NOTIONAL ATTITUDES (ON WISHING, SEEKING AND FINDING)*

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NOTIONAL ATTITUDES

Our knowledge, beliefs, doubts, etc., concern primarily logical *constructions* of propositions. If we assume that iterating 'belief attitudes' is valid, i.e., that the agent is perfectly introspective, he knows what he knows, believes, etc., then the so-called *propositional* attitudes are actually *hyperintensional* attitudes, i.e., they are relations of an agent to the *construction–concept* (of a proposition) expressed by the embedded clause. Their implicit counterparts, relations of an agent to the proposition denoted by the embedded clause, are just idealised cases of an agent with unlimited inferential abilities.

On the other hand, our wishes, intentions, seeking, (attempts at) finding, etc., concern (in empirical cases) particular *intensions* (offices, properties, propositions), and the so-called *notional* attitudes (to empirical notions) are (despite calling them notional) not hyperintensional. In the paper we formulate some criteria for notional attitudes and examine basic categories of them. In general, notional attitudes are relations-in-intension between an agent and an object to which the agent is *intentionally related.* Even relations of an agent to a proposition can be notional ones, in case there is no salient constructional counterpart, the attitude is not influenced by agent's inferential abilities. Another problem we meet is the ambiguity of sentences expressing notional attitudes. These statements can often be read both in the *de dicto* and in the *de re* way. Yet, a common feature that characterizes notional attitudes is using the construction of the respective intension in the *de dicto* way. The respective intension is mentioned; referring on such a situation, the reporter may use any of the equivalent constructions (concepts) of the intension, but not just a co-referring notion of another intension. Still, unlike the cases of relations of an agent to an individual when the respective individual office serves just as a pointer to the individual, when referring on notional attitudes the use of the respective *notion* of the intension is indispensable, which might perhaps justify calling such attitudes *notional,* though they actually are *intensional*. We also show that a passive form of a sentence cannot be

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usually read in the *de dicto* way (unless an idiom is used). Thus a common belief at the equivalence of the active and passive form of a statement is generally not justified.

Key words: Propositional / notional attitudes, hyper-intensions, *de dicto / de re* supposition, passive vs. active form.

1. Introduction

Some expressions are "sensitive" not only to the denotation of a related expression E but also to the logical structure, or more precisely to the meaning of E, in such a way that substitution *salva veritate* of an L-equivalent expression E' denoting the same intension (/extension) fails. Such sensitive expressions are, e.g., some attitude verbs, like *knowing, believing, counting,* ..., but also anaphora pronouns [17], etc., and the respective (indirect "oblique") contexts are called *hyper-intensional* [4], [32]. The problem has been noticed already by G. Frege who realized that his standard semantic scheme fails in indirect contexts, which led him to contextualistic solution (reference in "normal" contexts and sense in indirect contexts). Moreover, Frege's conception had been extensionalistic, and the notion of a *sense* had not been explicated and thus logically tractable.

Due to the above sensitivity to a meaning (logical *structure*) of the embedded clause, the analysis of propositional attitudes has become a stumbling block for all the denotational semantic theories that take into account just the denoted entity. Since Frege's times, many logicians strove after logically handling *structured meanings,* to name at least Russell's structured propositions, Carnap's attempts at the formulation of a stronger criterion for the identity of belief, i.e. *intensional isomorphism* between substituted expressions, Cresswell's tuples, etc., etc. Still, none of these attempts carries conviction of a full adequacy and correctness.[1]

Pavel Tichý, the author of Transparent Intensional Logic (TIL), and Pavel Materna, its main protagonist, have presented a fundamental revision of Frege's semantic scheme[2], which makes it possible to adequately explicate the "behaviour" of expressions even in hyper-intensional contexts. Tichý's solution respects the distinction between the meaning (sense) of an expression

[1] See, e.g., [2], [4], [26] Carnap's and Cresswell's solutions have been critised by Tichý in [28] and by Materna in [22]. The inadequacy of Carnap's solution has been noticed already by Church (see [3]).

[2] See [28], [22].

and the object denoted by the expression; but it differs from most current conceptions (incl. Montague's [25]) in at least two points:

a) it logically handles the hyperintensional structure of the meaning, sense is explicated as a hyperintensional entity (construction)
b) No contextualism is present; expressions simply denote (*either* an intension / *or* an extension) *via* its sense. Empirical expressions denote always intensions, and where it seems that they denote extensions they only possess *de re* supposition.

Before presenting our main results applying TIL on the problem of notional attitudes, we first very briefly recapitulate TIL philosophy, and its main notions and definitions.

2. Transparent Intensional Logic

In contrast to standard formal theory[3], which starts with a naked syntax and only subsequently proceeds to a semantic interpretation in a model, TIL is 'transparent' not only that it is anti-contextualistic but also not formalistic. Notion of a naked formal expression as a pure graphic shape can be arrived at only through abstracting its sense from it. In terms of conceptual priority, TIL starts with sense-endowed expressions, which is to say that the "formal language" of TIL-constructions constitutes an "interpreted formalism". Every factor that is semantically salient is explicitly present in the respective formalism. This is evident, for instance, in the explicit typing of the theory, the types of TIL being exclusively objectual. So what qualifies the "formal language" of TIL as transparent, inherently interpreted, is that a naked shape can be introduced as an expression only if it is paired off with a construction constructing an object of a particular type.

Definition 1 (Simple types of order 1):
(An objectual) base is a collection of mutually disjoint nonempty sets.

i) Every member of the base is a *type over base.*
ii) Let α, β_1, ..., β_m be *types over base,* then $(\alpha\ \beta_1 \ldots \beta_m)$, i.e. the set of all *m*-ary (total and partial) functions with an argument (a tuple) $\langle b_1, \ldots, b_m \rangle$, where b_i $(1 \leq i \leq m)$ is a member of the type β_i, and at most one value of type α, is a *type over base.*
iii) Nothing is a *type over base* unless it so follows from i) - ii).

An object O (that is a member) of a type α is called an α-object, denoted O/α.

[3] Now we present some characteristics of TIL as formulated in a nice way by Jespersen in [20].

According to Zalta ([30]), Russellian propositions play the desired role of *complexes* that result by "plugging" objects into the gaps of properties and relations. In our opinion, properties, relations, i.e. functions in general, have no "gaps"; particular objects simply are members of (the arguments of) these flat functions. But we can accept the possible-world semantics of propositions[4], while the demand of *structured meanings* is met by another entity: Between an expression and the denoted flat object there is a structured *mode of presentation* (*construction* in our terminology) of the object, i.e. meaning (perhaps the Fregean sense) of the expression. It is a complex, a *procedure*[5] that consists in a *creation of a function* by abstracting over objects and/or in *applying the function* to its arguments. But particular (physical/abstract) objects cannot be "plugged" into such a (conceptual) procedure; they must always be presented in an (albeit primitive) way, i.e., their concepts are constituents of the procedure. There are two such primitive modes of presentations that fill in the objects into the construction: *variables* and *trivialisations*. The other two kinds of constructions working over these ones are more complex; they are *closure* (creating a function by abstraction) and *composition* (applying a function to an argument).

TIL language of constructions can be viewed as a typed λ-calculus whose terms are names of (denote) constructions. Due to the perfect "isomorphism" between terms and constructions it is idle to mention the terms, and we transparently talk about the constructions[6]. Thus, e.g., instead of claiming that '$\lambda x\ [^0\!> x\ ^0 0]$' denotes the construction $\lambda x\ [^0\!> x\ ^0 0]$ which constructs the class of positive numbers, we simply say $\lambda x\ [^0\!> x\ ^0 0]$ is the construction.

Definition 2 (Constructions):

i) *Variables* are ***constructions.*** Variables and constructions involving variables construct objects dependently on a valuation v, they v-construct.

[4] True, Russell's propositions are the sort of things you can plug objects in and out of, those propositions are *structured* entities, no mappings (see [26]), but then the notion of mapping is needed as well.

[5] We might perhaps stress that constructions are (*declarations* of) procedures but not their *executions*.

[6] TIL entertains two notions of transparency: a) referential transparency tout court, no recourse to contextualism and b) a TIL lambda-term serves as a 'transparent' window onto a construction, TIL is not formalistic.

ii) If X is an entity whatsoever, even a construction, then ^{0}X is a ***construction*** called *trivialisation*. Trivialisation ^{0}X constructs X without any change.

iii) If X_0 is a construction that *v*-constructs a function (mapping) F, i.e. an $(\alpha\, \beta_1...\beta_n)$-object, and $X_1, ..., X_n$ are constructions that *v*-construct β_1-, ..., β_n-objects $b_1, ..., b_n$, respectively, then $[X_0\ X_1...X_n]$ is a ***construction*** called *composition*. If F is defined on the argument $\langle b_1, ..., b_n\rangle$, then composition $[X_0\ X_1...X_n]$ *v*-constructs the value of F on $\langle b_1, ..., b_n\rangle$; otherwise it does not construct anything, it is *v-improper*.

iv) Let $x_1, ..., x_n$ be pair-wise distinct variables that range over types $\beta_1, ..., \beta_n$, and let X be a construction that *v*-constructs an α-object for some type α. Then $[\lambda x_1...x_n\ X]$ is a ***construction*** called *closure* (abstraction). It *v*-constructs the following function F (of the type $(\alpha\ \beta_1...\beta_n)$): Let v' be a valuation that differs from v at most by assigning objects $b_1, ..., b_n$ (of the respective types) to variables $x_1, ..., x_n$, respectively. Then the value of the function F on the argument $\langle b_1, ..., b_n\rangle$ is the object v'-constructed by X. If X is v'-improper, then F is *undefined* on the given argument.

v) Nothing is a ***construction*** unless it so follows from i) - iv).

Notes:

1. The simplest constructions are variables; they are open constructions that construct objects dependently on valuation (they *v*-construct). They are no letters, characters, '*x*', '*y*', '*z*', ... are *names* of variables.
2. Trivialisation consists in grasping an object and its "delivering" without any change. If X is an entity, then ^{0}X is a presentation of X without a "perspective". The term '^{0}X' might be likened to a constant of a formal language. But unlike such a formal constant symbol, which can be interpreted in many ways so as to denote different entities and thus actually not being a constant but a "variable construction", '^{0}X' rigidly denotes construction ^{0}X that constantly constructs X. A possible objection against such a conception might be: Well, your transparent approach is punctilious, but you lose the expressive power of model theories that enable us to examine common features of properties, relations and functions in particular models. Our answer is: Not at all; TIL transparent approach is more precise without losing anything; due to the infinite hierarchy of types we have at our disposal variables ranging over objects of any level, which makes it possible to render particular "models" by valuations of (higher-level) variables.

3. A composition corresponds to the traditional operation of *application* (of a function to an argument). (Only) composition may fail to construct anything, it may be (v-)improper, namely in two cases: *First,* the component X_0 constructs a function F and components $X_1, ..., X_n$ construct $\langle b_1, ..., b_n\rangle$, but F is not defined on this argument. *Second,* some of the components $X_0, X_1, ..., X_n$ fail to construct an object (are v-improper).
   (In case $X_1, ..., X_n$ do not construct objects of proper types to create an argument of F, the expression '$[X_0\ X_1...X_n]$' does not denote a construction.)
4. Closure (λ-abstraction) enables us to construct a function, and thus to analyse talking about the whole function (to "mention" it), not only talking about a particular value on a given argument (to "use" the function). Closure can never be improper, even if it constructs a (degenerated) function that is undefined on all its argument, like, e.g., $\lambda x[^0{:}\ x\ ^00]$.
5. Tichý's definition of constructions comprises also single and double execution. We sometimes also adjust the definition by adding tuple and projection constructions. Since there is no need for them in this paper, we do not introduce these constructions.

Quantifiers – general $\forall_\alpha$ and existential $\exists_\alpha$ – are functional objects of type $(o(o\alpha))$. We will write $\forall x$A, $\exists x$A instead of $[^0\forall_\alpha\,\lambda x\ A]$, $[^0\exists_\alpha\,\lambda x\ A]$, respectively. Quantifiers are "totalising", i.e. they always return a truth value when being applied to a class (even if the characteristic function of the class were undefined on some arguments), namely $[^0\forall_\alpha\,\lambda x\ A]$ returns True iff $[\lambda x\ A]$ constructs the whole type α (A constructs True for all x ranging over α), otherwise False, $[^0\exists_\alpha\,\lambda x\ A]$ constructs True iff $[\lambda x\ A]$ constructs a non-empty subset of α (A constructs True for some x), otherwise False. Singulariser ι_α is an object of type $(\alpha(o\alpha))$, and instead of $[^0\iota_\alpha\lambda x\ A]$ we will use ιxA (*the only x such that A)*; $[^0\iota_\alpha\lambda x\ A]$ constructs the only member of the class constructed by $[\lambda x\ A]$ iff $[\lambda x\ A]$ constructs a singleton, otherwise it is an improper construction. We will use a standard infix notation without trivialisation in case of using truth-value functions ($\wedge$, $\vee$, ...), less-than, greater-than and identity functions ($\geq$, $\leq$, $=$, ...), but we have to keep in mind that these are just abbreviations that conceal the self-contained meaning of the respective "logical symbols". When a construction C constructs an object of type α, we will often write $C \rightarrow \alpha$.

The bridge between an expression and a construction (logical analysis of the expression) is provided by a *principle of subject-matter,* which says, roughly, that an expression is about all and only those objects, incl. *construc-*

tions, which receive mention in the expression (see [13]). Constructions are mentioned, e.g., in *hyper-intensional* contexts where the meaning, i.e. the expressed construction plays a crucial role. Thus a construction/meaning is a "full-right entity" to talk about, and has to be of a definite (higher-order) type, which is not possible within the simple hierarchy of types.

3. Propositional attitudes

The problem of propositional attitudes has been a subject of much dispute in this journal[7], and we can claim that an adequate solution has been proposed. In general, propositional attitudes are expressed by verbs like *to believe, to think that, to know, to doubt,* etc., and they are relations-in-intension between an *individual* and *the structured meaning* of the embedded clause, viz. a construction, i.e. they are objects of type $(o\iota*_n)_{\tau\omega}$. In belief contexts we are thus dealing with attitudes to constructions that construct propositions (or truth values in mathematics); hence they are attitudes to *propositional constructions,* and what has been called "propositional attitudes" are just *constructional attitudes* to propositional constructions[8]. If we assume that iterated attitudes are valid, i.e. that the agent is perfectly introspective, he knows what he knows, believes, etc, then what is *known, believed,* etc. concerns primarily *meaning, i.e. concept, construction.* This solution provides an adequate explication of the substitution failure in belief contexts and does not lead to the paradox of (mathematical/logical) omniscience. The fact that the following argument is obviously not valid: Charles knows that Bratislava has 500 000 inhabitants

$$\frac{500\,000 = 2^5 \times 5^6}{\text{Charles knows that Bratislava has } 2^5 \times 5^6 \text{ inhabitants}}$$

is explained away. This argument uses a rule scheme of the form:

[7] See, e.g., [6], [7], [9], [10], [11].

[8] Russell, who coined the phrase 'propositional attitude', certainly took his propositions to be complex objects, so the TIL account of attitudes is closer to the source than, e.g., the possible worlds approach in this respect.

$$\text{(R)} \quad \frac{\begin{array}{c}[{}^0B_{wt}\ {}^0X\ {}^0C_1] \\ [{}^0\!=\ C_1\ C_2]\end{array}}{[{}^0B_{wt}\ {}^0X\ {}^0C_2],}$$

where B is an (propositional) attitude and C_1, C_2 are any constructions. Such a rule is not correct, because the second premise states only an equivalence of constructions (the identity of the constructed objects), but not the identity of the constructions themselves. The rule would be correct if constructions C_1 and C_2 not only constructed the same object but were also identical, i.e. the second premise would have to be[9]:

$$[{}^0\!=\ {}^0C_1\ {}^0C_2].$$

It means that if the agent X has an attitude to a construction expressed by a sentence *p*, it does not mean that X has this attitude also to sentences that express logical consequences of the sentence *p* (they logically follow from *p*). This is due to the fact that X may have limited inferential abilities. Hence if the agent knows that 5 + 7 = 12, then he does not have to know all the mathematical truths; or, if he knows, e.g., the axioms of arithmetic, he does not have to know all the (provable) truths of arithmetic. Otherwise we would have to suppose that the language user is a logical and mathematical genius, that if he knows any mathematical truth then he knows also all its logical consequences, all the truths. This would not be in accordance with our intuitions and with the "principle of *Non-omniscience*".

On the other hand, however, the demand of the identity of constructions seems to be very restrictive. Identical constructions have to construct the same, but not only that; they must also be "built up" from the very same constituents, subconstructions, in the *very same way*. Thus Charles' believing that A and B would not lead to his believing that B and A, because ${}^0[A \wedge B] \neq {}^0[B \wedge A]$. Actually, this solution deprives the agent of *any* inferential abilities.

We might eliminate this restrictiveness by conceiving attitudes to *empirical* embedded clauses as "*implicitly*" relating the agent to propositions denoted by these clauses (to states-of-affairs). This would, however, mean that only two non-realistic (idealised) types of a language user are considered: either a logical and mathematical idiot (the former case), or a perfect language user who is a logical and mathematical genius (the latter case), and in this restricted sense omniscient [10]. Thus an '*explicit*' propositional atti-

[9] For details, see [22], [31, p 76]. The latter is, however, flawed just in this argument.

tude would be distinguished from an '*implicit*' (or 'unconscious') one [14]. Restrictive consequences of the constructional approach are reduced: in case of attitudes to empirical clauses we take into account also agent's relation to the denoted states-of-affairs (type $(o\iota o_{\tau\omega})_{\tau\omega}$), not only to their constructions (type $(o\iota{*}_1)_{\tau\omega}$). The relation of an implicit propositional attitude is thus closed under the relation of logical consequence (the agent is in a way "analytically" omniscient – if he, for instance, knows that *p*, then he also knows all the logical consequences of *p*, though he does not have to be aware of it, to know it "explicitly", i.e. to know all the constructions that identify *p*).

4. Notional attitudes

We have seen that when knowing, believing, doubting, etc., something – some proposition P, the agent is related to the proposition P only via the meaning of the respective clause C denoting P, and the substitution *salva veritate* of an equivalent clause C' denoting the same proposition P may fail, because the agent is not able to perform the respective inference (logical and / or mathematical operation) on the meaning of C.

When analysing notional attitudes, we ask a similar question, namely *to which kind of object is the agent related*? And trying to answer this question, the substitution test should always justify the answer. But in this case there is another preliminary, more fundamental question:

Which attitudes should be, in general, called notional ones?

At first sight, the answer might seem to be simple. Well, these are attitudes to some *notion*, i.e. *concept*, but not a concept (i.e. construction) of a proposition. We will show that the answer is not as simple, and actually, since we doubt that an exhaustive answer can be given, in this brief study we just formulate some criteria and categories of notional attitudes.

a) Attitudes to mathematical notions

Consider the sentence

(1) Charles calculates 2 + 5.

To which object is Charles related? It cannot be the denotation of 2 + 5, for Charles does not calculate 7. It cannot be the respective expression '2 + 5' as well (as might sententialists claim), because Charles can calculate 2 + 5 when being at the age of 5 (not knowing any such expression, or even a term of a formal language) and playing with the balls of an abacus[10]. He is related to the respective *construction*, trying to perform the procedure and to find out, which object (number) is thus constructed. (Of course, we can describe the agent's activity using sentence (1), which does not, however, mean that he was related to an expression.) Hence Calc(ulating) is an object of type $(o\ \iota\ {*_1})_{\tau\omega}$ and the analysis of (1) is:

(1') $\lambda w \lambda t\, [{}^0\mathrm{Calc}_{wt}\ {}^0\mathrm{Ch}\ {}^0[{}^0{+}\ {}^02\ {}^05]]$ (Trivialisation of $[{}^0{+}\ {}^02\ {}^05]$ is indispensable)

A similar way of reasoning can be used when analysing the sentence

(2) Charles is thinking of the greatest prime.

Since there is no greatest prime, Charles cannot be thinking of the denotation of this expression, he is related to the meaning of it (he is probably trying to find out whether the respective concept is empty, i.e., whether it does, or does not identify any number). Hence Th(inking) is here again an object of type $(o\ \iota\ {*_1})_{\tau\omega}$.[11] To analyse the sentence, we have to realise that the meaning of the simple expression 'prime' is the *concept* of the class of prime numbers, which cannot be just ^{0}prime. This expression has been introduced to the mathematical language by a linguistic definition as an abbreviation:
Prime (numbers) $=_{df}$ The class of natural numbers that have exactly two factors.
The respective concept is (N(atural) / $(o\tau)$, D(ivisible by) / $(o\tau\tau)$, Card / $\tau(o\tau)$):

$$\lambda x\, ([{}^0\mathrm{Nat}\ x] \wedge ([{}^0\mathrm{Card}\ \lambda y\, [{}^0\mathrm{Nat}\ y] \wedge [{}^0\mathrm{D}\ x\ y]] = {}^02))$$

Abbreviating, for the sake of simplicity, this concept by Prime, we get the analysis of (2):

(2') $\lambda w \lambda t\, [\ {}^0\mathrm{Th}_{wt}\ {}^0\mathrm{Ch}\ {}^0[\iota z\, ([\mathrm{Prime}\ z] \wedge \forall z'([\mathrm{Prime}\ z'] \supset (z \geq z')))]\ ]$.

Generalising, we can claim that attitudes to *mathematical* notions are objects of type $(o\iota{*_n})_{\tau\omega}$, n being mostly equal to 1.[12]

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[10] Of course, when the procedure of calculating gets more and more complicated, *executing* such a procedure without a proper notation is hardly imaginable. The importance of a symbolic notation, images, etc., in mathematics is stressed in [1]. Still, the sentence *does not say anything* about the *way* in which calculating is being performed.

[11] In general, 'think' is a strongly polymorphic expression, see [5].

[12] After all, *constructions* are the subject matter of mathematics, see Tichý's arguments in [29].

b) Attitudes to empirical notions

In this case the situation is rather more complicated, and we will show that in empirical contexts these attitudes are generally not hyper-intensional. First, simple relations (-in-intension) of an individual to another individual (of type $(o\iota\iota)_{\tau\omega}$), like, e.g., kicking, being in love, touching, talking to, etc., should not be considered as falling under the category of notional attitudes. For instance, in the following sentence

(3) Charles is talking to the Mayor of Dunedin

'talking' denotes a relation-in-intension of an individual to an individual – T(alk) / $(o\iota\iota)_{\tau\omega}$, though no particular individual is mentioned in (3). The office MD / $\iota_{\tau\omega}$ of the Mayor of Dunedin serves only as a 'pointer' to the unspecified individual, and its construction (composed of the constructions of M(ayor) / $(\iota\iota)_{\tau\omega}$ and D(unedin) / ι) occurs *de re*. [13]

(3') $\lambda w\lambda t\ [{}^0T_{wt}\ {}^0Ch\ [\lambda w^*\lambda t^*\ [{}^0M_{w^*t^*}\ {}^0D]]_{wt}\ ]$.

The two *de re* principles, i.e. the principle of existential presupposition and the principle of inter-substitutivity of coreferential expressions are valid. The Mayor of Dunedin has to exist, so as the sentence had *any* truth-value (existential presupposition), and the substitution *salva veritate* of a co-referential expression is possible. If Mr X is the Mayor, then Charles is talking to Mr X:

(S) $\lambda w\lambda t\ [{}^0T_{wt}\ {}^0Ch\ [\lambda w^*\lambda t^*\ [{}^0M_{w^*t^*}\ {}^0D]]_{wt}\ ]$
$\lambda w\lambda t\ ([\lambda w^*\lambda t^*\ [{}^0M_{w^*t^*}\ {}^0D]]_{wt} = {}^0X)$
$\therefore \lambda w\lambda t\ [{}^0T_{wt}\ {}^0Ch\ {}^0X]$[14].

It may even be the case that Charles is talking to Mr X without knowing that this person is the Mayor of Dunedin (occupies the office), yet we may report on such a situation with perfect truth using (3). Hence, describing such a situation, the *notion* of the respective office is dispensable and (3) should *not* be considered as an example of a *notional attitude,* because Charles' attitude of talking is related to Mr. X, an individual, not to the respective office (the notion of which is thus dispensable).

Consider, on the other hand, the sentence

(4) Charles would like to talk to the Mayor of Dunedin.

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[13] For the exhaustive study on *de dicto / de re*, see, e.g., [12].

[14] This argument is valid, for the rule like (R) can now be correctly used; there is no trivialisation of C_1, C_2

Here we meet the problem of an ambiguity, which is often the case of (notional) attitude verbs. The sentence (4) may inform on situation similar to the case (3), Charles would simply like to talk to Mr X without any concern in his office, and the reporter just uses this office as a pointer to Mr X. Hence 'would like to talk' denotes an object of type $(o\iota\iota)_{\tau\omega}$, and it is not the case of a notional attitude. But there is another, more interesting, and may be more adequate reading of (4). Our Charles may be discontent with public relations in the city of Dunedin, and he demands to meet and talk to the Mayor, not having any idea whoever he is, or even if there is one at all. Neglecting for a moment the meaning of 'would like', and denoting WLT / $(o\ \iota\ \iota_{\tau\omega})_{\tau\omega}$ the object denoted by 'would like to talk', we get:

(4') $\lambda w\lambda t\ [{}^0\mathrm{WLT}_{wt}\ {}^0\mathrm{Ch}\ [\lambda w^*\lambda t^*\ [{}^0\mathrm{M}_{w^*t^*}\ {}^0\mathrm{D}]]\ ]$.

This time the construction of the office MD, namely $[\lambda w^*\lambda t^*\ [{}^0\mathrm{M}_{w^*t^*}\ {}^0\mathrm{D}]]$, occurs *de dicto*, the substitution argument (S) cannot be applied, i.e., we cannot deduce that Charles intends to talk to Mr X. It may even happen that the Mayer of Dunedin does not exist (there is no Mr X holding the office), and yet (4) may be true. There is no existential commitment here.

Hence the agent is related to the office itself, and reporting on such a situation the respective *notion* of the office is indispensable. In our opinion, such an attitude should be classified as a *notional attitude* (to the office of the Mayor). Still, a question arises: Should such an attitude not be analysed in a way analogous to propositional attitudes, that is, as a relation of the agent to the respective *concept*, i.e. *construction* of the office? We do not think so. Unlike knowledge, belief, doubting, etc., which concern primarily the *mode* in which the respective proposition is presented to the agent (whose deductive abilities, inferences, etc., i.e. in general agent's knowledge, are strongly related to (depend on) the respective construction), *intentional* activities of the agent are primarily related to the office itself, regardless the way in which the office is reported to. If Charles intends to talk to the Pope, then he intends to talk to the Head of Roman Catholic Church, and vice versa. Even if Charles were an ignorant not knowing that the Pope and the Head of Roman Catholic Church are one and the same office, and demanded meeting the Pope, the speaker might truly report on the situation using the 'Head of Roman Catholic Church'.

Anyway, we have to return to the analysis of (4). The proposed analysis (4') is not the best one; it does not follow 'Parmenides Principle' of subject matter. Well, its constituents are concepts of those objects the sentence (4) talks about, and *only* of them, but not of *all* of them. We have to take into account semantically self-contained subexpressions 'would like to' and 'talk to', and construct the WLT object by composing their denotations. First, 'talk to' has to denote an object T / $(o\iota\iota)_{\tau\omega}$, a relation of an individual to an *individual* (we cannot, of course, talk to an office). Second, construction of the office MD, namely $[\lambda w^*\lambda t^* [^0M_{w^*t^*}\ ^0D]]$, still has to occur *de dicto*, as explained above. How can we overcome this discrepancy? There are two ways out; We can construct the property of talking to the Mayor of Dunedin

$$\lambda w\lambda t\ \lambda x\ [^0T_{wt}\ x\ [\lambda w^*\lambda t^*\ [^0M_{w^*t^*}\ ^0D]]_{wt}\ ],$$

and analyse the sentence as claiming that Charles would like to obtain this property. Hence 'would like' denotes an object WL / $(o\ \iota\ (o\iota)_{\tau\omega})_{\tau\omega}$:

$$(4'')\quad \lambda w\lambda t\ [^0WL_{wt}\ ^0Ch\ [\lambda w\lambda t\ \lambda x\ [^0T_{wt}\ x\ [\lambda w^*\lambda t^*\ [^0M_{w^*t^*}\ ^0D]]_{wt}\ ]]\ ]$$

Another possibility might be as follows (WL' / $(o\ \iota\ o_{\tau\omega})_{\tau\omega}$):

$$(4''')\quad \lambda w\lambda t\ [^0WL'_{wt}\ ^0Ch\ [\lambda w\lambda t\ [^0T_{wt}\ ^0Ch\ [\lambda w^*\lambda t^*\ [^0M_{w^*t^*}\ ^0D]]_{wt}\ ]])\ ]\ ],$$

which can be read as *Charles would like (wishes that) he would talk to the Mayor of Dunedin*. He has a relation to the respective proposition, namely of wishing that the proposition were (would be) true.

The latter is, however, rather free reformulation of the original sentence. (It means that the situation is such that he will probably not talk but he wishes that he would talk.) Still, the analysis using WL' would be necessary when analysing sentences mentioning two agents, like:

Charles would like (wishes) that Peter would talk to the Mayor of Dunedin.

$$\lambda w\lambda t\ [^0WL'_{wt}\ ^0Ch\ \lambda w\lambda t\ [^0T_{wt}\ ^0Peter\ [\lambda w^*\lambda t^*\ [^0M_{w^*t^*}\ ^0D]]_{wt}\ ]])\ ].$$

There is a question now whether WL' should be characterised as an (implicit) propositional attitude, or as a notional attitude to a proposition (of wishing the proposition to be true). Unlike explicit propositional attitudes (of knowing, believing, ...), agent's intentions, wishes, etc., are not sensitive to the *way* in which the proposition is reported to and these attitudes are closed under the relation of the logical consequence. If Charles wishes the above, then he, for instance, also wishes that there is somebody whom he would like Peter to talk to (we have to use a variable c ranging over offices $\iota_{\tau\omega}$, to obtain a correct inference):

$$\lambda w\lambda t\ [^0WL'_{wt}\ ^0Ch\ \lambda w\lambda t\ \exists c\ [^0T_{wt}\ ^0Peter\ c_{wt}])\ ].$$

Note that both (4'') and (4''') meet the demand of *de dicto* supposition of the (concept of) Mayor of Dunedin – $[\lambda w^*\lambda t^* [^0M_{w^*t^*}\ ^0D]]$, because both the (construction of) property of talking to the Mayor and the (construction of) proposition that Charles talks (will talk) to the Mayor are *de dicto,* which can be easily checked by performing respective α-equivalent transformations ("renaming" λ-bound variables, see [12]):

$(4'')\alpha\ \ \lambda w\lambda t\ [^0WL_{wt}\ ^0Ch\ [\lambda w_1\lambda t_1\ \lambda x\ [^0T_{w1t1}\ x\ [\lambda w^*\lambda t^*\ [^0M_{w^*t^*}\ ^0D]]_{w1t1}\ ]\ ]\ ]$

$(4''')\alpha\ \lambda w\lambda t\ [^0WL'_{wt}\ ^0Ch\ [\lambda w_1\lambda t_1\ [^0T_{w1t1}\ ^0Ch\ [\lambda w^*\lambda t^*\ [^0M_{w^*t^*}\ ^0D]]_{w1t1}\ ]])$
$]\ ],$

from which we obtain (performing β_r-equivalent reduction, see [12]):

$(4'')\alpha\beta\quad \lambda w\lambda t\ [^0WL_{wt}\ ^0Ch\ [\lambda w_1\lambda t_1\ \lambda x\ [^0T_{w1t1}\ x\ [^0M_{w1t1}\ ^0D]\ ]\ ]\ ]$

$(4''')\alpha\beta\quad \lambda w\lambda t\ [^0WL'_{wt}\ ^0Ch\ [\lambda w_1\lambda t\ [^0T_{w1t1}\ ^0Ch\ [^0M_{w1t1}\ ^0D]]]])\ ]\ ]$

The concept of the Mayor, i.e. $[\lambda w^*\lambda t^* [^0M_{w^*t^*}\ ^0D]]$ is not composed with ("applied to") the left-most *w,t*-pair ("reporter's perspective"). Using WL and T, the relation WLT of Charles to the office MD is defined as

$[^0WLT_{wt}\ ^0Ch\ [\lambda w^*\lambda t^*\ [^0M_{w^*t^*}\ ^0D]] \equiv [^0WL_{wt}\ ^0Ch\ [\lambda w\lambda t\ \lambda x\ [^0T_{wt}\ x$
$[\lambda w^*\lambda t^*\ [^0M_{w^*t^*}\ ^0D]]_{wt}]\ ]\ ],$

and using WL' and T, we have

$[^0WL'T_{wt}\ ^0Ch\ [\lambda w^*\lambda t^*\ [^0M_{w^*t^*}\ ^0D]] \equiv$
$[^0WL'_{wt}\ ^0Ch\ \lambda w\lambda t\ [^0T_{wt}\ ^0Ch\ [\lambda w^*\lambda t^*\ [^0M_{w^*t^*}\ ^0D]]_{wt}\ ]])\ ].$

Let us briefly return to the *de re* reading of our sentence (4). We have seen that on its *de re* reading 'would like to talk' denotes a WLT^r object of type $(o\iota\iota)_{\tau\omega}$. Thus the *de re* analysis of (4) is as follows:

$(4^{r})\ \lambda w\lambda t\ [^0WLT^r_{wt}\ ^0Ch\ [\lambda w^*\lambda t^*\ [^0M_{w^*t^*}\ ^0D]]_{wt}].$

But a more fine-grained analysis has to take into account also the objects denoted by 'would like' (WL/WL') and 'talk' (T). We have seen that WL (WL') is a relation-in-intension to a property (proposition) and combining these together with T results in the *de dicto* supposition of the construction $[\lambda w^*\lambda t^*\ [^0M_{w^*t^*}\ ^0D]]$ (concept of the Mayor) both in (4'') and (4'''). The way out is now not as easy. We might reformulate the sentence into (4passive) The Mayor of Dunedin is the man to whom Charles would like to talk to and denoting by WChT ($/(o\iota)_{\tau\omega}$) the property of individuals ('being wanted by Charles to talk to'), we obtain:

$$\lambda w \lambda t\ [{}^0\mathrm{WChT}_{wt}\ [\lambda w^* \lambda t^*\ [{}^0\mathrm{M}_{w^*t^*}\ {}^0\mathrm{D}]]_{wt}],$$

which can be read as 'the mayor of Dunedin has the property WChT'. To construct this property, we can choose WL (or WL'): $\lambda w \lambda t\ \lambda x\ [{}^0\mathrm{WL}_{wt}\ {}^0\mathrm{Ch}\ [\lambda w \lambda t\ \lambda y\ [{}^0\mathrm{T}_{wt}\ y\ x]]]$

Using the latter instead of ^{0}WChT, we get:

$$(4^{r''})\ \lambda w \lambda t\ [\ [\lambda w \lambda t\ \lambda x\ [{}^0\mathrm{WL}_{wt}\ {}^0\mathrm{Ch}\ [\lambda w \lambda t\ \lambda y\ [{}^0\mathrm{T}_{wt}\ y\ x]]]]_{wt}\ [\lambda w^* \lambda t^*\ [{}^0\mathrm{M}_{w^*t^*}\ {}^0\mathrm{D}]]_{wt}\ ],$$

or β_i-reduced

$$(4^{r''}\beta)\ \lambda w \lambda t\ [\lambda x\ [{}^0\mathrm{WL}_{wt}\ {}^0\mathrm{Ch}\ [\lambda w \lambda t\ \lambda y\ [{}^0\mathrm{T}_{wt}\ y\ x]]]\ [{}^0\mathrm{M}_{wt}\ {}^0\mathrm{D}]\ ],$$

which is the correct analysis of the *de re* reading of the sentence (4)[15].

Remark: Note that the passive form of the sentence (4) cannot be read in the *de dicto* way. Thus the common belief[16] at the equivalence of the active and passive form of a statement is not justified. The active form usually expresses the *de dicto* reading, whereas the passive form corresponds to the *de re* reading. While in case of notional attitudes one might object that even the passive form might be read *de dicto* (as, e.g., in 'the doctor was sent for', where 'the doctor' denotes an office / property that is not just a pointer, for the whole expression denotes an attitude of an anonymous agent to the office / property), it is not the case of propositional attitudes, where the passive form is exclusively *de re*.[17]

We will not deal with the problem of 'would like', or generally wishes, intentions, any more, because its detailed solution is out of the scope of this study[18].

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[15] Further β-reducing is not possible, the obtained construction would not be equivalent to ($4^{r''}\beta$), see [12].

[16] See, e.g., Frege's Begriffsschrift §3 where Frege says that the meaning of a sentence in active and its passive counterpart is more or the less the same, or "Der Gedanke", p. 64.

[17] Cf., e.g., 'X believes that the F is G' and 'The F is believed by X to be G'. The latter (*de* re) is not equivalent to the former (*de dicto*), see [12]

[18] For details see, e.g., [24]. However, the problem of (future, past) tenses connected with wishes, intentions, etc., is neglected in this book, and the possibility of analysing wishes as relations to offices or properties is not considered.

Imagine a similar situation, when

(5) Charles would like to marry a princess.

There are again two readings of (5), namely the *de re* reading and the *de dicto* reading. The former, namely there is a (certain) princess that Charles wishes to marry, does not give rise to (notional) attitude to the property of being a princess (Princess / $(o\iota)_{\tau\omega}$, M(arry) / $(o\iota\iota)_{\tau\omega}$):

$\lambda w \lambda t\, \exists x\, (\, [{}^0\mathrm{Princess}_{wt}\, x] \wedge [{}^0\mathrm{WL}_{wt}\, {}^0\mathrm{Ch}\, [\lambda w \lambda t\, \lambda y\, [{}^0\mathrm{M}_{wt}\, y\, x]]\,),$

(There is a princess and Charles wishes to obtain the property of being married to her) or alternatively

$\lambda w \lambda t\, \exists x\, (\, [{}^0\mathrm{Princess}_{wt}\, x] \wedge [{}^0\mathrm{WL'}_{wt}\, {}^0\mathrm{Ch}\, [\lambda w \lambda t\, [{}^0\mathrm{M}_{wt}\, {}^0\mathrm{Ch}\, x]]\,)$

(There is a princess and Charles wishes he were married to her).

The second reading: Charles' intention concerns primarily the property of being a princess, not a particular princess, and using a similar way of reasoning as above, we get
(WLM / $(o\, \iota\, o\iota_{\tau\omega})_{\tau\omega}$, Princess / $(o\iota)_{\tau\omega}$):

(5') $\lambda w \lambda t\, [{}^0\mathrm{WLM}_{wt}\, {}^0\mathrm{Ch}\, {}^0\mathrm{Princess}],$

which is again a typical example of a *notional* attitude, because the '*notion*' of the property Princess is indispensable here. A more fine-grained analysis results either in

(5'') $\lambda w \lambda t\, [{}^0\mathrm{WL}_{wt}\, {}^0\mathrm{Ch}\, [\lambda w \lambda t\, \lambda y\, \exists x\, (\, [{}^0\mathrm{Princess}_{wt}\, x] \wedge [{}^0\mathrm{M}_{wt}\, y\, x]\,)$

(Charles wishes to obtain the property of being married to some princess), or in

(5''') $\lambda w \lambda t\, [{}^0\mathrm{WL'}_{wt}\, {}^0\mathrm{Ch}\, [\lambda w \lambda t\, \exists x\, (\, [{}^0\mathrm{Princess}_{wt}\, x] \wedge [{}^0\mathrm{M}_{wt}\, {}^0\mathrm{Ch}\, x]\,)]$

(Charles wishes he were married to some princess).

Consider another example:

(6) Charles wants to become the president of USA

Now there is no *de re* reading of (6), Charles cannot become George W. Bush; 'wants to become' denotes a relation WB of an individual to an individual office, which can be constructed by composing the object W(ant to) /

$(o\ \iota\ (o\iota)_{\tau\omega})_{\tau\omega})$ and B(ecome) / $(o\ \iota\ \iota_{\tau\omega})_{\tau\omega}$, and we have again a case of *notional* attitude (P(resident of USA) / $\iota_{\tau\omega}$):

(6') $\lambda w \lambda t\, [{}^0W_{wt}\, {}^0Ch\, [\lambda w \lambda t\, \lambda x\, [{}^0B_{wt}\, x\, {}^0P]]]$,

${}^0P$ (president) occurs *de dicto*, and WB is defined by the equivalence:

$$[{}^0WB_{wt}\, {}^0Ch\, {}^0P] \equiv [{}^0W_{wt}\, {}^0Ch\, [\lambda w \lambda t\, \lambda x\, [{}^0B_{wt}\, x\, {}^0P]]]$$

(Note that B(ecome) itself is a typical case of a notional attitude; sentence *Charles became the President of USA* would be analysed as follows: $\lambda w \lambda t\, [{}^0B_{wt}\, {}^0Ch\, {}^0P]$.)

Summarising, we can characterise expressions like 'would like to talk to (the) F', 'wish to meet (the) F', 'try to marry (the) F', etc., as being ambivalent. On their *de dicto* reading they denote objects of type $(o\ \iota\ \iota_{\tau\omega})$ or $(o\ \iota\ (o\iota)_{\tau\omega})$, which fall under the category of notional attitudes. There is not an existentional presupposition of the respective sentence (the F does not have to exist) and substitution *salva veritate* of an expression F' co-referential to F is not possible. In general, *intentions, attempts, wishes*, and so like (expressed by the verbs like 'would like to', 'want to', 'wish to', 'intend to', 'try to', 'seek to') are relations (-in-intension) between an individual and an *intension*, which is either a property of individuals or a proposition. Such relations are (even in the latter case) characterised as the case of *notional* attitudes, because there is no constructional counterpart of them.

5. Seeking and finding

Attitudes of seeking and finding have been dealt with and analysed using TIL in, e.g., [19], [8], [17]. Since none of these is an exhaustive study on the problem of seeking and finding, and some arguments claimed there are even flawed, our intention is to summarise, complete and in a way correct these results.

We do not use 'look for' or 'seek' to talk about going to get something that we know what it is and where it is (at that case we use 'fetch' or 'pick up'). In other words, we cannot seek something (somebody) the identity of which is well known to us. Hence, e.g., Charles can be looking for the author of Waverly, the policeman can be seeking the murderer, etc., if they do not know who the author (the murderer) is, and they are trying to find out who he is, who occupies the respective office. The agent is related to the office, and we have another example of a typical *notional attitude*:

(7) Charles is seeking the murderer of X.

Assigning types to semantically self-contained subexpressions: Ch / ι, S(eek) / $(o\ \iota\ \iota_{\tau\omega})_{\tau\omega}$, M(urderer) / $(\iota\ \iota)_{\tau\omega}$, X / ι, we get the analysis

(7') $\lambda w \lambda t\ [{}^0S_{wt}\ {}^0Ch\ \lambda w \lambda t\ [{}^0M_{wt}\ {}^0X]]$

The concept of the murderer ($\lambda w \lambda t\ [{}^0M_{wt}\ {}^0X]]$) occurs *de dicto*, there is no existential presupposition, Charles might conduct the search even if Mr.X were not actually murdered, and the substitution *salva veritate* of a co-referential expression is not allowed; if the gardener is the murderer, it does not follow that Charles is seeking the gardener.

Still, 'looking for' and 'seeking' are again homonymous expressions, though. We can easily say that Václav Havel is looking for Dagmar. Does it mean that this search is an object of type $(o\ \iota\ \iota)_{\tau\omega}$, a relation of an individual to an *individual*? No, it does not. This kind of search is different from the activity of seeking as stipulated above, for the existential question never arises for individuals, and Václav Havel certainly knows exactly which individual Dagmar Havel is. But Václav does not know *where* Dagmar is, he is trying to locate her, to find the current place of her. Let M be a particular place on the Earth. Letting aside the problem of type of the object M (let it be, for instance, a (continuous) set of 3-D co-ordinates with respect to the centre of Earth – $(o\tau\tau\tau)$) and assigning a type μ to M, we can see that this search is again an attitude, this time to the μ-office[19].

More precisely: V(aclav) / ι, D(agmar) / ι, L(ook for) / $(o\ \iota\ \mu_{\tau\omega})_{\tau\omega}$, P(lace of) / $(\mu\ \iota)_{\tau\omega}$:

$\lambda w \lambda t\ [{}^0L_{wt}\ {}^0V\ [\lambda w \lambda t\ [{}^0P_{wt}\ {}^0D]]\ ]$, $\quad ([\lambda w \lambda t\ [{}^0P_{wt}\ {}^0D]] \rightarrow \mu_{\tau\omega})$

The sentence (7) is also ambiguous. It might be the case that the identity of the murderer of X is well known, let it be Mr.Y (police announced "Y, the murderer, is wanted"), and Charles the policeman wants to find the place where Y is concealed:

(7'') $\lambda w \lambda t\ [{}^0L_{wt}\ {}^0Ch\ \lambda w^* \lambda t^*\ [{}^0P_{w^*t^*}\ [\lambda w \lambda t\ [{}^0M_{wt}\ {}^0X]]_{w^*t^*}\ ]]$ ($[\lambda w \lambda t\ [{}^0M_{wt}\ {}^0X]]$ – *de dicto*)

Note that though the office of the murderer serves as a pointer to Y, its concept occurs *de dicto* in (7''), and there is again no existential presupposition even on this reading of sentence (7). Charles may be trying to find out

[19] This solution has been first proposed by Gahér in [17].

who the murderer is or *where* the murderer is (or both), in either cases the office of the murderer can be vacant. Charles may be trying to find or locate the murderer of X even if he does not know whether X had been murdered. Hence sentence (7) never implies that Charles is looking for Y.

But consider a *passive variant* of (7)

(7*) The murderer of X is looked for by Charles.

Now we are tempted to deduce that Mr.Y is looked for by Charles, which is correct. 'Looking for' in this passive form means trying to locate $(o\ \iota\ \mu_{\tau\omega})_{\tau\omega}$ and we have a *de re* case of the notional attitude:

(7*') $\lambda w \lambda t\ [\lambda x\ [{}^0L_{wt}\ {}^0Ch\ \lambda w^* \lambda t^*\ [{}^0P_{w^*t^*}\ x]]\ [\lambda w \lambda t\ [{}^0M_{wt}\ {}^0X]]_{wt}\ ]$ ($[\lambda w \lambda t\ [{}^0M_{wt}\ {}^0X]]$ – *de re*)

Consider the classic

(8) Schliemann sought the site of Troy.

When Schliemann began his activity of search, he did not *know* whether Troy existed, though he had been pretty convinced of its existence. Hence Troy cannot be analysed as an individual, let it be for the sake of simplicity an office T / $\iota_{\tau\omega}$ (sought – S / $(o\ \iota\ \mu_{\tau\omega})_{\tau\omega}$, site of – P / $(\mu\iota)_{\tau\omega}$)

(8') $\lambda w \lambda t\ [{}^0S_{wt}\ {}^0Sch\ [\lambda w^* \lambda t^*\ [{}^0P_{w^*t^*}\ {}^0T_{w^*t^*}\ ]]\ ]$

Again, both the concept of the site of Troy and that of Troy occur *de dicto*. If Burbank were the site of Troy, then despite Kaplan[20] Schliemann would not have been seeking Burbank. And even if he happened to come to the very location and stumbled at the ruins of Troy, he would ignore the place until he would have realised the connection between that and the offices. On the other hand, he might have successfully sought the site of Troy without ever standing at the respective place. He might have had an access to some sources which he knew to be truthful, and simply put two and two together and claim which place on the Earth the site of Troy is.

Summarising, activities of seeking or looking for relate an individual to an office (a $\iota_{\tau\omega}$-object or $\mu_{\tau\omega}$- object), and according to the above preliminary characteristics these are *notional* attitudes of types $(o\ \iota\ \mu_{\tau\omega})_{\tau\omega}$ or $(o\ \iota\ \iota_{\tau\omega})_{\tau\omega}$.

[20] As Jespersen rightly says in [19], citing Kaplan's [21].

When we seek something or look for something, we are trying to find an occupant of the office (that does not have to exist, the office may be vacant)[21].

Now, the search may be successful, which means that the seeker becomes the finder, the agent finds what he was looking for, or the search may be unsuccessful, the agent may fail in his effort, he does not find it. Hence if Charles were looking for the murderer of X, then one of the two following sentences has to be true and the other false:

(9) Charles found the murderer of X

(10) Charles did not find the murderer of X.

The latter may become true in two situations: Either Charles' competency did not cope up with the murderer, or the murderer did not exist, X had not been murdered. It means that sentences on (intended) finding do not have existential presupposition on the holder of the respective office, and finding cannot relate an individual to an individual but to the office (that had been sought). These are again *notional* attitudes of types $(o\ \iota\ \iota_{\tau\omega})_{\tau\omega}$ or $(o\ \iota\ \mu_{\tau\omega})_{\tau\omega}$. (If there were the existential presupposition and the murderer did not exist, then neither (9) nor (10) had any truth value, Strawson [27].) Of course, if the search has been successful, i.e. (9) is true, then the murderer has to exist (existential commitment).

(9') $\lambda w \lambda t\ [{}^0F_{wt}\ {}^0Ch\ \lambda w \lambda t\ [{}^0M_{wt}\ {}^0X]]$

(10') $\lambda w \lambda t\ \neg[{}^0F_{wt}\ {}^0Ch\ \lambda w \lambda t\ [{}^0M_{wt}\ {}^0X]]$

F(inding) is here an object of type $(o\ \iota\ \iota_{\tau\omega})_{\tau\omega}$ and the concept of the murderer $(\lambda w \lambda t\ [{}^0M_{wt}\ {}^0X]])$ possesses *de dicto* supposition. The two *de re* principles do not hold: In particular, if Mr.Y is the murderer of X, from (9') it does not follow that Charles found (located) Y. Having performed successful search on the murderer ($\iota_{\tau\omega}$-office) only entails Charles' finding out *who* is the murderer, but not *where* he is[22].

Similarly, it is true that Schliemann found the site of Troy, but another scenario is thinkable: If Troy did not exist, then it would be true that Schliemann's search were not successful, i.e. that

(11) Schliemann did not find the site of Troy

[21] 'Looking for' is, however, used in English only in case that the existence of the sought object is guaranteed, otherwise we use 'seeking'.

[22] This, in particular, is a correction of Jespersen, see [19].

(11') $\lambda w \lambda t \neg [{}^0F_{wt}\ {}^0Sch\ [\lambda w^* \lambda t^*\ [{}^0P_{w^*t^*}\ {}^0T_{w^*t^*}\ ]]\ ]$ $(F / (o\ \iota\ \mu_{\tau\omega})_{\tau\omega})$.

There is, however, another type of finding. Charles, on his way home, may trip over a stone, pick it up, and only on his coming home he finds out that the stone is the most beautiful diamond he has ever seen. We can report on the situation using (12):

(12) Charles found the most beautiful diamond.

This time Charles is not related to the office of the most beautiful diamond, he did not intend to find it, he did not even look for it. We can see that (12) is ambiguous: it might express a notional attitude to the office of the most beautiful diamond, if Charles were looking for it before (intended finding F of type $(o\ \iota\ \iota_{\tau\omega})_{\tau\omega}$ as above), or it may express a simple relation to an individual: This time (unintended) finding by chance is an object F' of type $(o\ \iota\ \iota)_{\tau\omega}$, and we have (MB / $(\iota\ (o\iota))_{\tau\omega}$ – most beautiful, D(iamond) / $(o\iota)_{\tau\omega}$):

(12') $\lambda w \lambda t\ [{}^0F'_{wt}\ {}^0Ch\ [\lambda w \lambda t\ [{}^0MB_{wt}\ {}^0D_{wt}]]_{wt}\ ]$

We can see that the construction of the most beautiful diamond occurs *de re*, i.e. the two *de re* principles hold. In particular, if the most beautiful diamond is Charles' most favourite stone (that made him rich), then Charles found his most favourite stone that made him rich.

Sentences on seeking and finding are systematically ambiguous, for 'seeking' ('looking for') and 'finding' are homonymous. The former may denote an object $S_1/(o\iota\iota_{\tau\omega})_{\tau\omega}$ in case the seeker is trying to find out *who* occupies the respective office, or an object $S_2/(o\iota\mu_{\tau\omega})_{\tau\omega}$ in case the seeker is trying to find out *where* the respective individual (that may even be the occupant of the related office) is. Anyway, both S_1 and S_2 can be characterised as *notional attitudes*. On the other hand 'finding' may simply express an incidental (chance) finding[23], in this case it denotes an object F_1 of type $(o\ \iota\ \iota)_{\tau\omega}$, which is not a notional attitude. In case that finding (or possibly not finding) has been preceded by a search[24] $S_1/(o\iota\iota_{\tau\omega})_{\tau\omega}$ or $S_2/(o\iota\mu_{\tau\omega})_{\tau\omega}$, then it can be characterised as a notional attitude F_2 / $(o\iota\iota_{\tau\omega})_{\tau\omega}$ or F_3 / $(o\iota\mu_{\tau\omega})_{\tau\omega}$, respectively.

[23] 'találni' in Hungarian language, see Jespersen [19]

[24] 'megtalálni' in Hungarian, see Jespersen [19]

6. Conclusion

Our knowledge, beliefs, doubts, etc. concern primarily *constructions*. If we assume that iterating such attitudes is valid, i.e., that the agent is perfectly introspective, he knows what he knows, believes, etc., then the so-called *propositional* attitudes are actually *hyperintensional* attitudes, i.e. relations of an agent to the *construction–concept* (of a proposition) expressed by the embedded clause, i.e. they are objects of type $(o\iota *_n)_{\tau\omega}$. Their implicit counterparts, relations (of type $(o\iota o_{\tau\omega})_{\tau\omega}$) of an agent to the proposition denoted by the embedded clause, are just idealised cases of an agent with unlimited inferential abilities. On the other hand, our wishes, intentions, attempts, etc., concern (in empirical cases) particular *intensions* (offices, properties, propositions), and the so-called *notional* attitudes (to empirical notions) are (despite calling them notional) not hyperintensional, they are objects of type $(o\iota\alpha_{\tau\omega})_{\tau\omega}$, for any type α. Even relations of type $(o\iota o_{\tau\omega})_{\tau\omega}$ of an agent to a proposition can be notional ones, in case there is no salient constructional counterpart, the attitude is not influenced by agent's inferential abilities. The respective intension is mentioned, it means that its concept must occur *de dicto*, and referring on such a situation, the reporter may use any of the equivalent constructions (concepts) of the respective intension. Still, unlike the case of relations of an agent to an individual when the office serves just as a pointer to the individual, when speaking about notional attitudes using the respective *notion* of the intension is indispensable, which might perhaps justify calling such attitudes *notional,* though they actually are *intensional.*

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