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Dimensions of number invariance

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The magic paradigm was used to examine preschoolers' abilities to make judgments requiring a number-related invariance concept. Findings from previous, similar studies have suggested number invariance schemes in 3- and 4-year-olds. The stimuli in those investigations, however, contained a redundancy of number and relative numerosity. When these dimensions were separated in the present study, fewer children exhibited conservation-like performance than in the redundant-cue task and, contrary to previous findings, most did not justify their responses with reference to number. The results were consistent with the suggestion that preschoolers do make use of invariance concepts in the magic paradigm, but that these need not always represent number invariance per se.

It long was assumed that children do not understand number invariance until the age of 6 or 7 years (Beilin, 1968; Piaget, 1952, 1968). Recent studies employing the "magic paradigm" (Gelman, 1972a), however, have indicated that 3- and 4-year-old children are able to make correct judgments of relative magnitude (and explain their responses in terms of number) in the face of irrelevant perceptual transformations (e.g., Bullock & Gelman, 1977; Gelman, 1972a; Gelman & Tucker, 1975; Silverman, Rose, & Phillis, 1979; Smither, Smiley, & Rees, 1974).[1]

The magic paradigm is a two-choice discrimination task with a single transfer trial. The child first learns to discriminate between designated "winner" and "loser" arrays. On the transfer, or magic, trial, a surreptitious transformation of the winner is made in which, for example, the length/density of the array is changed (Gelman, 1972a) or one element is changed in color or identity (Gelman & Tucker, 1975). All of the magic paradigm studies with which we are familiar, however, have maintained the relative numerosities (i.e., more and less) of the stimuli from training to the transfer trial (cf. Brainerd & Howe, 1978). Thus, correct transfer trial responses could be based either on number invariance or on an understanding of relative numerosity. This suggestion is consistent with Gelman's (1972b) description of the importance of learned quantity estimators (that may be redundant with number) and with results from Bullock and Gelman's (1977) investigation of numerical relation abilities in preschoolers. The latter study used the magic paradigm in requiring children to choose either a three- or a four-item array as a winner after a training phase involving one- and two-item arrays. They found that children as young as 2.5 years were able to make number-based relational judgments insofar as most chose the array of the appropriate magnitude relation (greater or lesser) on the transfer trial. This result reflects the operation of some kind of magnitude criterion (Marschark, 1977; Siegel, 1972), but not one of number invariance per se. This is not to say, of course, that young children do not have some concept of number invariance. The frequency of number-related justifications for magic trial choices, in fact, clearly seems to indicate such an ability (Gelman, 1972a; Gelman & Tucker, 1975). Previous magic paradigm studies simply have not isolated the effects of number invariance from other possible response schemes.

The present experiment also involved the magic paradigm, but the use of a conflict transfer trial allowed distinction of number invariance and relative numerosity as the bases of correct responding. After a training phase involving arrays of two and three large black dogs,

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the magic trial involved arrays of three and four small white dogs. If correct transfer trial responding were based on number invariance, the three-dog array would be selected, despite perceptual changes in the stimuli. If correct responding were based on the relative numerosity of the (larger) three-dog array during training, however, the four-dog array would be chosen on the transfer trial. Possible age differences in response criteria were investigated by using two experimental groups of preschoolers, one composed of older 2-year-olds and younger 3-year-olds and the other composed of older 3-year-olds and 4-year-olds. Since the two response alternatives described above were to be taken as indicating different response schemes, a control group of subjects of intermediate ages also was included. They saw the same training stimuli as the two experimental groups, but the magic trial involved arrays of three small white dogs and two large white dogs. This provided a check on the method of training and evidence as to whether subjects in our population would exhibit number invariance-like performance in the absence of a conflicting transfer trial.

METHOD

Stimuli

The stimuli consisted of several hard-rubber toy dogs. The large dogs measured approximately 6.5 x 2 x 2.5 cm, and the small dogs approximately 2.5 x 1 x 1.5 cm. The dogs were placed approximately 3 cm apart on 14 x 16 cm blue pads and were covered by blue cardboard boxes, 16 x 18 x 7 cm.

Subjects

A sample of 34 children, ranging in age from 2 years 5 months to 5 years, was split into two equal groups on the basis of age. The 17 younger children had a mean age of 2 years 11 months (range = 2 years 5 months to 3 years 4 months). The 17 older ones had a mean age of 4 years (range = 3 years 5 months to 5 years). The control group was composed of eight children with a mean age of 3 years 6 months (range = 3 years to 4 years).

Procedure

Children were tested individually in a room provided by the school. All of the authors served as experimenters. Each was quite familiar to the particular children he/she tested and spent time with the children prior to the experiment to build up subject interest in the task. The task was then introduced by showing the children the arrays and asking if they knew what the animals were. The several children who identified them as cats played the game with "cats" instead of "dogs." In being told how they could win the game, children were shown that the three-dog array was the winner and that the two-dog array was the loser. They were told that the arrays would be covered and moved around on the table to "hide the winner," which they were then to try to find. Finding the winner was rewarded with a dried bean to be placed in a plastic box. The beans made a popping sound and ricocheted when forced through a hole in the box, and all children were eager to obtain them. Further, the overt enthusiasm of the experimenters throughout the task kept the children attentive and talkative.

During the training phase, the experimenter shuffled the boxes and asked the child to point to the one thought to cover the winner. The experimenter then lifted the cover and asked if the exposed array was the winner. If the array was correctly identified as such, the child was rewarded. If the two-item array was identified (incorrectly) as the winner, the child was shown both arrays and was told again which was the winner and which the loser, and the next trial began. If the two-item array was uncovered and the child correctly said that it was not the winner, the array was re-covered and the experimenter asked for the location of the winner. Pointing to the correct array then was rewarded with a bean. Pointing to the loser array again was followed by feedback, as in the case of an incorrect identification. A minimum of eight training trials were involved, as extensive pilot testing had shown this to be sufficient for children to learn the task. A criterion of four consecutive correct answers immediately prior to the transfer trial ensured that children had learned the task and were paying attention.

The ninth trial was the transfer, or "magic," trial. The arrays of two and three large black dogs were surreptitiously replaced with arrays of three and four small white dogs. From the child's perspective, the transfer trial was the same as the previous ones until, after shuffling, the experimenter lifted both boxes and asked which was the winner (Silverman et al., 1979). On this trial, as well as on the first and fourth training trials, children were asked (before feedback) why they identified an array as the winner. Performance on the "magic" transfer trial was the dependent variable of primary interest for all subjects. Answers to response-choice justification questions and any evidence of counting behavior also were were recorded.

RESULTS AND DISCUSSION

Considering the control group first, only one of the eight subjects required more than eight training trials to reach criterion. All eight chose the three-item array as the winner on the transfer trial. Although none of the Trial 1 or Trial 4 choices was justified with reference to magnitude or number, three referred to number on the transfer trial. This indicates that the training phase was appropriate for producing number invariance-like responding. It is unclear, however, whether the five subjects who did not justify their transfer trial choices were responding on the basis of number or relative numerosity. As with previous studies in which number and relative magnitude were confounded, it is also possible that even the three children who referred to number here were actually responding on the basis of a relative rather than an absolute (number invariance) criterion. The results of three other children, described below, are consistent with that suggestion.

Children in the younger experimental group required a mean of 10.2 training trials to reach criterion; those in the older group required a mean of 8.6 trials. This difference was reliable by a one-tailed test [$t(15) = 1.94$, $p < .05$]. On the transfer trial, 10 subjects in the younger group chose the three-dog array, and 7 chose the four-dog array. Only 4 of the older children chose the three-dog array, however, whereas 13 chose the four-dog array. When corrected for continuity, these differences were only marginally reliable [$\chi^2(1) = 3.03$, $.05 < p < .10$]. Nevertheless, these results indicated that while children in the younger group were equally likely to base their transfer trial responses on absolute (number invariant) or relative numerosity, over 75% of the older group used relative numerosity as the basis for their judgments. There were no apparent age trends within the two groups. The younger subjects choosing the three-dog array were all between 2 years 8 months and 3 years 2 months; the older subjects doing so were between 3 years 10 months and 4 years.

Before considering these results further, two other findings should be noted. Although only two children (ages 3 years 2 months and 3 years 11 months) were observed to count the stimuli spontaneously (cf. Gelman, 1972b), five others counted them when asked why they chose a particular array. One of these (age 3 years 1 month) counted only on Trial 4, but the others (ages 2 years 11 months, 3 years 2 months, and 4 years 6 months, 4 years 10 months) did so on both Trials 1 and 4 as well as on the magic trial. The two older subjects, however, picked the four-item array as the winner on the transfer trial. Thus, even if their counting does indicate an attempt to determine whether the changes in the arrays corresponded to an actual change in number (Gelman & Tucker, 1975), this cannot be taken as evidence of a number invariance strategy. Further, only one (age 4 years) of the other 30 subjects was able to explain her transfer trial response choice adequately, saying that she chose the four-dog array because it had "more" (i.e., a relative numerosity response).

The present finding that, overall, the experimental subjects were at least as likely to deal with the stimulus arrays in a relational ($3 > 2$ and $4 > 3$) manner as in an absolute (number invariance) manner seems likely to be the result of the conflicting nature of the transfer trial. When considered in comparison with the unanimous, absolute responding in the nonconflict control group, this result might be taken as suggesting a relative instability of the number invariance concept in preschoolers. Given the finding that the older subjects were more likely than the younger ones to use the relational response scheme, however, these results more likely indicate that number invariance is only one of perhaps several hierarchically ordered schemes available to children in this paradigm.

These findings suggest that the number invariance-like results of children in previous studies might have been due, in fact, to relative numerosity schemes. Unfortunately, the previous studies described above have not separated responses like "It's the winner because it has three" from those like "It's the winner because it has more." Reanalyses of some of those data, however, might further clarify the results obtained here.

In conclusion, the present study supported Gelman's (1972a, 1972b) view that preschool children have simple invariance rules. They suggest, however, that the basis for responding correctly in the magic paradigm can be due to the operation of more than one magnitude-related skill. Preschoolers' learned quantity estimators (Gelman, 1972b) appear to be flexible enough to produce optimal performance across a variety of stimulus component configurations. Since different response schemes appear to be activated by stimulus and procedural variations beyond those used by Gelman and her colleagues and Silverman et al. (1979), the magic paradigm may prove to be a valuable tool for delineating both the numerical abilities of young children and the conditions likely to elicit them.

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NOTE

1. Although Gelman and her colleagues have claimed only that such findings indicate "number invariance schemes," others (e.g., Silverman et al., 1979, p. 30) have suggested that they indicate actual conservation ability.

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