EXPERIMENTAL TESTS OF ISOMETRY HYPOTHESES

1 INTRODUCTION

There has been a century-old controversy about the conventionality of isometry hypotheses—propositions that assert the constancy of a specific physical magnitude of a system under prescribed conditions. Is there any possible empirical test of sentences such as 'the standard metre has a constant length', or 'the standard clock has a constant period'? This kind of question is at the bottom of discussions about the congruence of space-time intervals, the homogeneity of time and space, the intrinsic metric of space-time, and the constancy of universal parameters and laws. Several outstanding philosophers have discussed this issue, and most of them agree that isometry hypotheses are conventional. Such is the verdict of Mach, Helmholtz, Poincaré, Reichenbach, Carnap, and Grünbaum. In recent times, the special problem of the isochrony of clocks has been discussed by Christensen [1977], and Roxburgh [1977]. The first author has claimed that isochrony hypotheses may be experimentally proved, and provided a test for the isochrony of clocks. The second author has criticised Christensen's method, and claimed that isometry hypotheses are conventional. I shall try to prove that isometry hypotheses are testable (hence not conventional), although they cannot be definitely proved.

2 STRUCTURE AND IMPORTANCE OF ISOMETRY HYPOTHESES

Isometry hypotheses are statements that can be put under the general form: 'If a_1 and a_2 are two states of a physical system a that belongs to a definite class A; and if the system can be transformed from the state a_1 to a_2 through a process g belonging to a definite class G of physical processes; then the value of the particular physical magnitude M is the same for both a_1 and a_2.'

Let us consider the following proposition:

(*H*.1) Solid metallic rods of constant temperature and submitted to constant external forces have constant length.

Here, *length* is the relevant magnitude M. This hypothesis may either be interpreted as referring to (*i*) systems of the class A of 'solid metallic rods of constant temperature and submitted to constant external forces', G being the whole class of physical processes; or to (*ii*) systems of the class A of 'solid metallic rods', G being the class of physical processes where the system is kept at constant temperature and submitted to constant external forces. The second choice allows the application of the hypothesis to a wider set of physical systems; actually, it seems that the first choice would render the proposition useless, since it is not likely that there exists any solid metallic rod of constant temperature and submitted to constant external forces during its whole existence.

A different elucidation of isometry hypotheses could be chosen, but the above form seems to include all relevant cases hitherto discussed. It applies even to propositions such as:

(*H*.2) The standard metre has a constant length.

If no condition is added to this proposition, it states both that there is just one standard metre and that any system belonging to the class A of standard metres has a constant length whatever the kind of transformation endured by this system.

If every physical transformation takes some time to occur, and if time travel is not possible, then any isometry hypothesis can only be applied to compare a quantity related to physical states that do not coexist or that are considered at different times.[1]

Isometry hypotheses are very important assumptions in classic measurement theory,[2] and most of its applications are due to this role. They are required in long-range fundamental comparisons between systems that are separated by large spatial and temporal intervals (*e.g.* the comparison of the mass of a body in London to that of another body in the Moon). They are also needed in most fundamental measurements of systems greater than the standard (*e.g.* the measurement of a time interval of some years using the sidereal day as a standard).

Although the isometry of space-time intervals has received most of the attention from philosophers, other kinds of isometry are explicitly or implicitly used in several empirical procedures. It is usually assumed that the total mass, volume, and electric charge of a body do not change when it is submitted to plastic deformation or when its parts are separated or attached in different ways; it is also supposed that the electric resistance and length of a piece of wire do not change when it is 'gently' bent or coiled (with a curvature radius much greater than the diameter of the wire). The ubiquity of isometry hypotheses in quantitative reasoning is remarkable.

3 STANDARD SYSTEMS

Within classical quantitative measurement theory each system, at each instant, has one and only one value for each magnitude. If only one single system is chosen as standard, measurement results will always be univocal at each time, and it would seem that only a conventional isometry hypothesis would be needed for metrological purposes. Any system could be freely chosen as standard, and only convenience considerations would guide this decision. But, as will be shown below, the freedom of choice between different kinds of standard systems is not complete. It must be guided by testable isometry hypotheses.

(*i*) *Standards as sets of similar systems.* In the early development of the metrical system, the standards of time, mass, and length, were single systems. The time standard was the period of the earth's rotation, and the unit of time was a conventionally chosen submultiple of this period. But even in the early nineteenth century not every standard was a single system.

[1] Graves and Roper [1965] have discussed the relevance of time travel for the testing of the constancy of standards.

[2] The expression 'classical theory of measurement' refers to the non-operational approach as exemplified by Campbell [1957].

The standard of temperature interval was the difference of temperature between the normal boiling point and the normal freezing point of pure water—*any sample* of pure water. Similarly, the unit of time is, since 1967, a multiple of the 'periods of the radiation corresponding to the transition between the two hyperfine levels of the ground state of the cesium-122 atom'.[1] According to this definition, the period of *any* cesium-133 atomic clock may be used as standard.

The use of sets of similar systems as standards presents several advantages over single standards. They can be properly reproduced, and therefore, if the present standards are lost or damaged, the old measurements do not loose their empirical connection with new measurements. Besides, the use of standard sets ensures the possibility of testing their constancy by cross-comparison. The constancy of the same magnitude of a set of systems is not a conventional assumption, if these systems can be intercompared. If experiment shows that subsets of the standard set of systems undergo mutual variations, then this choice of the standard does not lead to univocal results, and must be rejected because it would conflict with the desiderata of classical measurement.

(*ii*) *Single standards*. The constancy of the relevant magnitude of a unique standard is usually constructed as a conventional assumption. So, the constancy of the length of the French metre is assumed to be mere convention. But although the final specification of a single standard takes the form of a convention, the process of choice of physical standards has never been blind. Actually the choice of a metallic bar and of constant temperature in the specification of the French metre has been guided by the belief in the truth of (*H*.1). This hypothesis is testable, since it refers to classes of similar systems. It has never been refuted, and thus it provides a good reason for the acceptance of the French metre as invariant. If (*H*.1) had been refuted, the French metre would not be accepted by scientists as a valid length standard. It is for an analogous reason that scientists would not accept a wood stick as a length standard: because the following hypothesis:

(*H*.3) Wood rods kept at constant temperature and submitted to constant external forces have constant length.

is known to be false, as intercomparison of different wood sticks shows. Since empirical knowledge may be invoked to justify or to deny the validity of single standards, their choice is not considered by scientists as conventional.

(*iii*) *Independent standards of the same magnitude*. As a matter of historical fact, it sometimes occurs that two independent classes of systems are supposed to obey independent isometry hypotheses concerning the same physical quantity. To a good approximation, material length standards and atomic length standards are both supposed to be invariant in time, under

[1] *Cf.* Hellwig, Evenson, and Wineland [1978].

prescribed conditions. In such cases, the mutual agreement of the two underlying isometry hypotheses may be tested: we may search for relative variations of the two kinds of length standards. Such external tests of isometry hypotheses seem to be admitted by every author. But it must be stressed that external tests are not necessary to refute isometry hypotheses, and that they are weaker than internal tests: if both material length standards and atomic length standards are shown to stand internal isometry tests, but vary relative to one another, we have no reason to choose one of them as a "good" standard and to reject the other. But if an internal test refutes the isometry hypothesis underlying a standard choice, we know that this standard is inadequate.

4 ROLE OF AUXILIARY HYPOTHESES IN ISOMETRY TESTS

In most tests of isometry hypotheses, use will be made of auxiliary hypotheses that allow us to decide whether or not the tested system and the process that it undergoes conform to the conditions of the hypothesis. An exception could be provided by hypotheses such as:

(*H*.4) Everything has constant length.

But in most cases, a negative outcome of an isometry test may be interpreted either as a refutation of the hypothesis itself or of the auxiliary assumptions. This situation is known to occur in every test of relevant scientific hypotheses. But the *quantitative* assumptions necessary to test some isometry hypothesis (such as the constancy of temperature and force, in *H*.1) may give rise to a specific criticism. The constancy of physical quantities can only be warranted by auxiliary *isometry* hypotheses. If the test of every isometry hypothesis requires the assumption of other isometry hypotheses, we have a vicious circle, since we shall have to assume the very kind of thing that is being tested.

To avoid this objection it would be necessary either to show that some accepted isometry hypotheses may be tested without the help of auxiliary isometry assumptions, and that with the aid of these accepted isometry hypotheses the other isometry hypotheses could be tested; or to show that the assumption of the constancy of some quantities is not required in a fundamental way for the testing of such isometry hypotheses.

There are at least two ways of circumventing this difficulty, in some cases. The first way is the substitution of constancy requirements by the particular case of null values of the considered parameters. In order to test (*H*.1), for instance, it would be easier to envisage an experimental situation such that external forces and temperature are negligible (that is: even a hundredfold increase would produce no noticeable change in the length of the test bodies) than to keep non-negligible constant values of such parameters.

If this technique cannot be applied, a second trick may be used. Suppose that the magnitude M has initially the same value for all bodies of a set A, and after a time a relative disagreement is noticed. If this change is ascribed to the lack of constancy of a set of parameters P, and if the changes of M due to changes of P are known to be reversible, then we may control and change the value of P, and try to bring back the equality of M for this set. If this is not possible, the relative changes of M cannot be explained away as produced by changes of P, and the isometry hypothesis has been refuted.

We have seen that at least in some cases the existence of quantitative conditions within isometry hypotheses does not lead to vicious circles. A conclusive refutation of isometry hypotheses is possible when the negligible-value condition is feasible, or when the conditions that must be kept constant are controllable and their effects are reversible. When none of these tricks may be applied, it may occur that the isometry hypothesis cannot be submitted to conclusive refutation. But this will then be due to matter-of-fact limitations, and not to *a priori* knowable limitations of isometry hypotheses.

5 THE AGING OF CLOCKS

If an isometry hypothesis has passed an internal experimental test, it has been confirmed but not proved. If furthermore it is compatible with the additional accepted ideas of the field (theories and empirical generalisations) it may also be accepted as true. Indefinite doubts and vague conjectures can always be maintained regarding confirmed isometry hypotheses, whatever the kind of performed test. But when doubts about the validity of a particular hypothesis take a definite form, they deserve attention and allow the formulation of new tests.

Let us suppose that a set of clocks has passed an internal test of isochrony, but we suspect that something inside them is aging and changing their rates. Christensen [1977] has suggested the following test:

Stop a subset A_1 of this class of clocks. Let the remaining class (A_2) go on working. The clocks that belong to A_2 will continue to "age", while those of A_1 are kept "new". After a time, we put the A_1 clocks to work again. If now there is no more synchrony between the subsets, and if this result is consistent in several experiments where the clocks are stopped at different phases of their cycles, this will refute the hypothesis of the isochrony of the A-clocks.

If, after this experiment, the clocks are still synchronous, the isochrony hypothesis has been confirmed—but not proved. Christensen seems to think that no non-isochronous kind of clock would be able to pass this test (which, after all, is just another instance of internal test of isometry). Roxburgh [1977] has shown, by a counter example, the possibility of existence of two kinds of clocks such that both pass Christensen's test but the period of one of these classes is steadily changing relative to the other kind. They therefore cannot be both isochronous, and hence Christensen's test is not able to refute the isochrony of every kind of non-isochronous clock. Roxburgh concludes that isochrony statements are conventional. This would be true if isochrony hypotheses could never be submitted to empirical test—and this is not the case.

Christensen's test will be able to identify non-isochronous clocks whenever the periods of these clocks are changing due to an internal cause that can be interrupted when we stop the clock. Roxburgh's counter-example describes clocks where, besides the cyclic phenomenon used to count time, there is another independent motion that steadily changes the period of the clock. If the cyclic motion is arrested but the other motion goes on, the aging of the clock cannot be arrested, and Christensen's test fails to detect the lack of isochrony of such clocks. If the two kinds of motion could be interrupted in the test, the isochrony of Roxburgh's clocks might be refuted.

Any other class of clocks that can be used as counter-example of Christensen's argument will have this feature: there is an aging process that changes the period of the clock and that does not stop when the timing process is arrested. We may think, for instance, of radioactive clocks where the period would be related to the frequency of disintegrations of a radioactive sample. If this frequency is high enough, fluctuations will be irrelevant. We presently know that this rate will not be constant; but suppose that we do not know this, and that we want to test the isochrony of these clocks. They would pass Christensen's test, since the decay of the radioactive material does not stop when we stop the counting device.

Such kinds of clocks that pass Christensen's test may be submitted to further experiments whenever we have a definite suspicion about their isochrony. Suppose that the non-isochrony of the clocks is produced by an internal aging effect (something that only depends on the internal working of the system, such as Roxburgh's clocks and radioactive clocks). If this is the case, their aging can be slowed down by a relativistic process. Put a subset A_1 of these clocks in a rocket and send it away at a very high speed, or put them in a region of very high gravitational potential. In both cases, according to the theory of relativity, *all* kinds of internal processes occurring in these clocks will be slowed, and their aging will be smaller than that of the control group A_2.[1] After a time, the clocks that were sent in the rocket or submitted to a higher gravitational potential are brought back, and compared to the control group. If there was any internal cause of monotonic period change, the clocks will now be observed to have different periods. If the cause of non-isochrony is not monotonic, several similar tests (with varying delays) may be necessary to detect the changes of period. If this kind of test confirms the constancy of the period of the clocks, then either the theory of relativity is wrong, or there is no internal cause of aging in these clocks.

[1] I have used the standard prediction concerning the relativistic 'twin paradox'. Some authors do not agree that there is any differential aging, and in this case the test would not work. See Sachs [1974].

If a class of clocks has passed the above tests, it may be held as an isochronous class unless external influences are suspected to change their rate. If this conjecture takes a definite form, it may also be submitted to experimental tests. In some cases an elaborate theory may be required for the planning of such a test (*e.g.* if we want to study a possible influence of the expansion of the universe on the rate of clocks, relativistic cosmology will be necessary). But none of these difficulties discriminates particular problems related to the testability of isometry hypotheses. They occur in the test of any significant scientific hypothesis.

6 DISCUSSION

In this paper we have used epistemological terms such as 'refutation', 'confirmation', 'convention'. I have tried to show that the choice of standards is not conventional, since it must be grounded on isometry hypotheses, and isometry hypotheses are not conventional. The underlying epistemological assumption is this: if the choice of something is guided by something that is not conventional, this choice is not conventional. This and several other implicit or explicit epistemological assumptions which were used are liable to criticism. Such ideas are controversial, as most epistemological concepts and laws. I will therefore try to state my conclusions in such a way that they do not depend critically on the meaning of these epistemological concepts.

Instances of isometry hypotheses have been discussed, and experiments relevant to their acceptance have been described. In the same way as scientists may invoke experimental facts as contrary to scientific hypotheses, experimental facts may be invoked to criticise the discussed isometry hypotheses. Agreement about the acceptability of such hypotheses is as easy or as difficult to attain as in the case of other kinds of hypotheses. I believe that every isometry hypothesis that refers to classes of systems and that has ever been used in scientific work can be submitted to experimental tests. If instances can be shown of general and useful but untestable isometry hypotheses, this will probably be a peculiarity of these instances; untestability is not a general characteristic of isometry hypotheses.

Anything that can be affirmed of all isometry hypotheses can also be stated about a class of accepted physical laws, since there are accepted physical laws that may be included in the class of isometry hypotheses. The second postulate of relativity and all conservation laws (of charge, momentum, energy, and so on) are obviously isometry hypotheses. Any universal quantitative law that can be written under the form:

$$f(M_1, M_2, \ldots, M_n) = K,$$

where M_i is the value of a physical quantity, and K is a constant physical magnitude, might also be considered an isometry hypothesis. Whoever holds that all isometry hypotheses are conventional must also accept that these physical laws are conventional. We may perhaps say that typical isometry hypotheses used by scientists are as testable and as conventional as typical scientific laws.

Isometry hypotheses that refer to a unique standard are not directly testable, and their statement may be considered as conventional. They therefore differ from uniquely specified standards, which exist in many copies. But the hypothesis of constancy of unique standards can only be said to have scientific grounds if there is a general isometry hypothesis that includes it as a particular case (science is not a study of individuals, and statements about particular systems are only used as auxiliary sentences in science). The choice of any kind of physical standard is not blind, but grounded on testable general isometry hypotheses, and if these underlying hypotheses are found wrong, the standard is concluded to be inadequate. Hence, even seemingly arbitrary statements such as 'the French metre has a constant length' may be rejected (or accepted) by scientists depending on the result of experiments. If, notwithstanding all this, someone still wants to say that isometry hypotheses are conventional, I think that he must also acknowledge that all hypotheses are conventional.[1]

ROBERTO DE A. MARTINS
Universidade Federal do Paraná

REFERENCES

CAMPBELL, N. R. [1957]: *Foundations of Science*. New York: Dover.

CHRISTENSEN, F. [1977]: 'How to Establish Non-Conventional Isochrony', *British Journal for the Philosophy of Science*, **28**, pp. 49–54.

GRAVES, J. C., and ROPER, J. E. [1965]: 'Measuring Measuring Rods', *Philosophy of Science*, **32**, pp. 39–56.

HELLWIG, H., EVENSON, K. M., and WINELAND, D. J. [1978]: 'Time, Frequency and Physical Measurement', *Physics Today*, **31**, (12), pp. 23–30.

ROXBURGH, I. W. [1977]: 'The Conventionality of Uniform Time', *British Journal for the Philosophy of Science*, **28**, pp. 172–7.

SACHS, M. [1974]: 'On Dingle's Controversy about the Clock Paradox and the Evolution of Ideas in Science', *International Journal of Theoretical Physics*, **10**, pp. 321–31.

[1] This work has been supported by the Brazilian National Council for Scientific and Technological Development (C.N.Pq.).