Does a Computer have an Arrow of Time?

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Abstract

In [Sch05a], it is argued that Boltzmann's intuition, that the psychological arrow of time is necessarily aligned with the thermodynamic arrow, is correct. Schulman gives an explicit physical mechanism for this connection, based on the brain being representable as a computer, together with certain thermodynamic properties of computational processes. [Haw94] presents similar, if briefer, arguments.

The purpose of this paper is to critically examine the support for the link between thermodynamics and an arrow of time for computers. The principal arguments put forward by Schulman and Hawking will be shown to fail. It will be shown that any computational process that can take place in an entropy increasing universe, can equally take place in an entropy decreasing universe.

This conclusion does not automatically imply a psychological arrow can run counter to the thermodynamic arrow. Some alternative possible explanations for the alignment of the two arrows will be briefly discussed.

Contents

1   Introduction

In part of his response to Zermelo's reversibility objections to statistical mechanics, Boltzmann[Bol95] suggested it was possible (indeed, inevitable) to have extended regions of space, and time, that were entropy decreasing, but that living beings within those regions would be unable to perceive the difference:

> For the universe, the two directions of time are indistinguishable, just as in space there is no up and down. However, just as at a particular place on the earth's surface we call "down" the direction toward the center of the earth, so will a living being in a particular time interval of such a single world distinguish the direction of time toward the less probable state from the opposite direction (the former toward the past, the latter toward the future)

Authors such as[Rei71, Hor87] have developed this idea while others[Skl93, Ear06, Mau02] are critical.

As noted in [Skl85][Chapter 12], the perception of 'up' and 'down' can be directly traced to particular physical processes in different creatures (and specifically in the case of humans, the effect of the gravitational field on the fluid of the inner ear). While it may seem implausible that there could be an equivalent organ, which monitors the local entropy gradient, and informs the brain in which direction time is flowing, there remains the possibility that there is still something about the general functioning of the brain that can only take place in the direction of entropy increase.

In a recent paper Schulman[Sch05a] claims to identify such a function from the general thermodynamic properties of computations, as physical processes. He gives a detailed comparison of the components of a computer with the features of the psychological arrow to show

> the extent to which a computer . . . can be said to possess a psychological arrow. My contention is that the parallels are sufficiently strong as to leave little room for an independant psychological arrow.

He then appeals to Landauer's Principle[Lan61] to show that the intrinsic arrow of computational processes must be aligned with the thermodynamic arrow. As a result a computer is

> without an independant arrow of time, retaining the past/future distinction by virtue of its being part of a mechanistic world with a thermodynamic arrow in a particular direction.

Similar suggestions to Schulman's can be found in [Haw94]

> when a computer records something in memory, the total entropy increases. Thus computers remember things in the direction of time in which entropy increases. In a universe in which entropy is decreasing in time, computer memories will work backward.

It is argued in this paper that neither Hawking nor Schulman's arguments hold. The structure is as follows. First (Section 2) we will state how we will treat the thermodynamic arrow of time, and what we mean when we refer to an 'entropy increasing universe' and an 'entropy decreasing universe'. Then (Section 3) we consider what it takes for a physical process to embody a computation and the effect of a time reversal of this physical process. The processes that result from this temporal reversal are not equivalent to the processes that can represent a computation. We then show the key result that equivalent operations to the time reversed processes can be constructed, so the time reverse of those equivalent operations is a computation in a time reversed universe (Section 3.4) that is equivalent to the original computation. This demonstrates the physical possibility of such processes in entropy decreasing universes, and gives us a model to further study the possibilities of computation under such circumstances.

In Section 3.5, we examine the derivation of Landauer's Principle in an entropy decreasing universe. We find that the physical assumptions required for an entropy decreasing universe result in a reversal of the inequality that occurs in the usual statements of Landauer's Principle. Rather than necessitating entropy increases, when taking place in an entropy decreasing universe logical operations necessitate entropy decreases. In retrospect this will seem rather obvious.

Finally (Section 4) we consider the question of whether systems which gather, process and utilise information, are simply more likely to arise in entropy decreasing or entropy increasing universes. We examine this from the point of view of volume of state space arguments, to see if there is, all else being equal, any reason to expect that entropy decreasing universes are inherently hostile to the gathering and retention of information. We find that, perhaps surprisingly, they are not. We conclude that, on the basis of statistical mechanical arguments alone, we have no grounds for linking any computational arrow of time to the thermodynamic arrow of time.

Given the clear manner in which our own information processing seems aligned to the thermodynamic arrow, this may seem puzzling. We will briefly consider some possible explanations of this link, but which would require more complex arguments to justify. A surprising conclusion might be that, if the psychological arrow of time is necessarily aligned with the thermodynamic arrow, then it cannot be logically supervenient upon computational states. Alternatively, if the psychological arrow of

time is logically supervenient upon information processing, then it must be logically independant of the thermodynamic arrow.

2 The Thermodynamic Arrow

First it is necessary to make clear what is meant by an entropy increasing universe and an entropy decreasing universe.

The state space of the universe is formed from the product of the state spaces of a large number of smaller systems $\Omega = \prod_i \otimes \Omega_i$ and a measure, μ, on regions of the state space. It will be usually only be necessary to consider grouping the subsystems into a small number of distinct, larger subsystems, j, with most of the small subsystems grouped into a single 'environment', E:

$$\Omega_j = \prod_{i \in j} \otimes \Omega_i \tag{1}$$

$$\Omega_E = \prod_{i \in E} \otimes \Omega_i \tag{2}$$

$$\Omega = \Omega_E \prod_j \otimes \Omega_j \tag{3}$$

The dynamics are described by an invertible, measure preserving flow $\phi^{(t)}$ on the state space. For any region $\Delta \subseteq \Omega$ then $\mu(\phi^{(t)}(\Delta)) = \mu(\Delta)$, and there exists a map $\phi^{-(t)}$ such that $\phi^{-(t)} \circ \phi^{(t)}(\Delta) = \phi^{(t)} \circ \phi^{-(t)}(\Delta) = \Delta$.

2.1 Entropy increasing universe

An entropy increasing universe has a microstate that starts in a very small and special region $\Delta_0 \subseteq \Omega$. It is assumed that the dynamics of the flow on the state space is such that, over time, this region spreads out over the state space. As the measure is preserved, this can only happen by the region developing a very elongated and filamentary structure. As part of the special nature of the initial region, it will be assumed that the fine detail of this elongated and filamentary structure can be ignored for any future evolution of the system.

The initial region is a direct product of regions over the subsystems:

$$\Delta_0 = \prod_i \otimes \Delta_i$$

After the system has evolved, it will not, in general be the case that the evolved region $\phi^{(t)}(\Delta_0)$ is a direct product of regions over the subsystems.

We will assume that the state space Ω_i of each subsystem, i, is divided into distinct subregions $\omega_{i,j}$, such that $\cup_j \omega_{i,j} = \Omega_i$. The integer x_i identifies a subregion ω_{i,x_i} so that the array of integers $\underline{x} = (x_1, \ldots, x_j, \ldots)$ can be used to represent a direct product of subregions

$$\omega_{\underline{x}} = \prod_j \otimes \omega_{i,x_i} \tag{4}$$

The sets $\left\{ \overline{\Delta}_{i,x_i} \right\}$ and $\left\{ \overline{\Delta}_E \right\}$ are sets of all the regions that satisfy:

$$\overline{\Delta}_{i,x_i} \subseteq \omega_{i,x_i} \tag{5}$$

$$\overline{\Delta}_E \subseteq \Omega_E \tag{6}$$

for which there exits a set of $\underline{x}$ such that

$$\phi^{(t)}(\Delta_0) \quad \subseteq \quad \cup_{\underline{x}} \overline{\Delta}_{\underline{x}} \otimes \overline{\Delta}_E \tag{7}$$

$$\overline{\Delta}_{\underline{x}} \quad = \quad \prod_i \otimes \overline{\Delta}_{i,x_i} \tag{8}$$

The coarse graining of $\phi^{(t)}(\Delta_0)$ will be defined as the smallest superset of $\phi^{(t)}(\Delta_0)$, that can still be expressed as a union of direct products of subregions of the $\omega_{i,j}$ and a direct product of that union with a subregion of the environment. This implies the subregions $\Delta_{i,x_i} \in \left\{ \overline{\Delta}_{i,x_i} \right\}$ and $\Delta_E \in \left\{ \overline{\Delta}_E \right\}$ where $\forall \overline{\Delta}_{i,x_i}, \overline{\Delta}_E$

$$\cup_{\underline{x}} \Delta_{\underline{x}} \otimes \Delta_E \quad \subseteq \quad \cup_{\underline{x}} \overline{\Delta}_{\underline{x}} \otimes \overline{\Delta}_E \tag{9}$$

$$\Delta_{\underline{x}} \quad = \quad \prod_j \otimes \Delta_{i,x_i} \tag{10}$$

In an entropy increasing universe, we assume that the microscopic correlations that develop due to $\phi^{(t)}$ play no role in the future evolution of the system. In effect, this means that we may make the coarse grained replacement

$$\phi^{(t)}(\Delta_0) \rightarrow \cup_{\underline{x}} \Delta_{\underline{x}} \otimes \Delta_E \tag{11}$$

for all future evolution of the system.

The requirement that the initial state Δ_0 is such that it produces all these results, for all realistic maps $\phi^{(t)}$, will be referred to as the initial boundary condition, and the resulting evolution as being in an entropy increasing universe. For the purposes of this paper it will be assumed that these conditions can be met.

When looking at the interactions of localised systems at times long after the initial boundary condition, but long before complete thermalisation (which occurs at some future time t_{th}), this is represented by:

1. No initial microscopic correlations between macroscopic subsystems;

2. Thermal states are represented by Gibbs distributions at the start of any interaction.

3. Microscopic correlations develop between the subsystems;

4. The sum of the Gibbs entropies of the marginal distributions of the macroscopic subsystems, increases;

5. The microscopic correlations become, for all practical purposes, inaccessible and may be coarse grained away;

2.2 Time reversal and symmetry

For clarity, we now state explicitly what we will mean by time reversal and time symmetries.

For the time reversal of the dynamics, we first need the notion of the time reversal of the state space. This is not unproblematic (see [Alb01][Chapter 1], for example) but for the purposes of this article let us assume that there is no disagreement over

the time reverse of a state in our state space. The time reversal of the state space is a map $\Delta^T = T(\Delta) \subset \Omega$ such that $\mu(\Delta^T) = \mu(\Delta)$ and $\Delta = T \circ T(\Delta)$. For subsystems $T(\prod_i \otimes \Delta_i) = \prod_i \otimes T(\Delta_i)$ and for subspaces $T(\cup_n \Delta_n) = \cup_n T(\Delta_n)$. We also note if $A \subset B$ then $A^T \subset B^T$ and for all state spaces Ω we consider here $\Omega^T = \Omega$.

The time reversal of the dynamics about the time $t = t_0$, corresponding to $t \to t_0 - t$, will be defined as the map

$$\phi_{Tt_0}^{(t)}(\Delta) = T \circ \phi^{(2t_0-t)} \circ \phi^{-(2t_0)} \circ T(\Delta) \tag{12}$$

Two special cases may be more familiar. Firstly, for $t_0 = 0$ we have

$$\phi_{T0}^{(t)}(\Delta) = T \circ \phi^{(-t)} \circ T(\Delta)$$

Secondly, for a transformation $\phi^{(2t_0)}$, which takes place over the time period $0 < t < 2t_0$, then a reversal at $t = t_0$ has the transformation

$$\phi_{Tt_0}^{(2t_0)}(\Delta) = T \circ \phi^{-(2t_0)} \circ T(\Delta)$$

It is important to note one cannot use the coarse grained description $\cup_{\underline{x}} \Delta_{\underline{x}} \otimes \Delta_E$, defined in the previous section, for the time reversed dynamics. This coarse graining is valid, in the original dynamics, only for later times so is valid only for earlier times in the time reversed dynamics.

We now define time reversal invariance and time translation invariance of the dynamics, although unless explicitly stated, we will not be assuming any of these invariances hold. We explicitly state them so that it may be clear where we have not needed to assume them.

The dynamics are time reversal invariant at t_0 iff

$$\phi_{Tt_0}^{(t)}(\Delta) = \phi^{(t)}(\Delta) \tag{13}$$

Weak time translation invariance is defined as

$$\forall t > 0, s > 0 \quad \phi^{(t)} \circ \phi^{(s)}(\Delta) = \phi^{(t+s)}(\Delta) \tag{14}$$

and strong time translation invariance as

$$\forall t, s \quad \phi^{(t)} \circ \phi^{(s)}(\Delta) = \phi^{(t+s)}(\Delta) \tag{15}$$

Strong time translation invariance implies[1] $\phi^{-(t)} = \phi^{(-t)}$, and this in turn implies $\phi_{Tt_0}^{(t)}(\Delta) = T \circ \phi^{(-t)} \circ T(\Delta)$ for all t_0.

If a dynamics is time reversal invariant at all times, it is necessarily strong time translation invariant:

$$\left(\forall t_0 \quad \phi_{Tt_0}^{(t)}(\Delta) = \phi^{(t)}(\Delta) \right) \Rightarrow \left(\forall t, s \quad \phi^{(t)} \circ \phi^{(s)}(\Delta) = \phi^{(t+s)}(\Delta) \right) \tag{16}$$

If a dynamics is strong time translation invariant and time reversal invariant at a single time, then it is necessarily time reversal invariant at all times.

$$\left(\left(\forall t, s \quad \phi^{(t)} \circ \phi^{(s)}(\Delta) = \phi^{(t+s)}(\Delta) \right) \& \left(\exists t_0 | \phi_{Tt_0}^{(t)}(\Delta) = \phi^{(t)}(\Delta) \right) \right) \Rightarrow \left(\forall t_0 \quad \phi_{Tt_0}^{(t)}(\Delta) = \phi^{(t)}(\Delta) \right) \tag{17}$$

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[1] As stated previously, $\phi^{(t)}$ is invertible.

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2.3 Entropy decreasing universe

In an entropy decreasing universe, we postulate the existence of a future boundary condition, that at some future time τ, the state of the universe will be in the region of state space Δ_0^T. The most general means of doing this is to find the time reversal of the dynamics at $\tau/2$, $\phi_{T\tau/2}^{(t)}$, impose Δ_0 as the initial boundary condition on this dynamics, then perform a second time reversal at $\tau/2$, on the evolution of Δ_0. If the dynamics are time reversal invariant at $\tau/2$, then of course this simplifies to $\phi_{T\tau/2}^{(t)} = \phi^{(t)}$.

   We now find that the coarse graining works in reverse. Over the course of the evolution of the system, fine grained structure, of an elongated and filamentary kind, appears. This fine grained structure played no role in the evolution of the system prior to its appearance. However, its appearance allows the region of state space to evolve into smaller regions that its initial, coarse grained, appearance would have indicated. In thermodynamic terms, this can be characterised by a universal tendency for heat to spontaneously flow out of the environment and cause masses to be raised through gravitational potentials.

   When looking at the interactions of localised systems at times long before the future boundary condition, $t = \tau$, but long after the universe has come out of complete thermalisation, $t = \tau - t_{th}$, this will be represented by the reversed set of conditions:

1. A high degree of initial microscopic correlations between macroscopic subsystems.

2. Microscopic correlations disappear over the course of the interaction;

3. The sum of the Gibbs entropies of the marginal distributions of the macroscopic subsystems, is decreasing;

4. The microscopic correlations which disappear, played no role in the earlier evolution of the system. In the future evolution of the system, new microscopic correlations come into play;

5. Thermal states are represented by Gibbs distributions at the *end* of any interaction.

2.4 Time symmetric boundary conditions

Schulman[Sch97] has considered the problem of universes with two time boundary conditions. Although the possibility of such a universe remains questionable[Zeh05, Sch05b], it will be useful to consider such a situation here. In these conditions there is a requirement both that the universe begins in the special initial region of state space Δ_0, and at a remote future time τ ends in the special final region of state space Δ_0^T.

   A simple time reversal is not sufficient to deal with this. The possible trajectories of the system are those that pass through $\phi^{(\tau)}(\Delta_0) \cap T(\Delta_0)$ at $t = \tau$. Equivalent conditions are $\Delta_0 \cap \phi^{-(\tau)} \circ T(\Delta_0)$ at $t = 0$ or $\phi^{(\tau/2)}(\Delta_0) \cap T \circ \phi_{T\tau/2}^{(\tau/2)}(\Delta_0)$ at $t = \tau/2$.

Schulman argues that, provided the time span $\tau/2$ is much greater than the complete thermalisation time t_{th}, then during the epoch $0 < t < t_{th}$ the universe will be indistinguishable from an entropy increasing universe, and during the epoch $\tau - t_{th} < t < \tau$ the universe will be indistinguishable from an entropy decreasing one.

3 The Computation Arrow

A physical computation is a physical embodiment of a combination of logical operations. A logical operation is, conventionally, a mathematical operation which takes a finite number of distinct input states and maps them to a finite number of output states. Conventionally, the input logical state uniquely determines the output logical state, but there may be many input states corresponding to the same output state. If this is the case, the operation is called logical irreversible[Lan61].

> We shall call a device logically irreversible if the output of a device does not uniquely define the inputs.

If each β output state has only one possible α input state, then the operation is logically reversible.

The basic operations we need to consider are the NOT operation and the RESET TO ZERO (RTZ) operations (see Tables 1 and 2)[2]. The RTZ operation is perhaps less familiar than logical operations such as AND, OR. Nevertheless, all standard logical operations can be built from suitable combinations of these two operations, and they are the most widely studied logical operations from the point of view of thermodynamics.

NOT	
IN	OUT
0	1
1	0

RTZ	
IN	OUT
0	0
1	0

IDN	
IN	OUT
0	0
1	1

Table 1: Logical NOT   Table 2: Reset to Zero   Table 3: Logical Identity

The physical embodiment of a logical operation is a physical process, that starts with the system in one of a finite number of distinct regions of state space and evolves the system into one of a finite number of distinct regions of state space. The distinct regions of state space represent the input and output logical states. The same regions can (and often will) represent both an input and an output state. The process embodies the logical operation precisely when states in the region of state space corresponding to an input logical state always end in the region of state space corresponding to the output logical state that results from the action of the logical operation upon that input logical state.

To understand this, we will take a state space $\Omega = \Omega_S \otimes \Omega_E$, which is the product of the logical processing system Ω_S and environment Ω_E state spaces. In an entropy increasing universe, we assume the environment is initially in some region $E_0 \subset \Omega_E$

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[2]For completeness we include the identity or DO NOTHING operation, IDN, in Table 3.

and there are no correlations with the system. Each logical state α is represented by a region of the state space of the system $\{A_\alpha \subset \Omega_S\}$, such that $A_\alpha \cap A_\beta = \emptyset$ for $\alpha \neq \beta$. It is usually the case, and we will assume it here, that the input and output states of a logical operation are time reversal invariant subspaces: $A_\alpha^T = A_\alpha$ and $A_\beta^T = A_\beta$.

If the logical operation L maps logical states $\alpha \xrightarrow{L} \beta$, then the dynamic map $\phi_L^{(t_L)}$, acting over the duration t_L, embodies that operation if, and only if, $\forall \alpha \xrightarrow{L} \beta$

$$\phi_L^{(t_L)}(A_\alpha \otimes E_0) \subseteq A_\beta \otimes \Omega_E \tag{18}$$

At the end of the physical operation, the system and environment will be located in the region:

$$\Delta_{t_L} = \phi_L^{(t_L)}(\Delta_0) = \cup_\alpha \phi_L^{(t_L)}(A_\alpha \otimes E_0) \subseteq \cup_\beta A_\beta \otimes \Omega_E \tag{19}$$

In an entropy increasing universe, we assume that microscopic correlations between the system and the environment play no future role. If we are not considering time reversals, therefore, for future evolutions of the system we can replace Δ_{t_L} with

$$\Delta'_{t_L} = \cup_\beta A_\beta \otimes E_{t_L} \tag{20}$$

where $\forall E' \subseteq \Omega_E$ such that $\Delta_{t_L} \subseteq \cup_\beta A_\beta \otimes E'$, then

$$\Delta_{t_L} \subseteq \cup_\beta A_\beta \otimes E_{t_L} \subseteq \cup_\beta A_\beta \otimes E' \tag{21}$$

3.1 Temporal reversal

The temporal reversal of the physical operation, at time $\frac{1}{2}t_L$, involves the system and environment starting in the region of state space $\Delta_{t_L}^T$, and the evolution $\phi_{TL}^{(t)}(\Delta) = T \circ \phi_L^{(t_L-t)} \circ \phi_L^{-(t_L)} \circ T(\Delta)$.

Note that

$$T \circ \phi_L^{(t_L)}(A_\alpha \otimes E_0) \subseteq A_\beta \otimes \Omega_E \tag{22}$$

and

$$\phi_{TL}^{(t_L)}\left(T \circ \phi_L^{(t_L)}(A_\alpha \otimes E_0)\right) = A_\alpha \otimes E_0^T \subseteq A_\alpha \otimes \Omega_E \tag{23}$$

$\phi_{TL}^{(t_L)}$ has acted as a map from the system being in one of the regions of state space corresponding to a logical state β to being in a region of state space corresponding to a logical state α. However, in logically irreversible operations, there may be more that one α which was mapped to β by the operation L. It does not follow that there exists some α for which

$$\phi_{TL}^{(t_L)}((A_\beta \otimes \Omega_E) \cap \Delta_t) \subseteq A_\alpha \otimes \Omega_E \tag{24}$$

In general, time reversing a logically reversible operation does result in another logically reversible operation. Time reversing a logically irreversible operation results in a physical process which does not appear to resemble a logical operation at all.

3.2 Indeterministic operations

To better understand the consequences of the time reversal of logically irreversible operations, we need to widen the class of operations we are considering, to include *indeterministic*[3] operations[Mar05]:

> We shall call a device logically indeterministic if the input to a device does not uniquely define the outputs.

The time reversal of the logically reversible IDN and the NOT operations result in the IDN and NOT operations, respectively. Time reversal of logically irreversible RTZ, however, results in the indeterministic operation Unset From Zero (UFZ) in Table 4. Note that the operation UFZ does fulfil the requirement of logical reversibility, above. For completeness, we also add the indeterministic, irreversible operation Randomise (RND) in Table 5.

UFZ

IN	OUT
0	0
0	1

Table 4: Unset From Zero

RND

IN	OUT
0	0
0	1
1	0
1	1

Table 5: Randomise

A computation is not simply a sequence of operations. It is an ordered sequence of particular logical operations. If a Universal Turing Machine is constructed out of a collection of physical processes implementing a particular set of logically deterministic operations, the time reversal of those physical processes certainly does not produce the same set of operations. If the Universal Turing Machine was constructed using deterministic, logically irreversible operations, the time reversal would not include any logically irreversible operations but would include indeterministic operations. This would not be a Universal Turing Machine.

Logically irreversible operations may be simulated by logically reversible operations, but under time reversal this still does not recover the original computation. The logically reversible simulation of the RTZ operation is given in Table 6, and its time reversal in Table 7.

IN		OUT	
0	0	0	0
1	1	1	0

Table 6: Simulating RTZ

IN		OUT	
0	0	0	0
1	0	1	1

Table 7: Simulating UFZ

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[3]While indeterministic operations can be well defined, and can be embodied by physical processes, it has been argued that indeterministic operations do not count as logical operations[SLGP07], although indeterministic operations are required for computational complexity classes such as BPP, and so form a part of computational logic. As this point is not important for the discussion here, we will reserve 'logical operation' for logically deterministic operations in this paper, and refer to logically indeterministic operations as simply 'indeterministic operations'.

The time reversal is not a reversible simulation of RTZ, it is a deterministic simulation of UFZ. Although, in this case, both simulations can be achieved by the same logical operation (the $CNOT$ gate), the particular operation that is being simulated changes. A sequence of operations simulating irreversible operations becomes a sequence of operations simulating indeterministic operations. If the Universal Turing Machine was constructed using deterministic, logically reversible operations, simulating logically irreversible operations, the time reversal would not include any simulations of logically irreversible operations but would include simulations of indeterministic operations. This would still not be a Universal Turing Machine. The time reversal of a Universal Turing Machine is not a Universal Turing Machine. So it would appear that a computation, as a physical process, may have an arrow of time.

3.3 Logical reversal

We will now define the logical reversal of an operation, L, as the operation, RL, which has the same mapping on the logical states, as the time reversal of a physical implementation of that operation.

We do this by defining the proportion (according to a measure μ) of initial states

1. that start in logical state α

$$W_L(\alpha) = \frac{\mu\left((A_\alpha \otimes \Omega_E) \cap \Delta_0\right)}{\mu\left(\Delta_0\right)}$$

2. that end in logical state β given they started in α;

$$W_L(\beta|\alpha) = \frac{\mu\left((A_\beta \otimes \Omega_E) \cap \phi_L^{(t_L)}(A_\alpha \otimes \Omega_E) \cap \Delta_{t_L}\right)}{\mu\left((A_\alpha \otimes \Omega_E) \cap \Delta_0\right)}$$

3. that start in logical state α and end in logical state β;

$$W_L(\alpha, \beta) = \frac{\mu\left((A_\beta \otimes \Omega_E) \cap \phi_L^{(t_L)}(A_\alpha \otimes \Omega_E) \cap \Delta_{t_L}\right)}{\mu\left(\Delta_0\right)}$$

4. that end in logical state β;

$$W_L(\beta) = \frac{\mu\left((A_\beta \otimes \Omega_E) \cap \Delta_{t_L}\right)}{\mu\left(\Delta_0\right)}$$

5. and that started in logical state α, given that they ended in logical state β

$$W_L(\alpha|\beta) = \frac{\mu\left((A_\beta \otimes \Omega_E) \cap \phi_L^{(t_L)}(A_\alpha \otimes \Omega_E) \cap \Delta_{t_L}\right)}{\mu\left((A_\beta \otimes \Omega_E) \cap \Delta_{t_L}\right)}$$

For logically deterministic operations

$$W_L(\alpha|\beta) \in \{0, 1\}$$

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while for logically reversible operations

$$W_L(\beta|\alpha) \in \{0,1\}$$

We do not include input or output states with measure zero, so $W_L(\alpha) \neq 0$ and $W_L(\beta) \neq 0$. If $W_L(\alpha|\beta) = 0$ for the measure μ, it will be zero for all other measures, absolutely continuous with μ, that are preserved by the dynamics. Equivalent statements also hold true for $W_L(\alpha|\beta) = 1$, $W_L(\beta|\alpha) = 0$ and $W_L(\beta|\alpha) = 1$.

When we consider the temporal reversal TL of the physical process, we get states starting in logical states β, and ending in logical states α, with proportions

$$
\begin{aligned}
W_{TL}(\beta) &= \frac{\mu\left((A_\beta \otimes \Omega_E) \cap \Delta_{t_L}\right)}{\mu\left(\Delta_{t_L}\right)} \\
W_{TL}(\alpha|\beta) &= \frac{\mu\left(\phi_{TL}^{(t_L)}(A_\beta \otimes \Omega_E) \cap (A_\alpha \otimes \Omega_E) \cap \Delta_0\right)}{\mu\left((A_\beta \otimes \Omega_E) \cap \Delta_{t_L}\right)} \\
W_{TL}(\alpha,\beta) &= \frac{\mu\left(\phi_{TL}^{(t_L)}(A_\beta \otimes \Omega_E) \cap (A_\alpha \otimes \Omega_E) \cap \Delta_0\right)}{\mu\left(\Delta_{t_L}\right)} \\
W_{TL}(\alpha) &= \frac{\mu\left((A_\alpha \otimes \Omega_E) \cap \Delta_0\right)}{\mu\left(\Delta_{t_L}\right)} \\
W_{TL}(\beta|\alpha) &= \frac{\mu\left(\phi_{TL}^{(t_L)}(A_\beta \otimes \Omega_E) \cap (A_\alpha \otimes \Omega_E) \cap \Delta_0\right)}{\mu\left((A_\alpha \otimes \Omega_E) \cap \Delta_0\right)}
\end{aligned}
$$

It is straightforward to show that as

$$W_{TL}(\beta) = W_L(\beta)$$

then

$$W_{TL}(\alpha) = W_L(\alpha)$$

and

$$W_{TL}(\beta|\alpha) = W_L(\beta|\alpha)$$

It is also clear, by definition, that the temporal reversal of TL is just L:

$$TTL \equiv L$$

We will now define the reversal operation RL, of L, as a map from the logical states β to the logical states α,

$$\{\beta\} \xrightarrow{RL} \{\alpha\}$$

in the same time direction as L, with a dynamic map $\phi_{RL}^{(t_L)}$ such that $W_{RL}(\alpha|\beta) = W_L(\alpha|\beta)$:

$$
\begin{aligned}
W_{RL}(\beta) &= \frac{\mu\left((A_\beta \otimes \Omega_E) \cap \Lambda_0\right)}{\mu\left(\Lambda_0\right)} \\
W_{RL}(\alpha|\beta) &= \frac{\mu\left(\phi_{RL}^{(t_L)}(A_\beta \otimes \Omega_E) \cap (A_\alpha \otimes \Omega_E) \cap \Lambda_{t_L}\right)}{\mu\left((A_\beta \otimes \Omega_E) \cap \Lambda_0\right)} \\
W_{RL}(\alpha,\beta) &= \frac{\mu\left(\phi_{RL}^{(t_L)}(A_\beta \otimes \Omega_E) \cap (A_\alpha \otimes \Omega_E) \cap \Lambda_{t_L}\right)}{\mu\left(\Lambda_0\right)}
\end{aligned}
$$

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$$W_{RL}(\alpha) = \frac{\mu\left((A_\alpha \otimes \Omega_E) \cap \Lambda_{t_L}\right)}{\mu(\Lambda_0)}$$

$$W_{RL}(\beta|\alpha) = \frac{\mu\left(\phi_{RL}^{(t_L)}(A_\beta \otimes \Omega_E) \cap (A_\alpha \otimes \Omega_E) \cap \Lambda_{t_L}\right)}{\mu\left((A_\alpha \otimes \Omega_E) \cap \Lambda_{t_L}\right)}$$

where the system is initial in the region $\Lambda_0 = \cup_\beta A_\beta \otimes E_0$ and ends in the region $\Lambda_{t_L} = \phi_{RL}^{(t_L)}(\Lambda_0)$. Again, it is straightforward that

$$W_{RL}(\beta) = W_L(\beta)$$

leads to

$$W_{RL}(\alpha) = W_L(\alpha)$$

and

$$W_{RL}(\beta|\alpha) = W_L(\beta|\alpha)$$

By definition

$$RRL \equiv L$$

There is a straightforward method for constructing ϕ_{RL}:

1. Partition each β region into (α, β) subregions, $A_\beta = \cup_\alpha A_{(\alpha|\beta)}$, with $A_{(\alpha|\beta)} \cap A_{(\alpha'|\beta)} = \emptyset$, $\alpha \neq \alpha'$ such that

$$\frac{\mu(\alpha, \beta)}{\mu(\beta)} = W_L(\alpha|\beta)$$

2. The evolution of the system must prevent transitions between the subregions

$$\phi(A_{(\alpha|\beta)}) \cap \phi(A_{(\alpha'|\beta')}) = \emptyset \ \ \forall \alpha \neq \alpha', \beta \neq \beta'$$

3. Define regions A'_α by joining the α subregions together, from different β regions

$$A'_\alpha = \cup_\beta A_{(\alpha|\beta)}$$

and remove barriers to transitions between subregions with the same α value.

4. Evolve the distinct α regions to their final location in state space:

$$A_\alpha = \phi(A'_\alpha)$$

Further refinements are necessary for thermodynamic optimisation. Explicit physical processes by which the operations UFZ and RND can be constructed and optimised are given in [Mar05] and for generic operations in [Mar07].

3.4 Computational reversal

We will now consider sequences of operations, in a normal entropy increasing universe. We will not specify the particular set of operations. Our objective is not to consider the properties of a particular sequence of logical operations, or even of any sequence of logical operations intended for a particular purpose. We wish to consider the properties of any process that can be defined exclusively in terms of logical operations acting upon sets of logical states.

In this general situation, we start with a set of logical states $\{\alpha_0\}$. This is acted on by some logical operation L_0, and mapped to the output states $\{\alpha_1\}$. As we are in an entropy increasing universe, we may assume that any microscopic correlations that have developed between the information processing apparatus and the environment play no role in the future evolution of the system. The logical operation L_1 then maps the states $\{\alpha_1\}$ to the states $\{\alpha_2\}$, and so on.

This leads to the sequence $S_1\{L_i\}$:

$$\{\alpha_0\} \xrightarrow{L_1} \{\alpha_1\} \xrightarrow{L_2} \ldots \xrightarrow{L_i} \{\alpha_i\} \xrightarrow{L_{i+1}} \ldots \xrightarrow{L_f} \{\alpha_f\}$$

In the entropy decreasing universe that results from a time reversal at a point in the distant future, the sequence becomes $S_2\{TL_i\}$:

$$\{\alpha_f\} \xrightarrow{TL_f} \{\alpha_{f-1}\} \xrightarrow{TL_{f-1}} \ldots \xrightarrow{TL_{i+1}} \{\alpha_i\} \xrightarrow{TL_i} \ldots \xrightarrow{TL_1} \{\alpha_0\}$$

As noted before, the sequence of operations $S_2\{TL_i\}$, involving the time reversed TL operations, will not, in general, resemble the same computational process as $S_1\{L_i\}$.

Now construct a physical system, in the original entropy increasing universe, with initial logical states $\{\alpha_f\}$, a measure μ such that the physical representation of the states have weights $W_{RL_f}(\alpha_f) = W_{L_f}(\alpha_f)$, and the reversal operations $\{RL_i\}$, such that $W_{RL_i}(\alpha_{i-1}|\alpha_i) = W_{L_i}(\alpha_{i-1}|\alpha_i)$. This leads to the sequence $S_3\{RL_i\}$:

$$\{\alpha_f\} \xrightarrow{RL_f} \{\alpha_{f-1}\} \xrightarrow{RL_{f-1}} \ldots \xrightarrow{RL_{i+1}} \{\alpha_i\} \xrightarrow{RL_i} \ldots \xrightarrow{RL_1} \{\alpha_0\}$$

The time reversal of the universe containing the sequence $S_3\{RL_i\}$, gives the sequence $S_4\{TRL_i\}$:

$$\{\alpha_0\} \xrightarrow{TRL_1} \{\alpha_1\} \xrightarrow{TRL_2} \ldots \xrightarrow{TRL_i} \{\alpha_i\} \xrightarrow{TRL_{i+1}} \ldots \xrightarrow{TRL_f} \{\alpha_f\}$$

However, in follows from the definitions above, that $TRL_i \equiv RTL_i \equiv L_i$, so $S_4\{TRL_i\}$ is

$$\{\alpha_0\} \xrightarrow{L_1} \{\alpha_1\} \xrightarrow{L_2} \ldots \xrightarrow{L_i} \{\alpha_i\} \xrightarrow{L_{i+1}} \ldots \xrightarrow{L_f} \{\alpha_f\}$$

Sequence $S_4\{TRL_i\}$ is exactly the same set of logical operations as $S_1\{L_i\}$, performed in the same order, and on the same set of logical states. $S_4\{TRL_i\}$ takes place in an entropy decreasing universe.

For any computational process consisting of a sequence of logical operations on a set of logical states, in an entropy increasing universe, the same computational process is possible in an entropy decreasing universe. Although we were able to conclude in Section 3.2, above, that computational processes may have an intrinsic arrow, it does not appear to be the case that this arrow must be aligned with the thermodynamic arrow.

3.5 Landauer's Principle

Landauer's Principle is used as the basis for almost all conclusions regarding the thermodynamic properties of physical computation, yet the conclusion of the previous section seems to run counter to many widespread statements of this Principle:

> To erase a bit of information in an environment at temperature T requires dissipation of energy $\geq kT \ln 2$. [Cav90, Cav93]

> in erasing one bit ... of information one dissipates, on average, at least $k_B T \ln (2)$ of energy into the environment. [Pie00]

> a logically irreversible operation must be implemented by a physically irreversible device, which dissipates heat into the environment [Bub01]

> erasure of one bit of information increases the entropy of the environment by at least $k \ln 2$ [LR03][pg 27]

> any logically irreversible manipulation of data ... must be accompanied by a corresponding entropy increase in the non-information bearing degrees of freedom of the information processing apparatus or its environment. Conversely, it is generally accepted that any logically reversible transformation of information can in principle be accomplished by an appropriate physical mechanism operating in a thermodynamically reversible fashion. [Ben03]

> Computations are accompanied by dissipation ... Landauer has shown that computation requires irreversible processes and heat generation.[Sch05a]

It is Landauer's Principle on which Schulman basis the alignment of the thermodynamic and the computational arrows of time.

If Landauer's Principle is truly regarded as "the basic principle of the thermodynamics of information processing"[Ben03], how does this reconcile with the argument of the previous Section, that exactly the same information processing operations can take place in an entropy decreasing, as an entropy increasing universe? Does the computer act as a kind of Maxwell's Demon, dissipating heat against overall the anti-entropic direction?

The answer is, straightforwardly, no. As has been noted many times before[EN99, Mar02, Nor05], Landauer's Principle is not really a principle. It is a theorem, of statistical mechanics, derived[Pie00, Tur06, SLGP07, Mar07] on the assumption that the computation is taking place in an entropy increasing universe. All justifications of Landauer's Principle, from [Lan61] onwards, make this assumption. We will briefly review the derivation of Landauer's Principle in an entropy increasing universe, to see how the derivation turns out in an entropy decreasing universe.

3.5.1 Entropy increase

The states of the physical system embodying logical state α will be represented by density matrix ρ_α, and β by ρ_β. We assume[4] that the input logical states $\{\alpha\}$ and

---

[4]This is normal practice in the thermodynamics of computation. In [Mar07] this assumption, called uniform computing, is relaxed. The essential conclusions of this Section are not affected.

output logical states $\{\beta\}$ are represented by states of physical systems with the same entropy S and mean energies U, so that $\forall \alpha, \beta$:

$$S = -k\,\text{Tr}\left[\rho_\alpha \ln\left[\rho_\alpha\right]\right] = -k\,\text{Tr}\left[\rho_\beta \ln\left[\rho_\beta\right]\right] \tag{25}$$
$$U = \text{Tr}\left[H_S \rho_\alpha\right] = \text{Tr}\left[H_S \rho_\beta\right] \tag{26}$$

The input logical states occur with probability P_α, and the logical operation is defined by the probabilities $P(\beta|\alpha)$.

In an entropy increasing universe, we make the following assumptions:

1. The evolution of the system and environment is described by Hamiltonian dynamics, composed of internal energies of the system H_S and environment H_E, together with an interaction potential V_{SE}:

$$H = H_S \otimes I_E + I_S \otimes H_E + V_{SE}$$

2. The environment is initially in a Gibbs canonical state, at some temperature T, and there are no initial correlations between the system and the environment.

$$\rho_E(T) = \frac{e^{-H_E/kT}}{\text{Tr}\left[e^{-H_E/kT}\right]} \tag{27}$$
$$\rho_0 = \sum_\alpha P(\alpha)\rho_\alpha \otimes \rho_E(T) \tag{28}$$

3. The interaction energy between system and environment is negligible both before

$$\text{Tr}\left[V_{SE}\rho_0\right] \approx 0$$

and after

$$\text{Tr}\left[V_{SE}e^{-\imath Ht}\rho_0 e^{\imath Ht}\right] \approx 0$$

the interaction.

For the Hamiltonian H to embody the logical operation:

$$\text{Tr}_E\left[e^{-\imath Ht}\rho_\alpha \otimes \rho_E(T)e^{\imath Ht}\right] = \sum_\beta P(\beta|\alpha)\rho_\beta$$

It is a well known calculation[Gib02, Tol38, Par89, Pie00, Mar07] to show, using:

$$\rho_I = \sum_\alpha P(\alpha)\rho_\alpha$$
$$\rho_t = e^{-\imath Ht}\rho_0 e^{\imath Ht}$$
$$P(\beta) = \sum_\alpha P(\beta|\alpha)P(\alpha)$$
$$\rho_F = \text{Tr}_E\left[\rho_t\right] = \sum_\beta P(\beta)\rho_\beta$$
$$\rho'_E = \text{Tr}_S\left[\rho_t\right]$$

that two inequalities follow:

$$\text{Tr}\left[\rho_I \ln\left[\rho_I\right]\right] + \text{Tr}\left[\rho_E(T) \ln\left[\rho_E(T)\right]\right] \geq \text{Tr}\left[\rho_F \ln\left[\rho_F\right]\right] + \text{Tr}\left[\rho'_E \ln\left[\rho'_E\right]\right] \tag{29}$$
$$\text{Tr}\left[\rho'_E\left(\ln\left[\rho'_E\right] + \frac{H_E}{kT}\right)\right] \geq \text{Tr}\left[\rho_E(T)\left(\ln\left[\rho_E(T)\right] + \frac{H_E}{kT}\right)\right] \tag{30}$$

which combine to give

$$\sum_{\alpha} P(\alpha) \ln P(\alpha) - \sum_{\beta} P(\beta) \ln P(\beta) \geq \frac{\text{Tr}\left[H_E \rho_E(T)\right]}{kT} - \frac{\text{Tr}\left[H_E \rho_E'\right]}{kT} \tag{31}$$

This yields the standard form of Landauer's Principle, in an entropy increasing universe:

$$\Delta Q \geq -\Delta H k T \ln(2)$$

where ΔQ is the expectation value for the heat generated in an environment at temperature T and ΔH is the change in Shannon information over the course of the operation

$$\Delta H = \sum_{\alpha} P(\alpha) \log_2 P(\alpha) - \sum_{\beta} P(\beta) \log_2 P(\beta)$$

For logically deterministic, reversible computations, it is always the case that $\Delta H = 0$. These operations do not need to generate heat. On the other hand, for logically deterministic, irreversible operations $\Delta H < 0$ and so the heat generated in the environment is always positive. This is the basis of the claim that logically irreversible operations must be entropy increasing[5].

3.5.2 Entropy decrease

In an entropy decreasing universe, we would still make the assumptions that the input logical states $\{\alpha\}$ and output logical states $\{\beta\}$ are represented by physical systems with the same entropy and mean energies. The logical state α is represented by the density matrix ρ_α, and β by ρ_β, as before. The input logical states occur with probability P_α, and the logical operation is defined by the probabilities $P(\beta|\alpha)$.

We continue to assume:

1. The evolution of the system and environment is described by Hamiltonian dynamics.

$$H' = H_S' \otimes I_E + I_S \otimes H_E' + V_{SE}'$$

2. The interaction energy between system and environment is negligible both before

$$\text{Tr}\left[V_{SE}'\rho_0\right] \approx 0$$

and after

$$\text{Tr}\left[V_{SE}'e^{-\imath H't}\rho_0 e^{\imath H't}\right] \approx 0$$

the interaction.

but the imposition of a future boundary condition must require the local conditions to be:

3. *After* the operation the environment is in a Gibbs canonical state, at some temperature T, and there are no final microscopic correlations between the system and the environment.

---

[5]In [Mar02, Mar05, Mar07] it is argued that even this heat generation is not necessarily thermodynamically irreversible.

Now, for the Hamiltonian H' to fulfil these conditions and embody the logical operation it is necessary that

$$\mathrm{Tr}_E\left[e^{\imath H' t}\rho_\beta \otimes \rho_E(T)e^{-\imath H' t}\right] = \sum_\alpha \frac{P(\beta|\alpha)P(\alpha)}{\sum_{\alpha'} P(\beta|\alpha')P(\alpha')}\rho_\alpha$$

and

$$\rho_t = \sum_{\beta,\alpha} P(\beta|\alpha)P(\alpha)\rho_\beta \otimes \rho_E(T)$$

Using:

$$
\begin{aligned}
\rho_0 &= e^{\imath H' t}\rho_t e^{-\imath H' t} \\
\rho_I &= \mathrm{Tr}_E\left[\rho_0\right] = \sum_\alpha P(\alpha)\rho_\alpha \\
\rho'_E &= \mathrm{Tr}_S\left[\rho_0\right] \\
P(\beta) &= \sum_\alpha P(\beta|\alpha)P(\alpha) \\
\rho_F &= \mathrm{Tr}_E\left[\rho_t\right] = \sum_\beta P(\beta)\rho_\beta
\end{aligned}
$$

the two inequalities become

$$
\begin{aligned}
\mathrm{Tr}\left[\rho_F \ln\left[\rho_F\right]\right] + \mathrm{Tr}\left[\rho_E(T)\ln\left[\rho_E(T)\right]\right] &\geq \mathrm{Tr}\left[\rho_I \ln\left[\rho_I\right]\right] + \mathrm{Tr}\left[\rho'_E \ln\left[\rho'_E\right]\right] \\
\mathrm{Tr}\left[\rho'_E\left(\ln\left[\rho'_E\right] + \frac{H'_E}{kT}\right)\right] &\geq \mathrm{Tr}\left[\rho_E(T)\left(\ln\left[\rho_E(T)\right] + \frac{H'_E}{kT}\right)\right]
\end{aligned}
$$

which combine to give

$$-\sum_\alpha P(\alpha)\ln P(\alpha) + \sum_\beta P(\beta)\ln P(\beta) \geq \frac{\mathrm{Tr}\left[H'_E \rho_E(T)\right]}{kT} - \frac{\mathrm{Tr}\left[H'_E \rho'_E\right]}{kT}$$

Paying careful attention to the fact that $\rho_E(T)$ is now the *final* state of the environment the statistical mechanical calculation leads to:

$$\Delta Q \leq -\Delta H kT \ln(2)$$

where ΔQ is the expectation value for the heat generated in an environment.

For logically deterministic, irreversible operations $\Delta H < 0$ and so the heat generated in the environment is *less than* the positive number $-\Delta H kT \ln(2)$. For logically deterministic, reversible computations, $\Delta H = 0$ as before, but this now just means the heat generation must be *less than* zero. In an entropy decreasing universe, the derivation of Landauer's Principle yields a *maximum* heat generation. If less than the maximum heat is generated, then there will have been an uncompensated decrease in the entropy of the universe.

This is, of course, exactly what we should have expected! In entropy decreasing universes, the physical processes which embody computations are, generically, entropy decreasing processes. There is no contradiction between the statistical mechanical basis of Landauer's Principle, and the conclusions of Section 3.4.

4 The Correlation Arrow

It has been argued in the previous Sections that, although a computer may possess a computational arrow, it's functioning as a physical process does not imply the alignment of that arrow with the thermodynamic arrow. The argument was based upon all the same computational operations that can take place in an entropy increasing universe being physically possible in an entropy decreasing universe. This still leaves open the possibility that it is much more likely for systems to develop which process information in the same direction as entropy increase, than systems which process information in the direction of entropy decrease.

Turning to this question, the arguments will seem less concrete than in the previous sections. This is a consequence of the need to consider if cosmological boundary conditions, over the lifetime of the universe, on the state of the whole universe, may have influences on the localised behaviour of systems, operating over short timescales, at a time in between, and very far from, either initial or final state of the universe. It is unclear how secure the chain of reasoning involved in understanding such influences can be (see [Ear06], for example, for a sceptical view).

How might such an argument be constructed? Hawking[Haw94] suggests:

> If one imposes a final boundary condition . . . one can show that the correlation between the computer memory and the surroundings is greater at early times than at late times. In other words, the computer remembers the future, but not the past.

The acquisition of information requires an increase in the correlation between the computer and its surroundings. A future boundary condition, as interpreted in Section 2.3, requires correlations to decrease in time. To explore this requires a move beyond the consideration of a computer as an information processor. We must take into account the nature of the information that the system processes. It is a system that acquires new information about it's surroundings and interacts with its surroundings conditional upon the information it has acquired. Such behaviour has been characterised as an Information Gathering and Utilising System, or $IGUS$.

4.1 Information Gathering and Utilising Systems

The behaviour of an $IGUS$ may be described as:

1. There is a correlation between the macroscopic states of the internal states of an $IGUS$ with macroscopic states of its surroundings.

2. These macroscopic correlations occurred through an interaction of the system with the surroundings, in the past. At an earlier point in time the macroscopic correlations did not exist. The existing correlations are screened off by an earlier interaction.

3. New macroscopic correlations develop over time through conditional interactions. These can change the macroscopic internal states of the system conditional upon the states of the surroundings, or change the states of the surroundings, conditional upon the internal states of the system.

4. Any macroscopic correlations between the current state of the system and future states of its surroundings, are screened off by the existing correlations and interactions between system and environment that take place between the present and the future time.

The argument of Hawking is that such behaviour is compatible with an initial boundary condition, but incompatible with a future boundary condition.

We can examine this in two equivalent ways. The first is to consider an $IGUS$ in an entropy increasing and in an entropy decreasing universe. The second way is to consider the time reversal of these two scenarios. This will give a information processing system which is the logical reversal of an $IGUS$, in an entropy decreasing and in an entropy increasing universe, respectively. We refer to the logical reversal of an $IGUS$ as an $RIGUS$. The statement that an entropy decreasing universe is incompatible with the operation of an $IGUS$ is equivalent to the statement that an entropy increasing universe is incompatible with an $RIGUS$.

The question needing answering is whether an entropy increasing universe prefers systems resembling an $IGUS$ over systems resembling an $RIGUS$. If so the same argument should support the existence of an $RIGUS$ compared to an $IGUS$ in an entropy decreasing universe.

The behaviour of an $RIGUS$ will appear as:

1. There is a correlation between the macroscopic states of the internal states of an $RIGUS$ with macroscopic states of its surroundings.

2. These macroscopic correlations will disappear through a conditional interaction of the system with the surroundings, at some point in future. At a later point in time the macroscopic correlations will not exist.

3. There decrease in macroscopic correlations over time is through conditional interactions with the surroundings. These can change the macroscopic internal states of the system conditional upon the states of the surroundings, or change the states of the surroundings, conditional upon the internal states of the system.

4. Any macroscopic correlations between the current state of the system and past states of its surroundings is screened off by the existing correlations and interactions between the past time and the present.

Fortunately we do not need to construct explicit models for an $IGUS$ or an $RIGUS$. All we need to know is that either system must be constructed out of the kind of operations described in the previous sections.

It is now necessary to draw a distinction between the environmental degrees of freedom of a heat bath, and the macroscopic states of the surroundings that a computer might be correlated with. The set $\{A_i\}$ refer to the internal logical states of the $IGUS$. The macroscopically distinct regions of the surroundings are $\{B_i\}$. We represent the inaccessible regions of the environment by a separate subsystem Ω_E, which has no macroscopically distinguishable subregions. The overall state of the universe at time t is represented by Δ_t.

4.2 Growth in correlations

Acquisition of knowledge is represented in the following terms. At a time t_1 the computer is in the blank state represented by A_0, while the surroundings are in one of the regions B_i. The region of state space is

$$\Delta_{i,t_1} = B_i \otimes A_0 \otimes E_{t_1} \tag{32}$$

and the overall possible region is

$$\Theta_{t_1} = \cup_i B_i \otimes A_0 \otimes E_{t_1} \tag{33}$$

The acquisition of information requires an evolution between t_1 and t_2 for which:

$$\Delta_{i,t_2} = \phi^{(t_2)} \circ \phi^{-(t_1)}(\Delta_{i,t_1}) \subseteq B_i \otimes A_i \otimes \Omega_E \tag{34}$$

In an entropy increasing universe, we replace this by the coarse graining $B_i \otimes A_i \otimes E_{i,t_2} \supseteq \Delta_{i,t_2}$, for which

$$B_i \otimes A_i \otimes E_{i,t_2} \subseteq B_i \otimes A_i \otimes \overline{E}_{i,t_2} \tag{35}$$

for all E'_{i,t_2} such that:

$$\Delta_{t_2} \subseteq B_i \otimes A_i \otimes \overline{E}_{i,t_2} \subseteq B_i \otimes A_i \otimes \Omega_E \tag{36}$$

The overall region is

$$\Theta_{t_2} = \cup_i \phi^{(t_2)} \circ \phi^{-(t_1)}(B_i \otimes A_0 \otimes E_{t_1}) \tag{37}$$

which has a coarse graining $\cup_i B_i \otimes A_i \otimes E_{t_2} \supseteq \Theta_{t_2}$, such that

$$\cup_i B_i \otimes A_i \otimes E_{t_2} \subseteq B_i \otimes A_i \otimes \overline{E}_{t_2} \tag{38}$$

for all $\overline{E}_{t_2}$ such that:

$$\Theta_{t_2} \subseteq \cup_i B_i \otimes A_i \otimes \overline{E}_{t_2} \subseteq \cup_i B_i \otimes A_i \otimes \Omega_E \tag{39}$$

Now let us consider the reverse procedure, that would indicate the existence of an *RIGUS*. Start in $\Delta'_{i,t_1} = B_i \otimes A_i \otimes E'_{t_1}$ and perform the evolution

$$\Delta'_{i,t_2} = \phi'^{(t_2)} \circ \phi'^{-(t_1)}(\Delta'_{i,t_1}) \subseteq B_i \otimes A_0 \otimes \Omega_E \tag{40}$$

This leads to the coarse graining

$$\Delta'_{i,t_2} \subseteq B_i \otimes A_0 \otimes E'_{i,t_2} \tag{41}$$

and the overall region

$$\Theta'_{t_1} = \cup_i B_i \otimes A_i \otimes E'_{t_1} \tag{42}$$

evolves into

$$\Theta'_{t_2} = \cup_i \phi^{(t_2)} \circ \phi^{-(t_1)}(B_i \otimes A_i \otimes E'_{t_1}) \tag{43}$$

which has a coarse graining $\cup_i B_i \otimes A_0 \otimes E'_{t_2} \supseteq \Theta'_{t_2}$, such that

$$\cup_i B_i \otimes A_0 \otimes E'_{t_2} \subseteq B_i \otimes A_i \otimes \overline{E}'_{t_2} \tag{44}$$

for all $\overline{E}'_{t_2}$ such that:

$$\Theta'_{t_2} \subseteq \cup_i B_i \otimes A_i \otimes \overline{E}'_{t_2} \subseteq \cup_i B_i \otimes A_i \otimes \Omega_E \tag{45}$$

4.2.1 Measures on marginals

We now ask whether the requirement that an $RIGUS$ starts in a correlated state, and removes those correlations, is less compatible with an entropy increasing universe than an $IGUS$. We will assume that $\mu(E'_{t_1}) = \mu(E_{t_1})$ and that the internal states of the $IGUS$ and $RIGUS$ have equivalent measures: $\mu(A_0) = \mu(A_i)$.

First consider the measure of the initial states:

$$\mu(\Theta_{t_1}) = \mu(\Theta'_{t_1}) \tag{46}$$

An immediate consequence is that volume of state space arguments will not be able to show preference for an $IGUS$ over an $RIGUS$ on the basis of one or the other being simply more likely to occur.

From the measure preserving nature of the evolution of the $IGUS$ we have

$$\mu(\Delta_{i,t_1}) = \mu(\Delta_{i,t_2}) \tag{47}$$

while the coarse graining gives

$$\mu(B_i)\mu(A_0)\mu(E_{t_1}) \leq \mu(B_i)\mu(A_i)\mu(E_{i,t_2}) \tag{48}$$

Similarly

$$\mu(\Theta_{t_1}) = \mu(\Theta_{t_2}) \tag{49}$$

which when coarse grained gives

$$\sum_i \mu(B_i)\mu(A_0)\mu(E_{t_1}) \leq \sum_i \mu(B_i)\mu(A_i)\mu(E_{i,t_2}) \leq \sum_i \mu(B_i)\mu(A_i)\mu(E_{t_2}) \tag{50}$$

Using $\mu(A_0) = \mu(A_i)$, we get:

$$\mu(E_{t_1}) \leq \frac{\sum_i \mu(B_i)\mu(E_{i,t_2})}{\sum_i \mu(B_i)} \leq \mu(E_{t_2}) \tag{51}$$

While this might indicate an increase in entropy, we can easily get similar results for the $RIGUS$. The measures for the reverse interaction are

$$\mu(\Delta'_{i,t_1}) = \mu(\Delta'_{i,t_2}) \tag{52}$$

while the coarse graining gives

$$\mu(B_i)\mu(A_i)\mu(E'_{t_1}) \leq \mu(B_i)\mu(A_0)\mu(E'_{i,t_2}) \tag{53}$$

Similarly

$$\mu(\Theta'_{t_1}) = \mu(\Theta'_{t_2}) \tag{54}$$

which when coarse grained gives

$$\sum_i \mu(B_i)\mu(A_i)\mu(E'_{t_1}) \leq \sum_i \mu(B_i)\mu(A_0)\mu(E'_{i,t_2}) \leq \sum_i \mu(B_i)\mu(A_0)\mu(E'_{t_2}) \tag{55}$$

and $\mu(A_0) = \mu(A_i)$, gives:

$$\mu(E'_{t_1}) \leq \frac{\sum_i \mu(B_i)\mu(E'_{i,t_2})}{\sum_i \mu(B_i)} \leq \mu(E'_{t_2}) \tag{56}$$

It is clear that this $RIGUS$ interaction is just as entropy increasing as the $IGUS$ interaction. The direct growth in macroscopic correlations of an $IGUS$ is no more indicative of entropy increase than the reduction in macroscopic correlations associated with an $RIGUS$.

4.2.2 Micro- and macro-correlations

The loss of microcorrelation with the environment is responsible for the increase in entropy. This happens both for the macroscopically correlating interactions of an $IGUS$ and its reverse, $RIGUS$. What of the macroscopic correlations themselves? These are the correlations which are supposed to be forbidden to develop within an entropy decreasing universe.

While it is certainly true that the measure over the marginals increases during information acquisition:

$$\sum_i \mu(B_i) \sum_j \mu(A_j) \geq \sum_i \mu(B_i)\mu(A_i) = \sum_i \mu(B_i)\mu(A_0) \tag{57}$$

(where we continue to assume $\mu(A_i) = \mu(A_0)$) this is a qualitatively different kind of increase to that associated with microcorrelations. The coarse graining over the microcorrelations, that results in entropy increase, is associated with the inaccessibility of these microcorrelations. If the microscopic correlations were still accessible (in the manner of a spin-echo experiment) no entropy increase could be said to have occurred.

In the case of the macrocorrelations, however, it is essential that the correlations be accessible. It is precisely because the coarse grained state is $\cup_i B_i \otimes A_i \otimes E_{t_2}$ and *not* $\cup_i B_i \otimes \cup_j A_j \otimes E_{t_2}$, that the $IGUS$ is said to have information about its surroundings. It is the correlation that represents information, that enables to $IGUS$ to utilise that information in its interactions and future behaviour.

The transition:

$$\cup_i B_i \otimes A_i \otimes E_{t_2} \rightarrow \cup_i B_i \otimes \cup_j A_j \otimes E_{t_2} \tag{58}$$

would represent a decorrelation, that would destroy the information that the $IGUS$ held about the state of its surroundings. So the equivalent operation to the increase in entropy associated with losing microcorrelations, is not associated with an acquisition of information, but with its loss.

Let us consider the process by which such decorrelation occurs. In an entropy increasing universe, each thermodynamically irreversible operation increases the entropy of surroundings, and the environment. Noise causes the switching of computers internal states, or switching of the environment. An $IGUS$ must maintain the relevance of its information by protected against changes and checking the accuracy of its information. As the environmental degrees of freedom become saturated, the existence of noise cannot be protected against and decorrelation becomes irreversible. The computer ceases to be able to function, as the universe approaches a maximum entropy heat death.

Now it is precisely the fact that such irreversible decorrelation does *not* occur (except on very large timescales), that normally makes the information gathered useful. The utilisation of acquired information requires the existence of stable macroscopic correlations, so that the overall distribution cannot be replaced by the direct product of their marginal distributions. By contrast, the increase in thermodynamic entropy is due to the loss of microscopic correlations than means the macroscopically distinct distribution can be replaced by the direct product of their marginal distributions. The role played by correlations in macroscopic information and microscopic entropy is of a quite different nature.

4.3 No interaction, no correlation

We can examine this further by considering a simple system, with two states of the environment B_i and two states of an $IGUS$, A_i. If we suppose the system goes through the following stages:

$$A_0 \otimes B_0 \rightarrow A_0 \otimes (B_0 \oplus B_1) \rightarrow (A_0 \otimes B_0) \oplus (A_1 \otimes B_1) \rightarrow (A_0 \oplus A_1) \otimes (B_0 \oplus B_1) \quad (59)$$

Initially the system is in the low entropy, uncorrelated state. The environment evolves into one of two possible states. The system then measures the state of the environment, becoming correlated. Eventually decorrelation leads to heat death.

The reverse, $RIGUS$, would involve:

$$A_0 \otimes B_0 \rightarrow (A_0 \otimes B_0) \oplus (A_1 \otimes B_1) \rightarrow A_0 \otimes (B_0 \oplus B_1) \rightarrow (A_0 \oplus A_1) \otimes (B_0 \oplus B_1) \quad (60)$$

At first sight, this evolution seems implausible. We start with the low entropy, uncorrelated state. Correlations spontaneously appear. The $RIGUS$ removes these correlations, before noise, once again, leads to a heat death.

The problem in constructing a justification for eliminating the $RIGUS$ evolution on entropic grounds is that:

$$\mu(A_0)\mu(B_0) \leq \mu(A_0)\mu(B_0) + \mu(A_1)\mu(B_1) = \mu(A_0)(\mu(B_0) + \mu(B_1)) \leq (\mu(A_0) + \mu(A_1))(\mu(B_0) + \mu(B_1)) \quad (61)$$

the two intermediate states between the uncorrelated and the decorrelated states can have the same measure.

Our intuition says that $A_0 \otimes (B_0 \oplus B_1)$ will occur first rather than $(A_0 \otimes B_0) \oplus (A_1 \otimes B_1)$. The spontaneously correlated state would require all initial states in $A_0 \otimes B_0$ to evolve into either $A_0 \otimes B_0$ or $A_1 \otimes B_1$. To achieve this it is necessary for a correlated interaction to take place. If it is the case that at $t = 0$, there is no correlation, and the two systems do not interact (or share interaction with any combination of intermediary systems) between $t = 0$ and $t = \tau$, then

$$\phi^{(\tau)}(A_0 \otimes B_0) = \phi^{(\tau)}(A_0) \otimes \phi^{(\tau)}(B_0) \quad (62)$$

Whatever else might be the case, such an evolution cannot possibly induce a correlation.

If it seems surprising that such a conclusion can be drawn so rapidly after the negative conclusion of the previous section, it is important to notice the different. The entropic argument was based upon measures upon state space regions. This argument is based upon a restriction upon allowed evolutions of the combined system.

At first sight this might seem to provide the answer, neatly and simply. In an entropy increasing universe, the existence of macroscopic correlation at some intermediate time requires the existence of a macroscopic correlating interaction at an earlier time. By contrast, in an entropy decreasing universe, the existence of macroscopic correlations at the intermediate time requires the existence of a macroscopic decorrelating interaction in the future. This appears to bear out Hawkings' claim that macrocorrelations must decrease.

However, there are problems when one considers more complicated situations than the two state systems considered here. In an entropy increasing universe,

microcorrelations must develop, so it seems that the restriction of equation 62 is too strong. Once we allow microscopic correlations to be developing, it is less clear what condition on the dynamics is necessary to ensure an $RIGUS$ is less likely than an $IGUS$.

It might also seem implausible that the non-existence of an $RIGUS$ here and now, can genuinely because of a boundary condition in the remote past. All the condition implies is that, given the existence of a macroscopic correlation now, that there must have been, some time between now and the start of the universe, a macroscopic interaction. It does not even guarantee that the systems which are correlated, now, are the ones that interacted in the past - only that there must have been an interaction in the past that has had causal influences upon the two systems now.

In the time reversed situation, the future boundary condition is supposed to prevent the operation of an $IGUS$. However, all the future boundary condition actually guarantees is that, at some point in the future there must be a macroscopic interaction to remove the correlation. It does not guarantee that this interaction must involve the system currently correlated to its surroundings. Given the timescale involved for the future boundary condition to apply, there seems a long way to go to show that a remote final condition is sufficient to rule out the existence of $IGUS$ systems in an entropy decreasing universe. However, if true, this implies that a remote boundary condition has a more direct effect upon possible states now than just through the conditions given in Sections 2.1 and 2.3.

The argument now begins to resemble attempts to base the causal fork asymmetry on entropic arguments[6]. The literature on this topic is too large to consider here (see [Rei71, Hor87, Alb01, Loe07] and [Pri96, Ear06, Fri07] for criticisms). However, to question how clear the argument is from a remote boundary condition to situations now, we will simply consider two scenarios. The first will be Schulman's two time boundary condition, where both initial and future boundary condition constraints exist. The second will be a situation where a local entropy gradient exists, but without either Initial or future boundary condition. While these scenarios may be regarded as implausible, their purpose is to examine if there are gaps in the arguments based upon remote boundary conditions.

4.3.1 Two time boundary conditions

Suppose that we are in a two time boundary condition universe such as Schulman proposes, but for which the thermalisation time is much greater than half the lifespan of the universe. In such a situation one might find an overlap between the entropy increasing and decreasing portions of space-time. It may then be possible for a complex system, operating in a thermodynamically reversible manner, to operate in both temporal halves of the universe. Now suppose such a system is a computer is designed to work very close the thermodynamic reversibility, and can swap from a power source suitable for an entropy increasing universe to a power source suitable for an entropy decreasing universe.

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[6]Note, this cannot have been Hawking's intent, at least, as earlier in [Haw94] he speaks dismissively of causality and the arguments of [Rei71].

Why is it the case that, when the computer enters the entropy decreasing times-pan, it ceases to operate as an $IGUS$? All we can say is that, ultimately, any information it gathers, must be lost again before the universe reaches its final low entropy state. That seems to leave a large amount of time over which it is able to function! Of course, such a scenario also allows the possibility of an equivalent $RIGUS$ existing in the entropy increasing period of time. The emergence of such a $RIGUS$ may be taken as an indicator that a future boundary condition exists. However, there seems no direct reasoning, from thermodynamics, to tell us how far in the future is such a boundary condition located. If this is the case, we equally cannot tell how long an $IGUS$ will be able to continue to operate in an entropy decreasing universe.

4.3.2 Asymmetry without boundary conditions

The crossover, from a entropy increasing to decreasing universe, raises additional problems, if we are to consider the interactions between an $IGUS$ and an $RIGUS$ in the same region of space-time. We can remove this problem by considering another, rather exotic, situation, which questions whether a remote boundary condition could possibly be responsible for the absence of $RIGUS$ systems.

Consider a system, identical to the solar system except in two respects: the sun is not a sun, but a boundary that absorbs, scatters and emits photons and particles into the solar system, with exactly the same profile as our sun does; and around the solar system (just around the Oort cloud) there is another closed boundary, that absorbs, scatters and emits photons and particles into the solar system with the same profile as the radiation crossing a hypothetical surface enclosing our solar system. Now suppose that this completely enclosed system has been in this state indefinitely far into the past, and will be in this state indefinitely far into the future.

Such a system is explicitly time asymmetric. The profile of the radiation being absorbed, scattered and emitted on the two boundaries is quite different when viewed in a time reverse direction. In a normal time direction the solar boundary emits low entropy radiation, some of which falls upon an earth-like planet and is reradiated in a higher entropy form. Most of the solar radiation, along with most of the earthly re-radiation is eventually absorbed by the Oort boundary, which radiates a negligibly small amount of radiation back (largely concentrated at small points) apart from a roughly symmetric emission and absorbtion of radiation at the cosmic microwave background frequency. Reversing the time direction will produce a quite different profile of emission and absorbtion on the two boundaries.

Let us ignore issues, such as the question of the long term stability of the solar system and so forth, which are not directly relevant to the present day thermo-dynamics of our solar system. For much of the history of life on our earth, there has been a reasonably stable non-equilibrium state, maintained by the local entropy gradient between the radiation falling on earth, from our sun, and re-radiated out again. The enclosed solar system will be in a stable non-equilibrium state much like our solar system, including the earth-like planet. It would seem reasonable to expect conditions on the earth-like planet to resemble conditions on our earth.

The principal argument of this paper has been that there appears nothing in the thermodynamics of the local conditions on the earth-like planet that prevents

the existence of an $RIGUS$. The no-interaction no-correlation argument suggests that a remote initial boundary condition prevents it on our earth. However, in the enclosed solar system, there is no remote initial boundary condition. If an $RIGUS$ is still not possible in the enclosed solar system, it must be the case that there is something about the local entropy gradient that prevents it, rather than an initial boundary condition.

If the remote initial boundary condition has an influence on the state of our earth only through the entropy difference between the incoming and outgoing radiation, then the cause of the absence of an $RIGUS$ on earth must be the same as on the enclosed earth-like planet. We have found no explanation in terms of the local entropy gradient to prevent an $RIGUS$, so if the local entropy gradient screens off the effect of a remote initial boundary condition, then such a condition cannot provide an argument against the existence of an $RIGUS$ on our earth.

Alternatively, the remote initial boundary condition may have a direct effect on the conditions on earth today that is not screened off by the local entropy gradient. In this case it may prevent the existence of an $RIGUS$ on our earth, but leaves the possibility of an $RIGUS$ on the enclosed earth-like planet. It is hard to see what kind of process could supply such a direct effect, or how this would lead to conditions begin so radically different on the enclosed earth-like planet, but one possibility might be the asymmetry of electromagnetic radiation, between advanced and retarded waves.

5   NESS, not QSES. Complexity, not information

Any process that is a sequence of Quasi-Static Equilibrium States (QSES) can, in principle, be connected by thermodynamically reversible processes (it is this that enables us to determine the entropy difference between them). Let us consider a specific example: the paradigmatic ice cube melting in a glass of water, and the film of this being run backwards.

There is nothing about the two states: an ice cube in glass; and a glass of water; that tells us one must come before the other. It is entirely possible in an entropy increasing universe, for the ice cube to be in the future of the glass of water. There are entropy increasing processes by which a glass of water can be turned into a glass containing an ice cube. In the limiting case, of reversible quasistatic processes, we can go back and forward between ice cube and water, thermodynamically reversibly.

The same is equally true in an entropy decreasing universe. In such a universe there would also be entropy decreasing processes by which glasses of water could be converted into ice cubes in glasses and ice cubes in glasses converted into glasses of water.

The asymmetry in the process, with which we are familiar, is not the fact that an ice cube is succeeded by water, but is in the process by which it happens. It is the non-equilibrium nature of the process that reveals the entropic direction. It is the fact that the ice cube is in the process of *melting* that tells us the 'correct' direction of the film.

The generalisation of the arguments sections 3 is that any process, which can be defined solely in terms of a (deterministic or probabilistic) succession of QSES, can occur in an entropy increasing universe and in an entropy decreasing universe. What

distinguishes the two universes is not a possible succession of QSES, but rather the processes by which the transitions between the states can take place. This suggests that, if one is to find connections to an entropic arrow of time, we should not be looking at the QSES that are the thermodynamic limit for information processing systems. Any process defined solely in terms of such states can occur in either entropic direction.

The existence of Non-Equilibrium Steady States (NESS), on the other hand, are not time symmetric. Complex biochemical structures that arise in far from equilibrium conditions are associated with fundamentally time asymmetric, entropy increasing processes. The time reverse of these processes in entropy decreasing universes will lead to a different sequence of NESS, entropy decreasing processes. These complex structures are also the building blocks from which the biological processes are constructed that are necessary to house the information gathering and utilising systems.

A generalisation of this may be conjectured: any time asymmetry that is supposed to be a consequence of the thermodynamic time asymmetry, cannot be expressed solely in terms of sequences of QSES. If we are to find stable states whose time asymmetry is a consequence of thermodynamics, their properties must come from NESS, not QSES. This suggests that the ideas of complexity, rather than information, are needed.

6   Conclusion

The argument of this paper is that an arrow of time associated with information processing systems cannot be deduced from thermodynamic arguments. The thermodynamic arrow is insufficient to entail the computational arrow. Any sequence of logical operations in an entropy increasing universe is physical possible in an entropy decreasing universe. Landauer's Principle, as it is commonly stated, assumes statistical mechanical principles that are equivalent to being in an entropy increasing universe. If one changes those assumptions, so that one is in an entropy decreasing universe, a critical inequality in Landauer's Principle in reversed. The physical implementation of logical operations, which increase entropy, do so, not by virtue of any inherent properties of the logical operation, but by virtue of being in an entropy increasing universe. If the same logical operation is performed in an entropy decreasing universe, it is entropy decreasing. As a result, entropy decreasing universes are not inherently hostile to the acquisition, persistence or utilisation of information.

In principle, the operation of acquiring information can be made thermodynamically reversible. This is precisely one of the main insights of Landauer's work on the thermodynamics of computation: a measurement can take place without generating heat (see [LR90, LR03] and many references within).

> Landauers principle, while perhaps obvious in retrospect, makes it clear that information processing and acquisition have no intrinsic, irreducible thermodynamic cost[Ben03]

If the acquisition of information can take place in a thermodynamically neutral

manner, it can take place in an entropy decreasing as easily as an entropy increasing universe.

While any information gathering and utilising system will ultimately cease to function in an entropy decreasing universe that reaches a final extremal entropy state, this doesn't seem sufficient to rule out such systems[7]. Firstly, the decrease in entropy is due to the decorrelation that comes about from losing microcorrelations. It is of a different kind to the macrocorrelations that arise during the acquisition of information. Secondly, on the timescales during which information gathering and utilising systems work, between the low or high entropy extremal starting and ending points, there seems nothing to directly prefer $IGUS$ over $RIGUS$. Thirdly, if the effect of the initial or future boundary conditions is screened off by the local entropy gradient, the no correlation, no interaction argument does not seem to be applicable, as such an entropy gradient can exist in a situation with no initial or final boundary condition.

The suggestion is made that an entropic arrow of time will never be found in processes that can be defined solely in terms of a succession of Quasi-Static Equilibrium States. Information processing can be so defined. If the psychological arrow of time is to be aligned with the thermodynamic arrow, it cannot be through the information processing properties of the brain. It may be through the biochemical structures that arise in Non-Equilibrium Steady State processes, but if so, it is certainly not through any information processing characterisation of such structures. This would seem to imply that at least one aspect of conscious experience cannot be logically supervenient on the states of a computer. If instead the psychological arrow of time does indeed arise out of information processing properties, this would mean that the psychological arrow is logically independant of the thermodynamic arrow of time.

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[7]Quite aside from the fact that it must also ultimately cease to function in an entropy *increasing* universe.

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