Intrinsic Explanation and Field's Dispensabilist Strategy

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Russell Marcus

Department of Philosophy, Hamilton College

198 College Hill Road

Clinton NY 13323

rmarcus1@hamilton.edu

(315) 859-4056 (office)

(315) 381-3125 (home)

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Abstract:

Philosophy of mathematics for the last half-century has been dominated in one way or another by Quine's indispensability argument. The argument alleges that our best scientific theory quantifies over, and thus commits us to, mathematical objects. In this paper, I present new considerations which undermine the most serious challenge to Quine's argument, Hartry Field's reformulation of Newtonian Gravitational Theory.

## §1: Introduction

Quine argued that we are committed to the existence mathematical objects because of their indispensable uses in scientific theory. In this paper, I defend Quine's argument against the most popular objection to it, that we can reformulate science without reference to mathematical objects. I interpret Quine's argument as follows:[1]

(QIA) QIA.1: We should believe the theory which best accounts for our empirical experience.
QIA.2: If we believe a theory, we must believe in its ontic commitments.
QIA.3: The ontic commitments of any theory are the objects over which that theory first-order quantifies.
QIA.4: The theory which best accounts for our empirical experience quantifies over mathematical objects.
QIA.C: We should believe that mathematical objects exist.

An instrumentalist may deny either QIA.1 or QIA.2, or both. Regarding QIA.1, there is some debate over whether we should believe our best theories. Regarding QIA.2, one might take a fictionalist interpretation of the referents of a theory. I shall not consider instrumentalist responses to QIA. Also, I shall not oppose QIA.3, Quine's procedure for determining the ontic commitments of theories.[2]

Debate over QIA has focused mostly on QIA.4, spurred in large part by Hartry Field's claim that we can remove reference to mathematical objects from our best theory. Field 1980 provided two dispensabilist reformulations of Newtonian Gravitational Theory (NGT). A

[1] Quine alludes to an indispensability argument in many places; among them are Quines 1939, 1948, 1951, 1955, 1958, 1960, 1978, and 1986.

[2] Though, that the statements of a theory mean that there are mathematical objects need not entail that they refer to mathematical objects, let alone that by using the theory we must believe that there are mathematical objects, or even that mathematical objects exist.

second-order reformulation dispensed with Quine's quantification over sets in favor of quantification over space-time points. A first-order reformulation referred instead to space-time regions. There are technical questions about whether Field's reformulation is adequate for NGT, and whether such strategies are available for other current and future scientific theories.[3] Here, I grant that Field's reformulation, or something in its spirit, is successful.

My concern is whether such a reformulation, even if adequate, is a better theory than the standard one, for the purposes of QIA. The superiority of dispensabilist reformulations is important because the indispensability argument relies on the claim, at QIA.1-2, that we find our ontic commitments in our best theory. Field defends the superiority of his reformulation on the basis of a principle of intrinsic explanation. I argue that this principle is false, and that the standard theory will be better than its dispensabilist counterpart. Thus, I reject Field's claim that QIA.4 is false, not because reference to mathematical objects is ineliminable from science, but because the reformulated theories are not our best theories.

The value of dispensabilist reformulations has been questioned before. Colyvan 2001 argues that the standard theory is more attractive than Field's reformulation, but attractiveness is a vague and malleable criterion. Pincock (forthcoming) argues that the standard theory is better confirmed, but his claim seems implausible, since the reformulation makes fewer commitments. Burgess and Rosen 1997 argues that since dispensabilist reformulations are not preferred by practicing scientists, they are no better. But, the practicing scientist wants a useful theory to

[3] Burgess and Rosen 1997 elegantly collects the slew of reformulation strategies published in the wake of Field's monograph. Most of them replace standard mathematical theories with modal reformulations. I focus on Field since he explicitly defends the principle of intrinsic explanation.

produce empirical results, and is mainly unconcerned with ontic commitments. I hope to present a better explanation of why Field's reformulation is not a better theory than the standard one.

## §2: Intrinsic and Extrinsic Explanations and Theories

Field's argument for the superiority of his reformulation depends on a general preference for intrinsic explanations over extrinsic ones.[4]

> If in explaining the behavior of a physical system, one formulates one's explanation in terms of relations between physical things and numbers, then the explanation is what I would call an *extrinsic* one. It is extrinsic because the role of the numbers is simply to serve as labels for some of the features of the physical system: there is no pretence that the properties of the numbers influence the physical system whose behavior is being explained. (Field 1985: 192-3)

According to Field, numbers are extrinsic to physics, while physical objects and space-time regions are intrinsic. Numbers are extrinsic to geometry, too, while line segments and their ratios are intrinsic. The application of 'intrinsic' within mathematics proper begs several questions about relationships among mathematical theories. Are real numbers intrinsic or extrinsic to the theory of natural numbers? Are sets extrinsic to category theory? Such questions should make us wary of the intrinsic/extrinsic distinction.

These questions aside, Field relies on the presumption that mathematical objects do not influence physical systems to classify them as extrinsic to physical theory. "If, as at first blush appears to be the case, we need to invoke some real numbers... in our explanation of why the

[4] Field uses 'intrinsic' and 'extrinsic' to apply to entities, theories, and explanations. The application to entities is basic, since he classifies explanations and theories depending on the objects involved. They are intrinsic if they make no demand for extrinsic objects.

moon follows the path that it does, it isn't because we think that the real number plays a role as a *cause* of the moon's moving that way..." (Field 1980: 43)

Field's preference for intrinsic explanations is a broad methodological principle.

> Extrinsic explanations are often quite useful. But it seems to me that whenever one has an extrinsic explanation, one wants an intrinsic explanation that underlies it; one wants to be able to explain the behavior of the physical system *in terms of the intrinsic features of that system*, without invoking extrinsic entities (whether mathematical or non-mathematical) whose properties are irrelevant to the behavior of the system being explained). If one cannot do this, then it seems rather like magic that the extrinsic explanation works. (Field 1985: 193; see also Field 1980: 44.)

Call this principle PIE: we should prefer intrinsic explanations over extrinsic ones. PIE accounts for Field's preference for synthetic physical theories, like his reformulation of NGT based on physical geometry, over analytic ones, which rely on real numbers and their relations.[5]

Field's focus on explanations, rather than theories, is puzzling since his project is clearly a response to QIA, which is formulated in terms of theories, not explanations. Compare PIE to an analogous principle PIT: we should prefer intrinsic theories to extrinsic ones. QIA has force because of its reliance on Quine's demand that we find our ontic commitments in our best theory. To reformulate the indispensability argument in terms of explanations would force the indispensabilist to argue that we determine our commitments by consulting our explanations. A less controversial tactic, which I will adopt, is to focus on PIT, rather than PIE.

[5] PIE also supports Field's argument for substantivalist space-time and his hostility to modal reformulations of science.

§3: Field's Motivation for PIT: Hilbert's Intrinsic Geometry

I discern three arguments for PIT in Field's work. I assess the "magic" argument in §5. I consider an implicit Okhamist argument in §4. First, I evaluate Field's argument that the importance of Hilbert's 1899 reformulation of Euclidean geometry, which inspired Field's project, can be explained by PIT: Hilbert's axiomatization is superior because it is intrinsic. But, I think that we can understand Hilbert's success without adopting PIT as a general principle which supports Field's response to QIA.

The projects of axiomatizing mathematics in the late nineteenth century were motivated by diverse factors, two of which stand out: the oddities of transfinite set theory, and the development of non-Euclidean geometries.[6] In both cases, mathematical ontology was extended contentiously, but without obvious inconsistency. Rigor in the form of axiomatic foundations was sought for the controversial new theories.

Euclidean formulations of geometry had used real numbers to represent lengths of line segments and triples of real numbers to represent points. Hilbert's new axiomatization referred to regions of geometric space in lieu of real numbers, and used the geometric properties of betweenness, segment congruence, and angle congruence in the way that real numbers, and their ordering, were used in analytic versions. Hilbert constructed representation and uniqueness theorems which assured the adequacy of his so-called synthetic theory.

We can understand why Hilbert would prefer synthetic geometry over analytic versions without appealing to PIT. Here is what Hilbert says about his motivation:

[6] Hilbert mentions both in a letter to Frege discussing his motivation for axiomatizing geometry. (Frege 1980: Letter IV/4)

> I wanted to make it possible to understand those geometrical propositions that I regard as the most important results of geometric inquiries: that the parallel axiom is not a consequence of the other axioms, and similarly Archimedes' axiom, etc. I wanted to answer the question whether it is possible to prove the proposition that in two identical rectangles with an identical base line the sides must also be identical, or whether as in Euclid this proposition is a new postulate. I wanted to make it possible to understand and answer such questions as why the sum of the angles in a triangle is equal to two right angles and how this fact is connected with the parallel axiom..." (Frege 1980: 38-9)

We can best interpret Hilbert's motivation as purely mathematical, rather than ontological. Hilbert's account makes it clear that he wanted to clarify relations within geometry. By relying on geometric relationships to explain geometric phenomena, we avoid worries about the consistency of analysis, for example, and we can better explain geometric entailments. Hilbert provides no implication that his new theory is better for the purposes of revealing ontology, which is how Field uses his formulation of NGT. Furthermore, there are no benefits of parsimony in Hilbert's program. Hilbert only shows that real numbers are avoidable in the axiomatization of geometry, not that they are eliminable overall.

Regarding more general ontological questions, our main worry within mathematics is antinomy, not parsimony, and contradictions may be more easily discovered in the large than in the small. For a simple example, consider the set-theoretic translation of any reductio proof in number theory, e.g. that the square root of two is irrational. The contradiction in set-theoretic language will be extremely obscure, and impossible to recognize. The problem only worsens in more sophisticated mathematics. The superiority of Hilbert's axiomatization for the purposes of revealing geometric relations does not entail its superiority in discovering contradictions.

Different axiomatizations serve different purposes. For ontological purposes, it is the conjunction of Hilbert's construction with analysis, which maps geometric structures onto those of number theory, which really interests us. We need not invoke a general principle, PIT, to explain the utility of Hilbert's reformulation, and insist that his intrinsic, synthetic theory is superior in all ways to the analytic formulation.

## §4: Unification and Extrinsic Theories

Since we need not appeal to PIT to see the virtues of Hilbert's reformulation, Field's response to QIA is under-motivated. In addition, PIT, which seeks to isolate theories according to their intrinsic elements, opposes the standard scientific goal of unifying theories.

For an example of the virtues of extrinsic theories within mathematics, consider how the fundamental theorem of calculus bridges geometry and algebra. Algebra could easily be seen as extrinsic to geometry, but uniting them yields a more comprehensive, and more fruitful, theory. More importantly, we can prove more in an extrinsic second-order theory than we can in an incomplete first-order theory, like first-order arithmetic, itself.

In science, consider how welcome bridge laws between physics and chemistry or biology would be. The objects of biology are extrinsic to physics, but we still seek bridge laws. In fact, the classification of objects or theories as intrinsic or extrinsic seems fairly flexible. Consider how an Aristotelian would deem theories about planets and stars as extrinsic to principles covering terrestrial objects.

Our desire to unify theories shows PIT, as a general principle, to be false. Applied specifically to the mathematics used in science, PIT is false, also. The unification of mathematics with physics yields a simpler theory, a point which Field grants by arguing that we can continue to use standard science since mathematics is conservative. And, the isolation of scientific theory from mathematics, especially on the basis of a dispensabilist reformulation, denies that there are any interesting relations among mathematical and physical objects.

For example, it is a mathematical property of a three-membered set that it has exactly three two-membered subsets which do not contain the empty set. This property accounts for why we can, with a red marble, a blue marble and a green marble, form exactly three different-looking pairs of marbles. An intrinsic explanation of this fact would force us to interpret the mathematical property as a logical or empirical property. We can describe this mathematical fact, and more complex ones, as logical or empirical only at the cost of losing its relevant features. Also, inferences which are easily derived using mathematics become impossibly complex when written as logical derivations.[7] The only benefit arising from eliminating mathematics is parsimony.

Desire for parsimony proceeds from a principle applicable to empirical elements of theories: do not multiply physical entities without good reason. When constructing empirical theories, it is important not to posit more than that which accounts for the phenomena.

In contrast, we explore the mathematical universe with a desire to multiply entities. It is a virtue to be mathematically plenitudinous, as long as we avoid antimony. Set theorists proudly present discoveries of distinct new cardinals. Kripke models for modal logic have ameliorated mathematical worries about modality. Worries about the introduction of new mathematical entities, as with transfinites, or complex numbers, focus on their consistency, or the rigor with which they are introduced.

PIT would reduce ontology at the expense of perspicuity, fruitfulness, and coherence

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[7] See Boolos 1987 for details of the complexity.

with other theories. And it is not even clear that the simpler ontology is preferable. "It is at least very difficult to find any unequivocal historical or other evidence of the importance of economy of abstract ontology as a scientific standard for the evaluation of theories." (Burgess and Rosen 1997: 206)

Given a dispensabilist reformulation of standard science, we have two competing scientific theories: the intrinsic dispensabilist one, and the extrinsic standard one. The intrinsic theory is preferable only if one has a prior disposition to nominalism.

## §5: The End of Intrinsic Theories

In defending his general principle, Field granted the utility of extrinsic explanations, but argued that they seem like magic if there is no underlying intrinsic explanation presupposed. I presume that Field's idea is that explanations and theories of physical phenomena should be possible which only refer to entities which are active in producing those phenomena. Why should one insist on this?

The obvious defense of the general demand for intrinsic theories comes from linking theories with ontic commitment, as Quine does. Our theories should refer only to relevant objects in order to avoid errant commitments. But if we have the additional commitments already, the extrinsic theory involves us in nothing untoward, and simplifies and unifies our theory. We do not want mistakenly to impute causal powers to mathematical objects by using the extrinsic mathematical theory within physics. But, merely noting that mathematical objects are non-spatio-temporal easily blocks any confusion. There is nothing magical about the utility of mathematics in physical science, since mathematics provides a theoretical apparatus for all possible physical states of affairs.[8]

Explanations are less edifying if they are restricted to isolated, intrinsic objects. Resnik applies the preference for intrinsic theories to economics.

> The Expected Utility Theorem, which underwrites the use of utility functions, establishes that if an agent's preference ordering satisfies certain conditions then it can be represented by a real valued function which is unique up to positive linear transformations. From this it is usually argued that there is no need to presuppose ill understood utilities in accounting for behavior which maximizes expected utility because an account can be given directly in terms of preferences. (Resnik 1983: 515)

Resnik's says that an intrinsic account, in terms of preferences, is desirable because utilities are "ill understood." But, if they were better understood than preferences, then the account would go the other way. The principle underlying Resnik's preference is not PIT, but that we should explain things we do not understand in terms of things we do understand.

We can appreciate both intrinsic and extrinsic theories. The situation is like the relation between mathematical realists and intuitionists, from a realist perspective. The realist can appreciate the distinction between constructive and nonconstructive proofs, without concluding that only constructive proofs tell us what exists.

Philosophers with nominalist prejudices may see PIT as a common sense principle, and so may have neglected to recognize a gap in Field's argument against QIA. There also may be other reasons to reject QIA, or merely to prefer a theory which does not refer to mathematical objects. But the principle of intrinsic explanations, or theories, can not do this work.

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[8] This is Mark Balaguer's account; see Balaguer 1998, p 143.

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