neo-classical coherence theory of truth, pragmatist theory of truth, consensus theory of truth, constructivist theory of truth, verificationist theory of truth, redundancy/prosentential theory of truth, Tarski's semantic theory of truth, and the performative theory of truth, and maybe more

**Deflationary Theory of Truth:** a type of theory of truth that states that the word "truth" is not needed because it is empty in meaning

Nihilism: a theory which states that nothing is true

**Principle of Bivalence:** a principle that states that "either A or not-A" and holds in the domain of classical logic and thus this textbook

Truth matters for logic, because in order to have either a sound, cogent, or good IBE argument, each premise has to be true, at least, as chapters 2, 3, and 4 discuss. There are different **theories of truth** in the contemporary philosophical literature, each stating what it is for a statement to be true and/or false, and/or what it is for a proposition to have the truth value TRUE and/or FALSE (Glanzberg, 2021). In what follows, here are these different theories of truth in general. Most theories discussed rest upon the **truthmaking principle**: for every truth, there is something existent that makes it true (David 2022).

The most well-known theory of truth, it seems, is the correspondence theory of truth, remnants of which are present in Aristotle's and St. Thomas Aquinas' thought at least. According to the **correspondence theory of truth**, also known as the classical view of truth, a statement is true if and only if it corresponds to reality, or the way things are. Otherwise, it is false. In other words, a statement is true if and only if it corresponds to the facts; otherwise, the statement is false. 81 The statement does not have to be identical with or to a fact. As Aristotle states,

"To say that what is is not, or that what is not is, is false and to say that what is is, and that what is not is not, is true. (M 4.7)" (Melchert & Morrow 2019, 187).

Above, saying something is or is not is the statement, and "that what is" and "that what is not" refers to reality, the way things are, or the facts. Again, recapitulating what is discussed in chapter 1, "fact" here is anything, such as an event or state of affairs, that obtains or exists in the actual world, presumably either the physical or non-physical actual world.

In accordance with the truthmaking principle, here, it is correspondence to reality, or the way things are, or the facts, that makes a true statement true. What is on the left of the words "if and only if" is the antecedent for the consequent on the right, and what is on the right of the words "if and only if" is the antecedent for the consequent on the left. There is an antecedent and consequent on both sides, but the direction is always from antecedent to consequent for each conditional statement. The antecedent is the part of a conditional statement that contains a sufficient condition, and the consequent is the part of the conditional statement that contains a necessary condition.

Here is an example of a true statement according to this theory.

The statement "the universe began almost 14 billion years ago with a massive expansion from a single point" is true if and only if it corresponds to reality; otherwise, it is false.

<sup>&</sup>lt;sup>81</sup> Regarding the previous three sentences, see Glanzberg (2021), David (2022), and Melchert & Morrow (2019), pg. 187.

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This statement is true because it corresponds to reality. It corresponds to the reality of the Big Bang theory, which is based upon observations related to scientific laws such as Hubble's law of space expansion and also related to cosmic microwave background radiation. Thus, the conditional statement, from the antecedent on the left to the consequent on the right, is fulfilled. The statement corresponding to the evidence behind the Big Bang theory is a sufficient condition for it being true. It is also a necessary condition for it being true. This statement is part of the Big Bang theory, which could only have come about through such empirical observations related to Hubble's law and cosmic microwave background radiation. Thus, the conditional statement from the antecedent on the left to the consequent on the right is fulfilled. Because both conditional statements are fulfilled, this statement is true.

Here is an example of a false statement according to this theory.

The statement "*The Secret Garden* is written with the stream-of-consciousness technique" is true if and only if it corresponds with reality; otherwise, it is false.

This statement is false because one of the conditional statements, namely the one from left to right, is not fulfilled. This statement does not correspond with reality because it is not a fact that *The Secret Garden* is written with the stream-of-consciousness technique. This technique involves replicating a character's thought processes usually with incoherent leaps in thought and no punctuation at-large in the written work, but, instead, no such thing is present in *The Secret Garden*.

A more general version of the correspondence theory of truth is: a truthbearer is true if and only if it corresponds with a truthmaker (David 2022). Here, "**truthbearer**" may refer to beliefs, thoughts, ideas, judgments, statements, assertions, utterances, sentences, or propositions (ibid.). "**Truthmaker**" may refer to facts, states of affairs, events, things, tropes, or properties (ibid.).

There are some slight modern variations on the correspondence theory of truth, sometimes each dubbed a "**neo-classical correspondence theory of truth**" (Glanzberg 2021). One version is an **ontological thesis**, or a thesis relating to the idea of existence, of the correspondence theory of truth: a statement is true if and only if there *exists* a fact to which it corresponds; otherwise, it is false (ibid.). Another version, somewhat present in Bertrand Russell's and Ludwig Wittgenstein's works, takes the correspondence between a true statement and reality or fact to be a sort of structural congruence. For example, the statement "Anastasia loves Artyom" is true if and only if there exists someone named Anastasia, and there exists someone named Artyom, and Anastasia is related to Artyom through being in love with him. Another version, put forth by the philosopher J. L. Austin, views the correspondence as a sort of semantic correlation. For example, the statement "Anastasia loves Artyom" is true if and only if strue if and only if both "Anastasia" and "Artyom" and "Anastasia being in love with Artyom" correlate with some state of affairs that exists. Other versions treat the correspondence or mis-correspondence as a sort of agreement with facts or states of affairs. For example,

a truthbearer x is true if and only if x agrees with some fact; a truthbearer x is false if and only if x disagrees with some fact,

a truthbearer x is true if and only if x corresponds with some fact that exists; a truthbearer x is false if and only if x corresponds with some fact that does not exist,

a truthbearer x is true if and only if x corresponds with some state of affairs that obtains in the actual world; a truthbearer x is false if and only if x corresponds with some state of affairs that does not obtain in the actual world (David 2022),

a truthbearer is true if it represents a fact; a truthbearer is false if it does not represent a fact (Glanzberg 2021).

According to the **identity theory of truth**, put forth by Bertrand Russell and G. E. Moore initially, a statement's proposition has the truth value TRUE if and only if it is a fact; otherwise, it has the truth value

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FALSE. In other words, a true proposition is identical to a fact (Glanzberg, 2021). Historically, it is considered to be a version of the correspondence theory of truth (ibid.). In accordance with the truthmaking principle, identity with a fact is what makes a proposition true. According to this theory, propositions, rather than statements, are the primary bearers of truth (ibid.). Truth is a property of propositions rather than statements, and it is a simple unanalyzable property, which means that truth is a type of "raw" property and that the facts, whatever they are, are taken as given (ibid.). Put simply, the facts just are true propositions, and true propositions just are facts (ibid.). There is no difference between the truth and the facts (ibid.). The goal of the identity theory is to secure a connection between language, mind, and world (Gaskin, 2021). This is a broader presentation of the identity theory of truth, as some philosophers hold either:

(1) a proposition has the truth value TRUE if it is a fact, otherwise it has the truth value FALSE,

or,

(2) a proposition has the truth value TRUE **only if** it is a fact, otherwise it has the truth value FALSE,

and not,

(3) a proposition has the truth value TRUE **if and only if** it is a fact, otherwise it has the truth value FALSE (Gaskin, 2021).

Here, what follows the word "if" to the right at (1) is the antecedent, and what does not is the consequent. What follows the words "only if" to the right at (2) is the consequent, and what does not is the antecedent. What is on the left of the words "if and only if" at (3) is the antecedent for the consequent on the right, and what is on the right of the words "if and only if" at (3) is the antecedent for the consequent on the left. There is an antecedent and consequent on both sides, but the direction is always from antecedent to consequent. The antecedent is part of an "if…then…", "only if", or "if and only if" conditional statement that contains the sufficient condition, and the consequent is part of an "if…then…", "only if", or "if and only if", or "if and only if" conditional statement that contains the necessary condition. This textbook will assume, in line with the broader interpretation, that (3) is the correct presentation. For example,

the proposition *the offspring of a polar bear and a grizzly bear may be called a "pizzly bear"* has the truth value TRUE if and only if it is a fact, otherwise it has the truth value FALSE.

This proposition is of the statement "the offspring of a polar bear and a grizzly bear may be called a 'pizzly bear." It has the truth value TRUE because it is a fact. It is a fact that the offspring of a polar bear and a grizzly bear may be called a "pizzly bear," among other names such as "grolar bear," "zebra bear," "grizzlar," and "nanulak." Thus, the conditional statement going from the antecedent on the right to the consequent on the left is fulfilled. Additionally, it is a fact, because it has the truth value TRUE. This hybrid animal was discovered in 2006, and was potentially called a "pizzly" by a CBC News online reporter (CBC News, 2006). That is when this proposition became a fact. Thus, the conditional statement going from the antecedent on the left to the consequent on the right is fulfilled. Because both conditional statements are fulfilled, the proposition has the truth value TRUE. In other words:

The offspring of a polar bear and a grizzly bear may be called a "pizzly bear." (T)

Here is an example of a false proposition under the broad identity theory of truth.

The proposition *Ellen DeGeneres was born in Austin, Texas* has the truth value TRUE if and only if it is a fact, otherwise it has the truth value FALSE.

This proposition is of the statement "Ellen DeGeneres was born in Austin, Texas." It has the truth value FALSE because it is not a fact. Instead, Ellen DeGeneres was born in Metairie, Louisiana. The

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conditional from the antecedent on the left to the consequent on the right is not fulfilled. Because the conditional statements in both directions are not fulfilled, then, this proposition has the truth value FALSE. In other words,

Ellen DeGeneres was born in Austin, Texas. (F)

According to the **coherence theory of truth**, a truthbearer is true if and only if it is part of a coherent system of truthbearers and/or truthmakers; otherwise, it is false (Glanzberg 2021). In accordance with the truthmaking principle, what makes a truthbearer true is being part of a coherent system of truthbearers and/or truthmakers. Ideally, this "system of truthbearers and/or truthmakers" is the significant whole set of truthbearers and truthmakers knowable by an **omniscient being**, who knows every truthbearer and truthmaker, who rationally orders each proposition as TRUE or FALSE, and who is capable of knowing truths that people normally wouldn't be capable of (Young 2018). On some views, the set of truthbearers and/or truthmakers in question is the largest, consistent set believed by actual, currently-living people, or it consists in the set of truthbearers and/or truthmakers when the limit of human inquiry is reached (ibid.). There are two main options for what counts as "coherence": entailment and mutual explanatory support (ibid.). So, either

a truthbearer is true if and only if it is logically (or loosely) entailed by members of the set of truthbearers and/or truthmakers,

or,

a truthbearer is true if and only if there is mutual explanatory support between it and the other members of the set of truthbearers and/or truthmakers (ibid.).

Here, "**logical entailment**" means that the truthbearer follows from combining members of the set of truthbearers and/or truthmakers somehow through logical rules, such as modus ponens and modus tollens, both of which are discussed in chapter 2. Logical entailment is stronger than consistency, so it is not sufficient that all of the truthbearers and truthmakers are true in at least one possible world with no contradiction resulting (Glanzberg 2021).<sup>82</sup> "**Mutual explanatory support**" means each truthbearer or truthmaker in the set of truthbearers and/or truthmakers is explained by at least one other truthbearer or truthmaker in the same set. **Circular explanations** are acceptable under the coherentist theory of truth, because truth under this theory is a systematic coherence of the whole (Glanzberg 2021). Systematic coherence may involve circular explanations, where one truthbearer or truthmaker explains another, and that same other truthbearer or truthmaker explains the former. Note that coherence here, in any case, involves a relation between the content of the truthbearer on the left and the world that the truthbearers or truthmakers on the right say something about (ibid.). Truthbearers on the right may also constitute the world in an ideal way (ibid.).

Here is an example of a true truthbearer under this theory.

The truthbearer "the truth is something human beings treasure" is true if and only if it is loosely entailed by members of the set of truthbearers and/or truthmakers.

This truthbearer is true because it is loosely entailed by the facts, or truthmakers, "knowledge is something human beings treasure" and "knowledge requires truth." These facts provide the relation from content to world, and would be known by an omniscient being ideally. Loosely speaking, whatever knowledge requires would also be treasured. So, the conditional statement from the antecedent on the right to the consequent on the left is fulfilled. Additionally, this loose entailment by these two facts is a necessary condition for the truthbearer in question being true. In fact, the truth is something human beings treasure. It automatically follows that the facts "knowledge is something human beings treasure" and "knowledge is something human beings treasure".

<sup>&</sup>lt;sup>82</sup> See also Carneades.org (2015).

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requires truth" loosely entail it as an explanation why at least. Because both conditional statements in both directions are fulfilled, the truthbearer in question, then, is true.

The **neo-classical coherence theory of truth** is more modern and differs slightly from the coherence theory of truth, but is essentially the same. Under his theory, coherence involves a relation from the content of a truthbearer to the content of a truthbearer; there is no relation to the world involved, as there is with the coherence theory of truth.

Here is an example of a true truthbearer under this neo-classical theory.

The truthbearer "my senses are reliable" is true if and only if there is mutual explanatory support between it and other members in the set of truthbearers.

This truthbearer is true because there is mutual explanatory support between it and other truthbearers knowable by an omniscient being, ideally. Such truthbearers include "the digital clock says that it is 4:07 PM now," "I see a tree with green leaves," "I taste water," "I smell smoke indicating that there is some type of fire," etc. Such truthbearers give mutual explanatory support for the senses being reliable, because they each describe instances of my senses being reliable. Thus, the conditional statement from the antecedent on the right to the consequent on the left is fulfilled. Also, there is mutual explanatory support, because "my senses are reliable" is true. The truthbearers on the right mutually explain each other and the one on the left because my senses are in fact reliable. Because both consequents in both directions are fulfilled, the truthbearer in question is true.

According to the **pragmatist theory of truth**, a truthbearer is true if and only if it is useful; otherwise, it is false. In accordance with the truthmaking principle, what makes a truthbearer true here is usefulness in a nutshell. There are different ways in which a truthbearer can be useful. It can be the result of inquiry, whether scientific, ethical, practical, or legal etc. It can be whatever is satisfactory to believe, or the at-large agreed upon opinion. It can be whatever works in practice. It can consist in a commitment. It can be a solution to a problem. It can be what is expedient. According to the philosopher Charles Sanders Pierce (1839-1914),

a truthbearer is true if and only if it eventually gains acceptance by withstanding endless inquiry; otherwise, it is false (Capps 2019). The true truthbearer, normally a belief, would furthermore be free of all doubt (ibid.).

According to the philosopher William James (1842-1910), "a truthbearer being true just is its verification process (ibid.)." According to the philosopher John Dewey (1859-1952), "a judgment is true and warrantly assertible if and only if it is the solution to a problem in an inquiry (ibid.)."

Here is an example of a true truthbearer under this theory.

The truthbearer *adults need 7-8 hours of sleep per night consistently* is true if and only if it is useful; otherwise, it is false.

The proposition "adults need 7-8 hours of sleep per night consistently" is true because it is useful. In general, adults find that sleeping 7-8 hours every night helps them feel refreshed, rebuilds worn-down muscles, and improves performance, etc. (Kruger et. al., 2016). This is useful in that it is the result of scientific inquiry and it is expedient. So, the conditional statement from right to left is fulfilled. Additionally, this proposition being useful is a necessary condition for it being true. If sleeping 7-8 hours per night consistently for adults doesn't bring the benefits of feeling refreshed, rebuilding worn-down muscles, and improving performance, etc., then there is no purpose or need to do as such and thus the proposition is false. The conditional statement from left to right is fulfilled. Because the conditional statements are filled in both directions, the truthbearer in question is true.

Here is an example of a false truthbearer under this theory.

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The truthbearer *the phrase "with squirrel" means "to be expecting a baby in 6 months time"* is true if and only if it is useful; otherwise, it is false.

The proposition here "the phrase 'with squirrel' means 'to be expecting a baby in 6 months time" is false because it is not useful, in that the phrase "with squirrel" is no longer used with the meaning "to be expecting a baby in 6 months time" because it is an obsolete word in English (McCoy, 2016). Because the conditional statement from left to right is not fulfilled, the truthbearer in question is false.

According to the **consensus theory of truth**, a truthbearer is true if and only if there is a consensus, either actual or ideal, among the general population. According to the truthmaking principle, it is a consensus among the general population that makes a truthbearer true. Here, an **actual consensus** is a consensus in the actual world, physical or non-physical. An **ideal consensus** is a hypothetical consensus that would be true if something were the case. The **general population** here is at least 97.72%<sup>83</sup> of the currently living population on Earth. Here is an example of a true truthbearer under this theory.

The proposition *all human beings are physically mortal* is true if and only if there is a consensus, either actual or ideal, among the general population.

This proposition is true because there is an ideal consensus at least among the general population. If someone were to survey all of the currently living human population on Earth, at least 97.72% of the people would agree that all human beings are physically mortal, because, according to historical records, all human beings in the past have physically died and it is observed that human beings throughout the world physically die every day. The conditional statement from right to left is fulfilled. Additionally, a consensus among the general population is a necessary condition for this proposition being true. If the general population of human beings did not admit that they are physically mortal, then there would be some piece of information missing, rendering the proposition false. The conditional statement from left to right is fulfilled. Because both conditional statements in both directions are fulfilled, the truthbearer in question is true.

Here is an example of a false truthbearer according to this theory.

The proposition *the Earth is flat* is true if and only if there is a consensus, either actual or ideal, among the general population.

This proposition is false because there is no consensus, actual or ideal, among the population that the Earth is flat. There is no actual consensus, because less than 97.72% of the people worldwide believe that the Earth is flat. There is no ideal consensus, because if everyone were somehow shown the pictures taken from space by NASA, more than 97.72% of people would believe the Earth was round and not flat. They would not think that the pictures are part of a conspiracy theory. Because the conditional statement from left to right, then, is not fulfilled, the truthbearer in question is false.

According to the **constructivist theory of truth**, a truthbearer is true if and only if it is constructed somehow by the scientific community at large through one of the various scientific methodologies; otherwise, it is false. In accordance with the truthmaking principle, it is construction by the scientific community through a scientific methodology that makes a truthbearer true. In a nutshell, this theory equates science with truth and pseudoscience with falsity. Here, **science** is what results from trying to prove a theory or idea wrong and failing, whereas **pseudoscience** is faulty science or otherwise that results from trying to prove a theory or idea or confirm it (because it is more likely that there will be evidence in support of a theory than against it).

<sup>&</sup>lt;sup>83</sup> This number, 97.72%, comes from the area under the curve from 0 to the mean plus two standard deviations on the normal distribution bell curve.

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Here is an example of a true truthbearer under this theory.

The truthbearer *black holes exist* is true if and only if it is constructed somehow by the scientific community at large through one of the various scientific methodologies; otherwise, it is false.

This proposition is true because it is constructed by the scientific community of physicists through observational evidence in space such as the detection of gravitational waves or gravitational influences, event horizons, the accretion of matter in the form of disks, etc. Such observational evidence in general indicates the existence of numerous black holes in the universe, prevents the idea that "black holes exist" from being proven wrong, and thus counts as science as defined above. So, the conditional statement from right to left is fulfilled. Such observational evidence constructed through scientific methodologies is also a necessary condition for the proposition being true. There would be significantly more doubt as to the truth of the proposition without it. So, the conditional statement from left to right is fulfilled. Because the conditional statement is fulfilled in both directions, the proposition in question is true.

Here is an example of a false truthbearer under this theory.

The truthbearer *the Great Pyramid of Giza was built by aliens* is true if and only if it is constructed somehow by the scientific community at large through one of the various scientific methodologies; otherwise, it is false.

This proposition is false because it is not constructed somehow by the scientific community at large through one of the various scientific methodologies. It is not constructed in this way because it amounts to pseudoscience as defined above. The proposition results from searching for evidence to try to prove a theory surrounding it, instead of trying to disprove it and failing. Such evidence includes that engineering ahead of its time would have been required, knowledge of the mathematical number Pi ( $\pi$ ) ahead of the Greeks and of the Golden Ratio Phi ( $\phi$ )<sup>84</sup> would have been required, it is unusually earthquake-proof, and that there are no mummies inside of it (Williams, 2017). This theory could be disproven by what is written in Herodotus' *Histories*, which describes how it was built for the Egyptian King Khufu.<sup>85</sup> The conditional statement from left to right is not fulfilled, so the truthbearer in question is false.

According to the **verificationist theory of truth**, a truthbearer is true if and only if it is verified in the actual world in some way; otherwise it is false. There are two ways in which the truthbearer could be verified in the actual world: **empirically** (*a posteriori*) or **conceptually** (*a priori*). Whatever is verified empirically is verified *a posteriori*, meaning "from what comes after [in the world]." Whatever is verified conceptually is verified at least by the five senses and common sense in the physical actual world. A truthbearer that is verified conceptually is verified at least by the five senses and common sense in the physical actual world. A truthbearer that is verified conceptually is verified at least by the five sense sort of verification in the actual world that makes a truthbearer true.

Here is an example of a true truthbearer under this theory.

<sup>&</sup>lt;sup>84</sup> This number  $\phi$  is also discussed in chapter 3, in connection with what it means to be probably true.

<sup>&</sup>lt;sup>85</sup> See Herodotus' *Histories*, Book 2, for further information.

<sup>&</sup>lt;sup>86</sup> The distinction between *a priori* and *a posteriori* is NOT taken to be mutually exclusive, and it is NOT taken to be jointly exhaustive. Something can be verified both empirically and conceptually, as Kripke (1981) writes: "something may belong in the realm of such statements that can be known *a priori* but still may be known by particular people on the basis of experience...anyone who has worked with a computing machine knows that the computing machine may give an answer to whether such and such a number is prime. No one has calculated or proved that the number is prime; but the machine has given the answer" (35). Transcendentalists would argue that something can be verified neither conceptually nor empirically in a unique way.

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The truthbearer  $e^{i\pi} + 1 = 0$  is true if and only if it is verified in the actual world in some way; otherwise, it is false.

This truthbearer is true because it is verified in the actual world by being verified by the mind in the non-physical actual world. In particular, it is verified by various math proofs involving a quotient of trigonometric and exponential expressions being equivalent to a constant, manipulation of power-series expansions on  $e^{i\pi}$ , from complex numbers being expressed in polar coordinates, and an extension of the definition of an exponential function from real to complex exponents.<sup>87</sup> So, the conditional statement from the antecedent on the right to the consequent on the left is fulfilled. Such proofs are also a necessary condition for the truthbearer being true, because mathematics, of which this equation is a part, requires equations to be verified as true with proofs. So, the conditional statement from the antecedent on the left to the consequent on the right is fulfilled. Because both conditional statements in both directions are fulfilled, the truthbearer in question in true.

Here is an example of a false truthbearer under this theory.

The truthbearer *a merely possible golden mountain is able to be climbed* is true if and only if it is verified in the actual world in some way; otherwise, it is false.<sup>88</sup>

This truthbearer is false because it cannot be verified in the actual world in some way. A merely possible golden mountain cannot be in the actual world, either physical or non-physical, so it cannot be verified in it. Because the conditional statement from the antecedent on the left to the consequent on the right, then, is not fulfilled, the truthbearer in question is false.

According to the **redundancy theory of truth**, equivalently the prosentential theory of truth, a truthbearer is true if and only if the truthbearer itself is true; otherwise, it is false. In other words, "x" is true if and only if "x," where x is a truthbearer; otherwise, x is false. This is what is called a **recursive definition of truth**, which uses the prosentence "is true" prior to the "if and only if." Prosentences are placeholders for sentences, just as pronouns are placeholders for their antecedents called nouns. This theory of truth, on some view, takes truth or fact to be a redundant concept with no connection to reality or thought. In other words, saying that "x is true" does not have any meaning or sense. It begs the question, "is x true?" In accordance with the truthmaking principle, it is the truthbearer itself in a way that makes a truthbearer true.

This theory of truth leads into the **liar's paradox**. Consider the case where the truthbearer x is false, or not true. Then, "x is not true" is true if and only if "x is not true." However, if "x is not true," then it cannot be the case that "x is not true" is true. Also, if "x is not true" is true, then it cannot be the case that "x is not true" is true. Also, if "x is not true" is true, then it cannot be the case that "x is not true" is true. Also, if "x is not true," then it cannot be the case that "x is not true." Either way, x would be both true and not true, which is a contradiction. The liar's paradox in turn leads into the **revision theory of truth**, which states that truth needs to be revised due to the liar's paradox.

Alfred **Tarski's semantic theory of truth** is slightly different from the redundancy theory of truth, and avoids the liar's paradox, because there is some connection between the truthbearer in question and reality or thought. According to Tarski's semantic theory of truth, a truthbearer "x" is true if and only if x; otherwise, x is false.<sup>89</sup> The x without quotations here indicates the connection between the truthbearer itself "x" and thought or reality. *X* here need not be a fact. According to the truthbearer true. For example,

<sup>&</sup>lt;sup>87</sup> See Appendix for more information.

<sup>&</sup>lt;sup>88</sup> Inspiration for this example comes from Williamson 2013, pgs. 19-20.

<sup>&</sup>lt;sup>89</sup> Saul Kripke also gives a similar semantic theory of truth, where truthbearers such as "The cat is big is true" or "This sentence is false" are excluded from having the property of truth in order to avoid the liar's paradox.

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a truthbearer "dolphins are animals" is true if and only if dolphins are animals; otherwise, this truthbearer is false.

Simply put, this truthbearer is true because dolphins are animals, so the conditional statement from the antecedent on the right to the consequent on the left is fulfilled. Also, dolphins being animals is a necessary condition for "dolphins are animals" to be true, because the truthbearer requires some sort of connection to reality or thought. The conditional statement from the antecedent on the left to the consequent on the right is fulfilled. Because both conditional statements in both directions are fulfilled, the truthbearer in question is true.

Also, a truthbearer "pigs fly" is true if and only if pigs fly; otherwise, this truthbearer is false.

This truthbearer is false because it is not the case that pigs fly, so the conditional statement from the antecedent on the left to the consequent on the right is not fulfilled. Thus, the truthbearer in question is false.

According to the **performative theory of truth**, ascribing truth to a truthbearer just is to license endorsement of belief in the truth of the truthbearer (Dowden and Swartz). In other words, a truthbearer being true just is a performative act of acceptance of truth on the part of the receiver. In accordance with the truthmaking principle, it is the receiver's performative act of acceptance that makes a truthbearer true. For example,

ascribing truth to a truthbearer "you are revealing embarrassing information about me" just is to license endorsement of belief in the truth of the truthbearer.

It is not as if this proposition itself has the property of truth somehow automatically, only that the recipient of this proposition could now believe in this proposition's truth if they were so inclined.

In another vein, **pluralist theories of truth** can include some combination of at least more than one of the following discussed above: correspondence theory of truth, neo-classical correspondence theory of truth, identity theory of truth, coherence theory of truth, neo-classical coherence theory of truth, pragmatist theory of truth, consensus theory of truth, constructivist theory of truth, verificationist theory of truth, redundancy/prosentential theory of truth, Tarski's semantic theory of truth, and the performative theory of truth, and maybe more. **Deflationary theories of truth** state that the word "truth" is not needed because it is empty in meaning. **Nihilism** states that nothing is true.

A version of the identity theory of truth is used in this textbook: a statement or truthbearer X is true if it is a fact; a statement or truthbearer X is false if it is not a fact. If some truthbearer may or may not be a fact, then its truth or falsity is irrelevant. Assuming that the **principle of bivalence** "either A or not-A" holds, as it does in the domain of classical logic and thus this textbook, each truthbearer is either true or not true (false). A truthbearer that may be either true or false, then, would have its truth or falsity as being irrelevant. This specific version is used because it is the least insane version out of the theories of truth. The facts are what is most tangible in thought. Each is common sense as one unit. Tangibleness in thought promotes sanity because the mind can easily touch it. Sanity and rationality go hand-in-hand because if one is rational, then one is sane, and, similarly, one is rational only if one is sane, so sanity is a necessary condition for rationality. The remaining theories of truth involve at least some degree of intangibleness, which could lead to insanity and thus irrationality in the long run. For more reasoning on why this specific theory of truth is used, see Appendix.

Now that we've discussed different theories of truth and examples of truthbearers that are true or false under them, let's do some exercises involving these different theories.

[This is where the bullet point summary and section 5.2 exercises will be inserted.]

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#### Section 5.3: Facts

Section objective: Understand what counts as a fact, what may count as a fact, and what does not count as a fact.

#### **Key Terms**

Common Knowledge: the set of accepted facts for a certain audience or group of individuals

**Mutual Knowledge:** the set of pieces of knowledge that members of a certain audience or group think they each know with a high degree of probability

**Emotionally-loaded Statement:** also known as an emotionally-conditioned statement, a type of statement that has some emotionally-loaded word(s)

**Emotionally-loaded Words:** types of words that have emotional meanings attached to them in addition to what they mean in the actual world

**Emotionally-loaded Meaning:** a type of meaning that indicates that something is good or bad in a nondescriptive way

**"Spatio-temporally Conditioned":** a phrase that mean "at least remotely touching the four-dimensional continuum in physics of space and time fused together;" if something is spatio-temporally conditioned, then it has both a space coordinate and a time coordinate in the actual world someway and somehow, although it does not have to consist exclusively in such coordinates or be exclusively a four-dimensional entity

Cognitive Biases: subconscious errors in thinking that lead to misinterpretations about information in the actual world

**Prejudice:** a type of cognitive bias that consists in preconceived opinions that are not based upon knowledge, reason, thought, evidence or experience

**Money Illusion:** also known as price illusion, a type of cognitive bias that occurs when individuals have the tendency to think of currency in terms of its raw value instead of its real value

Scientific Theory: a structure of ideas that explains why or how scientific phenomena occur, and explains and interprets facts

Scientific Law: a statement in science that explains a relationship between certain facts and describes phenomena

To recapitulate from section 5.2, this textbook operates on a version of the identity theory of truth: a statement or truthbearer X is true if it is a fact; a statement or truthbearer X is false if it is not a fact. Facts, then, and what they are will be crucial for truth. However, what, exactly, are facts?

Returning to section 1.2, a fact in general could be something such as "a state of affairs or event(s) that obtain in the actual world," where, again, the actual world includes both the physical actual world observed everyday through common sense and the five senses, and the non-physical actual world, which includes actual minds. For example, the statement

"The sky is blue."

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is a state of affairs that obtains in the physical actual world through common sense, and thus it is a fact.

More specifically, not all general statements or rules are necessarily facts. For instance, you may think that the following truthbearers are facts:

(a) Men are taller than women.

(b) In the English language, "i" always goes before "e" when they are next to each other.

However, (a) is not a fact because there have been a few women who are and have been 7-8 feet, which is above the approximate average man's height worldwide of 5 feet 10 inches, as well as certain groups of women, such as approximately the top 16% in a normal distribution curve of female heights, which tend to be taller than 16% of the shortest male heights on a normal distribution curve (Roser et. al., 2019). Instead, the modified truthbearer,

(a') Generally speaking, men are taller than women,

is a fact. Additionally, (b) is not a fact because there are exceptions in the English language that after the letter "c" "e" goes before "i," i.e. "perceive." Instead, the modified truthbearer,

(b') In the English language, "i" always goes before "e" when they are next to each other, except after "c,"

is a fact.

Facts are not necessarily part of common knowledge. **Common knowledge** is the set of accepted facts for a certain audience or group of individuals. For instance, the First Amendment to the Constitution, which protects the rights to freedom of religion and freedom of speech, would count as common knowledge to the group of U.S. citizens, because it is an accepted fact for this group. However, if some thing, event or state of affairs obtains or is in the actual world, physical or non-physical, independently of anyone's observations or mind, then it obtains or is in the actual world. It is not a part of common knowledge, because it cannot be an accepted fact for any group of individuals. Facts are not necessarily dependent on any individual's observations or mind, although they are often corroborated by some individual's observations or mind, which corroboration may be necessary for practical purposes. The facts, then, that are not necessarily part of common knowledge, are facts, but they are not accepted facts on any level. Facts need not be accepted. For example,

(a) Alien life exists on Jupiter's moon Europa,

may be a truthbearer that counts as a fact now independently of anyone's observations or mind. Although most scientists agree that life probably exists beneath Europa's ice crust, no one individual has verified this for certain. It is possible that there exists a set of unknown facts, among which (c) is a member.

Facts are not mutual knowledge. **Mutual knowledge** is similar to common knowledge, but not quite. Mutual knowledge is the set of pieces of knowledge that members of a certain audience or group think they each know with a high degree of probability. Facts, by contrast, would be known with certainty, and not with merely a high degree of probability. For example, you may think that

(a) According to Heisenberg's Uncertainty Principle, it is impossible to know simultaneously with a high-level of precision two properties of a particle.

is a fact, at least for scientists and physicists. However, Heisenberg's Uncertainty Principle is known to be a true law of quantum mechanics merely with a high degree of probability for scientists and physicists. All scientific laws, by definition, do not necessarily express absolute certainty, and they also do not necessarily express certainty.

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Facts are neither emotionally-loaded nor emotionally-conditioned. Statements that are either **emotionally-loaded** or emotionally-conditioned tend to have emotionally-loaded words. **Emotionally-loaded** words are types of words that have emotionally-loaded meanings attached to them in addition to what they mean in the actual world. Such **emotionally-loaded meanings** indicate that something is good or bad in a non-descriptive way. This emotionally-loaded meaning on top of the meaning of the word in the actual world is what makes emotionally-loaded words and statements irrelevant in classical logic. Statements in classical logic, which are composed of words, can be either true or false, and describe something about the actual world or reality. These statements are descriptive. Emotionally-loaded statements with emotionally-loaded meanings are non-descriptive, and therefore they are irrelevant to classical logic.

\*YELLOW = BBE

For example, consider the emotionally-loaded words: "atrocious," "dimwit."

In the actual world, "atrocious" means "of a very poor quality; extremely bad or unpleasant." Additionally, "atrocious" has an emotional meaning attached to it, namely a type of meaning that evokes feelings of "horrible disgust." Someone who strongly dislikes fried tarantula may state "this is atrocious" with facial features indicating horrible disgust to express the emotions associated with the word "atrocious." Likewise, in the actual world "dimwit" means "a stupid or silly person." It has the attached emotional meaning of feelings of unworthiness towards the individual being called a "dimwit" and perhaps also feelings of discontent on the part of the speaker towards this individual. In the cases of "atrocious" and "dimwit," their emotionally-loaded meanings are non-descriptive. In the case of "atrocious," its emotionallyloaded meaning indicates feelings of horrible disgust, implying that the individual in question who is "atrocious" is bad. In the case of "dimwit," its emotionally-loaded meaning indicates feelings of unworthiness towards the individual in question and perhaps feelings of discontent, implying that the individual in question who is a "dimwit" is bad.

These emotionally-loaded words could be parts of the following emotionally-loaded statement:

(a) Joe Biden is an atrocious dimwit.

However, (e) is not a fact. The additional feelings associated with "atrocious" and "dimwit" do not apply to Joe Biden in the actual world, and are irrelevant. Instead, (e) could be neutrally-translated as the following fact:

(b) Joe Biden appointed new judges to the Supreme Court in 2021, which actions certain individuals voiced disapproval of.

The facts are spatio-temporally conditioned. "**Spatio-temporally conditioned**," here, means "at least remotely touching the four-dimensional continuum in physics of space and time fused together."<sup>90</sup> If something is spatio-temporally conditioned, then it has both a space coordinate and a time coordinate in the actual world someway and somehow, although it does not have to consist exclusively in such coordinates or be exclusively a four-dimensional entity. The facts, or each fact, are as such because they are each common sense as one unit. Our common sense, with each of the five senses, operates in spacetime, and thus the facts also do. For example, you may think that

(a) "One habit of intelligent humans is being easily annoyed by people around them, but saying nothing in order to avoid a meaningless argument."

is a fact (Ward, 2022). However, it is not a fact. There is no clear space coordinate attached. It seems that this statement is too general. At least, it refers to the set of all intelligent humans and their habits

<sup>&</sup>lt;sup>90</sup> This definition assumes that special and/or general relativity in physics is correct.

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in 2022 throughout the space of the entire planet and universe, but the evidence supporting that it does refer to this set is not given.

The facts are extremely concrete and have precise wording. With facts, the wording matters. For instance, you may think that

(a) "The Italian banker Gilberto Baschiera is considered a modern-day Robin Hood."

is a fact (Ward, 2022). However, it is not a fact. The words "considered a modern-day Robin Hood" are somewhat less concrete and less precise than normal. What is meant by these words, exactly? Instead,

(a) The Italian banker Gilberto Baschiera diverted 1 million euros to poorer clients from wealthy ones so that the poorer clients could qualify for loans over the course of 7 years, all the while making no profit and avoiding jail in 2018 due to a plea bargain.

is the corresponding, more concrete and more precisely-worded fact (ibid.).

The facts are not necessarily dependent upon a certain interpretation of things, events, or states of affairs that occur or obtain in the actual world. They are normally objective entities that exist in the actual world no matter how some individual interprets them. For example, assume that both Deidre and Yokono are looking at a specific shade of blue color on a color wheel, and they state the following:

Deidre: "This shade of blue is Maya blue."

Yokono: "This shade of blue is Sky blue."

It turns out that the lighting was different for Deidre and Yokono each, interfering with their direct perception of the shade of blue, and that the shade of blue is really Baby blue. This fact, like others, would not be dependent upon either Deidre's or Yokono's interpretations. Both Deidre and Yokono's statements are not facts, by comparison.

The facts are free of **cognitive biases** and prejudices. Cognitive biases are subconscious errors in thinking that lead to misinterpretations about information in the actual world. **Prejudice** is a type of cognitive bias that includes preconceived opinions that are not based upon knowledge, reason, thought, evidence or experience. These definitions of "cognitive biases" and "prejudices" are rather simple for very complex phenomenon, which technically is outside the domain of reference of this textbook. Case in point, consider this large and comprehensive list of cognitive biases:

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Figure 5.1: Cognitive Biases Codex

You may consider the following truthbearer to be a fact.

(a) In general, a 2% cut in individual income with no change in monetary value is less desirable than a 2% rise in individual income where there is an increase in the prices of goods for that individual (PsychOut, 2020).

However, (l) is not a fact, because it is influenced by the cognitive bias known as money illusion. **Money illusion**, also known as price illusion, occurs when individuals have the tendency to think of currency in terms of its raw value instead of its real value. The raw value of "\$2.00" in the absence of economic change is \$2.00, whereas its real value could be \$1.85 in the context of inflation and higher prices overall than in the past. Returning to (l), a 2% rise in income where there is an increase in the prices of goods could have the same real value as a 2% cut in individual income with no changes in monetary value (ibid.). They could basically be rational equivalents (ibid.). So, (l) could be re-thought of as the fact:

(b) In general, a 2% cut in individual income with no change in monetary value is equally desirable as a 2% rise in individual income where there is an increase in the prices of goods for that individual.

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The facts are similar to raw data, but not quite because the data is still filtered through an observer. For example,

(a) "As of 2022, Shailene Woodley's net worth is [about \$9 million]" (CelebsMoney, 2022).

seems to be a fact, and it is so far. It is similar to a piece of raw data on the Internet. However, if the observer who looked at the information or data related to (n) happened to make an error or be negligent in some relevant way, then this piece of raw data is may not be a fact. In other words, assuming that Shailene Woodley's net worth in 2022 was correctly determined, (n) is a fact.

The facts are not necessarily equivalent to scientific theories or scientific laws, although they could be. Although scientific theories are justified and worthy of belief, and may be classified as true beliefs, they nevertheless are not the same type of entity as facts. **Scientific theories** are structures of ideas that explain why or how scientific phenomena occur, and explain and interpret facts. They are, in general, "broader" entities than facts. Facts are instead the data or concrete entities that may be a part of scientific theories. For example, you may think that

(b) According to the theory of eternal black holes, for some black hole regions in the future, a white hole region has to exist in the past.

is a fact. However, this postulation of the existence of a white hole region in the past is part of the scientific theory structure to explain the observed phenomena of black holes. According to the solution of the Einstein field equations, a white hole region has to exist in the past, even though a white hole has not been observed. Furthermore, there is some debate as to whether white holes exist or not, so (o) is not a fact. A **scientific law** is a statement in science that explains a relationship between certain facts and describes phenomena. Scientific laws are built upon facts, and are not the same as facts or a fact. Scientific laws, like scientific theories, are justified, and they differ from facts in that they are a different type of entity. For example, you may think that

(a) According to Zipf's law, given some corpus of natural language utterances, the frequency of any word is inversely proportional to its rank in the frequency table.

is a fact. However, Zipf's law is meant to describe a trend or behavior, and not a certain cause-effect relationship. Additionally, corpuses of natural language utterances in the Chinese language, for instance, may provide a counter-example, as each character in the language is significantly more distinct, and there are more of them than there are letters in most other languages. So, it is not a fact.

Now that we've thought about what the facts are, let's get into some exercises concerning facts, what may be a fact, and what is not a fact.

[This is where the bullet point summary and section 5.3 exercises will be inserted.]

[This is where a chapter 5 cumulative practice test will be inserted.]

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# Chapter 6

# **Informal Fallacies**

### Introduction

Patrick J. Hurley and Lori Watson's logic textbook, *A Concise Introduction to Logic*, is famous for its explanation and discussion of various informal logical fallacies. Such informal logical fallacies may occur in our everyday lives, from occurrence in advertising, to politics, to everyday speech and conversation, to name a few. This chapter gives its own discussion of the fallacies discussed in Hurley and Watson's textbook, plus explanations and instructions on how to refute each of these fallacies in real life, which may prove handy.

# **Objectives**

Section 6.1: Understand what a fallacy, formal fallacy, and an informal fallacy in general is.

**Section 6.1:** Understand the difference between a formal fallacy and an informal fallacy, and know how to distinguish between them in practice.

Section 6.1: Learn what some examples of informal fallacies are.

Section 6.2: Understand and identify the different fallacies of relevance and their sub-types.

Section 6.2: Know how to refute each fallacy of relevance in real life.

# Section 6.1: Informal Fallacies in General

Section objectives:

- Understand what a fallacy, formal fallacy, and an informal fallacy in general is.
- Understand the difference between a formal fallacy and an informal fallacy, and know how to distinguish between them in practice.
- Learn what some examples of informal fallacies are.

#### Key Terms

**Fallacy:** a defect in an argument that arises from either a mistake in reasoning or the creation of an illusion that makes a bad argument appear good; a defect in an argument that consists in something other than false premises alone

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Non sequitur (2): another name for a fallacy, which, translated from the Latin, means "it does not follow"

Formal fallacy: a defect in the form or structure of an unsound, invalid deductive argument only

**Undistributed Middle Fallacy:** a type of formal fallacy specifically of the form "All A are B. All C are B. Therefore, all A are C." in which the word "therefore" indicates that the conclusion "all A are C" follows

**Informal Fallacy:** a defect in the content of an argument, where such an argument could be an uncogent inductive argument that is at least either weak or does not meet the total evidence requirement, or an unsound, invalid deductive argument

**Content (Informal Fallacy):** what refers to the strength/weakness or total evidence requirement of an inductive argument, and the validity/invalidity of a few particular types of deductive arguments

Before explaining informal fallacies in general, it may be best to both discuss what a fallacy is and to distinguish formal fallacies from informal fallacies. A **fallacy** is a "defect in an argument that arises from either a mistake in reasoning or the creation of an illusion that makes a bad argument appear good" (Hurley 2018, 125). It is also known as a defect in an argument that consists in something other than false premises alone. In other words, loosely, it is a mistake in reasoning or some sort of illusion. Another name for a fallacy is a *non sequitur*, which, translated from the Latin, means "it does not follow." A fallacy is also called by this name because it involves a conclusion that does not follow from the premises in some way.<sup>91</sup>

A fallacy is not equivalent to a type of argument, but instead is contained within an argument, specifically within a bad argument. A fallacy is a defect *in* an argument, not a defective argument. The defect itself is the fallacy, not the argument itself. Fallacies are defects themselves that are contained within defective arguments. For example,

 $\phi_1$ : If a human being is a boy, then a human being is a human person. (T)

 $\phi_2$ : A human being is a human person. (T)

 $\varphi$ : A human being is a boy. (F)

is a simple example of a bad, or defective, argument. Although both premises  $\phi_1$  and  $\phi_2$  are true, because whoever is a boy also has to be a human person and a human being by definition is a human person, conclusion  $\varphi$  is clearly false, because at least some human beings are girls. The fallacy here is not equivalent to the bad argument, or the set of  $\phi_1$ ,  $\phi_2$ , and  $\varphi$ , but instead is equivalent to a defect in how  $\phi_1$ ,  $\phi_2$ , and  $\varphi$  are put together. A true  $\phi_2$  affirming the consequent of a true  $\phi_1$  results in a false  $\varphi$  due to a defect in how they were put together. The defect or fallacy here is affirming the consequent, whereas the defective or bad argument is the set of all the statements.

Bad deductive and inductive arguments may contain fallacies (IBEs may as well). If a deductive argument contains a fallacy, then it is unsound, because it will at least be invalid. If an inductive argument contains a fallacy, then it is uncogent, because it will at least either be weak or not meet the total evidence requirement. However, keep in mind that a bad argument containing a fallacy is not a necessary condition, in other words not required, for it being unsound or uncogent. If a deductive or inductive argument only has premises that are not all true, while meeting the other conditions required for a good deductive or inductive argument, then that deductive argument is still unsound and that inductive argument is still uncogent, because the condition of "all premises true" for a good argument is not met.

<sup>&</sup>lt;sup>91</sup> Cf. *Non sequitur* (1) from chapter 2.

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Now, let's move on from what a fallacy is to the difference between a formal fallacy and an informal one. A **formal fallacy** is a defect in the form or structure of an argument. Formal fallacies can only occur in bad deductive arguments and not bad inductive ones. Hence, an argument that contains a formal fallacy will not only be unsound, or a bad deductive argument, but it will also be invalid. For example,

 $\phi_1$ : All cockapoos are animals. (T)

 $\phi_2$ : All African lungfish are animals. (T)

 $\varphi$ : All cockapoos are African lungfish. (F)

is a simple example of an argument containing a formal fallacy. This type of formal fallacy is called "**the undistributed middle fallacy**." It is a formal fallacy specifically of the form "All A are B. All C are B. Therefore, all A are C." in which the word "therefore" indicates that the conclusion "all A are C" follows. The defect in the argument above is in the form or the structure of how the two premises fit together. Just because it is true that all cockapoos are animals and that all African lungfish are animals, it does not mean that we can put the word "animals" together in each statement to get the conclusion "all cockapoos are African lungfish," which is false.

By contrast, an **informal fallacy** is a defect in the content of an argument, and not the form or structure itself. **Content**, here, mainly refers to the strength/weakness or total evidence requirement of an inductive argument, and the validity/invalidity of a few particular types of deductive arguments. Informal fallacies could occur in either bad inductive or bad deductive arguments (and maybe also in bad IBEs). A bad inductive argument that contains an informal fallacy could be at least either weak or not meet the total evidence requirement, and thus be uncogent. A bad deductive argument that contains an informal fallacy would be invalid, and thus unsound.

There is currently no single all-encompassing theory concerning what constitutes informal fallacies. When trying to identify informal fallacies it may be helpful to know what the relevant things in the content of the premises is, or what the relevant words mean, or what their meaning depends upon. Some attach conclusions to emotions, or emotions to conclusions. Some appeal to degrading features of the arguer or person. Some appeal to superstition. Some appeal to mental laziness. Without giving an evaluation of emotions as either good or bad, such informal fallacies can be expressions of emotions apart from reasons. Instead of being about form or structure, informal fallacies are about how the premises within the form or structure fit together and connect to claim to prove the conclusion. Here are some examples of arguments with informal fallacies.

 $\phi_1$ : The apple that I just tasted is rotten.

 $\varphi$ : Probably, all of the apples at the party are rotten.

Intuitively, one can tell that there is something wrong with the content of this argument, which is inductive. Normally, the taste of one apple would not be sufficient evidence for determining the taste of all the apples at a party.

 $\phi_1$ : A harpist is a person.

 $\varphi$ : Probably, a good harpist is a good person.

There is likewise a defect in the content of this inductive argument here. Someone who is good at playing the harp could also be a criminal.

 $\phi_1$ : My mailman said that the Flyers are the best ice hockey team.

 $\varphi$ : Probably, the Flyers are the best ice hockey team.

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There is a defect in this argument's content as well. The mailman in this case may not be the best expert in ice hockey.

 $\phi_1$ : If I meet up with Bob, then I will keep on meeting up with him frequently.

 $\phi_2$ : If I will keep on meeting up with him frequently, then I will end up marrying him.

 $\varphi$ : If I meet up with Bob, then I will end up marrying him.

There is a defect in this argument's content, although it is valid and deductive. Meeting up with someone does not necessitate the marrying of that same someone.

It may be helpful to know how to distinguish between a formal fallacy and an informal fallacy in an argument in practice in general. Here's how:

- (1) Pay attention to the argument's form first. If it is claiming to be valid, such as with a necessarily or absolutely true conclusion, but there is something off with the argument, you can use the informal circles and dots test mentioned in chapter 2 to see if it is valid or has a form defect and is invalid. If it is not claiming to be valid, then move on to step 2.
- (2) Look at the content at and surrounding the premises in the argument. Ask yourself "does anything seem off about the evidence presented?" If you can answer "yes" to this question, then there may be an informal fallacy. If you cannot answer "yes" to this question, or you can answer "no," then there probably is no informal fallacy and no formal fallacy in the argument.

Now that we've discussed what fallacies are, what informal fallacies are as opposed to formal fallacies, and briefly how to distinguish between the two, here are some exercises on informal fallacies in general.

[This is where the bullet point summary and section 6.1 exercises will be inserted.]

# Section 6.2: Fallacies of Relevance (and How to Refute Them)

Section objectives:

- Understand and identify the different fallacies of relevance and their sub-types.
- Know how to refute each fallacy of relevance in real life.

#### **Key Terms**

Fallacy of Relevance: a logically irrelevant argument that seems to be psychologically relevant, in which the proposed inferential claim may be merely an emotional appeal

**Appeal to Force:** a defect in which the arguer threatens some form of psychological, mental, spiritual, or physical harm, either implicitly or explicitly, to the reader or listener in order to get them to accept the conclusion as true

**Appeal to Pity:** a defect in which the arguer attempts to support a conclusion by merely evoking pity from the reader or listener that is directed toward the arguer or some third party

**Appeal to the People:** a defect in which the arguer attempts to support a conclusion by evoking the desires to be loved, esteemed, admired, valued, recognized, and accepted by others from the reader or listener

Appeal to Popular Attitudes and Emotions: another name for the appeal to the people fallacy, indicating that in this fallacy popular attitudes and the emotions associated with them are manipulated to get others to accept undemonstrated claims

**Appeal to General Belief:** another name for the appeal to the people fallacy, indicating a type of bad argument where a claim is correct only because most people generally believe it

**Direct Approach (Appeal to the People):** when the arguer aims appeal for the conclusion at a crowd or group of people, by inciting a mob mentality and exciting the emotions and enthusiasm of the crowd

**Indirect Approach (Appeal to the People):** when the arguer aims appeal not at a crowd or group but at one or more individuals, by focusing on some aspect of the individual's relationship to the crowd

**Appeal to Fear:** a specific form of the direct approach of the appeal to the people fallacy, also known as fear mongering, is a defect in which the arguer uses a premise containing the fear of something put in the mind of the crowd in arguing for a conclusion

**Appeal to Tradition:** a type of the appeal to the people fallacy under the indirect approach that is of the basic form "this is a tradition/a lot of people have done this thing in a certain way for a long time...therefore, (you should accept that) the conclusion is true"

**Bandwagon Argument:** a type of the appeal to the people fallacy under the indirect approach that is of the basic form "you don't want to be left behind or left outside of the group...therefore, (you should accept that) the conclusion is true"

**Appeal to Vanity:** a type of the appeal to the people fallacy under the indirect approach that is of the basic form "you want to be esteemed, admired, pursued, or imitated...therefore, (you should accept that) the conclusion is true"

**Appeal to Snobbery:** a type of the appeal to the people fallacy under the indirect approach that is of the basic form "you want to be part of a select few or elite group...therefore, (you should accept that) the conclusion is true"

**Argument Against the Person:** a defect in which an arguer verbally or in words attacks a person directly or indirectly, which person has advanced a certain argument

*Ad hominem* Abusive: a type of the argument against the person in which the original arguer is verbally abused directly by the other arguer, and the defects are associated directly, either truthfully or not, with the person or their character involved

Ad hominem Circumstantial: a type of an argument against the person in which the original arguer is abused indirectly, and the defects are associated with certain circumstances that affect the original arguer

*Tu quoque*: a type of the fallacy of argument against the person in which the original arguer may be verbally abused directly or indirectly, and is accused of being a hypocrite or arguing in bad faith

**Hypocrite:** some person whose actions conflict with what they think or believe; accusations of being a hypocrite may be involved in the *tu quoque* fallacy

**Arguing in Bad Faith:** a type of arguing in which a person argues for a conclusion either they know or believe to be false, either intentionally or unintentionally; accusations of arguing in bad faith may be involved in the *tu quoque* fallacy

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Accident: a type of fallacy of relevance, that consists in the misapplication of a general rule stated in the premises to a specific case at least mentioned in the conclusion

**Straw Man:** a type of fallacy of relevance that is a defect in which the arguer claims to have demolished an opponent's argument, also known as "the real man," but in reality has only demolished their distorted, exaggerated, and/or extreme form of it, also known as "the straw man"

"The Real Man": the opponent's undistorted argument in reference to straw man that is claimed to be demolished

"The Straw Man": in a straw man, a distorted, exaggerated, and/or extreme form of the opponent's argument that may be demolished in reality

**Missing the Point:** a type of fallacy of relevance that is a defect in which the premises are claimed to entail one conclusion, but in fact they entail another conclusion

"Catchall" Fallacy: a type of fallacy that technically could be applied to all types of fallacies

**Red Herring**: a type of fallacy of relevance that is a defect in which the arguer (subtly) changes the subject and claims to have won the argument by drawing a conclusion or assuming a conclusion has been established about this different subject

The fallacies about to be discussed in section 6.2 may appear to you in real life. Because of this, it is important for you to identify them and refute them in real life if necessary, as generally follows.

The first group of fallacies to be discussed are the **fallacies of relevance**. These fallacies are logically irrelevant arguments that seem to be psychologically relevant (Hurley 2018, 129). For these types of fallacies, the proposed inferential claim may be merely an emotional appeal. If this is the case, then this argument is either valid, if it is deductive, or weak, if it is inductive. The proposed factual claim or premise(s) in arguments with these types of fallacies may be merely emotional appeals as well. If this is the case, then the argument would have at least one false premise, but again, having at least one false premise alone would not be indicative of a fallacy; the defect would have to consist in something other than just at least one false premise.

The first type of fallacy of relevance is the **appeal to force** (Latin name: *argumentum ad baculum* = "argument towards the 'stick""). This fallacy is a defect in which the arguer threatens some form of psychological, mental, spiritual, or physical harm, either implicitly or explicitly, to the reader or listener in order to get them to accept the conclusion as true (ibid.). Arguments with this type of fallacy may psychologically impede the reader or listener from recognizing a missing premise that would undermine the argument. The premises or factual claim contain evidence that someone might be harmed, and not genuine evidence that supports the conclusion. The inferential claim could be worded generally as follows: "because you might be harmed, (you should accept that) the conclusion is true." The essence of this fallacy is a threat.

- Factual Claim (Appeal to Force): evidence that someone might be harmed
- Inferential Claim (Appeal to Force): because you might be harmed, (you should accept that) the conclusion is (probably) true
- Essence (Appeal to Force): threat

Here are some examples of uncogent arguments with appeals to force.

 $\phi_1$ : I'll intimidate you if you don't give me your car.

 $\varphi$ : You should give me your car.

This argument contains an appeal to force, because the arguer is threatening psychological harm of intimidation in  $\phi_1$ . The factual claim  $\phi_1$  contains evidence that someone might be harmed, namely the reader or listener. The inferential claim here  $(\phi_1 \rightarrow \phi)$  reasons because the reader or listener might be intimidated by the arguer, the conclusion that the reader or listener should give the arguer their car is probably true.

 $\phi_1$ : 90% of people in your state have voted for the Democratic Party.

 $\phi_2$ : If you don't vote for the Democratic Party, then I'll give you the cold shoulder.

 $\varphi$ : You should vote for the Democratic Party in your state.

This argument contains an appeal to force, because the arguer is threatening the psychological harm of giving the cold shoulder in  $\phi_2$ . The factual claim  $\Delta = {\phi_1, \phi_2}$  contains evidence at  $\phi_2$  that the reader or listener might be harmed by having the cold shoulder given to them. The inferential claim  $(\Delta \rightarrow \phi)$  reasons because most people in the reader or listener's state voted for the Democratic Party, and if the reader or listener doesn't vote for the Democratic Party then they'll be harmed by being given the cold shoulder, it is probably true that they should vote for the Democratic Party.

Here is how to refute an argument that contains an appeal to force.

- (1) Turn attention either to the threat (to expose it if it's subtle) or away from the threat (to avoid harm). You could start of by saying something similar to "that is an appeal to force...," "regardless of your threat that \_\_\_\_\_...," "independently of your threat that \_\_\_\_\_...," just to name a few.
- (2) Give a good argument for why the conclusion is false, or give a good argument for why the conclusion is true, but for logically relevant reasons distinct from the threat.

Take the first example of an argument containing an appeal to force above. To refute this, one could state "regardless of your threat to intimidate me, I should not give you my car because you have not paid me yet what I requested for it." There are many ways to refute arguments like those with appeals to force --- the steps described above are merely a guideline.

The next type of fallacy of relevance is the **appeal to pity** (Latin name: *argumentum ad misericordiam* = "argument towards pity"). This fallacy is a defect in which the arguer attempts to support a conclusion by merely evoking pity from the reader or listener that is directed toward the arguer or some third party (Hurley 2018, 130). The premises or factual claim for arguments containing this fallacy have some evidence of a need to be pitied or a recipient of mercy. The inferential claim for such arguments could be formulated generally as follows: "because I (or we) need to be pitied or a recipient of mercy, the conclusion is true." The "essence" of this type of fallacy is misplaced pity.

- Factual Claim (Appeal to Pity): evidence of a need to be pitied or a recipient of mercy
- Inferential Claim (Appeal to Pity): because I (or we) need to be pitied or a recipient of mercy, the conclusion is (probably) true
- Essence (Appeal to Pity): misplaced pity

Note that bad arguments containing appeals to pity are distinct from arguments from compassion, discussed in section 3.3. Again, arguments from compassion are non-fallacious (but not necessarily cogent), and, like arguments containing appeals to pity, invoke pity or compassion, but they also supply *why* the person(s) in question are deserving of such pity or compassion. Specifically, they aim to show that the person(s) in question are victims of circumstances and are not responsible.

Here is an example of an uncogent argument containing an appeal to pity.

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 $\phi_1$ : Karlotta has six hungry children to feed, and her husband desperately needs an operation to save his eyesight.

 $\varphi$ : The position open in the finance department should be given to Karlotta.

This argument contains an appeal to pity because it involves misplaced pity in the form of giving a position in the finance department to Karlotta due to her family not doing well. The factual claim at  $\phi_1$  contains evidence of Karlotta needing to be a recipient of mercy, precisely because she has six hungry children to feed and her husband needs the operation. The inferential claim  $(\phi_1 \rightarrow \varphi)$  is of the form "because Karlotta needs to be a recipient of mercy by getting this job, the conclusion that the position open in the finance department should be given to Karlotta is probably true."

Here is how to refute an argument containing an appeal to pity.

(1) Turn the argument into an argument from compassion, by bringing in relevant evidence as to why the arguer or third party is a victim of circumstances and not responsible, OR give a good argument for why the conclusion is true or false, independently of the arguer's or third party's pathetic circumstances.

To refute the example of appeal to pity above in real life, one could say: "Karlotta has an advanced degree in finance and is qualified to hold the position regardless of the pathetic circumstances surrounding her family. Therefore, you should give it to her." This is the option of giving a good argument as to why the conclusion is true independently of Karlotta's pathetic circumstances.

The third fallacy of relevance is the **appeal to the people** (Latin name: *argumentum ad populum* = "argument towards the people"). This fallacy is a defect in which the arguer attempts to support a conclusion by evoking the desires to be loved, esteemed, admired, valued, recognized, and accepted by others from the reader or listener (Hurley 2018, 131). Its basic form is as follows.

Basic Form (Appeal to People): You want to be accepted/ included in the group/ loved/ esteemed/ admired/ valued/ recognized/ etc....therefore, (you should accept that) the conclusion is (probably) true.

The premises or factual claim for arguments with this type of fallacy would have evidence that the reader may not be loved, esteemed, admired, valued, recognized, and/or accepted by others. The inferential claim could be formulated as follows: "because you want to be accepted/ included in the group/ loved/ esteemed/ admired/ valued/ recognized/ etc...(you should accept that) the conclusion is true." The essence of this fallacy is popularity, in the absence of reasons or evidence.

- Factual Claim (Appeal to the People): evidence that the reader may not be loved, esteemed, admired, valued, recognized, and/or accepted by others
- Inferential Claim (Appeal to the People): because you want to be accepted/ included in the group/ loved/ esteemed/ admired/ valued/ recognized/ etc...(you should accept that) the conclusion is (probably) true
- Essence (Appeal to the People): popularity, in the absence of reasons or evidence

This fallacy has some alternate names for it. It is also known as the **appeal to popular attitudes and emotions**, because with this fallacy popular attitudes and the emotions associated with them are manipulated to get others to accept undemonstrated claims (Conway 2000, 141). This may be done by evoking racial fears, prejudices, patriotism, and desires to be part of certain group. It may also be known as the **appeal to general belief**, because with it the claim of an argument is correct only because most people generally believe it (Conway 2000, 140). Under this name, it could also count as a weak argument from authority, discussed in section 3.3, because there normally is no reason to think that "most people" count as a qualified authority with specific credentials.

There are two approaches that the arguer can take in employing this fallacy: the direct approach and the indirect approach. The **direct approach** of this fallacy is when the arguer aims appeal for the conclusion at a crowd or group of people (Hurley 2018, 131). The arguer incites a mob mentality by exciting the emotions and enthusiasm of the crowd (ibid.). The **indirect approach** of this fallacy is when the arguer aims appeal not at a crowd or group but at one or more individuals, by focusing on some aspect of the individual's relationship to the crowd (Hurley 2018, 132).

Other than an appeal to people under the direct approach in general, a specific form of the direct approach of this fallacy is the appeal to fear. An **appeal to fear**, also known as fear mongering, is a defect in which the arguer uses a premise containing the fear of something put in the mind of the crowd in arguing for a conclusion (Hurley 2018, 131). In other words, fear is evoked to get the crowd to accept the conclusion. Sometimes fear can be supported by solid evidence, but in this fallacy it is instead usually transferred through a message or rumors. Appeals to fear may be used during any social or political change. The basic form of an argument containing an appeal to fear is as follows.

Basic Form (Appeal to Fear): you don't want certain disastrous consequences to result...therefore, (you should accept that) the conclusion is (probably) true

Some specific forms of the indirect approach for this fallacy include the appeal to tradition and three standard techniques in the advertising industry: the bandwagon argument, appeal to vanity, and appeal to snobbery (Hurley 2018, 132-134). Their basic forms are as follows.

- Basic Form (Appeal to Tradition): this is a tradition/a lot of people have done this thing in a certain way for a long time...therefore, (you should accept that) the conclusion is (probably) true
- Basic Form (**Bandwagon Argument**): you don't want to be left behind or left outside of the group...therefore, (you should accept that) the conclusion is (probably) true
- Basic Form (Appeal to Vanity): you want to be esteemed, admired, pursued, or imitated...therefore, (you should accept that) the conclusion is (probably) true
- Basic Form (**Appeal to Snobbery**): you want to be part of a select few or elite group...therefore, (you should accept that) the conclusion is (probably) true

Here are some examples of inductive arguments with an appeal to the people under these different approaches and types.

Politician: Are you tired of paying too much for those loathsome taxes? Then vote for me. I'll grant you a cut.

The fallacy in this argument is the general direct approach for the appeal to people, because the politician, the arguer, is aiming the conclusion of "vote for me" to a group of voters at-large and not each individual within the group. The politician incites the emotion and enthusiasm of the crowd by asking a heated question about taxes and promises a cut on taxes. The politician appeals to the mob's mentality of valuing their money. Implicitly, there is the idea that most people would want a tax cut, so popularity is employed in this way without direct evidence. It has the basic form "you (the group) want to be esteemed as frugal by obtaining a tax cut and not paying for those loathsome taxes. Therefore, it is probably true that you should vote for me." In the factual claim, there is evidence that the group will be esteemed as praiseworthy in wanting the tax cut, because the taxes are described as "loathsome." In the inferential claim, the reasoning goes "because you (the group) want to be esteemed as frugal, then it is probably true that you should vote for me."

 $\phi_1$ : Americans have celebrated the Fourth of July with fireworks for a long time.

 $\varphi$ : They should continue to celebrate this holiday in this way.

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The fallacy in this argument is the appeal to tradition under the indirect approach. It is under the indirect approach because it is aimed by the arguer at the individuals who are Americans and who also celebrate the Fourth of July. It has the basic form "(a lot of) Americans have celebrated the Fourth of July with fireworks for a long time. Therefore, it is probably true that they should continue to celebrate this holiday in this way." The factual claim at  $\phi_1$  contains evidence that the Americans may not be accepted by others if they discontinue the fireworks tradition, because the fireworks tradition has gone on "for a long time." The inferential claim contains reasoning as follows: "because they the Americans may not be accepted by others by discontinuing to celebrate the Fourth of July with fireworks, it is probably true that they should continue to celebrate the holiday in this way."

"The Palace Hotel - Only for the Richest."

The fallacy in this argument is the appeal to snobbery under the indirect approach. It is under the indirect approach because it is aimed by the arguer or advertiser at a certain group of people who may qualify for what is being deemed to be "the Richest." It has the form: "you want to be part of a select group that is 'the Richest'. Therefore, it is probably true that you stay at The Palace Hotel." The implicit factual claim has evidence that the individuals which the advertiser targets may not be esteemed as being among "the Richest" if they do not stay at the Palace Hotel. The implicit inferential claim reasons that, because you want to be esteemed as being among "the Richest," it is probably true that you should stay at The Palace Hotel.

Here is how to refute in real life an argument with any type of an appeal to the people.

- (1) Identify or acknowledge the psychological or emotional play on one's desires.
- (2) Give a good argument for why the conclusion is either true or false in spite of the psychological or emotional play on one's desires.

For example, to refute the third example above in real life, one could say or think to oneself: "yes, I may want to stay at The Palace Hotel, because doing so may put me in the group of 'the Richest', but, regardless of this desire, I think that it is probably false that I should stay at The Palace Hotel, because I can obtain a better deal for my current needs at a different hotel nearby."

Another fallacy of relevance is the **argument against the person** (Latin name: *argumentum ad hominem* = "argument towards the man"). This fallacy is a defect in which an arguer verbally or in words attacks a person directly or indirectly, which person has advanced a certain argument (Hurley 2018, 134). The direct or indirect personal attack is logically irrelevant to the conclusion. Arguments in which personal comments or observations are logically relevant to the conclusion are not fallacious, for instance in some cases where the person under attack is in fact a hard-core criminal. Bad arguments with arguments against the person involve at least two arguers and one conclusion on which they take some sort of side. The premise(s) or factual claim for arguments with this fallacy have evidence of the defects associated with one of the arguers. The inferential claim for arguments with this fallacy can be formulated as follows: because the first arguer is associated with these certain defects, his/her/their conclusion is (probably) false. The "essence" of this fallacy is a personal attack.

- Factual Claim (Argument Against the Person): (potential) evidence of defects associated with one arguer
- Inferential Claim (Argument Against the Person): because the first arguer is associated with these certain defects, his/her/their conclusion is (probably) false
- Essence (Argument Against the Person): personal attack

There are three forms of the argument against the person: *ad hominem* abusive, *ad hominem* circumstantial, and *tu quoque* (Latin translation = "and you too"). The *ad hominem* abusive is a type of the argument against the person in which the original arguer is verbally abused directly by the other arguer, and the defects are associated directly, either truthfully or not, with the person or their character involved (ibid.). Here is an example of an argument with an *ad hominem* abusive.

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Arguer 1 - I think that Teju should be our class president because she has fine leadership skills.

Arguer 2 – Well, that's not accurate because you're an idiot.

This argument contains an argument against the person because what arguer 2 says to arguer 1 involves a personal attack on arguer 1. The two arguers are arguer 1 and arguer 2, and the conclusion on which they take sides is "Teju should be our class president." Arguer 2 calls arguer 1 an idiot after arguer 1 puts forth their argument. The factual claim of arguer 2's argument contains potential evidence of the defect of being an idiot associated with arguer 1. It is not known whether or not this association of a defect with arguer 1 is true or false, but this aspect does not matter for the identification of an argument against the person here. The inferential claim reasons as follows: "because arguer 1 is an idiot (according to arguer 2), their conclusion, that Teju should be our class president, is not accurate (or false)." Specifically, this argument against the person is an *ad hominem* abusive, because arguer 1 is verbally abused directly by arguer 2 by having the defect of being an idiot associated directly, whether truthfully or falsely, with arguer 1's person. In other words, arguer 1's person is directly attacked with the insult of being called an idiot, which insult is irrelevant to arguer 1's argument.

The *ad hominem* circumstantial is a type of an argument against the person in which the original arguer is abused indirectly, and the defects are associated with certain circumstances that affect the original arguer (Hurley 2018, 135). With this type of the argument against the person fallacy, the defects are NOT associated with the person of the arguer or their character. Here is an example of an argument with the *ad hominem* circumstantial.

 $\phi_1$ : Lucretius was a madman due to substance abuse.

 $\varphi$ : The ancient philosopher Lucretius' conclusion that the universe is ultimately composed of only matter and void is false.

This argument contains an argument against the person because what the arguer says about Lucretius involves a personal attack, namely at  $\phi_1$  that Lucretius was a madman due to substance abuse. The two arguers here are Lucretius and the person putting forth the above argument. The conclusion on which they take sides is "the universe is ultimately composed of only matter and void." The factual claim at  $\phi_1$  contains potential evidence of the defects associated with Lucretius, namely that he was a madman and was as such through substance abuse. The inferential claim reasons as follows: because Lucretius is associated with being a madman, his conclusion, that the universe is ultimately only composed of matter and void, is false. This argument against the person is an *ad hominem* circumstantial because the defect of being a madman is due to the circumstances surrounding Lucretius, namely that he participated in substance abuse, whether it is true or not. The substance abuse and being a madman pertains to the circumstances surrounding Lucretius and not his person or character.

The *tu quoque* is a type of the fallacy of argument against the person in which the original arguer may be verbally abused directly or indirectly, and is accused of being a hypocrite or arguing in bad faith (Hurley 2018, 135-136). Here, a **hypocrite** is some person whose actions conflict with what they think or believe. Here, **arguing in bad faith** is a type of arguing in which a person argues for a conclusion either they know or believe to be false, either intentionally or unintentionally. Here is an example of an argument containing a *tu quoque* fallacy.

 $\phi_1$ : Bob went deer-hunting in the past.

 $\varphi$ : Bob's conclusion that the deer should not be killed because they are living beings like human beings is false.

This argument contains an argument against the person because there is an implicit personal verbal attack against Bob by the revelation of Bob's actions of killing deer at premise  $\phi_1$  that conflicts with his

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conclusion that deer should not be killed. The two arguers here are Bob and the person who put forth the above argument. The conclusion on which they take sides is "the deer should not be killed because they are living beings like human beings." The factual claim contains the evidence of the defect of Bob having killed deer at  $\phi_1$ , which conflicts with his conclusion. The inferential claim reasons as follows: because Bob is associated with having killed deer in the past, his conclusion, that deer should not be killed because they are living beings, is false. This type of argument against the person is the *tu quoque*, because Bob is being indirectly verbally abused, in that the actions implicit at  $\phi_1$  conflict with his conclusion, and he is accused of being a hypocrite, because what he thinks in his conclusion, namely that deer should not be killed because they are living beings like human beings, conflicts with his actions, namely his past killing of deer through deer-hunting. Perhaps Bob has felt some remorse over what he has done in the past instead.

Here is how to refute any type of an argument containing an argument against the person in real life.

- (1) Verbally distance yourself from the attack. You could start off by saying "even if you think that I am as such...," "I may be a hypocrite, but regardless...," "I do not think that I am as incompetent as you say. Regardless,...," "true, I did do that, but independently of that...," or something else along those lines.
- (2) Give a good argument for why the conclusion is either true or false that is independent of the verbal attack.

For the first example above, the argument with the *ad hominem* abusive, arguer 1 could refute the argument there as follows: "even if you think that I am an idiot, my argument is still accurate because having fine leadership skills seems to be closest to the essence of being a successful class president."

Another type of fallacy of relevance is accident. **Accident** is the misapplication of a general rule stated in the premises to a specific case at least mentioned in the conclusion (Hurley 2018, 137). With this fallacy, one or many accidental features of the specific case make it an exception to the general rule. The premise(s) or factual claim here have an inclusion of a general rule. The inferential claim reasons as follows: because the general rule is widely applicable, it is concluded that it applies in this specific case. The "essence" of this fallacy is a mistake in generality.

- Factual Claim (Accident): inclusion of general rule
- Inferential Claim (Accident): because the general rule is widely applicable, it is concluded that is applies in a specific case
- Essence (Accident): mistake in generality

Here are some examples of uncogent arguments containing this fallacy.

 $\phi_1$ : In general, science says that men tend to have more muscle mass than women.

 $\phi_2$ : There is a woman, who is the current +87kg Class Weightlifting World Champion<sup>92</sup>, lifting really heavy things as part of her workout, nearby some muscular man also lifting heavy things as part of his workout.

 $\varphi$ : That muscular man has more muscle mass than that woman.

The fallacy in this argument here is accident because there is a mistake in generality of applying the general rule that men tend to have more muscle mass than women. The specific case in question is the woman, who is the current +87kg (Super Heavyweight) Class Weightlifting World Champion, having her

<sup>&</sup>lt;sup>92</sup> Currently, the 2021 Tashkent, Uzbekistan +87kg (Super heavyweight) Women's World Champion is Son Young-hee from South Korea.

muscle mass compared with that of some muscular man lifting heavy things nearby. The accidental feature that makes this case an exception to the aforementioned rule is that a) the woman in question is extremely skilled in lifting heavy things, because she is the current Super Heavyweight Weightlifting World Champion, and thus she is very likely to have more muscle mass than the majority of men, including the muscular man in question. The factual claim includes the general rule, that men tend to have more muscle mass than women, at  $\phi_1$ . The inferential claim reasons as follows: because the general rule that men tend to have more muscle mass than women is widely applicable, it applies in the case of comparing the current +87kg Class Women's Weightlifting World Champion to some muscular man lifting heavy things.

Here is how to refute in real life an argument containing accident.

- (1) Tell the arguer that the specific case is an exception to the general rule proposed. You could say something like the following: "I understand the general rule you are proposing, but this is an exceptional case..."
- (2) Explain why the specific case is such an exception, noting the relevant accidental features of the specific case. You could say something like the following: "this is an exceptional case because . Therefore, the application of this general rule to this specific case is false."

For the example of the argument with accident above, you could refute it as follows: "I understand the general rule you are proposing, but this is an exceptional case, because the woman in question is extremely skilled in lifting heavy things, as she is the current Super Heavyweight Weightlifting World Champion, and thus she is very likely to have more muscle mass than the majority of men, including the muscular man in question."

Another fallacy of relevance is straw man. **Straw man** is a defect in which the arguer claims to have demolished an opponent's argument, also known as "**the real man**," but in reality has only demolished their distorted, exaggerated, and/or extreme form of it, also known as "**the straw man**" (Hurley 2018, 138). In other words, with straw man, the arguer claims to take down the real man, but in reality has only taken down the straw man. The premise(s) or factual claim in arguments with this fallacy include a distorted, exaggerated, and/or extreme form of the opponent's argument. The inferential claim here reasons as follows: because the opponent's argument is distorted, exaggerated, and/or extreme its conclusion is false. The "essence" of this fallacy is a distortion.

- Factual Claim (Straw Man): a distorted, exaggerated, and/or extreme form of the opponent's argument.
- Inferential Claim (Straw Man): because the opponent's argument is "distorted, exaggerated, and/or extreme," its conclusion is false
- Essence (Straw Man): distortion

Here is an example of an uncogent argument containing a straw man.

 $\phi_1$ : Professor Lightfoot argues that free-market capitalism is the most moral economic system because there is no coercion and it involves a positive-sum game.

 $\phi_2$ : Within free-market capitalism, there is no coercion only because it's about greed, and hunger for money and power, and the positive-sum game only helps the rich and hurts the poor.

 $\varphi$ : Professor Lightfoot's conclusion is no good.

This argument contains a straw man, because Professor Lightfoot's argument at  $\phi_1$  ("the real man") is portrayed in a negative light and distorted at  $\phi_2$  ("the straw man"). The factual claim includes an exaggerated form of Professor Lightfoot's argument, in saying that there is no coercion because it's about greed, and hunger for money and power, and that the positive-sum game only helps the rich and hurts the

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poor. The inferential claim reasons as follows: because Professor Lightfoot's argument is "exaggerated" at  $\phi_2$ , his conclusion is false, or no good.

Here is how to refute an argument containing a straw man in real life.

- (1) Tell the arguer that the exaggerated argument (the straw man) that has been constructed is not your original position or argument (the real man). You could start of by saying something like the following: "this argument is a straw man that does not reflect my original position...."
- (2) Restate your original position, and explain how it is different from the arguer's straw man. You could say something along the following lines: "my original position is that\_\_\_\_\_...It differs from your straw man in that \_\_\_\_\_...Therefore, the conclusion of my original argument is not necessarily false."

For the example argument above, Professor Lightfoot could refute it by saying: "this argument is a straw man and does not reflect my original position that free-market capitalism is the most moral economic system. It differs from your straw man in that it does not necessarily say that no coercion and a positive-sum game are restricted to or reduced to those negative factors you mentioned."

Another fallacy of relevance is missing the point (Latin name: ignoratio elenchi = "ignorance of the proof"). **Missing the point** is a defect in which the premises are claimed to entail one conclusion, but in fact they entail another conclusion (Hurley 2018, 139). The missing the point fallacy is a sort of "**catchall**" fallacy, or a type of fallacy that technically could be applied to all types of fallacies, where the conclusion drawn misses the point, because technically all types of fallacies miss the point in some way. The premise(s) or factual claim here has evidence that supports some conclusion A. The inferential claim here reasons as follows: because of this evidence (that supports conclusion A), therefore conclusion B is true. The "essence" of this fallacy is irrelevancy.

- Factual Claim (Missing the Point): evidence that supports conclusion A
- Inferential Claim (Missing the Point): because of this evidence (that supports conclusion A), conclusion B is true.
- Essence (Missing the Point): irrelevancy

Here is an example of an unsound argument containing missing the point.

 $\phi_1$ : If she is a bachelorette, then she is unmarried.

 $\phi_2$ : She is a bachelorette.

 $\varphi$ : She is looking for a husband.

This argument contains a missing the point because the conclusion  $\varphi$  is irrelevant to the two premises  $\phi_1$  and  $\phi_2$ . The premises claim to entail the conclusion "she is looking for a husband," but the actual conclusion they entail, by a hypothetical syllogism, is "she is unmarried." The factual claim ( $\phi_1$  and  $\phi_2$ ) contains evidence that supports conclusion A, which is "she is unmarried." The inferential claim reasons as follows: because of  $\phi_1$  and  $\phi_2$  (that supports conclusion A), conclusion B, which is "she is looking for a husband," is true.

Here is how to refute an argument containing a missing the point in real life.

- 1) Tell the arguer that the conclusion does not follow from the premises..
- 2) Explain why. For a deductive argument, if it is invalid, then point out the missing premise(s) required or invalid form. For an inductive argument, if it is uncogent, point out how the principle of the uniformity of nature and/or total evidence requirement is not met.
#### 3) Point out the correct conclusion and briefly explain why.

For the example above, one could refute it as follows: "This conclusion does not follow from the premises, because it is a statement that is not part of the hypothetical syllogism. Even if she is a bachelorette, she may not be interested in looking for a husband, because she may want to remain single. The correct conclusion is 'she is unmarried,' because this follows in all possible worlds assuming the premises are true."

The final fallacy of relevance to be discussed is red herring. **Red herring** is a defect in which the arguer (subtly) changes the subject and claims to have won the argument by drawing a conclusion or assuming a conclusion has been established about this different subject (Hurley 2018, 140). A red herring can be "flashy" by leading the arguer and/or listener off track. The premise(s) or factual claim here contain evidence related to a different issue than the original. The inferential claim here reasons as follows: because of this evidence related to a different issue, therefore a conclusion related to this different issue is true, and I have won the original argument. The "essence" of this fallacy is that the arguer draws the reader and/or listener off the subject.

- Factual Claim (Red Herring): evidence related to a different issue from the original
- Inferential Claim (Red Herring): because of this evidence related to a different issue, therefore a conclusion related to this different issue is true, and I have won the original argument.
- Essence (Red Herring): the arguer draws the reader and/or listener off the subject

Here is an example of an uncogent argument that contains a red herring.

 $\phi_1$ : Aristotle's ideas support this point.

 $\phi_2$ : Aristotle argues that character virtue, reason, and the Supreme Good are all related to each other.

 $a_1$ : The Supreme Good must exist.

 $\varphi$ : Plato's argument for the existence of the Forms is a good one because there has to be at least some eternal, unchanging things for unity.

This argument contains a red herring because the arguer subtly changes the subject from Aristotle's to Plato's ideas and claims to have won in doing so. The arguer argues that Plato's argument is good, from the premises of Aristotle's ideas. The arguer draws the reader off the subject of Aristotle and onto Plato. The factual claim here is related to Aristotle's ideas at premises  $\phi_1$  and  $\phi_2$ , and sub-conclusion  $a_1$ , which is a different issue than the original goal of giving some conclusion on Plato's argument for the existence of the Forms. The inferential claim reasons as follows: because of this evidence related to Aristotle's ideas, therefore a conclusion related to Plato's argument for the existence of the Forms is true, and I (the arguer) have won the argument.

Here is how to refute an argument containing a red herring in real life.

- (1) Point out to the arguer that they have changed the subject. You could say: "we are now talking about a different subject matter, X. I thought we were talking about Y."
- (2) Give a good argument for why a conclusion related to the original subject matter is true (or false) that is relatively unrelated to the other subject matter.

For the example argument above, the reader could refute in return as follows: "We are now talking about a different subject matter, Plato's argument. I thought we were talking about Aristotle's ideas. It seems that character virtue also exists, then, if Aristotle's Supreme Good exists and character virtue has to be related to it."

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The fallacies of straw man, missing the point, and red herring may be confused with each other. The following diagram is supposed to help clear their similarities and differences up, and to help you further distinguish between them in practice.



Figure 6.1: Straw Man vs. Missing the Point vs. Red Herring

Here is a summary of the fallacies of relevance discussed in this section.

- (1) Appeal to Force: arguer threatens reader and/or listener
- (2) Appeal to Pity: arguer evokes pity from reader and/or listener
- (3) Appeal to the People: arguer plays on need for security, etc. of the reader and/or listener
- (4) Argument Against the Person: arguer 2 rejects the argument presented by arguer 1 by verbally attacking arguer 1
- (5) Accident: a general rule is misapplied to a specific case
- (6) Straw Man: arguer distorts opponent's position
- (7) Missing the Point: premises claimed to entail conclusion B actually entail conclusion A
- (8) Red Herring: arguer draws the reader and/or listener off the subject

Now that we've discussed the different types of fallacies of relevance and how to refute them in real life, let's do some exercises on these subjects.

[This is where the bullet point summary and section 6.2 exercises will be inserted.]

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[This is where a chapter 6 cumulative practice test will be inserted.]

## **Chapter 6 Bibliography/References**

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# Glossary

⊆: the symbol meaning "is a subset of"

 $\subset$ : the symbol that means that one set is a proper subset of another, and included within but not identical with it

**\supseteq:** the symbol that means "is a superset of"

 $\supset$ : the symbol that means that the set on the left is a proper superset of the one on the right, and encompassing but not identical with it

"---" (Set Theory): the symbol on top that is a denial or negation of what's underneath, and also refers to the complement of what's underneath

 $\rightarrow$ : the symbol that means "if A then B," or "B follows from A," or "assuming A, then B," where A is whatever is to the left side of the arrow and B is whatever is to the right side of the arrow

 $\rightarrow$  (Inferential Claim): a symbol which indicates the reasoning process within an inferential claim

∈: the symbol that is the memberships relation in set theory, meaning "is a member (or element) of "

" $\neq \phi \subset \Gamma$ ": a group of symbols that are another way of saying that the set or entity in question "is true," and for the argument  $\Gamma$  it means "is not equal to the empty set, which is a proper subset of the set of the argument  $\Gamma$ "

"=  $\emptyset$ ": symbols which mean that something is equivalent to the empty set

" $\neq \emptyset$ ": symbols which mean that something is not equivalent to the empty set

a priori: Latin words that mean "from what is before [in the mind]"

*a posteriori*: Latin words that mean "from what comes after [in the world]"

Abduction: the set of all abductive arguments

Abduction (Formal): the set A such that A:  $a_{h1}$ ,  $a_{h2}$ ,  $a_{h3}$ ,...,  $a_{hn}$ ;  $a_{m1}$ ,  $a_{m2}$ ,  $a_{m3}$ , ...,  $a_{mn}$ , where n is any number,  $a_{hn}$  is some abductive argument, and  $a_{mn}$  is some inference to the best explanation

**Abductive argument:** explanatory reasoning in generating, adopting, or discovering worthy scientific hypotheses in order to avoid contradictions or surprising observations and evidence, put forth by Charles Sanders Pierce, that is part of the logic of science

Accepted fact: a claim that everyone, at least in the intended audience, agrees with

Accident: a type of fallacy of relevance, that consists in the misapplication of a general rule stated in the premises to a specific case at least mentioned in the conclusion

Actual Consensus (Consensus Theory of Truth): a consensus in the actual world, physical or non-physical

Actual World: the physical world and universe we experience everyday, or could experience, with our five senses and with common sense, and which includes all present cases

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*Ad hominem* Abusive: a type of the argument against the person in which the original arguer is verbally abused directly by the other arguer, and the defects are associated directly, either truthfully or not, with the person or their character involved

*Ad hominem* Circumstantial: a type of an argument against the person in which the original arguer is abused indirectly, and the defects are associated with certain circumstances that affect the original arguer

Advice: a type of non-argument that makes a recommendation for the future

Affirming the Consequent: a fallacious argument of the form "If X, then Y. Y. Therefore, X."

**Analogical Argument:** a type of argument in which the factual claim contains evidence that at least one entity or object has n properties and at least one entity has n+1 properties, and the inferential claim includes reasoning from this evidence to another entity having the  $n + 1^{nth}$  property

Antecedent: part of an "if...then...", "only if", or "if and only if" conditional statement that includes a sufficient condition

**Appeal to Fear:** a specific form of the direct approach of the appeal to the people fallacy, also known as fear mongering, is a defect in which the arguer uses a premise containing the fear of something put in the mind of the crowd in arguing for a conclusion

**Appeal to Force:** a defect in which the arguer threatens some form of psychological, mental, spiritual, or physical harm, either implicitly or explicitly, to the reader or listener in order to get them to accept the conclusion as true

**Appeal to General Belief:** another name for the appeal to the people fallacy, indicating a type of bad argument where a claim is correct only because most people generally believe it

**Appeal to Pity:** a defect in which the arguer attempts to support a conclusion by merely evoking pity from the reader or listener that is directed toward the arguer or some third party

**Appeal to Popular Attitudes and Emotions:** another name for the appeal to the people fallacy, indicating that in this fallacy popular attitudes and the emotions associated with them are manipulated to get others to accept undemonstrated claims

**Appeal to the People:** a defect in which the arguer attempts to support a conclusion by evoking the desires to be loved, esteemed, admired, valued, recognized, and accepted by others from the reader or listener

**Appeal to Snobbery:** a type of the appeal to the people fallacy under the indirect approach that is of the basic form "you want to be part of a select few or elite group...therefore, (you should accept that) the conclusion is true"

**Appeal to Tradition:** a type of the appeal to the people fallacy under the indirect approach that is of the basic form "this is a tradition/a lot of people have done this thing in a certain way for a long time...therefore, (you should accept that) the conclusion is true"

**Appeal to Vanity:** a type of the appeal to the people fallacy under the indirect approach that is of the basic form "you want to be esteemed, admired, pursued, or imitated...therefore, (you should accept that) the conclusion is true"

**Arguing in Bad Faith:** a type of arguing in which a person argues for a conclusion either they know or believe to be false, either intentionally or unintentionally; accusations of arguing in bad faith may be involved in the *tu quoque* fallacy

Argument: a group of statements in which one or more of these statements claims to prove that another one of these is true

Argument (Sets): a set of at least two statements, where at least one member is a premise and at least one member is a conclusion

**Argument Against the Person:** a defect in which an arguer verbally or in words attacks a person directly or indirectly, which person has advanced a certain argument

Argument Based Upon Mathematics: a type of argument in which at least either the factual or inferential claim includes a mathematical derivation

**Argument For the Application of a Scientific Law:** a type of argument in which the factual claim includes references to some known scientific law and a circumstance in which this scientific law could be applied, and the inferential claim includes reasoning from applying this scientific law to this circumstance and saying that this application is probably true in the conclusion

**Argument For the Discovery of a Scientific Law:** a type of argument in which the factual claim includes a statement or statements about observed instances of a certain effect, and the inferential claim includes reasoning from these instances to the conclusion of a named law governing these instances being probably true

**Argument From Authority:** a type of argument in which the factual claim includes a citation of an authority or authorities' backing up a statement or statements, and the inferential claim includes reasoning from this authority's and authorities' support to the conclusion of the statement(s) being probably true

**Argument From Compassion:** a type of argument in which the factual claim includes some evidence that someone is a victim of circumstances, and the inferential claim includes reasoning from such evidence to the probably true conclusion that the person in question is deserving of some benefit or compassion in recompense

**Argument From Definition:** a type of argument in which at least either the factual claim or inferential claim includes some definition of a word or phrase

Argument From Example: an argument that looks like an illustration but isn't

Argument From Example (Factual and Inferential Claims): a type of argument in which the factual claim includes some example(s) and the inferential claim includes reasoning from such example(s) to some conclusion being probably true

**Argument From Signs:** a type of argument in which the factual claim includes the description of some sign(s), and the inferential claim includes reasoning from the description of some sign(s) to the conclusion that the description is probably true

Atypical Sample: a type of sample that is random

**Bad Argument**: a group of statements in which the premises do not objectively succeed in proving the conclusion to be true

**Bad Argument (Alternate)**: a type of argument in which either the factual claim or inferential claim is false

**Bad argument (Set theory)**: a type of argument in which it is not the case that both  $\Delta$  (or  $\Delta'$ ) and  $\Delta$  (or  $\Delta'$ )  $\rightarrow \psi$  are true

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**Bad Inference to the Best Explanation (Formal):** an inference to the best explanation in which it is not the case that both all premises,  $P_1 \dots P_n$  and  $P_{n+1}$ , are true and the IBE clause is not met

**Bad Inference to the Best Explanation (General):** an inference to the best explanation that does not succeed in establishing some link between explanation of the evidence and the (non-necessary) truth of the hypothesis or statement that best explains such evidence

**Bad Inference to the Best Explanation (Set Theory):** an inference to the best explanation in which it is not the case that both  $\{P_1 \dots P_n, P_{n+1}\} \subset \Delta \neq \emptyset \subset \Gamma$  (or  $\{P_1 \dots P_n, P_{n+1}; a_1, a_2, a_3, \dots a_n\} \subset \Delta' \neq \emptyset \subset \Gamma$ ), and IBE clause =  $\{\Delta \text{ (or } \Delta') \rightarrow \psi\} \neq \emptyset \subset \Gamma$ 

**Bandwagon Argument:** a type of the appeal to the people fallacy under the indirect approach that is of the basic form "you don't want to be left behind or left outside of the group…therefore, (you should accept that) the conclusion is true"

Being true as a whole: a property that applies to premises and sub-conclusions when they meet the total evidence requirement

Beliefs or Opinions: types of non-arguments that express what someone believes or thinks

Candidate Hypothesis: a statement that could explain the evidence

"Catchall" Fallacy: a type of fallacy that technically could be applied to all types of fallacies

**Categorical Syllogism:** a type of argument that consists in exactly two premises and one conclusion, each of which begin with either the words "All," "No," or "Some"

**Causal Relationship:** a type of relationship that exists between properties in a strong analogical argument if and only if one, or a group of them, is a necessary condition for another, or a group of others

**Causal Relationship (Sets):** a type of relationship that exists between properties if and only if one, or either the intersection or union of each of some attributes in a group, is a superset of, or identical to or encompassing, another property, or either the intersection or union of each of the others in a group

**Causal Relationship (Formal):** set Z is in a causal relationship with set Z' if and only if  $Z \supseteq Z'$ , and Set Z' is in a causal relationship with set Z if and only if  $Z' \supseteq Z$ , where  $Z = \{z_1 \cup z_2 \cup z_3 \cup ... \cup z_n \text{ OR } z_1 \cap z_2 \cap z_3 \cap ... \cap z_n\}$ ,  $Z' = \{z_{n+1} \cup z_{n+2} \cup z_{n+3} \cup ... \cup z_{n+n'} \text{ OR } z_{n+1} \cap z_{n+2} \cap z_{n+3} \cap ... \cap z_{n+n'}\}$ , where  $z_1 ... z_n, z_{n+1}, ... z_{n+n'}$  are properties and for all natural numbers n, n', n' > 0

**Circular Explanation:** where one truthbearer or truthmaker explains another, and that same other truthbearer or truthmaker explains the former

Class: either a set or a group of sets whose members as a whole share some property

**Class Statement:** a type of statement that says something about some class as a whole and that employs collective predication

**Cogent argument**: a good inductive argument that is both a strong argument, or has its inferential claim true, and has all true premises and the total evidence requirement met, or has its factual claim true

**Cogent argument (Set theory)**: and inductive argument in which **both**  $E \subseteq \Gamma$ ; **and** for all  $\phi_n$ ,  $a_n$  in  $\{\phi_1, \phi_2, \phi_3, \phi_4, \dots, \phi_n, \alpha_1, \alpha_2, \alpha_3, \dots, \alpha_n\} \subseteq \Delta'$  (or  $\Delta$ ),  $\phi_n$ ,  $a_n$  is true; **and** for each  $\phi_n$ ,  $\alpha_n$  in  $\{\phi_1, \phi_2, \phi_3, \phi_4, \dots, \phi_n, \alpha_1, \alpha_2, \alpha_3, \dots, \alpha_n\} \subseteq \Delta'$  (or  $\Delta$ ),  $\phi_n$  and  $\alpha_n$  are coherent with D, where D is the domain of the conclusion such that  $\psi \in D$ 

Cognitive Biases: subconscious errors in thinking that lead to misinterpretations about information in the actual world

**Coherence (Total evidence requirement)**: a property of the premises  $\phi_n$  and sub-conclusions  $\alpha_n$  in an inductive argument as part of the total evidence requirement, where each  $\phi_n$ ,  $\alpha_n$  fits with all of the evidence in D and no  $\phi_n$ ,  $\alpha_n$  contradicts any of the evidence in D

**Coherence Theory of Truth:** a type of theory of truth that states that a truthbearer is true if and only if it is part of a coherent system of truthbearers and/or truthmakers, otherwise, it is false, where coherence is either some form of entailment or mutual explanatory support, and the system of truthbearers and/or truthmakers is ideally the set of truthbearers and/or truthmakers each of which an omniscient being would know the truth of

Collective Predication: a type of predication in which an attribute or property is said of a whole class

Common Knowledge: the set of accepted facts for a certain audience or group of individuals

**Complement (Set Theory):** everything strictly speaking outside of a set in question, whether that is another set or sets or not

**Complex Argument**: a type of argument with more than one conclusion, usually divided into subconclusions and the final conclusion

**Composition:** a type of argument in which the factual claim includes some general statement(s) about the parts of something having a property, and the inferential claim includes reasoning from that general statement(s) to the whole thing or class having that same property in the probably true conclusion that is a class statement

**Conceptual Verification:** a type of verification that is *a priori* at least by the mind in the non-physical actual world

**Conclusion**: a statement in an argument that is claimed to be proven true by the evidence or reasons in the premises

**Conclusion (Proportional Syllogism)**: the third statement of a proportional syllogism that is of the form "O is P," where O is an object and P is a property

Conclusion Indicator: a word that provides a clue in identifying a conclusion

Conditional Statement: a type of non-argument that is an "if...then..." statement

**Conjunct:** "X" or "Y" in a conjunctive statement of the form "both X and Y"

**Conjunction (Argument):** a type of argument where, from the occurrence of each of a number of things, all (or at least more than one) of those same things are implied in the conclusion

Conjunctive Statement: a type of statement of the form "both X and Y," also known as a conjunction

**Consensus Theory of Truth:** a type of theory of truth that states that a truthbearer is true if and only if there is a consensus, either actual or ideal, among the general population

**Consequent:** part of an "if...then...", "only if", or "if and only if" conditional statement that includes a necessary condition

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**Constructivist Theory of Truth:** a type of theory of truth that states that a truthbearer is true if and only if it is constructed somehow by the scientific community at large through one of the various scientific methodologies; otherwise, it is false

**Content (Informal Fallacy):** what refers to the strength/weakness or total evidence requirement of an inductive argument, and the validity/invalidity of a few particular types of deductive arguments

"Correlation does not necessarily imply causation": the principle that two things being correlated in reality do not necessarily imply that one causes the other

**Correspondence Theory of Truth:** also known as the classical view of truth, a type of theory of truth, present in Aristotle's and St. Thomas Aquinas' writings at least, stating that a statement is true if it corresponds to reality, where reality is the way things are or the facts

**Correspondence Theory of Truth (General):** a type of theory of truth that states that a truthbearer is true if and only if it corresponds with a truthmaker

**Correspondence Theory of Truth (Ontological Thesis):** a version of the correspondence theory of truth, or a neo-classical theory of truth, that states that a statement is true if and only if there *exists* a fact to which it corresponds; otherwise, it is false

Declarative Sentence: a type of sentence that claims to say something about the actual world

Deduction: the set of all deductive arguments as a whole

**Deductive argument**: each and every thing or member of the set of deduction which involves the claim that the conclusion follows from the premises by necessity

**Deflationary Theory of Truth:** a type of theory of truth that states that the word "truth" is not needed because it is empty in meaning

**DeMorgan's Rules:** a type of argument with exactly one premise and one conclusion, in which either the premise is the denial of a disjunctive statement of the form "either A or B" and the conclusion is a conjunctive statement of the form "both not-A and not-B," or vice versa, or the premise is a statement that is the denial of the conjunction of A and B and the conclusion is a statement of the form "either not-A or not-B," or vice versa.

Denying the Antecedent: a fallacious argument of the form "If X, then Y. Not-X. Therefore, not-Y."

**Dilemma:** a type of argument, where the factual claim includes a disjunctive statement and two conditional statements, in which the antecedent of each is one of the disjuncts of the disjunctive statement, and the conclusion is a disjunctive statement consisting of the consequents of the two conditional statements in the factual claim

**Direct Approach (Appeal to the People):** when the arguer aims appeal for the conclusion at a crowd or group of people, by inciting a mob mentality and exciting the emotions and enthusiasm of the crowd

**Disjunct:** "X" or "Y" in a disjunctive statement of the form "either X or Y"

Disjunctive Statement: a type of statement of the form "either X or Y," also known as a disjunction

**Disjunctive Syllogism:** a type of argument that consists in exactly two premises and one conclusion, in which the factual claim contains a statement of the form "either A or B" and the negation of one of the disjuncts, and the conclusion contains the other disjunct

**Distributive Predication:** a type of predication in which an attribute or property is said of each and every member of a class

**Division:** a type of argument in which the factual claim includes some class statement about a whole thing or class having the same property, and the inferential claim includes reasoning from that class statement to the probably true conclusion that some part or parts of the whole thing in question each have the same property

**Domain of the Conclusion (Total evidence requirement)**: the set of the general subject matter that the conclusion statement is a member of, and also where the subject and predicate of the conclusion statement intersect

Emotionally-loaded Meaning: a type of meaning that indicates that something is good or bad in a nondescriptive way

**Emotionally-loaded Statement:** also known as an emotionally-conditioned statement, a type of statement that has some emotionally-loaded word(s)

**Emotionally-loaded Words:** types of words that have emotional meanings attached to them in addition to what they mean in the actual world

**Empirical Verification:** a type of verification that is *a posteriori* at least by the five senses and common sense in the physical actual world

**Empty Set**: represented by " $\emptyset$ ," "{}," and "{ $\emptyset$ }," the one and only set that cannot have any elements, is a subset of itself without being a member of itself, and is also a subset of every other set

"Evidence" (Total evidence requirement): anything in either the physical or non-physical actual world

Explanandum: the accepted fact to be explained by the explanans in an explanation

**Explanans**: the statement or statements that explains why the accepted fact is true in an explanation

Explanation: a type of non-argument that explains or sheds light on why an accepted fact is the case

**Expository passage**: a type of non-argument that starts with a topic sentence developed by the subsequent sentence or sentences

Fact: a state of affairs or event that obtains in the actual world

Factual claim: a stipulation in an argument that at least one of the statements must claim to give evidence or a reason

**Factual Claim (Complex IBE – Set Theory):** the subset  $\Delta' \subset \Gamma$ , where  $\Gamma$  is the argument with the inference to the best explanation, such that the set of premises  $\{P_1 \dots P_n, P_{n+1}\} \subset \Delta'$  and the set of subconclusions  $\{a_1, a_2, a_3, \dots, a_n\} \subset \Delta'$ 

Factual Claim (IBE): the condition of all premises being true

Factual Claim (Sets): for a simple argument, the subset consisting of all the statements that are premises, and for a complex argument, the subset consisting of all the statements that are either premises or subconclusions

**Factual Claim (Simple IBE – Set Theory):** the subset  $\Delta \subset \Gamma$ , where  $\Gamma$  is the argument with the inference to the best explanation, such that the set of premises  $\{P_1 \dots P_n, P_{n+1}\} = \Delta$ 

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**Fallacy:** a defect in an argument that arises from either a mistake in reasoning or the creation of an illusion that makes a bad argument appear good; a defect in an argument that consists in something other than false premises alone

Fallacy of Relevance: a logically irrelevant argument that seems to be psychologically relevant, in which the proposed inferential claim may be merely an emotional appeal

Falsity (Alternate): the state of being equivalent to the empty set

Formal fallacy: a defect in the form or structure of an unsound, invalid deductive argument only

**Good Inference to the Best Explanation (Formal):** an inference to the best explanation in which both all premises, or both  $P_1 \dots P_n$ , and  $P_{n+1}$ , are true and the IBE clause is met

**Good Inference to the Best Explanation (General):** an inference to the best explanation that succeeds in establishing some link between explanation of the evidence and the (non-necessary) truth of the hypothesis or statement that best explains such evidence

Good Inference to the Best Explanation (Set Theory): an inference to the best explanation in which both

 $\{P_1 \dots P_n, P_{n+1}\} \subset \Delta \neq \emptyset \subset \Gamma \text{ (or } \{P_1 \dots P_n, P_{n+1}; a_1, a_2, a_3, \dots a_n\} \subset \Delta' \neq \emptyset \subset \Gamma \text{), and IBE clause} = \{\Delta \text{ (or } \Delta') \rightarrow \psi\} \neq \emptyset \subset \Gamma$ 

**General Population (Consensus Theory of Truth):** at least 97.72% of the currently living population on Earth

General Rule: a type of rule that is not necessarily true, and is meant to apply in most but not all cases

**General Rule Argument:** a type of argument in which the factual claim includes a general rule and an individual or group that falls under the subject of the general rule, and the inferential claim includes reasoning from applying this general rule to the individual or group, concluding that it is probably true that the individual or group meets this general rule

**General Scientific Argument:** a type of argument that is a very general summary of scientific findings and may appear in thought processes or writings in reference to a larger body of more specific scientific findings, and that can be further classified as either an argument for the discovery of a scientific law or an argument for the application of a scientific law

**General Statement:** a type of statement that says something about each and every member of a class and that employs distributive predication

**Generalization:** a type of argument in which the factual claim includes a sample of a group having a property and the inferential claim includes reasoning from this sample of a group having a property to the entire group from which the sample was taken having that same property

**Generalized Causal Inference:** a type of argument in which either the factual claim includes some sort of cause, loosely defined, and the inferential claim includes reasoning from that cause to its effect, loosely defined, being probably true in the conclusion, or the factual claim includes some sort of effect, loosely defined, and the inferential claim includes reasoning from that effect to its cause, loosely defined, being probably true in the conclusion

**Golden Ratio**:  $\varphi = (1 + \sqrt{5})/2 = 1.618033988...$ 

**Good argument**: a group of statements in which the premises objectively succeed in proving the conclusion to be true

Good Argument (Alternate): a type of argument in which both its factual and inferential claims are true

**Good argument (Set theory)**: a type of argument in which both  $\Delta$  (or  $\Delta'$ ) and  $\Delta$  (or  $\Delta'$ )  $\rightarrow \psi$  are true

**Good Inference to the Best Explanation (Formal):** an inference to the best explanation in which both all premises, or  $P_1 \dots P_n$  and  $P_{n+1}$ , are true and the IBE clause is met

**Good Inference to the Best Explanation (General):** an inference to the best explanation that succeeds in establishing some link between explanation of the evidence and the (non-necessary) truth of the hypothesis or statement that best explains such evidence

Good Inference to the Best Explanation (Set Theory): an inference to the best explanation in which both

 $\{P_1 \dots P_n, P_{n+1}\} \subset \Delta \neq \emptyset \subset \Gamma \text{ (or } \{P_1 \dots P_n, P_{n+1}; a_1, a_2, a_3, \dots a_n\} \subset \Delta' \neq \emptyset \subset \Gamma \text{), and IBE clause} = [\Delta \text{ (or } \Delta') \rightarrow \psi] \neq \emptyset \subset \Gamma$ 

"group of them": in this textbook's definitions of "systemic relationship" and "causal relationship," what can either be the intersection or the union of the properties, more specifically the sets of the properties, in question

 $H_1, ..., H_n$ : the set of all the hypotheses you can think of as candidates to explain evidence E in a formalized inference to the best explanation, where each hypothesis in this group is a single statement

**Hypocrite:** some person whose actions conflict with what they think or believe; accusations of being a hypocrite may be involved in the *tu quoque* fallacy

**Hypothetical Syllogism**: a type of argument that consists in exactly two premises and one conclusion, and in which the factual claim includes a statement containing both a sufficient and necessary condition

**IBE Clause:** the clause in a formalized inference to the best explanation that states that, given evidence E and hypotheses  $H_1, \ldots, H_n$  of E, if  $H_i$  explains E better than any of the other hypotheses, infer that  $H_i$  is closer to the truth than any hypothesis  $H_1, \ldots, H_n$ 

**Ideal Consensus (Consensus Theory of Truth):** a hypothetical consensus that would be true if something were the case

**Identity Theory of Truth:** a type of theory of truth put forth by Bertrand Russell and G. E. Moore initially, stating that a statement's proposition has the truth value TRUE if and only if it is a fact; otherwise, it has the truth value FALSE

"if": a word that indicates that an antecedent follows

"if and only if": words that indicate that what is on the left is an antecedent for the consequent on the right, and what is on the right is an antecedent for the consequent on the left

Illustration: a type of non-argument that is an expository passage with one or more examples

**Impossible World**: a world outside of the set of all possible worlds where at least some logical contradiction is true

**Indirect Approach (Appeal to the People):** when the arguer aims appeal not at a crowd or group but at one or more individuals, by focusing on some aspect of the individual's relationship to the crowd

Induction: the set of all inductive arguments as a whole

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Inference: the observed reasoning process of an argument

**Inference to the Best Explanation (IBE):** the modern sense of "abductive argument;" explanatory reasoning in the justification of hypotheses etc., that is mainly part of logic and philosophy

**Inferential Claim**: a stipulation in an argument that there must be some claim that some statement follows from alleged evidence, which are the reason or reasons

Inferential Claim (IBE): the condition of the IBE clause being met

**Inferential Claim (IBE – Set Theory):** IBE clause =  $[\Delta (\text{or } \Delta') \rightarrow \psi]$ 

**Inferential Claim (Sets)**: the claim that the conclusion follows from the subset consisting of all the statements that are either premises or sub-conclusions, assuming these are true in all possible worlds

**Informal Circles and Dots Test**: a type of test where you draw circles and dots within and outside of each other in order to determine if an argument is valid or invalid

**Informal Fallacy:** a defect in the content of an argument, where such an argument could be an uncogent inductive argument that is at least either weak or does not meet the total evidence requirement, or an unsound, invalid deductive argument

Intelligent Being (Argument From Signs): in general, any type of agent with at least some potential to create or analyze

Intersection (Set Theory): the area inclusive of where two or more sets overlap

Intuitionistic Logic: a non-classical system of Logic that rejects the Law of the Excluded Middle

**Invalidity (For an Unsuccessful Deductive Argument**: the property such that the inferential claim ( $\beta$ ) is false

Large Sample: a type of sample in which the number of members *n* is at least equivalent to  $\frac{4Z^2\sigma^2}{W^2}$ , where

Z represents a score for a desired confidence interval,  $\sigma$  is the variance of the members, and W is twice the margin of error of the sample

Law of Non-Contradiction: a principle in Logic which states that contradictory propositions, which are those of the form "A and not-A," cannot be true in the same sense at the same time

Law of the Excluded Middle: a principle in Classical Logic which states that either a proposition or its negation is true

**Liar's Paradox:** the paradox upon considering the case where the truthbearer x is false, or not true, under the recursive definition of truth; then, "x is not true" is true if and only if "x is not true;" however, if "x is not true," then it cannot be the case that "x is not true" is true; also, if "x is not true" is true, then it cannot be the case that "x would be both true and not true, which is a contradiction

Logic: the study of arguments

**Logical Entailment (Coherence Theory of Truth):** what is part of the coherence theory of truth under a version of it where the truthbearer follows from combining members of the set of truthbearers and/or truthmakers somehow through logical rules, such as modus ponens and modus tollens

**Loosely associated statements**: a type of non-argument in which statements are about the same general subject without a claim to prove something

**Many-valued Logic**: a non-classical system of Logic outside the domain of Classical Logic which allows for valuations of statements other than "true" or "false"

**Major Premise (Proportional Syllogism)**: the first premise in the factual claim of a proportional syllogism that is of the form "X proportion of Y are P," where X is some fraction or percentage, Y is a set containing members, and P is a property

**Mathematical Induction:** more complicated deductive argument based upon mathematics, in which one counts down rather than up, namely some conjecture is both true for n=1 or n=0 and true for if n=k then n=k+1

Meaning: the things that a statement refers to and/or the sense of all the word in a statement taken together

**Member**: a thing in a set

Merely Possible World: a non-actual possible world that is not accessible from the actual world

**Missing the Point:** a type of fallacy of relevance that is a defect in which the premises are claimed to entail one conclusion, but in fact they entail another conclusion

**Mixed Hypothetical Syllogism**: a type of hypothetical syllogism in which only one of the premises is a conditional statement

**Money Illusion:** also known as price illusion, a type of cognitive bias that occurs when individuals have the tendency to think of currency in terms of its raw value instead of its real value

**Mutual Explanatory Support (Coherence Theory of Truth):** what is part of the coherence theory of truth under a version of it where each truthbearer or truthmaker in the set of truthbearers and/or truthmakers is explained by at least one other truthbearer or truthmaker in the same set

**Mutual Knowledge:** the set of pieces of knowledge that members of a certain audience or group think they each know with a high degree of probability

Natural Numbers: the whole numbers 0, 1, 2, 3, .... etc. counting to infinity

Necessary condition: a set of which a sufficient condition is a member

**Neo-classical Coherence Theory of Truth:** a type of theory of truth that is the same as the coherence theory of truth, except that coherence involves a relation from the content of a truthbearer to the content of a truthbearer; there is no relation to the world involved

**Neo-classical Correspondence Theory of Truth:** a theory of truth that is a slight modern variation on the correspondence theory of truth

Nihilism: a theory which states that nothing is true

*Non sequitur* (1): a conclusion that does not follow from the premise(s), literally translated from the Latin as "it does not follow"

Non sequitur (2): another name for a fallacy, which, translated from the Latin, means "it does not follow"

Non-argument: a passage in which there is no claim that anything is being proven true

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**Non-Random Sample:** a type of sample in which there are biases such as the following - non-equitable, non-balanced, or non-objective representation of all participants, improper gender, race, socioeconomic, etc. distribution, premature terminations of any sort, time-related factors, cause-effect mix-ups of any sort, cherry-picking or confirmation bias, ignorance of relevant parts or groups, or arbitrary rejections, favoritism, intentionally searching for correlations, observer selection, volunteer bias, etc.

**Non-statistical Generalization:** a type of generalization that uses a non-statistical sample, in which the members of the sample are not chosen

"not-" (Set Theory Lingo): a word and hyphen that means "it is not the case that"

**Omniscient Being (Coherence Theory of Truth):** a type of being who knows every truthbearer and/or truthmaker, who rationally orders each proposition as TRUE or FALSE, and who is capable of knowing truths that people normally wouldn't be capable of

"only if": words that indicate that a consequent follows

Ontological Thesis: a thesis relating to the idea of existence

**Performative Theory of Truth:** a type of theory of truth in which ascribing truth to a truthbearer just is to license endorsement of belief in the truth of the truthbearer

**Pluralist Theory of Truth:** a type of theory of truth that can include some combination of at least more than one of the following - correspondence theory of truth, neo-classical correspondence theory of truth, identity theory of truth, coherence theory of truth, neo-classical coherence theory of truth, pragmatist theory of truth, consensus theory of truth, constructivist theory of truth, verificationist theory of truth, redundancy/prosentential theory of truth, Tarski's semantic theory of truth, and the performative theory of truth, and maybe more

Possible World (General): a way the actual world is, could have been, or could be

Possible World (Specific): a member of or case in the set of all possible worlds, which is the set of all cases

**Pragmatist Theory of Truth:** a type of theory of truth that generally states that a truthbearer is true if and only if it is useful; otherwise, it is false

Predicate: in general, all the words including the verb and those that follow after in a statement

**Prediction:** a type of argument in which the factual claim includes at least one statement in the present or past about members of a group having some property or attribute, and the inferential claim includes reasoning from these members of a group having some property or attribute to some future statement about some member(s) of this group having the same property or attribute

**Prejudice:** a type of cognitive bias that consists in preconceived opinions that are not based upon knowledge, reason, thought, evidence or experience

Premise: a statement in an argument that claims to give evidence or a reason

Premise indicator: a word that provides a clue in identifying a premise

**Principle of Bivalence:** a principle that states that "either A or not-A" and holds in the domain of classical logic and thus this textbook

**Principle of the Uniformity of Nature**: things or events that are or occur in one spatiotemporal region tend to be similar to or also occur in others, assuming such things or events are true and/or factual

**"Probably true:"** words in the definition of strength that can mean either "true in 61.80% of all possible worlds" or "true in the actual world

**Proportional Syllogism:** a type of argument that consists in exactly three statements, in which the factual claim consists in exactly one statement, the major premise, about a proportion of members of a set having some property plus exactly one statement, the minor premise, about an individual or object being a member of that set, and the inferential claim consists in reasoning from those two statements to the third statement or probably true conclusion that the individual or object has the property in question

Proposition: the meaning of a statement that has a truth value

**Prosentences:** placeholders for sentences, such as "is true," just as pronouns are placeholders for their antecedents called nouns

**Pseudoscience (Constructivist Theory of Truth):** faulty science or otherwise that results from trying to prove a theory or idea or confirm it

**Pure Hypothetical Syllogism**: a type of hypothetical syllogism in which each of the two premises and the conclusion are conditional statements

**Qualified Authority:** anyone who both has expertise in the relevant field, and lacks bias and prejudice, and lacks a motive to lie and disseminate misinformation

**Random Sample:** a type of sample which involves selecting members or individuals each with equivalent probabilities and without bias

**Recursive Definition of Truth:** "x" is true if and only if "x," where x is a truthbearer; otherwise, x is false

**Red Herring**: a type of fallacy of relevance that is a defect in which the arguer (subtly) changes the subject and claims to have won the argument by drawing a conclusion or assuming a conclusion has been established about this different subject

**Redundancy Theory of Truth:** a type of theory of truth, equivalently the prosentential theory of truth, that uses a recursive definition of truth and states that a truthbearer is true if and only if the truthbearer itself is true; otherwise, it is false

**Report**: a type of non-argument that gives information about a topic or event

**Representative Sample:** a type of sample that is both large and random

Rightside-up "U" (Set Theory Lingo): a symbol that means "the union of" what is on the left and right

Science (Constructivist Theory of Truth): what results from trying to prove a theory or idea wrong and failing

Scientific Law: a statement in science that explains a relationship between certain facts and describes phenomena

Scientific Theory: a structure of ideas that explains why or how scientific phenomena occur, and explains and interprets facts

Set: a thing or a group of things

Sign (Argument From Signs): any kind of message produced by an intelligent being

Simple argument: a type of argument with one conclusion

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**Simplification:** a type of argument where, from all of a certain number of things, at least one these things or a certain number of each of these things is implied in the conclusion

**Small Sample:** a type of sample in which it is not the case that the number of members *n* is at least equivalent to  $\frac{4Z^2\sigma^2}{W^2}$ , where *Z* represents a score for a desired confidence interval,  $\sigma$  is the variance of the members, and *W* is twice the margin of error of the sample

**Sound Argument**: a good deductive argument, that is both valid, or its inferential claim is true, and all of its premises are true, or its factual claim is true

**"Spatio-temporally Conditioned":** a phrase that mean "at least remotely touching the four-dimensional continuum in physics of space and time fused together;" if something is spatio-temporally conditioned, then it has both a space coordinate and a time coordinate in the actual world someway and somehow, although it does not have to consist exclusively in such coordinates or be exclusively a four-dimensional entity

**Statement**: a declarative sentence, or a sentence component that can stand alone as a declarative sentence, that can be either true or false.

**Statistical Generalization:** a type of generalization that uses a statistical sample, in which the members of the sample are chosen

**Straw Man:** a type of fallacy of relevance that is a defect in which the arguer claims to have demolished an opponent's argument, also known as "the real man," but in reality has only demolished their distorted, exaggerated, and/or extreme form of it, also known as "the straw man"

**Strength**: the property of an argument whereby the conclusion of that argument is probably true, on the assumption that the factual claim is true in all possible worlds

**Strong Analogical Argument:** an analogical argument in which there is either a systemic or causal relationship between all n properties in total and the  $n + 1^{nth}$  property(ies)

Strong Analogical Argument (Formal): an analogical argument where either  $Z \subseteq Z'$  or  $Z \supseteq Z'$ 

Strong Argument From Authority: an argument from authority that includes only qualified authorities

Strong Generalization: a generalization that has a representative sample

**Strong Proportional Syllogism**: a proportional syllogism in which the conclusion follows from the factual claim with at least a 61.80% chance

Subject: in general, all the words that occur before the verb in a statement

Subset: a type of set which is either identical to or included within another set

Superset: a type of set which is either identical to or encompassing another set

Sufficient condition: a member of a set of a necessary condition

**Systemic Relationship:** a type of relationship that exists between properties in a strong analogical argument if and only if one, or a group of them, is a sufficient condition for another, or a group of others

**Systemic Relationship (Sets):** a type of relationship that exists between properties if and only if one, or either the intersection or union of each set of some properties in a group, is a subset of, or identical to or included within, another property, or either the intersection or union of each of the sets of others in a group

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Systemic Relationship (Formal): set Z is in a systemic relationship with set Z' if and only if  $Z \subseteq Z'$ , and Set Z' is in a systemic relationship with set Z if and only if  $Z' \subseteq Z$ , where  $Z = \{z_1 \cup z_2 \cup z_3 \cup ... \cup z_n \text{ OR } z_1 \cap z_2 \cap z_3 \cap ... \cap z_n\}$ ,  $Z' = \{z_{n+1} \cup z_{n+2} \cup z_{n+3} \cup ... \cup z_{n+n'} \text{ OR } z_{n+1} \cap z_{n+2} \cap z_{n+3} \cap ... \cap z_{n+n'}\}$ , where  $z_1 ... z_n$ ,  $z_{n+1}$ ,  $... z_{n+n'}$  are properties and for all natural numbers n, n', n' > 0

**Tarski's Semantic Theory of Truth:** a type of theory of truth that states that a truthbearer "*x*" is true if and only if *x*; otherwise, *x* is false

"The Real Man": the opponent's undistorted argument in reference to straw man that is claimed to be demolished

"The Straw Man": in a straw man, a distorted, exaggerated, and/or extreme form of the opponent's argument that may be demolished in reality

**Theoretical Virtues:** also known as explanatory virtues, the qualities that a hypothesis possesses that makes it better off than it otherwise would have been, which include: simplicity, generality (or universal applicability), coherence with scientific theories, and fit

**Theory of Truth:** a theory in the contemporary philosophical literature that states what it is for a statement to be true and/or false, and/or what it is for a proposition to have the truth value TRUE and/or FALSE

**Total evidence requirement**: all evidence relevant to an inductive argument cannot be left out by the premises as a whole

**Total evidence requirement (Set theory)**: a cogent's requirement for the premises and sub-conclusions that for each  $\phi_n$ ,  $\alpha_n$  in the set  $\{\phi_1, \phi_2, \phi_3, \phi_4, \dots, \phi_n, \alpha_1, \alpha_2, \alpha_3, \dots, \alpha_n\} \subseteq \Delta'$  (factual claim),  $\phi_n$  and  $\alpha_n$  are coherent with D, where the set D is the domain of the conclusion such that  $\psi \in D$ 

**Triple Bar (Set Theory):** a symbol that indicates equivalence, where equivalence indicates that what is on the left is a sufficient and necessary condition for what is on the right

Trolley Problem: a thought experiment in ethics

Truth value: a property assigned to a proposition or statement that is either true or false

Truth (Alternate): the state of not being equivalent to the empty set

Truthbearer: beliefs, thoughts, ideas, judgments, statements, assertions, utterances, sentences, or propositions

Truthmaker: facts, states of affairs, events, things, tropes, or properties

Truthmaking Principle: for every truth, there is something that makes it true

*Tu quoque*: a type of the fallacy of argument against the person in which the original arguer may be verbally abused directly or indirectly, and is accused of being a hypocrite or arguing in bad faith

**Uncogent argument**: a bad inductive argument in which it is not the case that it is both a strong argument, or has its inferential claim true, and has all true premises and the total evidence requirement met, or has its factual claim true

**Uncogent argument (Set theory):** an inductive argument in which either  $E = \emptyset \subseteq \Gamma$ ; or for some  $\phi_n, a_n$  in  $\{\phi_1, \phi_2, \phi_3, \phi_4, \dots, \phi_n, \alpha_1, \alpha_2, \alpha_3, \dots, \alpha_n\} \subseteq \Delta'$  (or  $\Delta$ ),  $\phi_n, a_n$  is false; or for some  $\phi_n, \alpha_n$  in  $\{\phi_1, \phi_2, \phi_3, \phi_4, \dots, \phi_n, \alpha_1, \alpha_2, \alpha_3, \dots, \alpha_n\} \subseteq \Delta'$  (or  $\Delta$ ),  $\phi_n$  or  $\alpha_n$  is not coherent with D, where D is the domain of the conclusion such that  $\psi \in D$ .

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**Undistributed Middle Fallacy:** a type of formal fallacy specifically of the form "All A are B. All C are B. Therefore, all A are C." in which the word "therefore" indicates that the conclusion "all A are C" follows

Union (Set Theory): the area inclusive of everything in the sets in question and in their intersections.

**Unqualified Authority:** anyone who either lacks expertise in the relevant field, or has bias or prejudice, or has a motive to lie or disseminate misinformation

**Unsound Argument**: a bad deductive argument, that is either invalid, has one or more false premises, or both

**Upside-down "U" (Set Theory Lingo):** a symbol that mean "the intersection of" what is on the left and the right

Unrepresentative Sample: a type of sample that is either small or non-random

Vacuous Truth: a statement that is true only because its subject is empty

"Vacuously True:" words meaning to be true of something that is empty, or to be true by default

Validity (For a Successful Deductive Argument): the property such that the inferential claim ( $\beta$ ) is true

Validity (General): the property of an argument whereby the conclusion of that argument is true in all possible worlds, on the assumption that the premises/sub-conclusions are true in all possible worlds

Verificationist Theory of Truth: a type of theory of truth that states that a truthbearer is true if and only if it is verified in the actual world in some way, either empirically or conceptually at least; otherwise it is false

Warning: a type of non-argument that alerts someone to some danger

Weak Analogical Argument: an analogical argument in which it is not the case that there is either a systemic or causal relationship between all n properties in total and the  $n + 1^{nth}$  property(ies)

Weak Argument From Authority: an argument from authority that includes at least one unqualified authority

Weak Analogical Argument (Formal): an analogical argument where it is not the case that either  $Z \subseteq Z'$  or  $Z \supseteq Z'$ 

Weak Generalization: a generalization that has an unrepresentative sample

**Weak Proportional Syllogism**: a proportional syllogism in which it is not the case that the conclusion follows from the factual claim with at least a 61.80% chance

**Weakness**: the property of an inductive argument whereby the conclusion of that argument is probably false, on the assumption that the factual claim is true in all possible worlds

**xNANDy Gate:** a real-life example of DeMorgan's Rules in electrical circuitry, that is equivalently broken into a negative x OR a negative y gate, and vice versa

**xNORy Gate:** a real-life example of DeMorgan's Rules in electrical circuitry, that is equivalently broken into a negative x AND negative y gate, and vice versa